ADHIKAANSH
ACADEMY
(IITJEE NEET IX X XI XII)

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MATHS NOTES
(CLASS 12TH)

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Chapter-11

Three Dimensional Geometry

Direction cosines of a line are the cosines of the angles made by the line with the positive direct ions of the coordinate axes.

- *If l, m, n* are the direct ion cosines of a line, then $l^2 + m^2 + n^2 = 1$

- Direct ion cosines of a line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

  $\dfrac{x_2 - x_1}{PQ}, \dfrac{y_2 - y_1}{PQ}, \dfrac{z_2 - z_2}{PQ}$

- Where $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

- Direction ratios of a line are the numbers which are proportional to the direct ion cosines of a line.

- If l, m, n are the direct ion cosines and a, b, c are the direct ion ratios of a line

  Then, $l = \dfrac{a}{\sqrt{a^2 + b^2 + c^2}}$, $m = \dfrac{b}{\sqrt{a^2 + b^2 + c^2}}$, $n = \dfrac{c}{\sqrt{a^2 + b^2 + c^2}}$

- Skew lines are lines in space which are neither parallel nor intersecting. They lie in different planes.

- Angle between skew lines is the angle between two intersecting lines drawn from any point (preferably through the origin) parallel to each of the skew lines.

- If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two lines; and θ is the acute angle between the two lines; then,

  $\cos\theta = \left| \dfrac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$

- Vector equation of a line that passes through the given point whose position vector is $\overline{a}$ and parallel to a given vector $\overline{b}$ is $\overline{r} = \overline{a} + \lambda\overline{b}$

Key Notes

- Equation of a line through a point (x_1, y_1, z_1) and having direct ion cosines l, m, n is

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

- The vector equation of a line which passes through two points whose posit ion vectors are $\bar{a}$ and $\bar{b}$ is $\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$

- Cartesian equation of a line that passes through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

- If θ is the acute angle between $\bar{r} = \bar{a}_1 + \lambda \bar{b}_1$ and $\bar{r} = \bar{a}_2 + \lambda \bar{b}_2$ then, $\cos\theta = \left| \dfrac{\bar{b}.\bar{b}_2}{\| \bar{b}_1 \| \bar{b}_2 \|} \right|$

- If $\dfrac{x - x_1}{l_1} = \dfrac{y - y_1}{m_1} = \dfrac{z - z_1}{n_1}$ and $\dfrac{x - x_2}{l_2} = \dfrac{y - y_2}{m_2} = \dfrac{z - z_2}{n_2}$ are the equations of two lines, then the acute angle between the two lines is given by $\cos\theta = \left| l_1 l_2 + m_1 m_2 + n_1 n_2 \right|$

- Shortest distance between two skew lines is the line segment perpendicular to both the lines.

- Shortest distance between $\bar{r} = \bar{a}_1 + \wedge \bar{b}_1$ and $\bar{r} = \bar{a}_2 + \wedge \bar{b}_2$ $\left| \dfrac{(\bar{b}_1 \times \bar{b}_2).(\bar{a}_2 - \bar{a}_1)}{|\bar{b}_1 \times \bar{b}_2|} \right|$

- Shortest distance between the lines: $\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}$ and $\dfrac{x - x_2}{a_1} = \dfrac{y - y_2}{b_1} = \dfrac{z - z_2}{c_1}$ is

$$\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2 + (a_1 b_2 - a_2 b_1)^2}}$$

- Distance between parallel lines $\bar{r} = \bar{a}_1 + \wedge \bar{b}_1$ and $\bar{r} = \bar{a}_2 + \wedge \bar{b}_2$ $\left| \dfrac{(\bar{b}) \times (\bar{a}_2 - \bar{a}_1)}{|\bar{b}_1|} \right|$

Key Notes

- In the vector form, equation of a plane which is at a distance d from the origin, and n ˆ is the unit vector normal to the plane through the origin is $\overline{} = d$

- Equation of a plane which is at a distance of d from the origin and the direction cosines of the normal to the plane as l, m, n is $lx + my + nz = d$.

- The equation of a plane through a point whose posit ion vector is a and perpendicular to the vector $\overline{N}$ $is (\overline{r} - \overline{a})$. N=0

- Equation of a plane perpendicular to a given line with direction ratios A, B, C and passing through a given point (x_1, y_1, z_1) is

- $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

- Equation of a plane passing through three non collinear points (x_1, y_1, z_1)

- (x_2, y_2, z_2) and (x_3, y_3, z_3) is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

- Vector equation of a plane that contains three non collinear points having position vectors

- $\overline{a}, \overline{b}$ and $\overline{c}$ is $(\overline{r} - \overline{a}).\left[(\overline{b} - \overline{a}) \times (\overline{c} - \overline{a}) \right] = 0$

- Equation of a plane that cuts the coordinates axes at $(a, 0, 0), (0, b, 0) \ and \ (0, 0, c)$ is $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$

- Vector equation of a plane that p asses thro ugh the in the section of planes $\overline{r}.\overline{n}_1 = d_1$ and $\overline{r}.\overline{n}_2 = d_2$ is $\overline{r}.(\overline{n}_1 + \lambda \overline{n}_2) = d_1 + \lambda d_2$ where λ is any nonzero constant.

- Cartesian equation of a plane that passes that passes through the intersection of two given planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is $(A_1x + B_1y + C_1z + D_1) + \lambda(A_2x + B_2y + C_2z + D_2 = 0$

- Two lines $\overline{r} = \overline{a}_1 + \lambda \overline{b}_1$ and $\overline{r} = \overline{a}_2 + \mu \overline{b}_2$ are coplanar if $(\overline{a}_2 - \overline{a}_1).(\overline{b}_1 \times - \overline{b}_2) = 0$

Key Notes

- In the Cartesian form above lines passing through the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2 z_2)$

- $$= \frac{y - y_2}{b_2} = \frac{z - z_2}{C_2}$$ are coplanar if $$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

- In the vector form, if θ is the angle between the two planes, $\overline{r}.\overline{n}_1 = d_1$ and $\overline{r}.\overline{n}_2 = d_2$, then

  $$\theta = \cos^{-1} \frac{\left| \overline{n}_1 . \overline{n}_2 \right|}{\left| \overline{n}_1 \right| \left| \overline{n}_2 \right|}$$

- The angle ϕ between the line $\overline{r} = \overline{a} + \lambda \overline{b}$ and the plane $\overline{r} . \hat{n} = d$ $$\sin \phi = \left| \frac{\overline{b} . \hat{n}}{\left| \overline{b} \right| \left| \hat{n} \right|} \right|$$

- The angle θ between the planes $A_1 x + B_1 y + C_1 z + D_1 = 0$ and $A_2 x + B_2 y + C_2 z + D_2 = 0$ is given by
  $$\cos \theta = \left| \frac{A_1 A_2 + + B_1 B_2 + C_1 C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

- The distance of a point whose position vector is $\overline{a}$ from the plane $\overline{r} . \hat{n} = d$ is $\left| d - \overline{a} . \hat{n} \right|$

- The distance from a point (x_1, y_1, z_1) to the plane $Ax + By + Cz + D = 0$ is $\left| \dfrac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$

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