Name:______________________ Date:__________

Logarithms Practice

Do Now: Solve the following equations.

1) $\log(3d) + \log(2d - 2) = \log(3d^2 + 2d - 4)$

$\log(6d^2 - 6d) = \log(3d^2 + 2d - 4)$

$6d^2 - 6d = 3d^2 + 2d - 4$

$3d^2 - 8d + 4 = 0$

$x: 12$
$+: -8$
$-6, -2$

$3d^2 - 6d - 2d + 4 = 0$

$3d(d-2) - 2(d-2) = 0$

$(3d-2)(d-2) = 0$

$3d-2=0$	$d-2=0$
$3d=2$	$d=2$
$d=2/3$	

Check:

$\log(3\cdot\frac{2}{3}) + \log(2(\frac{2}{3})-2) = \log(3(\frac{2}{3})^2 + 2(\frac{2}{3}) - 4)$ X

$\log(3\cdot 2) + \log(2(2)-2) = \log(3(2)^2 + 2(2) - 4)$ ✓

$\boxed{x=2}$

2) $\log_3(x^4 + 51x^2 - 27) - \log_3(2x^2 + 1) = 3$

$\log_3\left(\frac{x^4+51x^2-27}{2x^2+1}\right) = 3$

$3^3 = \frac{x^4+51x^2-27}{2x^2+1}$

$27 = \frac{x^4+51x^2-27}{2x^2+1}$

$27(2x^2+1) = x^4 + 51x^2 - 27$

$54x^2 + 27 = x^4 + 51x^2 - 27$

$x^4 - 3x^2 - 54 = 0$

$(x^2-9)(x^2+6) = 0$

$(x+3)(x-3)(x^2+6) = 0$

$x=-3$	$x=3$	$x^2=-6$
		$\cancel{x = \pm\sqrt{6}}$

Check:

$\log_3(3^4 + 51(3)^2 - 27) - \log_3(2(3)^2 + 1) = 3$ ✓

$\log_3((-3)^4 + 51(-3)^2 - 27) - \log_3(2(-3)^2 + 1) = 3$ ✓

$\boxed{x = \pm 3}$

Solve the following equations.

1) $\log(x-2) - \log(x+2) = \log(x-1) - \log(x+7)$

$\log\left(\frac{x-2}{x+2}\right) = \log\left(\frac{x-1}{x+7}\right)$

$\frac{x-2}{x+2} = \frac{x-1}{x+7}$

$(x-2)(x+7) = (x-1)(x+2)$

$x^2+5x-14 = x^2+x-2$

$4x = 12$

$x=3$

Check:

$\log(3-2)-\log(3+2)=\log(3-1)-\log(3+7)$ ✓

$\boxed{x=3}$

2) $\ln(2x^2) + \ln(x^2+10) = \ln(3x^4+27x^2-8)$

$\ln(2x^4+20x^2) = \ln(3x^4+27x^2-8)$

$2x^4+20x^2 = 3x^4+27x^2-8$

$x^4+7x^2-8=0$

$(x^2-1)(x^2+8)=0$

$(x+1)(x-1)(x^2+8)=0$

$x=-1 \quad x=1 \quad \cancel{x^2=-8} \quad \cancel{x=\pm\sqrt{-8}}$

$\boxed{x=\pm 1}$

Check:

$\ln(2(1)^2)+\ln(1^2+10)=\ln(3(1)^4+27(1)^2-8)$ ✓

$\ln(2(-1)^2)+\ln((-1)^2+10)=\ln(3(-1)^4+27(-1)^2-8)$ ✓

3) $\log_2(x-9) + \log_2(x+5) = 7$

$\log_2(x^2-4x-45)=7$

$2^7 = x^2-4x-45$

$128 = x^2-4x-45$

$x^2-4x-173=0$

$\frac{4\pm\sqrt{(-4)^2-4(1)(-173)}}{2(1)}$

$\frac{4\pm\sqrt{708}}{2}$

$\frac{4\pm 2\sqrt{177}}{2} = 2\pm\sqrt{177}$

Check:

$\log_2(2+\sqrt{177}-9)+\log_2(2+\sqrt{177}+5)=7$ ✓

$\log_2(2-\sqrt{177}-9)+\log_2(2-\sqrt{177}+5)=7$ ✗

$\boxed{x=2+\sqrt{177}}$

4) $\log(x^4+100x^2-156) - \log(x^2+1) = 2$

$\log\left(\frac{x^4+100x^2-156}{x^2+1}\right)=2$

$10^2 = \frac{x^4+100x^2-156}{x^2+1}$

$100(x^2+1) = x^4+100x^2-156$

$100x^2+100 = x^4+100x^2-156$

$x^4-256=0$

$(x^2-16)(x^2+16)=0$

$(x-4)(x+4)(x^2+16)=0$

$x=4 \quad x=-4 \quad \cancel{x=\pm 4i}$

Check:

$\log(4^4+100(4)^2-156)-\log(4^2+1)=2$ ✓

$\log((-4)^4+100(-4)^2-156)-\log((-4)^2+1)=2$ ✓

$\boxed{x=\pm 4}$

5) Solve for t in the equation $A = P\left(1+\frac{r}{n}\right)^{nt}$.

$$\frac{A}{P} = \left(1+\frac{r}{n}\right)^{nt}$$

$$\log\left(\frac{A}{P}\right) = \log\left(\left(1+\frac{r}{n}\right)^{nt}\right)$$

$$\log\left(\frac{A}{P}\right) = nt\log\left(1+\frac{r}{n}\right)$$

$$\frac{\log\left(\frac{A}{P}\right)}{\log\left(1+\frac{r}{n}\right)} = \frac{nt\log\left(1+\frac{r}{n}\right)}{\log\left(1+\frac{r}{n}\right)}$$

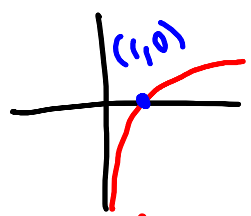
$$\frac{\frac{\log\left(\frac{A}{P}\right)}{\log\left(1+\frac{r}{n}\right)}}{n} = \frac{nt}{n}$$

$$t = \frac{\frac{\log\left(\frac{A}{P}\right)}{\log\left(1+\frac{r}{n}\right)}}{n}$$

$$\boxed{t = \frac{\log\left(\frac{A}{P}\right)}{n\log\left(1+\frac{r}{n}\right)}}$$

6) If $a > 1$, sketch the following graphs WITHOUT A CALCULATOR:

a) $f(x) = \log_a x$



Domain: $(0,\infty)$

Range: $(-\infty,\infty)$

Asymptote: $x=0$

X-Intercept: $(1,0)$

Left-end Behavior:

As $x \to 0$

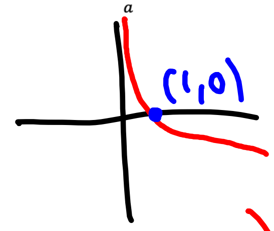
$f(x) \to -\infty$

Right-end Behavior:

As $x \to \infty$

$f(x) \to \infty$

b) $f(x) = \log_{\frac{1}{a}} x$



Domain: $(0,\infty)$

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X-Intercept: $(1,0)$

Left-end Behavior:

As $x \to 0$

$f(x) \to \infty$

Right-end Behavior:

As $x \to \infty$

$f(x) \to -\infty$

7) Given the exponential function, $7^{9x-1} = 10^x + 7$, how does taking the logarithm of each side of the equation allow you to find the solution? Use complete sentences.

When you log both sides, you can then use the properties of logarithms to write each exponent as a coefficient. This allows you to get the variable out as being an exponent and lets you solve for the variable.

8) Pierson invests \$10,000 in a savings account that is compounded continuously using the equation $A = 10{,}000e^{0.05t}$, where A is the amount of money Pierson has in his savings account after t years. Pierson will not deposit or withdrawal any of his money.

a) What is the amount in Pierson's savings account after 18 years?

$A = 10{,}000e^{0.05(18)}$

$A = 24596.031116$

$\boxed{\$24596.03}$

b) How many years will it take Pierson to save \$15,000? (Round to the nearest day).

$\frac{15000}{10000} = \frac{10000e^{0.05t}}{10000}$

$1.5 = e^{0.05t}$

$\ln 1.5 = \ln e^{0.05t}$

$\frac{\ln 1.5}{0.05} = \frac{0.05t}{0.05}$

$t = \frac{\ln 1.5}{0.05}$

$t = 8.10930216216$

$0.10930216216 \times 365 = 39.8952891$

$\boxed{\text{8 years and 40 days}}$

Solve the following equations.

9) $e^{7x-5} + 2 = 7{,}456$

$-2 \quad -2$

$e^{7x-5} = 7454$

$\ln e^{7x-5} = \ln 7454$

$7x - 5 = \ln 7454$

$7x = \ln 7454$

$\boxed{x = \frac{\ln 7454}{7}}$

10) $5^{3x-5} = 15^{x+10}$

$\log 5^{3x-5} = \log 15^{x+10}$

$(3x-5)\log 5 = (x+10)\log 15$

$3x\log 5 - 5\log 5 = x\log 15 + 10\log 15$

$3x\log 5 - x\log 15 = 10\log 15 + 5\log 5$

$x(3\log 5 - \log 15) = 10\log 15 + 5\log 5$

$\boxed{x = \frac{10\log 15 + 5\log 5}{3\log 5 - \log 15}}$

Solve the following equations.

11) $\log_2(2x^2+3x+40) - \log_2(2x^2+x) = 2$

$\log_2\left(\frac{2x^2+3x+40}{2x^2+x}\right) = 2$

$2^2 = \frac{2x^2+3x+40}{2x^2+x}$

$4(2x^2+x) = 2x^2+3x+40$

$8x^2+4x = 2x^2+3x+40$

$6x^2+x-40=0$

$6x^2+16x-15x-40=0$

$2x(3x+8)-5(3x+8)=0$

$(2x-5)(3x+8)=0$

$2x-5=0 \quad | \quad 3x+8=0$

$x=5/2 \quad | \quad x=-8/3$

$\boxed{x=-\frac{8}{3}, \frac{5}{2}}$

Check:

$\log_2(2(\frac{5}{2})^2+3(\frac{5}{2})+40) - \log_2(2(\frac{5}{2})^2+\frac{5}{2}) = 2$ ✓

$\log_2(2(-\frac{8}{3})^2+3(-\frac{8}{3})+40) - \log_2(2(-\frac{8}{3})^2+(-\frac{8}{3})) = 2$ ✓

12) $\log(h+5) + \log(h-2) = \log(-5h+10)$

$\log(h^2+3h-10) = \log(-5h+10)$

$h^2+3h-10 = -5h+10$

$h^2+8h-20=0$

$(h+10)(h-2)=0$

$h=-10 \quad | \quad h=2$

Check:

$\log(-10+5)+\log(-10-2) = \log(-5(-10)+10)$ ✗

$\log(2+5)+\log(2-2) = \log(-5(2)+10)$ ✗

$\boxed{\varnothing}$

13) $\ln(x^3+64) - \ln(x^2-4x+16) = \ln(2x)$

$\ln\left(\frac{x^3+64}{x^2-4x+16}\right) = \ln(2x)$

$\frac{x^3+64}{x^2-4x+16} = 2x$

$\frac{(x+4)\cancel{(x^2-4x+16)}}{\cancel{x^2-4x+16}} = 2x$

$x+4=2x$

$x=4$

$\boxed{x=4}$

Check:

$\ln(4^3+64) - \ln(4^2-4(4)+16) = \ln(2\cdot4)$ ✓

14) $\log(x+1) + \log(x+7) = \log(x^3-5x^2+19x+1)$

$\log(x^2+8x+7) = \log(x^3-5x^2+19x+1)$

$x^2+8x+7 = x^3-5x^2+19x+1$

$x^3-6x^2+11x-6=0$

Potential Zeroes: $\frac{\text{Factors of } 6}{\text{Factors of } 1}$ $\frac{\pm1, \pm2, \pm3, \pm6}{\pm1} = \pm1, \pm2, \pm3, \pm6$

1	1	-6	11	-6
	↓	1	-5	6
	1	-5	6	0

$x=1$

$x^2-5x+6=0$

$(x-3)(x-2)=0$

$x=3 \quad | \quad x=2$

$\boxed{x=1,2,3}$

Check:

$\log(1+1)+\log(1+7) = \log(1^3-5(1)^2+19(1)+1)$ ✓

$\log(2+1)+\log(2+7) = \log(2^3-5(2)^2+19(2)+1)$ ✓

$\log(3+1)+\log(3+7) = \log(3^3-5(3)^2+19(3)+1)$ ✓

Evaluate the following without using a calculator.

15) $8\log_{\frac{1}{2}} 8 - 10^{\log 4} + \ln e^{-2} + 9\log_{343} 7$

$8(-3) - 4 + (-2) + 9\left(\frac{1}{3}\right)$

$-24 - 4 - 2 + 3$

$\boxed{-27}$

16) $\ln[\log_2(\log \frac{1}{10000})]$

$\ln(\log_2(-4))$

$\boxed{\emptyset}$

Find the inverse of the following functions.

17) $y = \frac{3}{4}^{0.54x+9.73}$

$x = \frac{3}{4}^{0.54y+9.73}$

$\log_{3/4} x = 0.54y + 9.73$

$\log_{3/4} x - 9.73 = 0.54y$

$\boxed{\frac{\log_{3/4}(x) - 9.73}{0.54} = y}$

18) $y = 13^{2-x}$.

$x = 13^{2-y}$

$\log_{13} x = 2 - y$

$\log_{13}(x) - 2 = -y$

$-(\log_{13}(x) - 2) = y$

$\boxed{-\log_{13}(x) + 2 = y}$

19) Describe how $f(x) = \log x$ changes to form each of the following equations:

a) $g(x) = -\log(x - 7)$

Reflection over x-axis and translated 7 units right

b) $h(x) = \log(x + 2) - 9$

Translated 2 units left and 9 units down

c) $j(x) = -\log(x - 1) + 1$

Reflection over x-axis, translated 1 unit right and 1 unit up

Rewrite each of the following logarithms using the Change of Base Formula, then round to the nearest thousandth.

20) $\log_4 8$

$\frac{\log 8}{\log 4}$

1.500

21) $\log_{456} 120$

$\frac{\log 120}{\log 456}$

0.782

22) $\log_{\frac{1}{2}} \frac{1}{3}$

$\frac{\log 1/3}{\log 1/2}$

1.585

Solve the following equations.

23) $\log_2(x) + \log_2(x^2 + 8x - 4) = 5$

$\log_2(x^3+8x^2-4x)=5$

$x^3+8x^2-4x=32$

$x^3+8x^2-4x-32=0$

$x^2(x+8)-4(x+8)=0$

$(x^2-4)(x+8)=0$

$(x+2)(x-2)(x+8)=0$

$x=-2 \mid x=2 \mid x=-8$

$\boxed{x=2}$

heck:

$\log_2(-2)+\log_2((-2)^2+8(-2)-4)=5$ ✗

$\log_2(2)+\log_2(2^2+8(2)-4)=5$ ✓

$\log_2(-8)+\log_2((-8)^2+8(-8)-4)=5$ ✗

24) $\log_2(x^3 - 1) - \log_2(x^2 + 5) = \log_2(x - 1)$

$\log_2\left(\frac{x^3-1}{x^2+5}\right)=\log_2(x-1)$

$\frac{x^3-1}{x^2+5}=x-1$

$(x-1)(x^2+x+1)=(x^2+5)(x-1)$

$x^2+x+1=x^2+5$

$x=4$

$\boxed{x=4}$

Check:

$\log_2(4^3-1)-\log_2(4^2+5)=\log_2(4-1)$ ✓

25)) $\log_{\frac{1}{12}}(2x) + \log_{\frac{1}{12}}(x^3 - 1.5x^2 - 28x + 90) = -2$

$$\log_{\frac{1}{12}}(2x^4 - 3x^3 - 56x^2 + 180x) = -2$$

$$\left(\tfrac{1}{12}\right)^{-2} = 2x^4 - 3x^3 - 56x^2 + 180x$$

$$144 = 2x^4 - 3x^3 - 56x^2 + 180x$$

$$2x^4 - 3x^3 - 56x^2 + 180x \underline{-144} = 0$$

Possible zeroes: $\frac{\text{Factors of } 144}{\text{Factors of } 2} = \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 18, \pm 24, \pm 36, \pm 48, \pm 72, \pm 144, \pm 9, \pm 16}{\pm 1, \pm 2}$

$= \pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 4, \pm 6, \pm 8, \pm 12, \pm 18, \pm 9, \pm 16, \pm 144, \pm 4.5, \pm 24, \pm 36, \pm 48, \pm 72$

$$\begin{array}{c|ccccc} 2 & 2 & -3 & -56 & 180 & -144 \\ & \downarrow & 4 & 2 & -108 & 144 \\ \hline & 2 & 1 & -54 & 72 & 0 \end{array}$$

$x = 2$

Check:

$\log_{1/12}(2 \cdot 2) + \log_{1/12}(2^3 - 1.5(2)^2 - 28(2) + 90) = -2$ ✓

$\log_{1/12}(2 \cdot 4) + \log_{1/12}(4^3 - 1.5(4)^2 - 28(4) + 90) = -2$ ✓

$$\begin{array}{c|cccc} 4 & 2 & 1 & -54 & 72 \\ & \downarrow & 8 & 36 & -72 \\ \hline & 2 & 9 & -18 & 0 \end{array}$$

$x = 4$

$\log_{1/12}(2 \cdot (-6)) + \log_{1/12}((-6)^3 - 1.5(-6)^2 - 28(-6) + 90) = -2$ ✗

$\log_{1/12}(2 \cdot \frac{3}{2}) + \log_{1/12}((\frac{3}{2})^3 - 1.5(\frac{3}{2})^2 - 28(\frac{3}{2}) + 90) = -2$ ✓

$$2x^2 + 9x - 18 = 0$$
$$2x^2 + 12x - 3x - 18 = 0$$
$$2x(x+6) - 3(x+6) = 0$$
$$(x+6)(2x-3) = 0$$

$x = -6$ | $x = 3/2$

$$\boxed{x = \frac{3}{2}, 2, 4}$$