# CIRCLE

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## DEFINITIONS

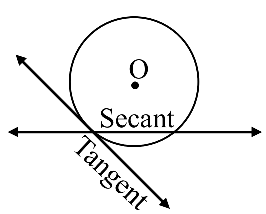
◈ **Secant :**

A line which intersects a circle in two distinct points is called a secant.

◈ **Tangent :**

A line meeting a circle only in one point is called a tangent to the circle at that point.

The point at which the tangent line meets the circle is called the point of contact.




## NUMBER OF TANGENTS TO A CIRCLE

(i) There is no tangent passing through a point lying inside the circle.

(ii) There is one and only one tangent passing through a point lying on a circle.

(iii) There are exactly two tangents through a point lying outside a circle.

## LENGTH OF TANGENT

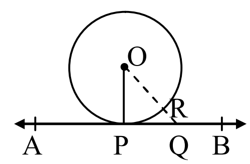
The length of the line segment of the tangent between a given point and the given point of contact with the circle is called the length of the tangent from the point to the circle.

## RESULTS ON TANGENTS

**Theorem 1 :**

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

**Given :** A circle with centre O and a tangent AB at a point P of the circle.




**To prove :** $OP \perp AB$.

**Construction :** Take a point Q, other than P, on AB. Join OQ.

**Proof :** Q is a point on the tangent AB, other than the point of contact P.

$\therefore$ Q lies outside the circle.

Let OQ intersect the circle at R.

Then, $OR < OQ$ [a part is less than the whole]...(i)

But, $OP = OR$ [radii of the same circle]. ....(ii)

$\therefore$ $OP < OQ$ [from (i) and (ii)].

Thus, OP is shorter than, any other line segment joining O to any point of AB, other than P.

In other words, OP is the shortest distance between the point O and the line AB.

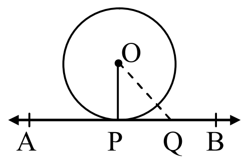
But, the shortest distance between a point and a line is the perpendicular distance.

$\therefore$ $OP \perp AB$.

**Theorem 2 : (Converse of Theorem 1)**

A line drawn through the end of a radius and perpendicular to it is a tangent to the circle.

**Given :** A circle with centre O in which OP is a radius and AB is a line through P such that $OP \perp AB$.




**To prove :** AB is a tangent to the circle at the point P.

**Construction :** Take a point Q, different from P, on AB. Join OQ.

**Proof :** We know that the perpendicular distance from a point to a line is the shortest distance between them.

$\therefore$ $OP \perp AB \Rightarrow OP$ is the shortest distance from O to AB.

$\therefore$ $OP < OQ$.

$\therefore$ Q lies outside the circle

[Θ OP is the radius and $OP < OQ$].

Thus, every point on AB, other than P, lies outside the circle.

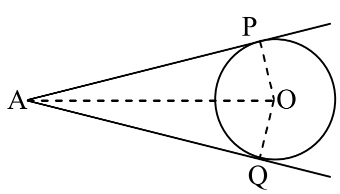
$\therefore$ AB meets the circle at the point P only.

Hence, AB is the tangent to the circle at the point P.

**Theorem 3 :**

The lengths of tangents drawn from an external point to a circle are equal.

**Given :** Two tangents AP and AQ are drawn from a point A to a circle with centre O.




**To prove :** AP = AQ

**Construction :** Join OP, OQ and OA.

**Proof :** AP is a tangent at P and OP is the radius through P.

$\therefore$ $OP \perp AP$.

Similarly, $OQ \perp AQ$.

In the right triangle OPA and OQA, we have

OP = OQ [radii of the same circle]

OA = OA [common]

$\therefore$ $\Delta OPA \cong \Delta OQA$ [by RHS–congruence]

Hence, AP = AQ.

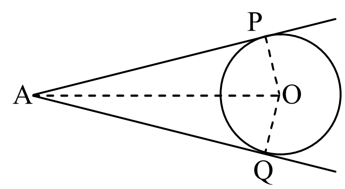
**Theorem 4 :**

If two tangents are drawn from an external point then

(i) They subtend equal angles at the centre, and

(ii) They are equally inclined to the line segment joining the centre to that point.

**Given :** A circle with centre O and a point A outside it. Also, AP and AQ are the two tangents to the circle.




**To prove :** $\angle AOP = \angle AOQ$ and $\angle OAP = \angle OAQ$.

**Proof :** In $\Delta AOP$ and $\Delta AOQ$, we have

AP = AQ [tangents from an external point are equal]

OP = OQ [radii of the same circle]

OA = OA [common]

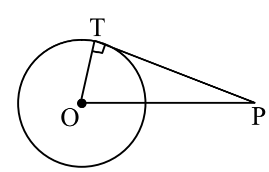
$\therefore$ $\Delta AOP$ $\Delta AOQ$ [by SSS–congruence].

Hence, $\angle AOP = \angle AOQ$ and $\angle OAP = \angle OAQ$.

## ❖ EXAMPLES ❖

**Ex.1** From a point P, 10 cm away from the centre of a circle, a tangent PT of length 8 cm is drawn. Find the radius of the circle.

**Sol.** Let O be the centre of the given circle and let P be a point such that




OP = 10 cm.

Let PT be the tangent such that PT = 8 cm.

Join OT.

Now, PT is a tangent at T and OT is the radius through T.

$\therefore \quad OT \perp PT.$

In the right $\Delta OTP$, we have

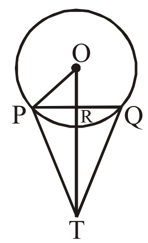
$OP^2 = OT^2 + PT^2$ [by Pythagoras' theorem]

$\Rightarrow \quad OT = \sqrt{OP^2 - PT^2} = \sqrt{(10)^2 - (8)^2}$ cm

$= \sqrt{36}$ cm = 6 cm.

Hence, the radius of the circle is 6 cm.

**Ex.2** In the given figure, PQ is a chord of length 8cm of a circle of radius 5cm. The tangents at P and Q intersect at a point T. Find the length TP.




**Sol.** Join OP and OT Let OT intersect PQ at a point R.

Then, TP = TQ and $\angle PTR = \angle QTR$.

$\therefore$ TR $\perp$ PQ and TR bisects PQ.

$\therefore$ PR = RQ = 4 cm.

Also, $OR = \sqrt{OP^2 - PR^2} = \sqrt{5^2 - 4^2}$ cm

$= \sqrt{25 - 16}$ cm $= \sqrt{9}$ cm = 3cm.

Let TP = x cm and TR = y cm.

From right $\Delta TRP$, we get

$TP^2 = TR^2 + PR^2$

$\Rightarrow x^2 = y^2 + 16 \quad \Rightarrow x^2 - y^2 = 16$ .... (i)

From right $\Delta OPT$, we get

$TP^2 + OP^2 = OT^2$

$\Rightarrow \quad x^2 + 5^2 = (y + 3)^2$ [$\Theta$ $OT^2 = (OR + RT)^2$]

$\Rightarrow \quad x^2 - y^2 = 6y - 16$ ....(ii)

From (i) and (ii), we get

$$6y - 16 = 16 \Rightarrow 6y = 32 \Rightarrow y = \frac{16}{3}.$$

Putting $y = \frac{16}{3}$ in (i), we get

$$x^2 = 16 + \left(\frac{16}{3}\right)^2 = \left(\frac{256}{9} + 16\right) = \frac{400}{9}$$

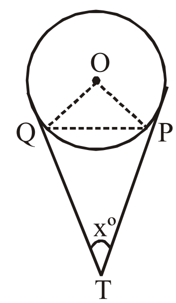
$$\Rightarrow \quad x = \sqrt{\frac{400}{9}} = \frac{20}{3}.$$

Hence, length TP = x cm = $\left(\frac{20}{3}\right)$ cm

= 6.67 cm.

**Ex.3** Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

**Sol.** **Given :** A circle with centre O and an external point T from which tangents TP and TQ are drawn to touch the circle at P and Q.




**To prove :** $\angle PTQ = 2\angle OPQ$.

**Proof :** Let $\angle PTQ = x^\circ$. Then,

$\angle TQP + \angle TPQ + \angle PTQ = 180^\circ$

[$\Theta$ sum of the $\angle$s of a triangle is 180°]

$\Rightarrow \quad \angle TQP + \angle TPQ = (180^\circ - x)$ ....(i)

We know that the lengths of tangent drawn from an external point to a circle are equal.

So, TP = TQ.

Now, TP = TQ

$\Rightarrow \angle TQP = \angle TPQ$

$= \frac{1}{2}(180^\circ - x) = \left(90^\circ - \frac{x}{2}\right)$

$\therefore \quad \angle OPQ = (\angle OPT - \angle TPQ)$

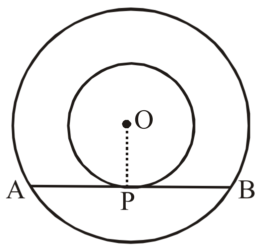
$= 90^\circ - \left(90^\circ - \frac{x}{2}\right) = \frac{x}{2}$

$\Rightarrow \angle OPQ = \frac{1}{2} \angle PTQ$

$\Rightarrow \angle PTQ = 2\angle OPQ.$

**Ex.4** Prove that in two concentric circles, the chord of the larger circle which touches the smaller circle, is bisected at the point of contact.

**Sol.** **Given :** Two circles with the same centre O and AB is a chord of the larger circle which touches the smaller circle at P.




**To prove :** AP = BP.

**Construction :** Join OP.

**Proof :** AB is a tangent to the smaller circle at the point P and OP is the radius through P.

$\therefore$ OP $\perp$ AB.

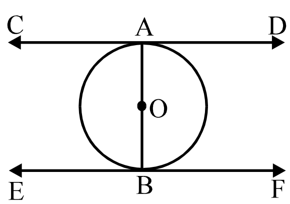
But, the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore$ OP bisects AB. Hence, AP = BP.

**Ex.5** Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

**Sol.** **Given :** CD and EF are the tangents at the end points A and B of the diameter AB of a circle with centre O.

**To prove :** CD || EF.




**Proof :** CD is the tangent to the circle at the point A.

$\therefore$ $\angle BAD = 90°$

EF is the tangent to the circle at the point B.

$\therefore$ $\angle ABE = 90°$

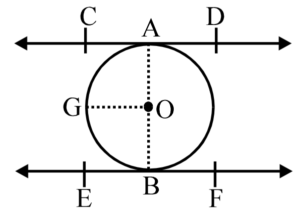
Thus, $\angle BAD = \angle ABE$ (each equal to 90°).

But these are alternate interior angles.

$\therefore$ CD || EF

**Ex.6** Prove that the line segment joining the point of contact of two parallel tangents to a circle is a diameter of the circle.

**Sol.** **Given :** CD and EF are two parallel tangents at the points A and B of a circle with centre O.




**To prove :** AOB is a diameter of the circle.

**Construction :** Join OA and OB.

Draw OG || CD

**Proof :** OG || CD and AO cuts them.

$\therefore$ $\angle CAO + \angle GOA = 180°$

$\Rightarrow$ $90° + \angle GOA = 180°$ [OA $\perp$ CD]

$\Rightarrow$ $\angle GOA = 90°$

Similarly, $\angle GOB = 90°$

$\therefore$ $\angle GOA + \angle GOB = (90° + 90°) = 180°$

$\Rightarrow$ AOB is a straight line

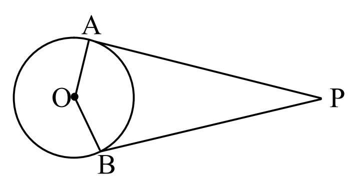
Hence, AOB is a diameter of the circle with centre O.

**Ex.7** Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the pointsof contact to the centre.

**Sol.** **Given :** PA and PB are the tangent drawn from a point P to a circle with centre O. Also, the line segments OA and OB are drawn.

**To Prove :** $\angle APB + \angle AOB = 180°$

**Proof :** We know that the tangent to a circle is perpendicular to the radius through the point of contact.




$\therefore$ PA $\perp$ OA $\Rightarrow \angle OAP = 90°$, and

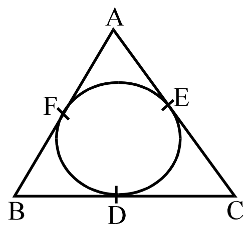
PB $\perp$ OB $\Rightarrow \angle OBP = 90°$.

$\therefore$ $\angle OAP + \angle OBP = 90°$.

Hence, $\angle APB + \angle AOB = 180°$

[Θ sum of the all the angles of a quadrilateral is 360°]

**Ex.8** In the given figure, the incircle of $\Delta ABC$ touches the sides BC, CA and AB at D, E, F respectively.




Prove that $AF + BD + CE = AE + CD + BF$

$$= \frac{1}{2} \text{ (perimeter of } \Delta ABC)$$

**Sol.** We know that the lengths of tangents from an exterior point to a circle are equal.

$\therefore$ $AF = AE$ .... (i) [tangents from A]

$BD = BF$ ..... (ii) [tangents from B]

$CE = CD$ ..... (iii) [tangents from C]

Adding (i), (ii) and (iii), we get

$(AF + BD + CE) = (AE + BF + CD) = k$ (say)

Perimeter of $\Delta ABC = (AF + BD + CE)$

$+ (AE + BF + CD)$

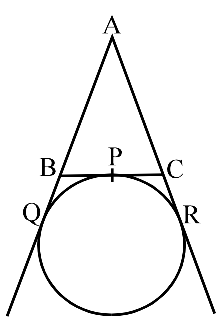
$= (k + k) = 2k$

$\therefore$ $k = \frac{1}{2}$ (perimeter of $\Delta ABC$).

Hence $AF + BD + CE = AE + CD + BF$

$= \frac{1}{2}$ (perimeter of $\Delta ABC$)

**Ex.9** A circle touches the side BC of a $\Delta ABC$ at P, and touches AB and AC produced at Q and R respectively, as shown in the figure.




Show that $AQ = \frac{1}{2}$(perimeter of $\Delta ABC$)

**Sol.** We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ $AQ = AR$ .... (i) [tangents from A]

$BP = BQ$ .... (ii) [tangents from B]

$CP = CR$ .... (iii) [tangents from C]

Perimeter of $\Delta ABC$

$= AB + BC + AC$

$= AB + BP + CP + AC$

$= AB + BQ + CR + AC$ [using (ii) and (iii)]

$= AQ + AR$

$= 2AQ$ [using (i)].

Hence, $AQ = \frac{1}{2}$ (perimeter of $\Delta ABC$)

**Ex.10** Prove that there is one and only one tangent at any point on the circumference of a circle.

**Sol.** Let P be a point on the circumference of a circle with centre O. If possible, Let PT and PT′ be two tangents at a point P of the circle.

Now, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

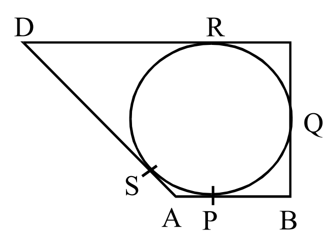
$\therefore$ $OP \perp PT$ and similarly, $OP \perp PT'$

$\Rightarrow$ $\angle OPT = 90°$ and $\angle OPT' = 90°$

$\Rightarrow$ $\angle OPT = \angle OPT'$

This is possible only when PT and PT′ coincide. Hence, there is one and only one tangent at any point on the circumference of a circle.

**Ex.11** A quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure.




Prove that $AB + CD = AD + BC$

**Sol.** We known that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ $AP = AS$ ....(i) [tangents from A]

BP = BQ ....(ii) [tangents from B]

CR = CQ ....(iii) [tangents from C]

DR = DS ....(iv) [tangents from D]

$\therefore$ AB + CD = (AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

[using (i), (ii), (iii), (iv)]

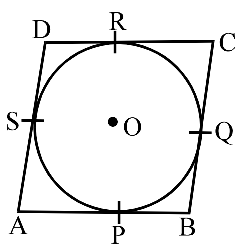
= (AS + DS) + (BQ + CQ)

= (AD + BC).

Hence, (AB + CD) = (AD + BC)

**Ex.12** Prove that the paralleogram circumscribing a circle, is a rhombus.

**Sol.**




**Given :** A parallelogram ABCD

circumsribes a circle with centre O.

**To prove :** AB = BC = CD = AD

**Proof :** we know that the lengths of tangents drawn from an exterior point to a circle are equal.

$\therefore$ AP = AS .... (i) [tangents from A]

BP = BQ .... (ii) [tangents from B]

CR = CQ .... (iii) [tangents from C]

DR = DS .... (iv) [tangents from D]

$\therefore$ AB + CD = AP + BP + CR + DR

= AS + BQ + CQ + DS

[From (i), (ii), (iii), (iv)]

= (AS + DS) + (BQ + CQ)

= AD + BC

Hence, (AB + CD) = (AD + BC)

$\Rightarrow$ 2AB = 2AD

[Θ opposite sides of a parallelogram are equal]

$\Rightarrow$ AB = AD

$\therefore$ CD = AB = AD = BC

Hence, ABCD is a rhombus

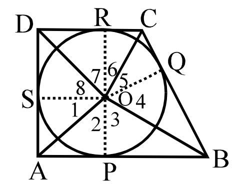
**Ex.13** Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**Sol.** **Given :** A quadrilateral ABCD circumscribes a circle with centre O.

**To Prove :** $\angle AOB + \angle COD = 180°$

and $\angle BOC + \angle AOD = 180°$

**Construction :** Join OP, OQ, OR and OF




**Proof :** We know that the tangents drawn from an external point of a circle subtend equal angles at the centre.

$\therefore$ $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$

and $\angle 7 = \angle 8$

And, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6$

$+ \angle 7 + \angle 8 = 360°$ [$\angle$s at a point]

$\Rightarrow 2(\angle 2 + \angle 3) + 2(\angle 6 + \angle 7) = 360°$ and

$2(\angle 1 + \angle 8) + 2(\angle 4 + \angle 5) = 360°$

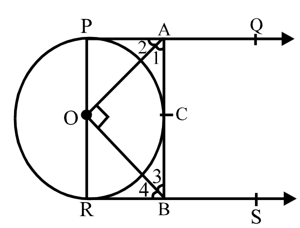
$\Rightarrow \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$ and

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$ and

$\angle AOD + \angle BOC = 180°$

**Ex.14** In the given figure, PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects PQ at A and RS at B.




Prove that $\angle AOB = 90^o$

**Sol.** **Given :** PQ and RS are two parallel tangents to a circle with centre O and AB is a tangent to the circle at a point C, intersecting PQ and RS at A and B respectively.

**To prove :** $\angle AOB = 90^\circ$

**Proof :** Since PA and RB are tangents to the circle at P and R respectively and POR is a diameter of the circle, we have

$\angle OPA = 90^\circ$ and $\angle ORB = 90^\circ$

$\Rightarrow \angle OPA + \angle ORB = 180^\circ$

$\Rightarrow PA \parallel RB$

We know that the tangents to a circle from an external point are equally inclined to the line segment joining this point to the centre.

$\therefore \quad \angle 2 = \angle 1$ and $\angle 4 = \angle 3$

Now, PA|| RB and AB is a transversal.

$\therefore \quad \angle PAB + \angle RBA = 180^\circ$

$\Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180^\circ$

$\Rightarrow 2\angle 1 + 2\angle 3 = 180^\circ$

$[\therefore \angle 2 = \angle 1$ and $\angle 4$ and $\angle 3]$

$\Rightarrow 2(\angle 1 + \angle 3) = 180^\circ$

$\Rightarrow \angle 1 + \angle 3 = 90^\circ$

From $\Delta AOB$, we have

$\angle AOB + \angle 1 + \angle 3 = 180^\circ$

[$\because$ sum of the $\angle$s of a triangle is $180^\circ$]

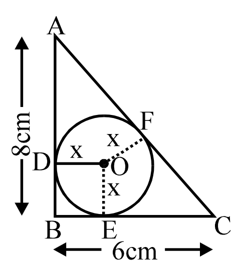
$\Rightarrow \angle AOB + 90^\circ = 180^\circ$

$\Rightarrow \angle AOB = 90^\circ$

Hence, $\angle AOB = 90^\circ$

**Ex.15** ABC is a right triangle, right angled at B. A circle is inscribed in it. The lengths of the two sides containing the right angle are 6 cm and 8 cm. Find the radius of the incircle.

**Sol.**




Let the radius of the in circle be x cm.

Let the in circle touch the side AB, BC and CA at D, E, F respectively. Let O be the centre of the circle.

Then, OD = OE = OF = x cm.

Also, AB = 8 cm and BC = 6 cm.

Since the tangents to a circle from an external point are equal, we have

AF = AD = $(8 - x)$ cm, and

CF = CE = $(6 - x)$ cm.

$\therefore$ AC = AF + CF = $(8 - x)$ cm + $(6 - x)$ cm
= $(14 - 2x)$ cm.

Now , $AC^2 = AB^2 + BC^2$

$\Rightarrow (14 - 2x)^2 = 8^2 + 6^2 = 100 = (10)^2$

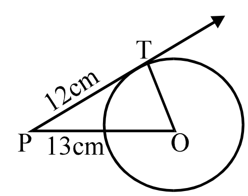
$\Rightarrow 14 - 2x = \pm 10 \qquad \Rightarrow x = 2$ or $x = 12$

$\Rightarrow x = 2$ [neglecting $x = 12$].

Hence, the radius of the in circle is 2cm.

**Ex.16** A point P is 13 cm from the centre of the circle. The length of the tangent drawn from P to the circle is 12cm. Find the radius of the circle.

**Sol.** Since tangent to a circle is perpendicular to the radius through the point of contact.




$\therefore \quad \angle OTP = 90^\circ$

In right triangle OTP, we have

$OP^2 = OT^2 + PT^2$

$\Rightarrow 13^2 = OT^2 + 12^2$

$\Rightarrow OT^2 = 13^2 - 12^2$

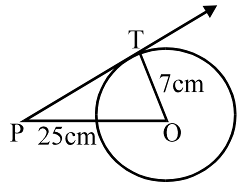
$= (13 - 12)(13 + 12) = 25$

$\Rightarrow OT = 5$

Hence, radius of the circle is 5 cm.

**Ex.17** Find the length of the tangent drawn from a point whose distance from the centre of a circle is 25 cm. Given that the radius of the circle is 7 cm.

**Sol.** Let P be the given point, O be the centre of the circle and PT be the length of tangent from P. Then, OP = 25 cm and OT = 7 cm.




Since tangent to a circle is always perpendicular to the radius through the point of contact.

$\therefore \quad \angle OTP = 90°$

In right triangle OTP, we have

$OP^2 = OT^2 + PT^2$

$\Rightarrow \quad 25^2 = 7^2 + PT^2$

$\Rightarrow \quad PT^2 = 25^2 - 7^2$

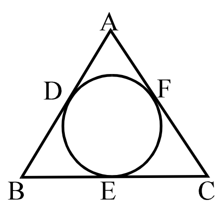
$= (25 - 7)(25 + 7)$

$= 576$

$\Rightarrow \quad PT = 24$ cm

Hence, length of tangent from P = 24 cm

**Ex.18** In Fig., if AB = AC, prove that BE = EC




**Sol.** Since tangents from an exterior point to a circle are equal in length

$\therefore$ AD = AF [Tangents from A]

BD = BE [Tangents from B]

CE = CF [Tangents from C]

Now,

AB = AC

$\Rightarrow$ AB – AD = AC – AD

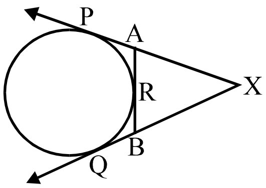
[Subtracting AD from both sides]

$\Rightarrow$ AB – AD = AC – AF [Using (i)]

$\Rightarrow$ BD = CF $\Rightarrow$ BE = CF [Using (ii)]

$\Rightarrow$ BE = CE [Using (iii)]

**Ex.19** In fig. XP and XQ are tangents from X to the circle with centre O. R is a point on the circle.




Prove that, XA + AR = XB + BR.

**Sol.** Since lengths of tangents from an exterior point to a circle are equal.

$\therefore$ XP = XQ .... (i) [From X]

AP = AR .... (ii) [From A]

BQ = BR .... (iii) [From B]

Now, XP = XQ

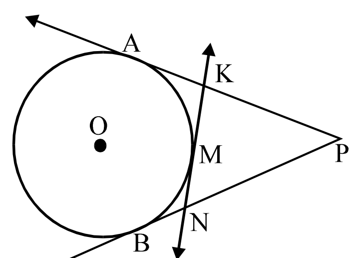
$\Rightarrow$ XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + BR

[Using equations (i) and (ii)]

**Ex.20** PA and PB are tangents from P to the circle with centre O. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that KN = AK + BN.

**Sol.** We know that the tangents drawn from an external point to a circle are equal in length.




$\therefore$ PA = PB .... (i) [From P]

KA = KM .... (ii) [From K]

and, NB = NM .... (iii) [From N]

Adding equations (ii) and (iii), we get

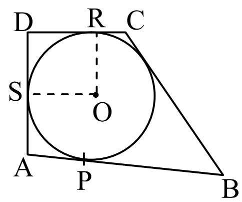
KA + NB = KM + NM

$\Rightarrow$ AK + BN = KM + MN $\Rightarrow$ AK + BN = KN

**Ex.21** ABCD is a quadrilateral such that $\angle D = 90°$. A circle (O, r) touches the sides AB, BC, CD and DA at P,Q,R and S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, find r.

**Sol.** Since tangent to a circle is perpendicular to the radius through the point.

$\therefore \quad \angle ORD = \angle OSD = 90°$




It is given that $\angle D = 90°$ Also, OR = OS. Therefore,

ORDS is a square.

Since tangents from an exterior point to a circle are equal in length.

$\therefore$ BP = BQ

CQ = CR and DR = DS

Now, BP = BQ

$\Rightarrow$ BQ = 27 [Θ BP = 27 cm (Given)]

$\Rightarrow$ BC – CQ = 27

$\Rightarrow$ 38 – CQ = 27 [Θ BC = 38 cm ]

$\Rightarrow$ CQ = 11cm

$\Rightarrow$ CR = 11cm [Θ CR = CQ ]

$\Rightarrow$ CD – DR = 11

$\Rightarrow$ 25 – DR = 11 [Θ CD = 25cm ]

$\Rightarrow$ DR = 14 cm

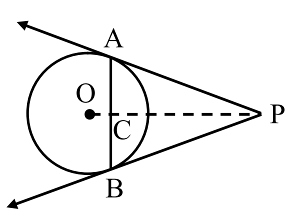
But, ORDS is a square.

Therefore, OR = DR = 14 cm

Hence, r = 14 cm

**Ex.22** Prove that the tangents at the extremities of any chord make equal angles with the chord.

**Sol.** Let AB be a chord of a circle with centre O, and let AP and BP be the tangents at A and B respectively. Suppose the tangents meet at P. Join OP. Suppose OP meets AB at C. We have to prove that $\angle$PAC = $\angle$PBC In triangles PCA and PCB, we have




PA = PB

$\begin{bmatrix} \Theta \text{Tangents from an external point} \\ \text{are equal} \end{bmatrix}$

$\angle$ APC = $\angle$ BPC

[Θ PA and PB are equally inclined to OP]

and, PC = PC [Common]

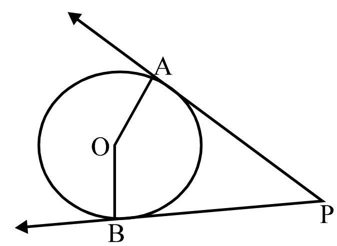
So, by SAS - criterion of congruence, we have

$\Delta$ PAC $\cong$ $\Delta$ PBC

$\Rightarrow \angle$ PAC = $\angle$ PBC

**Ex.23** In fig., O is the centre of the circle, PA and PB are tangent segments. Show that the quadrilateral AOBP is cyclic.

**Sol.**




Since tangent at a point to a circle is perpendicular to the radius through the point.

$\therefore$ OA$\perp$AP and OB$\perp$BP

$\Rightarrow \angle OAP = 90°$ and $\angle OBP = 90°$

$\Rightarrow \angle OAP + \angle OBP = 90° + 90° = 180°$ ....(i)

In quadrilateral OAPB, we have

$\angle OAP + \angle APB + \angle AOB + \angle OBP = 360°$

$\Rightarrow (\angle APB + \angle AOB) + (\angle OAP + \angle OBP) = 360°$

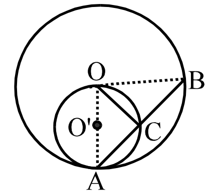
$\Rightarrow \angle APB + \angle AOB + 180° = 360°$

$\angle APB + \angle AOB = 180°$ ....(ii)

From equations (i) and (ii),we can say that the quadrilateral AOBP is cyclic.

**Ex.24** In fig., circles C(O,r) and C(O′, r/2) touch internally at a point A and AB is a chord of the circle C (O,r) intersecting C(O′, r/2) at C, Prove that AC = CB.

**Sol.** Join OA, OC and OB. Clearly, $\angle$OCA is the angle in a semi-circle.



$\therefore \quad \angle OCA = 90°$

In right triangles OCA and OCB, we have

OA = OB = r

$\angle OCA = \angle OCB = 90°$

and OC = OC

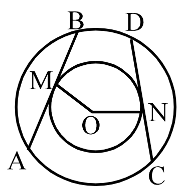
So, by RHS criterion of congruence, we get

$\Delta$ OCA $\cong$ $\Delta$ OCB

$\Rightarrow$ AC = CB

**Ex.25** In two concentric circles, prove that all chords of the outer circle which touch the inner circle are of equal length.

**Sol.** Let AB and CD be two chords of the circle which touch the inner circle at M and N respectively.




Then, we have to prove that

AB = CD

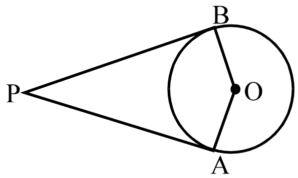
Since AB and CD are tangents to the smaller circle.

$\therefore$ OM = ON = Radius of the smaller circle

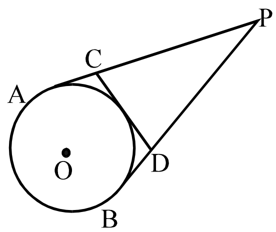
Thus, AB and CD are two chords of the larger circle such that they are equidistant from the centre. Hence, AB = CD.

# EXERCISE #1

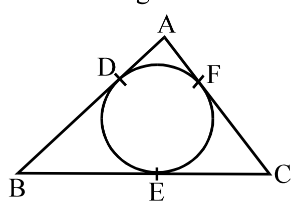
**Q.1** In the given figure, PA and PB are the tangent segments to a circle with centre O. Show that the points A, O, B and P are concyclic.



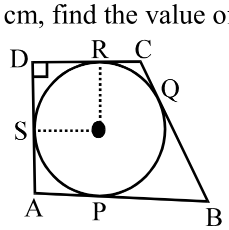

**Q.2** From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14cm, find the perimeter of Δ PCD.




**Q.3** A circle is inscribed in a ΔABC having AB = 10 cm, BC = 12 cm and CA = 8 cm and touching these sides at D, E, F respectively, as shown in the figure. Find AD, BE and CF.




**Q.4** In the given figure, ABCD is a quadrilateral in which ∠ D = 90°. A circle C(O, r) touches the sides AB, BC, CD and DA at P, Q, R , S respectively. If BC = 38 cm, CD = 25 cm and BP = 27 cm, find the value of r.

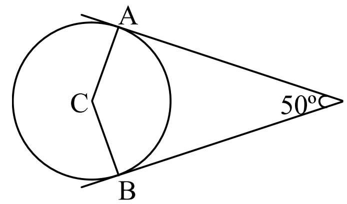



**Q.5** A point P is 7 cm from the centre of circle whose diameter is 8cm. How many tangents can be drawn to the circle ?

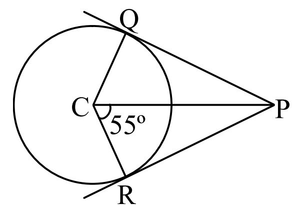
**Q.6** Find the distance between two parallel tangents to a circle whose radius is 4.5 cm.

**Q.7** A square circumscribe a circle of radius 5 cm. Find the length of a diagonal of the square.

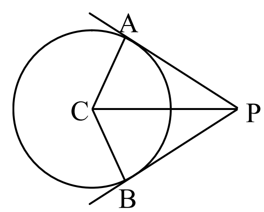
**Q.8** In the adjoining figure, PA and PB are tangents from P to a circle with centre C. If ∠APB = 50º, find ∠ACB.



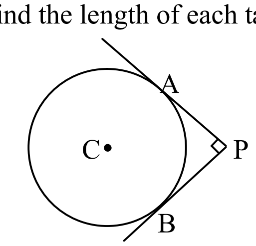

**Q.9** In the adjoining figure PQ and PR are tangents from P to a circle with centre O. If ∠POR= 55º, find ∠QPR




**Q.10** In the adjoining figure PA and PB are tangents from Q to a circle with centre C. If ∠APB = 80º, find ∠ACP.



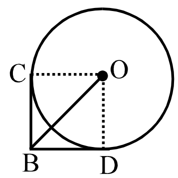

**Q.11** In the adjoining figure PA and PB are tangents from P to a circle with centre C. If the radius of the circle is 4 cm and PA ⊥ PB, then find the length of each tangent



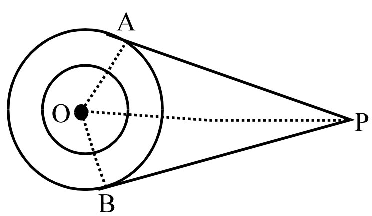

**Q.12** Find the length of tangent drawn to a circle with radius 7 cm from a point 25 cm away from the centre of the circle.

**Q.13** A point P is 26 cm away from the centre of a circle and the length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

**Q.14** Two tangent segments BC and BD are drawn to a circle with centre O such that $\angle CBD = 120°$. Prove that OB = 2BC.

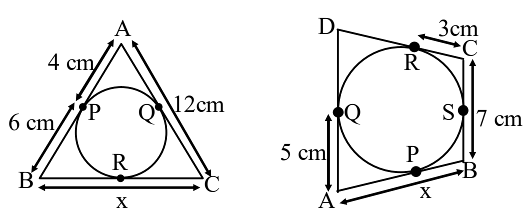



**Q.15** In the given figure, O is the centre of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to the outer and inner circle respectively. If PA = 10 cm, find the length of PB up to one place of decimal.




**Q.16** (a) In the figure (i) given below, triangle ABC is circumscribed, find x.

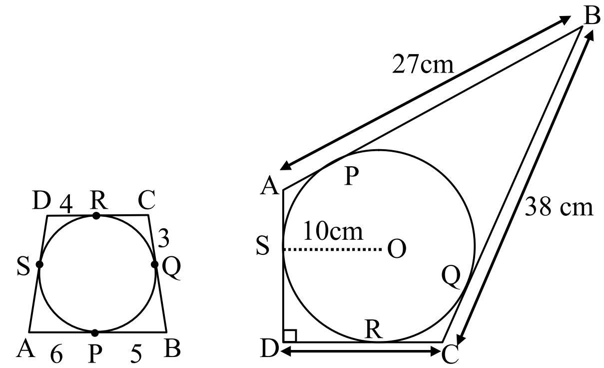
(b) In the figure (ii) given below, quadrilateral ABCD is circumscribed, find x.




(i) (ii)

**Q.17** (a) In the figure (i) given below, quadrilateral ABCD is circumscribed; find the perimeter of quadrilateral ABCD.

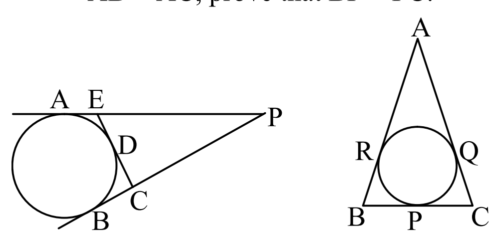
(b) In the figure (ii) given below. quadrilateral ABCD is circumscribed and AD⊥DC, find x if radius of incircle is 10 cm.




(i) (ii)

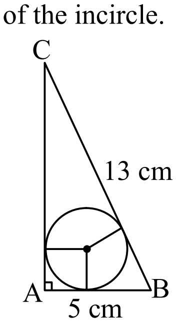
**Q.18** (a) In the figure (i) given below, from an external point P, tangents PA and PB are drawn to a circle. CE is a tangent to the circle at D. If AP = 15 cm, find the perimeter of the triangle PEC.

(b) In the figure (ii) given below, the incircle of $\Delta ABC$ touches the sides BC, CA and AB at points P, Q and R respectively. If AB = AC, prove that BP = PC.

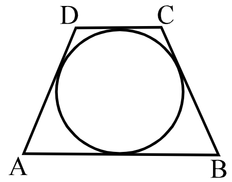



(i) (ii)

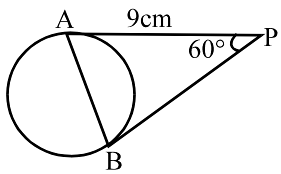
**Q.19** In the figure given below, ABC is a right angled triangle at A with sides AB = 5 cm and BC = 13 cm. A circle with centre O has been inscribed in the triangle ABC. Calculate the radius of the incircle.



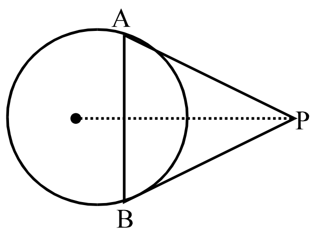

**Q.20** In the given figure, a circle touches all the four sides of a quadrilateral ABCD whose three sides are AB = 6 cm, BC = 7 cm and CD = 4cm. Find AD.




**Q.21** In the given figure, PA and PB are tangents such that PA = 9 cm and $\angle APB = 60°$. Find the length of chord AB.



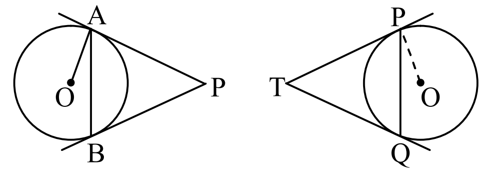

**Q.22** From a point P, two tangents PA and PB are drawn to a circle C (O, r). If OP = 2r, show that $\Delta APB$ is equilateral.




**Q.23** Prove that the opposite sides of a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

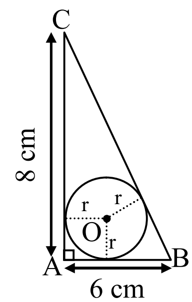
**Q.24** (a) In the figure (i) given below, PA and PB are tangents drawn from an external point P to a circle with centre O. Prove that $\angle APB = 2\angle OAB$.

(b) In the figure (ii) given below, PQ is a chord of length 8 cm of a circle with centre O. The tangents at P and Q intersect at T. If the radius of the circle is 5 cm, find the length PT.

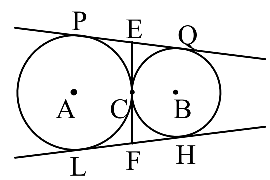



(i) (ii)

**Q.25** In the adjoining figure, ABC is a right angled triangle with AB = 6 cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of r, the radius of the inscribed circle.




**Q.26** In the adjoining figure two circles touch each other externally at C. Prove that the common tangent at C bisects the other two common tangents.




## Answer Key

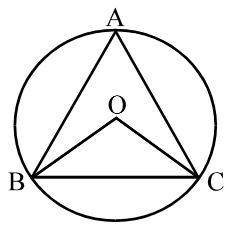
| | | | | |
|---|---|---|---|---|
| **2.** 28 cm | **3.** AD = 3cm , BE = 7cm, CF = 5cm | **4.** r = 14 cm | **5.** two | **6.** 9 cm |
| **7.** $10\sqrt{2}$ cm | **8.** 130º | **9.** 70º | **10.** 50º | **11.** 4 cm |
| **12.** 24 cm | **13.** 10 cm | **15.** 10.9 cm | **16.** (a) 14 cm ; (b) 9 cm | **17.** (a) 36 cm ; (b) 21 cm |
| **18.** (a) 30 cm; | **19.** 2 cm | **20.** 3 cm | **21.** 9 cm | **24.** (b) $6\frac{2}{3}$cm |
| **25.** 2 cm | | | | |

## EXERCISE # 2

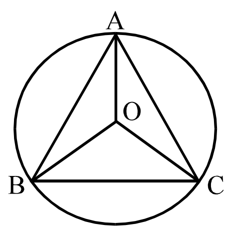
**Q.1** If the length of a chord of a circle is 16 cm and is at a distance of 15 cm from the centre of the circle, then find the radius of the circle (in cm).

**Q.2** The radius of a circle is 6 cm. Then find the perpendicular distance from the centre of the circle to the chord which is 8 cm in length.

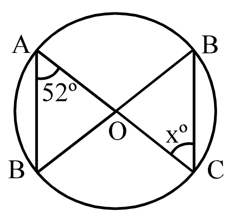
**Q.3** An equilateral triangle ABC is inscribed in a circle with centre O. Then find ∠BOC.




**Q.4** In the adjoining figure, O is the centre of the circle. If ∠ OBC = 25º, then find ∠ BAC.



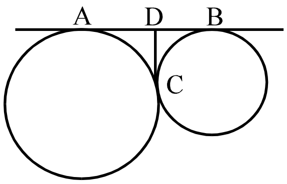

**Q.5** In fig. O is the centre of the circle. If ∠ BAC = 52º, then find ∠ OCD.



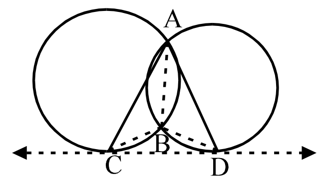

**Q.6** In a circle with centre O, AB and CD are two diameters perpendicular to each other. Then find the length of chord AC.

**Q.7** In a circle with centre O, the unequal chords AB and CD intersect each other at P. Then find, Δ APC and Δ DPB.

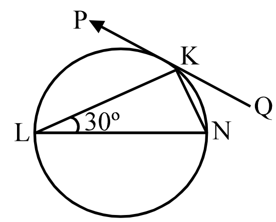
**Q.8** In the given figure, AB and CD are two common tangents to the two touching circles. If DC = 4 cm, then find AB.



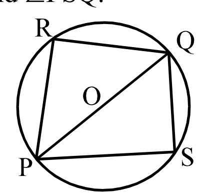

**Q.9** CD is a direct common tangent to two circles intersecting each other at A and B. Then find ∠ CAD + ∠ CBD.




**Q.10** In the adjoining figure, PQ is the tangent at K. If LN is a diameter and ∠ KLN = 30º, then find ∠ PKL.



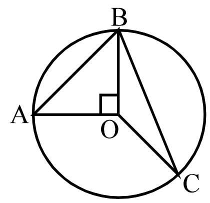

**Q.11** In the adjoining figure, POQ is the diameter of the circle. R and S are any two points on the circle. Then find relation between ∠PRQ and ∠PSQ.



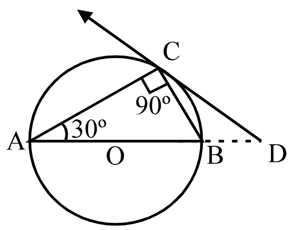

**Q.12** Two equal circles of radius r intersect such that each passes through the centre of the other. Then find the length of common chord.

**Q.13** If four sides of a quadrilateral ABCD are tangential to a circle, then find relation between AB, BC, CD, AD, BD.

**Q.14** In the adjoining figure, A, B, C are three points on a circle with centre O. If ∠ AOB = 90º and ∠ BOC = 120º, then find ∠ ABC.




**Q.15** AB is a diameter and AC is a chord of a circle such that ∠ BAC = 30º. The tangent at C intersects AB produced in D. Then find relation between BC & BD.




**Q.16** Find the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm.

**Q.17** Two circles of radii 20 cm and 37 cm intersect in A and B. If O and O' are their centres and AB = 24 cm, then find distance OO'.

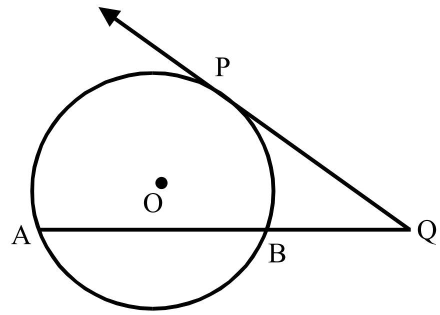
**Q.18** If two diameters of a circle intersect each other at right angles, then find the type of quadrilateral formed by joining their end points.

**Q.19** If ABC is an arc of a circle and ∠ ABC = 135º, then find the ratio of arc PQR to the circumference.

**Q.20** If one angle of a cyclic trapezium is triple the other, then find the greater one measures.

**Q.21** Find the angle in a major segment of a circle.

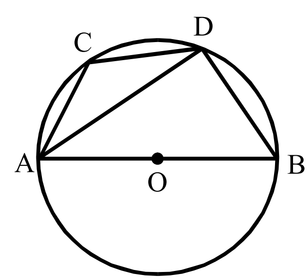
**Q.22** O is the centre of a circle. If tangent PQ = 12 cm and BQ = 8 cm, then find chord AB.




**Q.23** AB and CD are two parallel chords of a circle with centre O such that AB = 6 cm and CD = 12 cm. The chords are on the same side of the centre and the distance between them is 3 cm. Then find the radius of the circle.

**Q.24** In a circle of radius 17 cm, two parallel chords are drawn on opposite side of a diameter. The distance between the chords is 23 cm. if the length of one chord is 16 cm. Then find the length of the other.

**Q.25** In the adjoining figure, ∠ ADC = 140º and AOB is the diameter of the circle. Then find ∠ BAC.




**Q.26** If two circle are such that the centre of one lies on the circumference of the other, then find the ratio of the common chord of the two circles to the radius of any one of the circles.

**Q.27** If tangents QR, RP, PQ are drawn respectively at A, B, C to a circle circumscribing an acute angled Δ ABC so as to form another Δ PQR, then find ∠ RPQ.

**Q.28** Two circles touch externally. The sum of their areas is 130 π sq cm and the distance between their centres is 14 cm. Find the radius of the smaller circle.

**Q.29** Two circles touch internally. The sum of their areas is, 116 $\pi$ sq. cm and the distance between their centres is 6 cm. Find the radius of the larger circle.

**Q.30** Two circles touch each other internally. Their radii are 2 cm and 3 cm. Find the biggest chord of the outer circle which is outside the inner circle, is of length.

## Answer Key

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|---|---|---|---|---|---|
| **1.** 17 | **2.** $2\sqrt{5}$ cm | **3.** 120° | **4.** 65° | **5.** 52° | **6.** $\frac{1}{\sqrt{2}}$ AB |
| **7.** Similar | **8.** 8 cm | **9.** 180° | **10.** 60° | **11.** $\angle$ PRQ = $\angle$ PSQ | |
| **12.** $r\sqrt{3}$ | **13.** AB + CD = BC + AD | | **14.** 75° | **15.** BC = BD | |
| **16.** 10 cm | **17.** 51 cm | **18.** square | **19.** 3 : 8 | **20.** 135° | **21.** Less than 90° |
| **22.** 10 cm | **23.** $3\sqrt{5}$ cm | **24.** 30 cm | **25.** 50° | **26.** $\sqrt{3}$ : 1 | **27.** 180º – 2 $\angle$ BAC |
| **28.** 3 cm | **29.** 10 cm | **30.** $4\sqrt{2}$ cm | | | |