

Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper

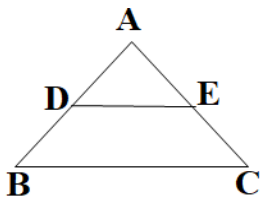
Time Allowed: 3 Hrs.

Maximum Marks : 80

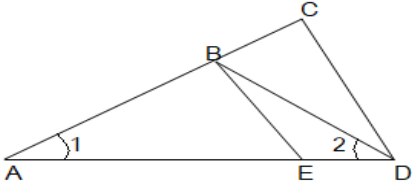
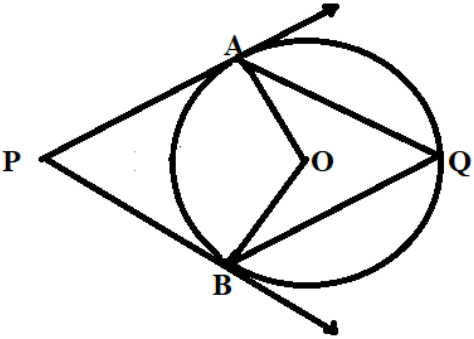
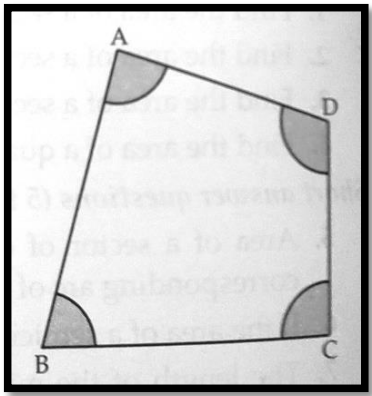
General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

| SECTION A | | |
|---|--|-------------------|
| Section A consists of 20 questions of 1 mark each. | | |
| S.NO | | MA RKS |
| 1 | Let a and b be two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If $HCF(a,b) = p^m q^n$ and $LCM(a,b) = p^r q^s$, then $(m+n)(r+s) =$ (a) 15 (b) 30 (c) 35 (d) 72 | 1 |
| 2 | Let p be a prime number. The quadratic equation having its roots as factors of p is (a) $x^2 - px + p = 0$ (b) $x^2 - (p+1)x + p = 0$ (c) $x^2 + (p+1)x + p = 0$ (d) $x^2 - px + p + 1 = 0$ | 1 |
| 3 | If α and β are the zeros of a polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then p is (a) -2/3 (b) 2/3 (c) 1/3 (d) -1/3 | 1 |
| 4 | If the system of equations $3x+y = 1$ and $(2k-1)x + (k-1)y = 2k+1$ is inconsistent, then k = (a) -1 (b) 0 (c) 1 (d) 2 | 1 |
| 5 | If the vertices of a parallelogram PQRS taken in order are P(3,4), Q(-2,3) and R(-3,-2), then the coordinates of its fourth vertex S are (a) (-2,-1) (b) (-2,-3) (c) (2,-1) (d) (1,2) | 1 |
| 6 | $\Delta ABC \sim \Delta PQR$. If AM and PN are altitudes of ΔABC and ΔPQR respectively and $AB^2 : PQ^2 = 4 : 9$, then AM: PN = (a) 3:2 (b) 16:81 (c) 4:9 (d) 2:3 | 1 |

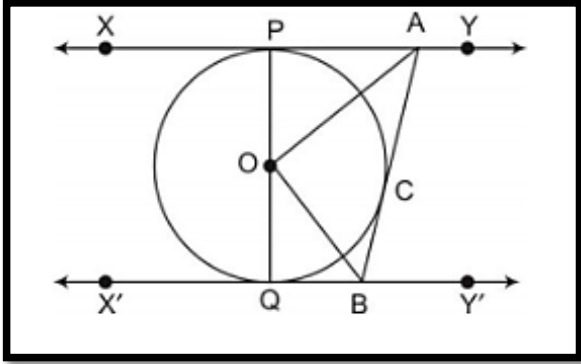
| | | | | | | | | | | | | | | |
|------------------|---|--------------|-------|-------|-------|-------|-------|------------------|----|----|----|----|---|---|
| 7 | If $x \tan 60^\circ \cos 60^\circ = \sin 60^\circ \cot 60^\circ$, then $x =$ (a) $\cos 30^\circ$ (b) $\tan 30^\circ$ (c) $\sin 30^\circ$ (d) $\cot 30^\circ$ | 1 | | | | | | | | | | | | |
| 8 | If $\sin \theta + \cos \theta = \sqrt{2}$, then $\tan \theta + \cot \theta =$ (a) 1 (b) 2 (c) 3 (d) 4 | 1 | | | | | | | | | | | | |
| 9 | In the given figure, $DE \parallel BC$, $AE = a$ units, $EC = b$ units, $DE = x$ units and $BC = y$ units. Which of the following is true? <div style="text-align: center;">  </div> (a) $x = \frac{a+b}{ay}$ (b) $y = \frac{ax}{a+b}$ (c) $x = \frac{ay}{a+b}$ (d) $\frac{x}{y} = \frac{a}{b}$ | 1 | | | | | | | | | | | | |
| 10 | ABCD is a trapezium with $AD \parallel BC$ and $AD = 4$ cm. If the diagonals AC and BD intersect each other at O such that $AO/OC = DO/OB = 1/2$, then $BC =$ (a) 6cm (b) 7cm (c) 8cm (d) 9cm | 1 | | | | | | | | | | | | |
| 11 | If two tangents inclined at an angle of 60° are drawn to a circle of radius 3cm, then the length of each tangent is equal to (a) $\frac{3\sqrt{3}}{2}$ cm (b) 3cm (c) 6cm (d) $3\sqrt{3}$ cm | 1 | | | | | | | | | | | | |
| 12 | The area of the circle that can be inscribed in a square of 6cm is (a) 36π cm ² (b) 18π cm ² (c) 12π cm ² (d) 9π cm ² | 1 | | | | | | | | | | | | |
| 13 | The sum of the length, breadth and height of a cuboid is $6\sqrt{3}$ cm and the length of its diagonal is $2\sqrt{3}$ cm. The total surface area of the cuboid is (a) 48 cm ² (b) 72 cm ² (c) 96 cm ² (d) 108 cm ² | 1 | | | | | | | | | | | | |
| 14 | If the difference of Mode and Median of a data is 24, then the difference of median and mean is (a) 8 (b) 12 (c) 24 (d) 36 | 1 | | | | | | | | | | | | |
| 15 | The number of revolutions made by a circular wheel of radius 0.25m in rolling a distance of 11km is (a) 2800 (b) 4000 (c) 5500 (d) 7000 | 1 | | | | | | | | | | | | |
| 16 | For the following distribution, <table border="1" style="margin-left: auto; margin-right: auto;"> <tbody> <tr> <td>Class</td> <td>0-5</td> <td>5-10</td> <td>10-15</td> <td>15-20</td> <td>20-25</td> </tr> <tr> <td>Frequency</td> <td>10</td> <td>15</td> <td>12</td> <td>20</td> <td>9</td> </tr> </tbody> </table> the sum of the lower limits of the median and modal class is (a) 15 (b) 25 (c) 30 (d) 35 | Class | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 | Frequency | 10 | 15 | 12 | 20 | 9 | 1 |
| Class | 0-5 | 5-10 | 10-15 | 15-20 | 20-25 | | | | | | | | | |
| Frequency | 10 | 15 | 12 | 20 | 9 | | | | | | | | | |

| | | |
|----|--|---|
| 17 | <p>Two dice are rolled simultaneously. What is the probability that 6 will come up at least once?</p> <p>(a) 1/6 (b) 7/36 (c) 11/36 (d) 13/36</p> | 1 |
| 18 | <p>If $5 \tan \beta = 4$, then $\frac{5 \sin \beta - 2 \cos \beta}{5 \sin \beta + 2 \cos \beta} =$</p> <p>(a) 1/3 (b) 2/5 (c) 3/5 (d) 6</p> | 1 |
| 19 | <p>DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option</p> <p>Statement A (Assertion): If product of two numbers is 5780 and their HCF is 17, then their LCM is 340</p> <p>Statement R(Reason) : HCF is always a factor of LCM</p> <p>(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)</p> <p>(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)</p> <p>(c) Assertion (A) is true but reason (R) is false.</p> <p>(d) Assertion (A) is false but reason (R) is true.</p> | 1 |
| 20 | <p>Statement A (Assertion): If the co-ordinates of the mid-points of the sides AB and AC of ΔABC are D(3,5) and E(-3,-3) respectively, then BC = 20 units</p> <p>Statement R(Reason) : The line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.</p> <p>(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)</p> <p>(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)</p> <p>(c) Assertion (A) is true but reason(R) is false.</p> <p>(d) Assertion (A) is false but reason(R) is true.</p> | 1 |

| SECTION B | | |
|---|--|--------------|
| Section B consists of 5 questions of 2 marks each. | | |
| S.No. | | Marks |
| 21 | If $49x+51y= 499$, $51 x+49 y= 501$, then find the value of x and y | 2 |
| 22 | <p>In the given figure below, $\frac{AD}{AE} = \frac{AC}{BD}$ and $\angle 1 = \angle 2$. Show that $\Delta BAE \sim \Delta CAD$.</p>  | 2 |
| 23 | <p>In the given figure, O is the centre of circle. Find $\angle AQB$, given that PA and PB are tangents to the circle and $\angle APB= 75^\circ$.</p>  | 2 |
| 24 | <p>The length of the minute hand of a clock is 6cm. Find the area swept by it when it moves from 7:05 p.m. to 7:40 p.m.</p> <p style="text-align: center;">OR</p> <p>In the given figure, arcs have been drawn of radius 7cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.</p>  | 2 |

| | | |
|-----------|--|----------|
| | | |
| 25 | <p>If $\sin(A+B) = 1$ and $\cos(A-B) = \frac{\sqrt{3}}{2}$, $0^\circ < A+B \leq 90^\circ$ and $A > B$, then find the measures of angles A and B.</p> <p style="text-align: center;">OR</p> <p>Find an acute angle θ when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$</p> | 2 |

| SECTION C | | |
|---|---|--------------|
| Section C consists of 6 questions of 3 marks each. | | |
| S.No | | Marks |
| 26 | Given that $\sqrt{3}$ is irrational, prove that $5 + 2\sqrt{3}$ is irrational. | 3 |
| 27 | If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of the polynomial $2x^2 - 5x - 3$, then find the values of p and q. | 3 |
| 28 | <p>A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr ; it would have taken 6 hours more than the scheduled time. Find the length of the journey.</p> <p style="text-align: center;">OR</p> <p>Anuj had some chocolates, and he divided them into two lots A and B. He sold the first lot at the rate of ₹2 for 3 chocolates and the second lot at the rate of ₹1 per chocolate, and got a total of ₹400. If he had sold the first lot at the rate of ₹1 per chocolate, and the second lot at the rate of ₹4 for 5 chocolates, his total collection would have been ₹460. Find the total number of chocolates he had.</p> | 3 |
| 29 | <p>Prove the following that-</p> $\frac{\tan^3\theta}{1 + \tan^2\theta} + \frac{\cot^3\theta}{1 + \cot^2\theta} = \sec\theta \operatorname{cosec}\theta - 2 \sin\theta \cos\theta$ | 3 |
| 30 | <p>Prove that a parallelogram circumscribing a circle is a rhombus</p> <p style="text-align: center;">OR</p> | 3 |

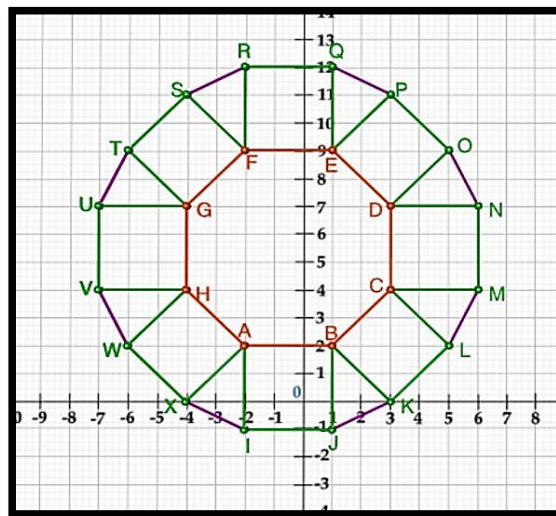
| | | |
|---|--|---------------------|
| | <p>In the figure XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B, what is the measure of $\angle AOB$.</p>  | |
| <p>31</p> | <p>Two coins are tossed simultaneously. What is the probability of getting</p> <ul style="list-style-type: none"> (i) At least one head? (ii) At most one tail? (iii) A head and a tail? | <p>3</p> |
| SECTION D | | |
| Section D consists of 4 questions of 5 marks each. | | |
| <p>S.No</p> | | <p>Marks</p> |
| <p>32</p> | <p>To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool?</p> <p style="text-align: center;">OR</p> <p>In a flight of 600km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr from its usual speed and the time of the flight increased by 30 min. Find the scheduled duration of the flight.</p> | <p>5</p> |
| <p>33</p> | <p>Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.</p> <p>Using the above theorem prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non parallel sides in the same ratio.</p> | <p>5</p> |

| 34 | <p>Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and the upper part is conical with the same base radius, but of height 2.1 m. If the canvas used to make the tents costs ₹120 per m², find the amount shared by each school to set up the tents.</p> <p style="text-align: center;">OR</p> <p>There are two identical solid cubical boxes of side 7cm. From the top face of the first cube a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to form a dome. Find</p> <p>(i) the ratio of the total surface area of the two new solids formed</p> <p>(ii) volume of each new solid formed.</p> | 5 | | | | | | | | | | | | | | | | | | | | | | |
|----------------|--|----------------|-----------|-------|---|---------|---|---------|---|---------|----|---------|----|---------|----|---------|---|---------|---|---------|---|----------|---|---|
| 35 | <p>The median of the following data is 525. Find the values of x and y, if the total frequency is 100</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Class interval</th> <th>Frequency</th> </tr> </thead> <tbody> <tr> <td>0–100</td> <td>2</td> </tr> <tr> <td>100–200</td> <td>5</td> </tr> <tr> <td>200–300</td> <td>x</td> </tr> <tr> <td>300–400</td> <td>12</td> </tr> <tr> <td>400–500</td> <td>17</td> </tr> <tr> <td>500–600</td> <td>20</td> </tr> <tr> <td>600–700</td> <td>y</td> </tr> <tr> <td>700–800</td> <td>9</td> </tr> <tr> <td>800–900</td> <td>7</td> </tr> <tr> <td>900–1000</td> <td>4</td> </tr> </tbody> </table> | Class interval | Frequency | 0–100 | 2 | 100–200 | 5 | 200–300 | x | 300–400 | 12 | 400–500 | 17 | 500–600 | 20 | 600–700 | y | 700–800 | 9 | 800–900 | 7 | 900–1000 | 4 | 5 |
| Class interval | Frequency | | | | | | | | | | | | | | | | | | | | | | | |
| 0–100 | 2 | | | | | | | | | | | | | | | | | | | | | | | |
| 100–200 | 5 | | | | | | | | | | | | | | | | | | | | | | | |
| 200–300 | x | | | | | | | | | | | | | | | | | | | | | | | |
| 300–400 | 12 | | | | | | | | | | | | | | | | | | | | | | | |
| 400–500 | 17 | | | | | | | | | | | | | | | | | | | | | | | |
| 500–600 | 20 | | | | | | | | | | | | | | | | | | | | | | | |
| 600–700 | y | | | | | | | | | | | | | | | | | | | | | | | |
| 700–800 | 9 | | | | | | | | | | | | | | | | | | | | | | | |
| 800–900 | 7 | | | | | | | | | | | | | | | | | | | | | | | |
| 900–1000 | 4 | | | | | | | | | | | | | | | | | | | | | | | |

| SECTION E | |
|---|---|
| Case study based questions are compulsory. | |
| 36 | A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, paintings etc. Shown below is a tiled floor in the archaeological Museum of Seville, made using squares, triangles and hexagons. |



A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern



Use the above figure to answer the questions that follow:

- (i) What is the length of the line segment joining points B and F? 1
- (ii) The centre 'Z' of the figure will be the point of intersection of the diagonals of quadrilateral WXOP. Then what are the coordinates of Z? 1
- (iii) What are the coordinates of the point on y axis equidistant from A and G? 2

OR

What is the area of Trapezium AFGH?

37

The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.



- (i) If the first circular row has 30 seats, how many seats will be there in the 10th row? **1**
- (ii) For 1500 seats in the auditorium, how many rows need to be there? **2**

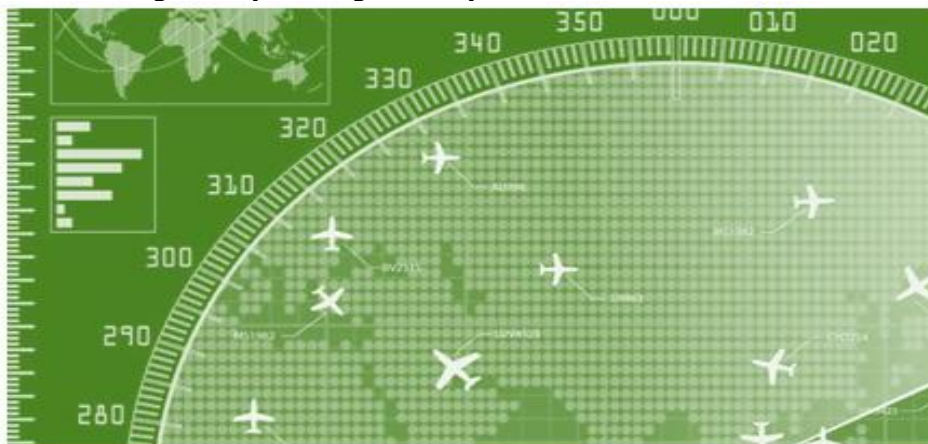
OR

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?

- (iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row? **1**

38

We all have seen the airplanes flying in the sky but might have not thought of how they actually reach the correct destination. Air Traffic Control (ATC) is a service provided by ground-based air traffic controllers who direct aircraft on the ground and through a given section of controlled airspace, and can provide advisory services to aircraft in non-controlled airspace. Actually, all this air traffic is managed and regulated by using various concepts based on coordinate geometry and trigonometry.



| | | |
|--|--|---|
| | <p>At a given instance, ATC finds that the angle of elevation of an airplane from a point on the ground is 60°. After a flight of 30 seconds, it is observed that the angle of elevation changes to 30°. The height of the plane remains constantly as $3000\sqrt{3}$ m. Use the above information to answer the questions that follow-</p> <p>(i) Draw a neat labelled figure to show the above situation diagrammatically.</p> <p>(ii) What is the distance travelled by the plane in 30 seconds?</p> <p style="text-align: center;">OR</p> <p>Keeping the height constant, during the above flight, it was observed that after $15(\sqrt{3} - 1)$ seconds, the angle of elevation changed to 45°. How much is the distance travelled in that duration.</p> <p>(iii) What is the speed of the plane in km/hr.</p> | <p>1</p> <p>2</p> <p>1</p> |
|--|--|---|

SAMPLE QUESTION PAPER
MARKING SCHEME
SUBJECT: MATHEMATICS- STANDARD
CLASS X

SECTION - A

| | | |
|------------|---|---|
| 1 | (c) 35 | 1 |
| 2 | (b) $x^2 - (p+1)x + p = 0$ | 1 |
| 3 | (b) $\frac{2}{3}$ | 1 |
| 4 | (d) 2 | 1 |
| 5 | (c) (2, -1) | 1 |
| 6 | (d) 2:3 | 1 |
| 7 | (b) $\tan 30^\circ$ | 1 |
| 8 | (b) 2 | 1 |
| 9 | (c) $x = \frac{ay}{a+b}$ | 1 |
| 10 | (c) 8cm | 1 |
| 11 | (d) $3\sqrt{3}$ cm | 1 |
| 12 | (d) $9\pi \text{ cm}^2$ | 1 |
| 13 | (c) 96 cm^2 | 1 |
| 14 | (b) 12 | 1 |
| 15 | (d) 7000 | 1 |
| 16 | (b) 25 | 1 |
| 17 | (c) $\frac{11}{36}$ | 1 |
| 18 | (a) $\frac{1}{3}$ | 1 |
| 19 | (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) | 1 |
| 20. | (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A) | 1 |

$$\begin{aligned}
&= \frac{\theta_1}{360} \times \pi \times 7^2 + \frac{\theta_2}{360} \times \pi \times 7^2 + \frac{\theta_3}{360} \times \pi \times 7^2 + \frac{\theta_4}{360} \times \pi \times 7^2 && \frac{1}{2} \\
&= \frac{(\theta_1 + \theta_2 + \theta_3 + \theta_4)}{360} \times \pi \times 7^2 \\
&= \frac{(360)}{360} \times \frac{22}{7} \times 7 \times 7 \text{ (By angle sum property of a triangle)} && \frac{1}{2} \\
&= 154 \text{ cm}^2 && \frac{1}{2}
\end{aligned}$$

- 25 $\sin(A+B)=1 = \sin 90$, so $A+B = 90$(i) 1/2
 $\cos(A-B)=\sqrt{3}/2 = \cos 30$, so $A-B= 30$(ii) 1/2
From (i) & (ii) $\angle A = 60^\circ$ 1/2
And $\angle B = 30^\circ$ 1/2

OR

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Dividing the numerator and denominator of LHS by $\cos\theta$, we get 1/2

$$\frac{1 - \tan\theta}{1 + \tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Which on simplification (or comparison) gives $\tan\theta = \sqrt{3}$ 1/2
Or $\theta = 60^\circ$ 1/2

SECTION - C

- 26 Let us assume $5 + 2\sqrt{3}$ is rational, then it must be in the form of p/q where p and q are co-prime integers and $q \neq 0$ 1
- i.e $5 + 2\sqrt{3} = p/q$ 1/2
- So $\sqrt{3} = \frac{p-5q}{2q}$(i) 1/2
- Since $p, q, 5$ and 2 are integers and $q \neq 0$, HS of equation (i) is rational. But LHS of (i) is $\sqrt{3}$ which is irrational. This is not possible. 1/2
- This contradiction has arisen due to our wrong assumption that $5 + 2\sqrt{3}$ is rational. So, $5 + 2\sqrt{3}$ is irrational. 1/2

- 27 Let α and β be the zeros of the polynomial $2x^2 - 5x - 3$
Then $\alpha + \beta = 5/2$ 1/2
And $\alpha\beta = -3/2$. 1/2
Let 2α and 2β be the zeros $x^2 + px + q$
Then $2\alpha + 2\beta = -p$ 1/2
 $2(\alpha + \beta) = -p$
 $2 \times 5/2 = -p$
So $p = -5$ 1/2
And $2\alpha \times 2\beta = q$ 1/2
 $4\alpha\beta = q$
So $q = 4 \times -3/2$
 $= -6$ 1/2

28 Let the actual speed of the train be x km/hr and let the actual time taken be y hours. 1/2
 Distance covered is xy km
 If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours i.e.,
 when speed is $(x+6)$ km/hr, time of journey is $(y-4)$ hours.
 \therefore Distance covered $= (x+6)(y-4)$
 $\Rightarrow xy = (x+6)(y-4)$
 $\Rightarrow -4x + 6y - 24 = 0$ 1/2
 $\Rightarrow -2x + 3y - 12 = 0$ (i)
 Similarly $xy = (x-6)(y+6)$
 $\Rightarrow 6x - 6y - 36 = 0$
 $\Rightarrow x - y - 6 = 0$ (ii) 1/2
 Solving (i) and (ii) we get $x=30$ and $y=24$ 1
 Putting the values of x and y in equation (i), we obtain
 Distance $= (30 \times 24)$ km $= 720$ km. 1/2
 Hence, the length of the journey is 720km.

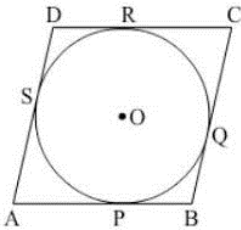
OR

Let the number of chocolates in lot A be x 1/2
 And let the number of chocolates in lot B be y
 \therefore total number of chocolates $= x + y$
 Price of 1 chocolate = ₹ $\frac{2}{3}$, so for x chocolates $= \frac{2}{3}x$
 and price of y chocolates at the rate of ₹ 1 per chocolate $= y$.
 \therefore by the given condition $\frac{2}{3}x + y = 400$ 1/2
 $\Rightarrow 2x + 3y = 1200$ (i)
 Similarly $x + \frac{4}{5}y = 460$ 1/2
 $\Rightarrow 5x + 4y = 2300$ (ii)
 Solving (i) and (ii) we get
 $x = 300$ and $y = 200$
 $\therefore x + y = 300 + 200 = 500$ 1
 So, Anuj had 500 chocolates. 1/2

29 LHS : $\frac{\sin^3\theta / \cos^3\theta}{1 + \sin^2\theta / \cos^2\theta} + \frac{\cos^3\theta / \sin^3\theta}{1 + \cos^2\theta / \sin^2\theta}$ 1/2

$$\begin{aligned}
&= \frac{\sin^3\theta/\cos^3\theta}{(\cos^2\theta + \sin^2\theta)/\cos^2\theta} + \frac{\cos^3\theta/\sin^3\theta}{(\sin^2\theta + \cos^2\theta)/\sin^2\theta} \\
&= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} && \frac{1}{2} \\
&= \frac{\sin^4\theta + \cos^4\theta}{\cos\theta\sin\theta} && \frac{1}{2} \\
&= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta} && \frac{1}{2} \\
&= \frac{1 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta} && \frac{1}{2} \\
&= \frac{1}{\cos\theta\sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta} \\
&= \sec\theta\operatorname{cosec}\theta - 2\sin\theta\cos\theta && \frac{1}{2} \\
&= \text{RHS}
\end{aligned}$$

30

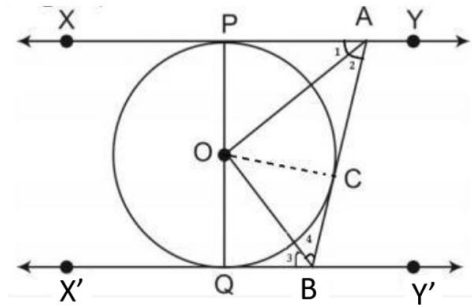


Let ABCD be the rhombus circumscribing the circle with centre O, such that AB, BC, CD and DA touch the circle at points P, Q, R and S respectively.

We know that the tangents drawn to a circle from an exterior point are equal in length.

$$\begin{aligned}
&\therefore AP = AS \dots\dots\dots(1) \\
&BP = BQ \dots\dots\dots(2) \\
&CR = CQ \dots\dots\dots(3) && 1 \\
&DR = DS \dots\dots\dots(4). \\
&\text{Adding (1), (2), (3) and (4) we get} \\
&AP+BP+CR+DR = AS+BQ+CQ+DS \\
&(AP+BP) + (CR+DR) = (AS+DS) + (BQ+CQ) \\
&\therefore AB+CD=AD+BC \dots\dots\dots(5) && 1 \\
&\text{Since } AB=DC \text{ and } AD=BC \text{ (opposite sides of parallelogram ABCD)} && \frac{1}{2} \\
&\text{putting in (5) we get, } 2AB=2AD \\
&\text{or } AB = AD. \\
&\therefore AB=BC=DC=AD \\
&\text{Since a parallelogram with equal adjacent sides is a rhombus, so ABCD is a} && \frac{1}{2} \\
&\text{rhombus}
\end{aligned}$$

OR



Join OC

In $\triangle OPA$ and $\triangle OCA$

$OP = OC$ (radii of same circle)

$PA = CA$ (length of two tangents from an external point) 1

$AO = AO$ (Common)

Therefore, $\triangle OPA \cong \triangle OCA$ (By SSS congruency criterion) $\frac{1}{2}$

Hence, $\angle 1 = \angle 2$ (CPCT) $\frac{1}{2}$

Similarly $\angle 3 = \angle 4$

$\angle PAB + \angle QBA = 180^\circ$ (co interior angles are supplementary as $XY \parallel X'Y'$) $\frac{1}{2}$

$$2\angle 2 + 2\angle 4 = 180^\circ$$

$$\angle 2 + \angle 4 = 90^\circ \text{-----(1)} \quad \text{span style="float: right;"> $\frac{1}{2}$$$

$$\angle 2 + \angle 4 + \angle AOB = 180^\circ \text{ (Angle sum property)}$$

Using (1), we get, $\angle AOB = 90^\circ$

- 31**
- (i) $P(\text{At least one head}) = \frac{3}{4}$ 1
 - (ii) $P(\text{At most one tail}) = \frac{3}{4}$ 1
 - (iii) $P(\text{A head and a tail}) = \frac{2}{4} = \frac{1}{2}$ 1

SECTION D

- 32** Let the time taken by larger pipe alone to fill the tank = x hours $\frac{1}{2}$
 Therefore, the time taken by the smaller pipe = $x+10$ hours

Water filled by larger pipe running for 4 hours = $\frac{4}{x}$ litres

Water filled by smaller pipe running for 9 hours = $\frac{9}{x+10}$ litres

We know that

$$\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$$

1

Which on simplification gives:

$$x^2 - 16x - 80 = 0$$

1

$$x^2 - 20x + 4x - 80 = 0$$

$$x(x-20) + 4(x-20) = 0$$

$$(x+4)(x-20) = 0$$

$$x = -4, 20$$

1

x cannot be negative.

$$\text{Thus, } x = 20$$

1/2

$$x + 10 = 30$$

1/2

Larger pipe would alone fill the tank in 20 hours and smaller pipe would fill the tank alone in 30 hours.

1/2

OR

Let the usual speed of plane be x km/hr

1/2

and the reduced speed of the plane be (x-200) km/hr

Distance = 600 km [Given]

According to the question,

(time taken at reduced speed) - (Schedule time) = 30 minutes = 0.5 hours.

1

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$$

Which on simplification gives:

$$x^2 - 200x - 240000 = 0$$

1

$$x^2 - 600x + 400x - 240000 = 0$$

$$x(x-600) + 400(x-600) = 0$$

$$(x-600)(x+400) = 0$$

$$x = 600 \text{ or } x = -400$$

1

But speed cannot be negative.

1/2

∴ The usual speed is 600 km/hr and

1/2

the scheduled duration of the flight is $\frac{600}{600} = 1$ hour

1/2

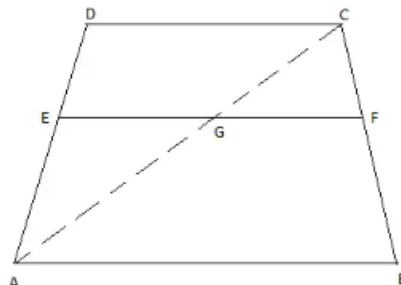
33 For the Theorem :

Given, To prove, Construction and figure

1 1/2

Proof

1 1/2



1/2

Let ABCD be a trapezium DC||AB and EF is a line parallel to AB and hence to DC.

To prove : $\frac{DE}{EA} = \frac{CF}{FB}$

Construction : Join AC, meeting EF in G.

Proof :

In $\triangle ABC$, we have

GF||AB

CG/GA=CF/FB [By BPT](1) 1/2

In $\triangle ADC$, we have

EG||DC (EF ||AB & AB ||DC) 1/2

DE/EA= CG/GA [By BPT](2)

From (1) & (2), we get,

$\frac{DE}{EA} = \frac{CF}{FB}$ 1/2

34. Radius of the base of cylinder (r) = 2.8 m = Radius of the base of the cone (r)

Height of the cylinder (h)=3.5 m

Height of the cone (H)=2.1 m.

Slant height of conical part (l)= $\sqrt{r^2+H^2}$

= $\sqrt{(2.8)^2+(2.1)^2}$ 1

= $\sqrt{7.84+4.41}$ 1

= $\sqrt{12.25} = 3.5$ m 1

Area of canvas used to make tent = CSA of cylinder + CSA of cone 1

= $2 \times \pi \times 2.8 \times 3.5 + \pi \times 2.8 \times 3.5$ 1

= 61.6+30.8 1

= 92.4m² 1

Cost of 1500 tents at ₹120 per sq.m

= 1500×120×92.4

= 16,632,000

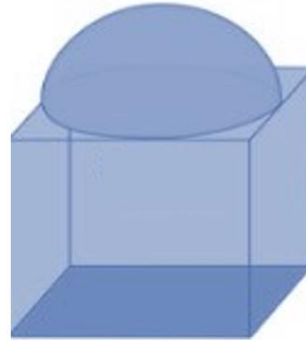
Share of each school to set up the tents = 16632000/50 = ₹332,640

OR

First Solid



Second Solid



(i) SA for first new solid (S_1):

$$\begin{aligned} &6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2 \\ &= 294 + 77 - 38.5 \\ &= 332.5 \text{ cm}^2 \end{aligned}$$

1

SA for second new solid (S_2):

$$\begin{aligned} &6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2 \\ &= 294 + 77 - 38.5 \\ &= 332.5 \text{ cm}^2 \end{aligned}$$

1

1

So $S_1 : S_2 = 1 : 1$

(ii) Volume for first new solid (V_1) = $7 \times 7 \times 7 - \frac{2}{3}\pi \times 3.5^3$

$$= 343 - \frac{539}{6} = \frac{1519}{6} \text{ cm}^3$$

1

Volume for second new solid (V_2) = $7 \times 7 \times 7 + \frac{2}{3}\pi \times 3.5^3$

$$= 343 + \frac{539}{6} = \frac{2597}{6} \text{ cm}^3$$

1

35 Median = 525, so Median Class = 500 – 600

$\frac{1}{2}$

| Class interval | Frequency | Cumulative Frequency |
|----------------|-----------|----------------------|
| 0–100 | 2 | 2 |
| 100–200 | 5 | 7 |
| 200–300 | x | 7+x |
| 300–400 | 12 | 19+x |
| 400–500 | 17 | 36+x |
| 500–600 | 20 | 56+x |
| 600–700 | y | 56+x+y |
| 700–800 | 9 | 65+x+y |
| 800–900 | 7 | 72+x+y |
| 900–1000 | 4 | 76+x+y |

$1\frac{1}{2}$

$$76+x+y=100 \Rightarrow x+y=24 \quad \dots(i)$$

1

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$\frac{1}{2}$

Since, $l=500$, $h=100$, $f=20$, $cf=36+x$ and $n=100$

Therefore, putting the value in the Median formula, we get;

$$525 = 500 + \frac{50 - (36+x)}{20} \times 100$$

$\frac{1}{2}$

$$\text{so } x = 9$$

$$y = 24 - x \text{ (from eq.i)}$$

$$y = 24 - 9 = 15$$

Therefore, the value of $x = 9$

$\frac{1}{2}$

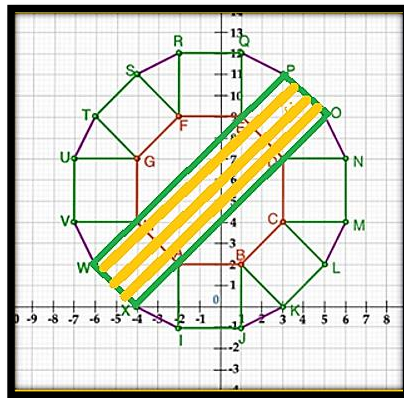
and $y = 15$.

$\frac{1}{2}$

- 36** (i) $B(1,2)$, $F(-2,9)$
 $BF^2 = (-2-1)^2 + (9-2)^2$
 $= (-3)^2 + (7)^2$
 $= 9 + 49$
 $= 58$
 So, $BF = \sqrt{58}$ units

1

(ii)



$$W(-6,2), X(-4,0), O(5,9), P(3,11)$$

$\frac{1}{2}$

Clearly WXOP is a rectangle

Point of intersection of diagonals of a rectangle is the mid point of the diagonals. So the required point is mid point of WO or XP

$$= \left(\frac{-6+5}{2}, \frac{2+9}{2} \right)$$

$\frac{1}{2}$

$$= \left(\frac{-1}{2}, \frac{11}{2} \right)$$

- (iii) $A(-2,2)$, $G(-4,7)$
 Let the point on y-axis be $Z(0,y)$
 $AZ^2 = GZ^2$

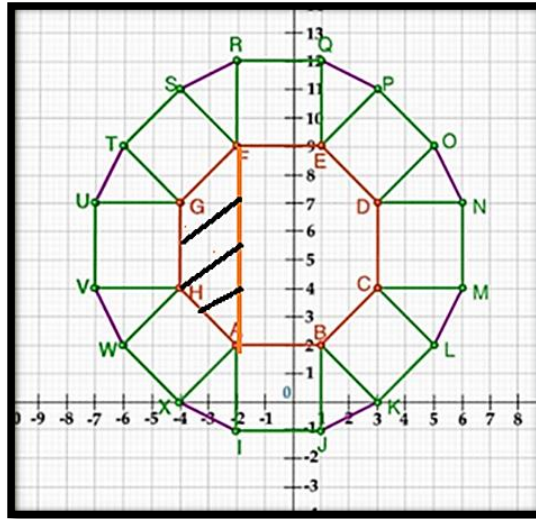
$\frac{1}{2}$

$\frac{1}{2}$

$$\begin{aligned}
 (0+2)^2 + (y-2)^2 &= (0+4)^2 + (y-7)^2 \\
 (2)^2 + y^2 + 4 - 4y &= (4)^2 + y^2 + 49 - 14y \\
 8 - 4y &= 65 - 14y \\
 10y &= 57 \\
 \text{So, } y &= 5.7 \\
 \text{i.e. the required point is } &(0, 5.7)
 \end{aligned}$$

1/2
1/2

OR



$$\begin{aligned}
 A(-2, 2), F(-2, 9), G(-4, 7), H(-4, 4) \\
 \text{Clearly } GH &= 7 - 4 = 3 \text{ units} \\
 AF &= 9 - 2 = 7 \text{ units} \\
 \text{So, height of the trapezium } AFGH &= 2 \text{ units} \\
 \text{So, area of } AFGH &= \frac{1}{2}(AF + GH) \times \text{height} \\
 &= \frac{1}{2}(7 + 3) \times 2 \\
 &= 10 \text{ sq. units}
 \end{aligned}$$

1/2
1/2
1/2
1/2

37. (i) Since each row is increasing by 10 seats, so it is an AP with first term $a = 30$, and common difference $d = 10$.

1/2

$$\begin{aligned}
 \text{No number of seats in } 10^{\text{th}} \text{ row} &= a_{10} = a + 9d \\
 &= 30 + 9 \times 10 = 120
 \end{aligned}$$

1/2

$$\begin{aligned}
 \text{(ii) } S_n &= \frac{n}{2}(2a + (n-1)d) \\
 1500 &= \frac{n}{2}(2 \times 30 + (n-1)10) \\
 3000 &= 50n + 10n^2 \\
 n^2 + 5n - 300 &= 0 \\
 n^2 + 20n - 15n - 300 &= 0 \\
 (n+20)(n-15) &= 0
 \end{aligned}$$

1/2

1/2

Rejecting the negative value, $n = 15$

1/2

1/2

OR

No. of seats already put up to the 10th row = S_{10}

1/2

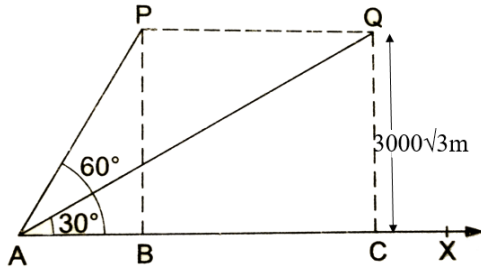
$$S_{10} = \frac{10}{2} \{2 \times 30 + (10-1)10\}$$

1/2

$= 5(60 + 90) = 750$ 1/2
 So, the number of seats still required to be put are $1500 - 750 = 750$ 1/2

(iii) If no. of rows = 17 1/2
 then the middle row is the 9th row
 $a_8 = a + 8d$
 $= 30 + 80$
 $= 110$ seats 1/2

38 (i)

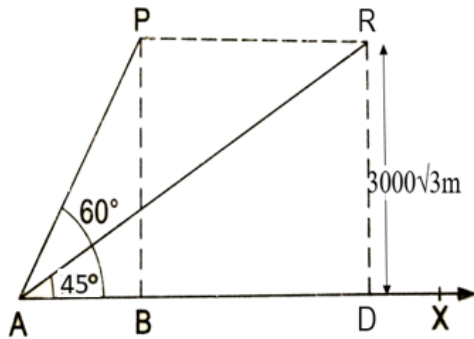


1

P and Q are the two positions of the plane flying at a height of $3000\sqrt{3}$ m.
 A is the point of observation.

(ii) In $\triangle PAB$, $\tan 60^\circ = PB/AB$
 Or $\sqrt{3} = 3000\sqrt{3}/AB$
 So $AB = 3000$ m 1
 $\tan 30^\circ = QC/AC$
 $1/\sqrt{3} = 3000\sqrt{3}/AC$
 $AC = 9000$ m 1/2
 distance covered = $9000 - 3000$
 $= 6000$ m. 1/2

OR



1/2

In $\triangle PAB$, $\tan 60^\circ = PB/AB$
 Or $\sqrt{3} = 3000\sqrt{3}/AB$
 So $AB = 3000$ m 1/2
 $\tan 45^\circ = RD/AD$
 $1 = 3000\sqrt{3}/AD$ 1/2

$$\begin{aligned} AD &= 3000\sqrt{3} \text{ m} \\ \text{distance covered} &= 3000\sqrt{3} - 3000 && \frac{1}{2} \\ &= 3000(\sqrt{3} - 1)\text{m}. \end{aligned}$$

$$\begin{aligned} \text{(iii) speed} &= 6000/30 && \frac{1}{2} \\ &= 200 \text{ m/s} \\ &= 200 \times 3600/1000 && \frac{1}{2} \\ &= 720\text{km/hr} \end{aligned}$$

$$\begin{aligned} \text{Alternatively: speed} &= \frac{3000(\sqrt{3} - 1)}{15(\sqrt{3} - 1)} && \frac{1}{2} \\ &= 200 \text{ m/s} \\ &= 200 \times 3600/1000 && \frac{1}{2} \\ &= 720\text{km/hr} \end{aligned}$$