

# **ADHIKAANSH ACADEMY (IITJEE NEET IX X XI XII)**

**RUN BY:**

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# **MATHS NOTES (CLASS 11<sup>TH</sup>)**



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## Chapter 5

# COMPLEX NUMBERS AND QUADRATIC EQUATIONS

### INTRODUCTION

$\sqrt{-36}$ ,  $\sqrt{-25}$  etc do not have values in the system of real numbers.

So we need to extend the real numbers system to a larger system.

Let us denote  $\sqrt{-1}$  by the symbol  $i$ .

$$\text{ie } i^2 = -1$$

A number of the form  $a+ib$  where  $a$  &  $b$  are real numbers is defined to be a complex number.

Eg  $2+i3$ ,  $-7+\sqrt{2}i$ ,  $\sqrt{3}i$ ,  $4+i$ ,  $5=5+0i$ ,  $-7=-7+0i$  etc

For  $z = 2+i5$ ,  $\text{Re } z = 2$  (real part)

and  $\text{Im } z = 5$  (imaginary part)

Refer algebra of complex numbers of text book pg 98

#### 1) Addition of complex numbers

$$\begin{aligned}(2+i3) + (-3+i2) &= (2-3) + i(3+2) \\ &= -1+5i\end{aligned}$$

#### 2) Difference of complex numbers

$$\begin{aligned}(2+i3) - (-3+i2) &= (2+3) + i(3-2) \\ &= 5 + i\end{aligned}$$

#### 3) Multiplication of two complex numbers

$$\begin{aligned}(2+i3)(-3+i2) &= 2(-3+i2) + i3(-3+i2) \\ &= -6+4i-9i+6i^2 \\ &= -6-5i-6 \quad (i^2 = -1) \\ &= -12-5i\end{aligned}$$

#### 4) Division of complex numbers

$$\begin{aligned}\frac{2+i3}{-3+i2} &= \frac{(2+i3)}{(-3+i2)} \times \frac{(-3-i2)}{(-3-i2)} \\ &= \frac{-6-4i-9i-6i^2}{(-3)^2-(i2)^2} \\ &= \frac{-6-13i+6}{9-(-1) \times 4} \\ &= \frac{-13i}{13} = -i\end{aligned}$$

### 5) Equality of 2 complex numbers

$$a+ib = c+id, \text{ iff } a=c \text{ \& } b=d$$

6)  $a+ib = 0$ , iff  $a=0$  and  $b=0$

Refer : the square roots of a negative real no & identities (text page 100,101)

### Formulas

a) IF  $Z=a+ib$  then modulus of  $Z$  ie  $|Z| = (a^2+b^2)^{1/2}$

b) Conjugate of  $Z$  is  $a-ib$

c) **Multiplicative inverse of  $a+ib$**  =  $\frac{a}{(a^2+b^2)} - \frac{ib}{(a^2+b^2)}$

### \*\*d) Polar representation of a complex number

$$a+ib = r(\cos \theta + i \sin \theta)$$

Where  $r = |Z| = (a^2+b^2)^{1/2}$  and  $\theta = \arg Z$  (argument or amplitude of  $Z$  which has many different values but when  $-\pi < \theta \leq \pi$ ,  $\theta$  is called principal argument of  $Z$ ).

### Trick method to find $\theta$

Step 1 First find angle using the following

- 1)  $\cos \theta = 1$  and  $\sin \theta = 0$  then angle = 0
- 2)  $\cos \theta = 0$  and  $\sin \theta = 1$  then angle =  $\pi/2$
- 3)  $\sin \theta = \sqrt{3}/2$  and  $\cos \theta = 1/2$  then angle =  $\pi/3$
- 4)  $\sin \theta = 1/2$  and  $\cos \theta = \sqrt{3}/2$  then angle =  $\pi/6$

Step 2: To find  $\theta$

- 1) If both  $\sin \theta$  and  $\cos \theta$  are positive then  $\theta =$  angle (first quadrant)
- 2) If  $\sin \theta$  positive,  $\cos \theta$  negative then  $\theta = \pi$ -angle (second quadrant)
- 3) If both  $\sin \theta$  and  $\cos \theta$  are negative the  $\theta = \pi$ +angle (third quadrant)
- 4) If  $\sin \theta$  negative and  $\cos \theta$  positive then  $\theta = 2\pi$ -angle (fourth quadrant)  
Or  $\theta = -$  (angle) since  $\sin (-\theta) = -\sin \theta$  and  $\cos (-\theta) = \cos \theta$
- 5) If  $\sin \theta = 0$  and  $\cos \theta = -1$  then  $\theta = \pi$

### \*\*e) Formula needed to find square root of a complex number

$$(a+b)^2 = (a-b)^2 + 4ab$$

$$\text{ie } [x^2 + y^2]^2 = [x^2 - y^2]^2 + 4x^2y^2$$

### e) Powers of $i$

$$\text{i) } i^{4k} = 1$$

$$\text{ii) } i^{4k+1} = i$$

$$\text{iii) } i^{4k+2} = -1$$

$$\text{iv) } i^{4k+3} = -i, \text{ for any integer } k$$

#### Examples:

$$i^1 = i, i^2 = -1, i^3 = -i \text{ and } i^4 = 1 \&$$

$$i^{19} = i^{16} \times i^3 = 1 \times -i = -i$$

g) Solutions of quadratic equation  $ax^2 + bx + c = 0$  with real coefficients  $a, b, c$  and  $a \neq 0$  are given by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , If  $b^2 - 4ac \geq 0$

$$\text{If } b^2 - 4ac < 0 \text{ then } x = \frac{-b \pm \sqrt{4ac - b^2} i}{2a}$$

Refer text page 102 the modulus and conjugate of a complex number properties given in the end. (i) to (v)

Ex 5.1

Q. 3\*(1 mark), 8\* (4 marks), 11\*\*, 12\*\*, 13\*\*, 14\*\*(4 Marks)

### Polar form (very important)

#### Ex 5.2

Q 2\*\*) Express  $Z = -\sqrt{3} + i$  in the polar form and also write the modulus and the argument of  $Z$

Solution Let  $-\sqrt{3} + i = r(\cos\theta + i\sin\theta)$

$$\text{Here } a = -\sqrt{3}, b = 1$$

$$r = (a^2 + b^2)^{1/2} = \sqrt{3 + 1} = \sqrt{4} = 2$$

$$-\sqrt{3} + i = 2\cos\theta + i \times 2\sin\theta$$

$$\text{Therefore } 2\cos\theta = -\sqrt{3} \text{ and } 2\sin\theta = 1$$

$$\cos\theta = -\sqrt{3}/2 \text{ and } \sin\theta = 1/2$$

Here  $\cos\theta$  negative and  $\sin\theta$  positive

Therefore  $\theta = \pi - \pi/6 = 5\pi/6$  (see trick method given above)

Therefore polar form of  $Z = -\sqrt{3} + i = 2(\cos 5\pi/6 + i\sin 5\pi/6)$

$|Z| = 2$  and argument of  $Z = 5\pi/6$  and  $-\sqrt{3} + i = 2(\cos 5\pi/6 + i\sin 5\pi/6)$

### Ex 5.2

Q (1 to 8)\*\* Note: Q 1)  $\theta = 4\pi/3$  or principal argument  $\theta = 4\pi/3 - 2\pi = -2\pi/3$

Q 5)  $\theta = 5\pi/4$  or principal argument  $\theta = 5\pi/4 - 2\pi = -3\pi/4$

eg 7\*\*, eg 8\*\*

### Ex 5.3

Q 1,8,9,10 (1 mark)

Misc examples (12 to 16)\*\*

### Misc exercise

Q 4\*\*,5\*\*,10\*\*,11\*\*,12\*\*,13\*\*,14\*\*,15\*\*,16\*\*,17\*,20\*\*

Supplementary material

eg 12\*\*

### Ex 5.4

Q (1 to 6)\*\*

## EXTRA/HOT QUESTIONS

1\*\* Find the square roots of the following complex numbers (4 marks)

- i.  $6 + 8i$
- ii.  $3 - 4i$
- iii.  $2 + 3i$  (HOT)
- iv.  $7 - 30\sqrt{2}i$
- v.  $\frac{3 + 4i}{3 - 4i}$  (HOT)

2\*\* Convert the following complex numbers in the polar form

- i.  $3\sqrt{3} + 3i$
- ii.  $\frac{1 - i}{1 + i}$

- iii.  $1 + i$
- iv.  $-1 + \sqrt{3}i$
- v.  $-3 + 3i$
- vi.  $-2 - i$

3. If  $a + ib = \frac{x+i}{x-i}$  where  $x$  is a real, prove that  $a^2 + b^2 = 1$  and  $b/a = 2x/(x^2-1)$  4marks

4 Find the real and imaginary part of  $i$ . (1 mark)

5 Compute :  $i + i^2 + i^3 + i^4$  (1 mark)

6 Solve the following quadratic equations (I mark)

i)  $x^2 - (\sqrt{2} + 1)x + \sqrt{2} = 0$

ii)  $2x^2 + 5 = 0$

7 Find the complex conjugate and multiplicative inverse of (4 mark)

i)  $(2 - 5i)^2$

ii)  $\frac{2 + 3i}{3 - 7i}$

8 If  $|Z| = 2$  and  $\arg Z = \pi/4$  then  $Z =$  \_\_\_\_\_. (1 mark)

### Answers

1) i)  $2\sqrt{2} + \sqrt{2}i, -2\sqrt{2} - \sqrt{2}i$

ii)  $2 - i, -2 + i$

iii)  $\frac{\sqrt{\sqrt{13} + 2}}{\sqrt{2}} + \frac{\sqrt{\sqrt{13} - 2}}{\sqrt{2}} i, \frac{\sqrt{\sqrt{13} + 2}}{\sqrt{2}} + \frac{\sqrt{\sqrt{13} - 2}}{\sqrt{2}} i,$

iv)  $5 - 3\sqrt{2}i, -5 + 3\sqrt{2}i$

v)  $3/5 + 4/5 i, -3/5 - 4/5 i$

2) i)  $6(\cos \pi/6 + i \sin \pi/6)$

ii)  $\cos(-\pi/2) + i \sin(-\pi/2)$

iii)  $\sqrt{2}(\cos \pi/4 + i \sin \pi/4)$

iv)  $2(\cos 2\pi/3 + i \sin 2\pi/3)$

iv)  $3\sqrt{2}(\cos 3\pi/4 + i \sin 3\pi/4)$

vi)  $2\sqrt{2}(\cos 5\pi/4 + i\sin 5\pi/4)$  or  $2\sqrt{2}[\cos(-3\pi/4) + i\sin(-3\pi/4)]$

4) 0,1

5) 0

6) i)  $\sqrt{2}, 1$

ii)  $\sqrt{\frac{5}{2}} i, -\sqrt{\frac{5}{2}} i$

7) i)  $-21 + 10i, \frac{-21}{541} - \frac{10}{541} i$

ii)  $\frac{-15}{58} - \frac{23i}{58}, \frac{3-7i}{2+3i}$

8)  $\sqrt{2} + i\sqrt{2}$



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