1. Real Numbers

Exercise 1.1

1. Question

If a and b are two odd positive integers such that a > b, then prove that one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even.

Answer

Given: a and b are two odd positive integers such that a>b.

To prove: out of the numbers $\frac{a+b}{2}, \frac{a\ -\ b}{2}$ one is odd and other is even.

Proof: a and b both are odd positive integers,Since,

$$\left(\frac{a+b}{2}\right)+\left(\frac{a-b}{2}\right)=\left(\frac{a+b+a-b}{2}\right)=a$$

which is an odd number as "a" is given to be a odd number.Hence one of the number out of $\frac{a+b}{2}, \frac{a\ -\ b}{2}$ must be even and other must be odd because adding two even numbers gives an even number and adding two odd numbers gives an even number. Here, $\frac{a+b}{2}$ will give an odd number and $\frac{a-b}{2}$ will give an even number.

EXAMPLE:

Take a = 7 and b = 3 such that a>b

Now, $\frac{a+b}{2}=\frac{7+3}{2}=\frac{10}{2}=5$ which is odd

And $\frac{a-b}{2}=\frac{7-3}{2}=\frac{4}{2}=2$ which is even

Conclusion: Out of out of the numbers $\frac{a+b}{2}, \frac{a\ -\ b}{2}$,$\frac{a+b}{2}$ is an odd number and $\frac{a-b}{2}$ is an even number.

2. Question

Prove that the products of two consecutive positive integers is divisible by 2.

Answer

Let the numbers are a and a-1

Product of these number: $a(a - 1) = a^2 - a$

Case 1: When a is even:

$a=2p$

then $(2p)^2 - 2p \Rightarrow 4p^2 - 2p$

$2p(2p-1)$ .... it is divisible by 2

Case 2: When a is odd:

$a = 2p+1$

then $(2p+1)^2 - (2p+1) \Rightarrow 4p^2 + 4p + 1 - 2p - 1$

$= 4p^2 + 2p \Rightarrow 2p(2p + 1)$ .... it is divisible by 2

Hence, we conclude that product of two consecutive integers is always divisible by 2

3. Question

Prove that the product of three consecutive positive integers is divisible by 6.

Answer

Let a be any positive integer and b=6. By division lemma there exists integers q and r such that

$a = 6q+r$ where $0 \le r < 6$ $a = 6q$, or $a = 6q+1$ or, $a=6q+2$ or, $a=6q+3$ or $a=6q+4$ or $a=6q+5$ Let n is any positive integer.

Since any positive integer is of the form 6q or, 6q + 1 or, 6q + 2 or, 6q + 3 or, 6q + 4 or, 6q + 5.

Case 1:

If $n = 6q$ then

$n(n + 1)(n + 2) = 6q (6q + 1)(6q + 2)$, which is divisible by 6.

Case 2:

If $n = 6q + 1$, then

$n(n + 1)(n + 2) = (6q + 1)(6q + 2) (6q + 3)$

$= (6q + 1)2(3q + 1)3(2q + 1)$

$= 6(6q + 1)(3q + 1)(2q + 1)$, which is divisible by 6.

Case 3:

If $n = 6q + 2$, then

$n(n + 1)(n + 2) = (6q + 2) (6q + 3)(6q + 4)$

$= 2(3q + 1)3(2q + 1)(6q + 4)$

= 6(3q + 1)(2q + 1)(6q + 4), which is divisible by 6.

Case 4:

If n = 6q + 3, then

n(n + 1)(n + 2) = (6q + 3) (6q + 4)(6q + 5)

= 3(2q + 1)2(3q + 2)(6q + 5)

= 6(2q + 1)(3q + 2)(6q + 5), which is divisible by 6.Case 5:

If n = 6q + 4, then

n(n + 1)(n + 2) = (6q + 4) (6q + 5)(6q + 6)

= (6q + 4) (6q + 5)6(q + 1)

= 6(6q + 4) (6q + 5)(q + 1), which is divisible by 6.

Case 6:

If n = 6q + 5, then

n(n + 1)(n + 2) = (6q + 5) (6q + 6)(6q + 7)

= (6q + 5) 6(q + 1)(6q + 7)

= 6(6q + 5) (q + 1)(6q + 7), which is divisible by 6.

Hence, n(n + 1)(n + 2) is divisible by 6

4. Question

For any positive integer, prove that $n^3 - n$ divisible by 6.

Answer

$n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$

For a number to be divisible by 6, it should be divisible by 2 and 3 both,

Divisibility by 3:

n – 1, n and n + 1 are three consecutive whole numbers.

By Euclid’s division lemma

n + 1 = 3q + r, for some integer k and r < 3

As, r < 3 possible values of ‘r’ are 0, 1 and 2.

If r = 0

n + 1 = 3q

⇒ n + 1 is divisible by 3

⇒ n(n – 1)(n + 1) is divisible by 3

⇒ $(n^3 - n)$ is divisible by 3

If r = 1

⇒ n + 1 = 3q + 1

⇒ n = 3q

⇒ n is divisible by 3

⇒ n(n − 1)(n + 1) is divisible by 3

⇒ $(n^3 - n)$ is divisible by 3

If r = 2

⇒ n + 1 = 3q + 2

⇒ n + 1 − 2 = 3q

⇒ n − 1 = 3q

⇒ n - 1 is divisible by 3

⇒ n(n − 1)(n + 1) is divisible by 3

⇒ $(n^3 - n)$ is divisible by 3

Divisibility by 2:

If n is even

Clearly, n(n − 1)(n + 1) is divisible by 2

If n is odd

⇒ n + 1 is even

⇒ n + 1 is divisible by 2

⇒ n(n − 1)(n + 1) is divisible by 2

Hence, for any value of n, n^3 - n is divisible by 2 and 3 both, therefore n^3 − n is divisible by 6.

5. Question

Prove that if a positive integer is of the form 6q + 5, then it is of the form 3q + 2 for some integer q, but not conversely.

Answer

let A = 6q + 5, be any number, where q is any positive integer.

Part 1: To show A is in the form of 3q + 2, where q is another integerA= 6q + 5 = 6q + 3 + 2 = 3(2q + 1) + 2 = 3q' + 2As, q is any positive integer, q' = 3q + 2 is also a positive integerand hence 6q + 5, is in form of 3q' + 5

Part 2: To show converse is not true, i.e. if a no is in the form of 3q + 2, then it may or may not be in the form of 6q + 5For example, consider:8 = 3(2) + 2 is in the 3q + 2 form, but it can't be expand in

6q + 5 form.

6. Question

Prove that the square of any positive integer of the form 5q + 1 is of the same form.

Answer

Let N = 5p + 1. Then,

According to the condition:

$N^2 = 25p^2 + 10p + 1 \Rightarrow 5(5p^2 + 2p) + 1 \Rightarrow 5A+1$

Where $A = 5p^2 + 2p$

Therefore N^2 is of the form 5m + 1.

7. Question

Prove that the square of any positive integer is of the form 3m or, 3m + 1 but not of the form 3m + 2.

Answer

we know, that any positive integer N is of the form 3q, 3q + 1 or, 3q + 2.

When N = 3q, then

$N^2 = 9q^2 = 3(3q)^2 = 3m$ where $m = 3q^2$

And, as q is an integer, $m = 3q^2$ is also an integer.

When N = 3q + 1,

then $N^2 = (3q + 1)^2 = 9q^2 + 6q + 1 \Rightarrow 3q(3q + 2) + 1 = 3m + 1$ where m = q(3q + 2)And, as q is an integer, m = q(3q + 2) is also an integer.

When N = 3q + 2, then $N^2 = (3q + 2)^2 = 9q^2 + 12q + 4 \Rightarrow 3(3q^2 + 4q + 1) + 1$

3m + 1 where, $m = 3q^2 + 4q + 1$

And, as q is an integer, $m = 3q^2 + 4q + 1$ is also an integer.

Therefore,

N^2 is of the form 3m, 3m + 1 but not of the form 3m + 2

8. Question

Prove that the square of any positive integer is of the form 4q or 4q + 1 for some integer q.

Answer

Since any positive integer n is of the form 2p or, 2p + 1

When n = 2p, then $n^2 = 4p^2 = 4a$ where $a = p^2$

When n = 2p + 1, then $n^2 = (2p + 1)^2 = 4p^2 + 4p + 1 \Rightarrow 4p(p + 1) + 1$

$\Rightarrow 4m + 1$ where $m = p(p + 1)$

Therefore square of any positive integer is of the form 4q or 4q + 1 for some integer q

9. Question

Prove that the square of any positive integer is of the form 5q, 5q + 1, 5q + 4 for some integer q.

Answer

Since any positive integer n is of the form 5p or 5p + 1, or 5p + 2 or 5m + 3 or 5p +4.

When n = 5p, then

$n^2 = (5p)^2 = 25p^2 \Rightarrow 5 (5p^2) = 5a$, where $a = 5p^2$

When n = 5p + 1,

then $n^2 = (5p + 1)^2 = 25p^2 + 10p + 1$

$\Rightarrow 5p(5p + 2) + 1 \Rightarrow 5a + 1$ Where $a = p(5p + 2)$

When n = 5p + 2,

then $n^2 = (5p + 2)^2 \Rightarrow 25p^2 + 20p + 4 \Rightarrow 5p(5p + 4) + 4$

$\Rightarrow 5a + 4$ where $a = p(5p + 4)$

When n = 5p + 3, then $n^2 = 25p^2 + 30p + 9$

$\Rightarrow 5(5p^2 + 6p + 1) + 4 \Rightarrow 5a + 4$ where $a = 5p^2 + 6m + 1$

When n = 5p + 4, then $n^2 = (5p + 4)^2 = 25p^2 + 40p + 16 \Rightarrow 5(5p^2 + 8m + 3) + 1 = 5a + 1$ where $a = 5p^2 + 8m + 3$

Therefore from above results we got that n2 is of the form 5q or, 5q + 1 or, 5q + 4.

10. Question

Show that the square of an odd positive integer is of the form 8q + 1, for some integer q.

Answer

To show: the square of an odd positive integer is of the form 8q + 1, for some integer q.

Solution: Let a be any positive integer and b=4. Applying the Euclid's division lemma with a and b=4 we havea = 4p + r where $0 \le r < 4$ and p is some integer,$\Rightarrow$ r can be 0,1,2,3$\Rightarrow$ a = 4p + 0 , a = 4p + 1 , a = 4p+ 2 , a = 4p + 3 , Since a is odd integer,So a = 4p + 1 or a = 4p + 3 So any odd integer is of the form a = 4p+ 1 or a = 4p+ 3 .Since any odd positive integer n is of the form 4p + 1 or 4p + 3.

When n = 4p + 1,

then $n^2 = (4p + 1)^2$

Apply the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (4p + 1)^2 = 16p^2 + 8p + 1$ ..... (1)

Take 8p common out of $16p^2 + 8p \Rightarrow 8p$

$(2p+ 1) + 1 = 8q + 1$ where $q = p(2p + 1)$

If $n = 4p + 3$, then $n^2 = (4p + 3)^2$

Apply the formula $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (4p + 3)^2 = 16p^2 + 24p + 9$

Now $16p^2 + 24p + 9$ can be written as $16p^2 + 24p + 8 + 1$

$\Rightarrow (4p + 3)^2 = 8(2p^2 + 3p + 1) + 1 = 8q + 1$ where $q = 2p^2 + 3p + 1$

From above results we got that n^2 is of the form $8q + 1$.

Note: To show that the square of an odd positive integer is of the form 8q + 1 We have started the question from taking b=4 initially because whenwe take square of any form of 4 such as 4p+1 we end up having the values which are the multiple of 8 as in (1).While attempting these types of questions always remember to start with the value which would end up giving the value the question demands.and follow the above steps.

11. Question

Show that any positive odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5, where q is some integer.

Answer

To prove: any positive odd integer is of the form 6q + 1 or, 6q + 3 or, 6q + 5, where q is some integer.

Solution: Let 'a' be any odd positive integer we need to prove that a is of the form 6q+1, or 6q+3, or 6q+5, where q is some integer.

Since a is an integer consider b = 6 another integer applying Euclid's division lemma there exist integers q and r such that we get,

$a = 6q + r$ for some integer $q \neq 0$, and $r = 0, 1, 2, 3, 4, 5$ since $0 \leq r < 6$.

Therefore according to question:

a = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5

However since a is odd so 'a' cannot take the values 6q, 6q+2 and 6q+4

(since all these are even integers, hence divisible by 2)

Therefore a = 6q + 1 , a = 6q + 3 , a = 6q + 5

Exercise 1.2

1. Question

Define HCF of two positive integers and find the HCF of the following pairs of numbers:

(i) 32 and 54 (ii) 18 and 24

(iii) 70 and 30 (iv) 56 and 88

(v) 475 and 495 (vi) 75 and 243

(vii)240 and 6552(viii)155 and 1385

(ix) 100 and 190 (x) 105 and 120

Answer

Definition of HCF (Highest Common Factor): The largest positive integer which divides two or more integers without any remainder is called Highest Common Factor (HCF) or Greatest Common Divisor or Greatest Common Factor (GCF).

(i) Prime factorization of 32 and 54 are:

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$54 = 2 \times 3 \times 3 \times 3$

From above prime factorization we got that the highest common factor of 32 and 54 is 2

(ii) Prime factorization of 18 and 24 are:

$18 = 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

From above prime factorization we got that the highest common factor of 18 and 24 is $3 \times 2 \Rightarrow 6$

(iii) Prime factorization of 30 and 70 are:

$30 = 2 \times 3 \times 5$

$70 = 2 \times 5 \times 7$

From above prime factorization we got that the highest common factor of 30 and 70 is $2 \times 5 \Rightarrow 10$

(iv) Prime factorization of 56 and 88 are:

$56 = 2 \times 2 \times 2 \times 7$

$88 = 2 \times 2 \times 2 \times 11$

From above prime factorization we got that the highest common factor of 56 and 88 is $2 \times 2 \times 2 \Rightarrow 8$

(v) Prime factorization of 475 and 495 are:

$475 = 5 \times 5 \times 19$

$495 = 3 \times 3 \times 5 \times 11$

From above prime factorization we got that the highest common factor of 475 and 495 is 5

(vi) Prime factorization of 75 and 243 are:

75 = 3 × 5 × 5

243 = 3 × 3 x 3 × 3 × 3

From above prime factorization we got that the highest common factor of 75 and 243 is 3

(vii) Prime factorization of 75 and 243 are:

240 = 2 × 2 ×2 × 2 × 3 × 5

6552 = 2 × 2 × 2 × 3 x 3 × 7 × 13

From above prime factorization we got that the highest common factor of 240 and 6552 is 2×2×2×3 ⇒ 24

(viii) Prime factorization of 155 and 1385 are:

155 = 5 × 31

1385 = 5 × 277

From above prime factorization we got that the highest common factor of 155 and 1385 is 5

(ix) Prime factorization of 100 and 190 are:

100 = 2 × 2 × 5 × 5

190 = 2 × 5 ×19

From above prime factorization we got that the highest common factor of 155 and 1385 is 2×5 ⇒ 10

(x) Prime factorization of 105 and 120 are:

105 = 3 × 5 × 7

120 = 2 × 2 × 2 × 3 × 5

From above prime factorization we got that the highest common factor of 105 and 120 is 3×5 ⇒ 15

2. Question

Use Euclid’s division algorithm to find the HCF of

(i) 135 and 225 (ii)196 & 38220

(iii) 867 & 255.

Answer

(i) Concept used : To obtain the HCF of two positive integers, say c and d, with c > d,we followthe steps below:Step 1 : Apply Euclid’s division lemma, to c and d. So, we find whole numbers, q and r such that c = dq + r, 0 ≤ r < d.Step 2 : If r = 0, d is the HCF of c and d. If r ≠ 0, apply the division lemma to d and r.Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.Now, We know that,

= 225>135

Applying Euclid’s division algorithm:(Dividend = Divisor × Quotient + Remainder)

225 = 135 ×1+90

Here remainder = 90,

So, Again Applying Euclid’s division algorithm

$135 = 90 \times 1 + 45$

Here remainder = 45,

So, Again Applying Euclid’s division algorithm

$90 = 45 \times 2 + 0$

Remainder = 0,

Hence,

HCF of (135, 225) = 45

(ii) Concept used : To obtain the HCF of two positive integers, say c and d, with $c > d$,we followthe steps below:Step 1 : Apply Euclid’s division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \le r < d$.Step 2 : If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.Now, We know that,

$38220 > 196$

So, Applying Euclid’s division algorithm

$38220 = 196 \times 195 + 0$ (Dividend = Divisor × Quotient + Remainder)

Remainder = 0

Hence,

HCF of (196, 38220) = 196

(iii) Concept used : To obtain the HCF of two positive integers, say c and d, with $c > d$,we followthe steps below:Step 1 : Apply Euclid’s division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \le r < d$.Step 2 : If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.Now, We know that,

$867 > 255$

So, Applying Euclid’s division algorithm

$867 = 255 \times 3 + 102$ (Dividend = Divisor × Quotient + Remainder)

Remainder = 102

So, Again Applying Euclid’s division algorithm

$255 = 102 \times 2 + 51$

Remainder = 51

So, Again Applying Euclid’s division algorithm

$102 = 51 \times 2 + 0$

Remainder = 0

Hence,

(HCF 0f 867 and 255) = 51

3. Question

Find the HCF of the following pairs of integers and express it as a linear combination of them

(i) 963 & 657 (ii) 592 & 252

(iii) 506 & 1155 (iv) 1288 & 575

Answer

(i) Using Euclid's Division Lemma

a = bq + r, (o ≤r<b)

963 = 657×1 + 306

657 = 306×2 + 45

306 = 45×6+36

45 = 36×1+9

36 = 9×4+0

∴ HCF (657, 963) = 9

linear form:657a + 306b = 9The above equation have many solutions, one of them is a = -15, b = 22i.e. 9 = 657(-15) + 306(22)

(ii) Using Euclid's Division Lemma

a = bq + r, (o ≤r<b)

592 = 252×2+88

252 = 88×2+76

88 = 76×1+12

76 = 12×6+4

12 = 4×3+0

∴ HCF (592, 252) = 4

linear form:592a + 252b = 4The above equation have many solutions, one of them is a = 77, b = -20i.e. 4 = 592(77) + 252(20)

(iii) Using Euclid's Division Lemma

a = bq + r, (o ≤r<b)

1155 = 506×2+143

$506 = 143 \times 3 + 77$

$143 = 77 \times 1 + 66$

$77 = 66 \times 1 + 11$

$66 = 11 \times 6 + 0$

$\therefore$ HCF (506, 1155) = 11

linear form: $506a + 1155b = 11$The above equation have many solutions, one of them is $a = 16$, $b = -7$i.e. $11 = 506(16) + 1155(-7)$

(iv) Using Euclid's Division Lemma

$a = bq + r$, $(o \le r < b)$

$1288 = 575 \times 2 + 138$

$575 = 138 \times 4 + 23$

$138 = 23 \times 6 + 0$

$\therefore$ HCF (1288, 575) = 23

linear form: $1288a + 575b = 23$

The above equation have many solutions, one of them is $a = 4$, $b = 9$

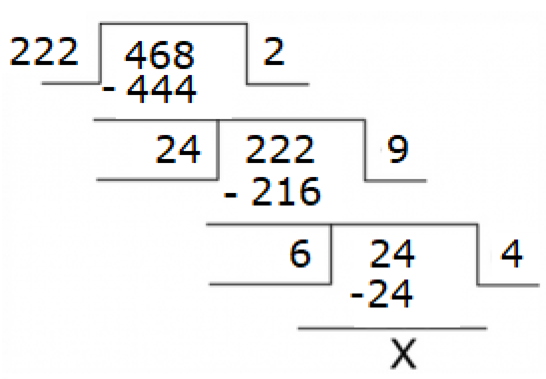
i.e. $23 = 1288(4) + 575(9)$

4. Question

Express the HCF of 468 and 222 as $468x + 222y$ where x, y are integers in two different ways.

Answer

HCF of 468 and 222 is found by division mehtod:



Therefore, HCF (468,222) = 6

Now, we need to express the HCF of 468 and 222 as $468x + 222y$ where x and y are any two integers.

Now, HCF i.e. 6 can be written as,

HCF = $222 - 216 = 222 - (24 \times 9)$

Writing 468 = 222 × 2 + 24, we get,

⇒ HCF = 222 - {(468 – 222 x 2) × 9}

⇒ HCF = 222 - {(468 ×9) – (222 × 2 × 9)}

⇒ HCF = 222 - (468 × 9) + (222 × 18)

⇒ HCF = 222 + (222 × 18) - (468 × 9)

Taking 222 common from the first two terms, we get,

⇒ HCF = 222[1 + 18] – 468 × 9

⇒ HCF = 222 × 19 – 468 × 9

⇒ HCF = 468 ×(-9) + 222 ×(19)

Let, say, x = -9 and y =19

Then, HCF = 468 ×(x) + 222 ×(y)

Therefore the HCF of 468 and 222 is written in the form of 468x + 222y where, -9 and 19 are the two integers.

5. Question

If the HCF of 408 and 1032 is expressible in the form 1032 m – 408 × 5, find m.

Answer

Prime factors of 408: 2 × 2 × 2 x 3 × 17

Prime factors of 1032: 2 × 2 ×2 × 3 × 43

HCF of 408 and 1032 = 2 × 2 × 2 × 3 = 24

According to question:

24 = 1032 m – 408 × 5

24 = 1032 m – 2040

2064 = 1032 m

∴ m = $\frac{2064}{1032}$ ⇒ 2

Therefore m = 2

6. Question

If the HCF of 657 and 963 is expressible in the form 657 x + 963 × -15, find x.

Answer

Prime factors of 657: 3 × 3 × 73

Prime factors of 963: 3 × 3 × 107

HCF of 657 and 963 = 3 × 3 = 9

According to question: the HCF of 657 and 963 is expressible in the form 657 x + 963 × -15

⇒ 9 = 657x + 963 × -15

⇒ 9 = 657x - 14445

⇒ 9 + 14445 = 657x

∴ x = 14454/657 = 22

Therefore x = 22

7. Question

Find the largest number which divides 615 and 963 leaving remainder 6 in each case.

Answer

To find: the largest number which divides 615 and 963 leaving remainder 6 in each case.

Solution: Let the HCF of 615 and 963 be x.

Since it is given remainder is 6 in each case, Therefore for the numbers to be completely divisible, 6 should be subtracted from both the numbers.

Therefore new numbers are:

615-6 = 609

963-6 = 957

Prime factors of 609 = 3 × 3 × 29

Prime factors of 957= 3 × 11 × 29

Therefore HCF of 609 and 957 is: 3 × 29 = 87

Hence the largest number which divides 615 and 963 leaving remainder 6 in each case is 87.

8. Question

Find the greatest number which divides 285 and 1249 leaving remainders 9 and 7 respectively.

Answer

The new numbers after subtracting remainders are:

285-9 = 276

1249-7 = 1242

Prime factors of 276 = 23 × 3 × 2

Prime factors of 1242 = 2 × 3 × 3 × 3 × 23

Therefore HCF of 276 and 1242 is: 2 × 3 × 23 = 138

Hence the greatest number which divides 285 and 1249 leaving remainder 9 and 7 respectively is138

9. Question

Find the largest number which exactly divides 280 and 1245 leaving remainders 4 and 3, respectively.

Answer

The new numbers after subtracting remainders are:

280-4 = 276

1245-3 = 1242

Prime factors of 276 = 23 × 3 × 2

Prime factors of 1242 = 2 × 3 × 3 × 3 × 23

Therefore HCF of 276 and 1242 is: 2 × 3 × 23 = 138

Hence the greatest number which divides 280 and 1245 leaving remainder 4 and 3 respectively is138

10. Question

What is the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively?

Answer

The new numbers after subtracting remainders are:

626-1 = 625

3127-2 = 3125

15628-3 = 15625

Prime factors of 625 = 5 × 5 × 5 × 5

Prime factors of 3125 = 5 × 5 × 5 × 5 × 5

Prime factors of 15625 = 5 × 5 × 5 × 5 × 5 ×x 5

Therefore HCF of 625, 3125 and 15625 is: 5 × 5 × 5 × 5= 625

Hence the largest number which divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively is 625

11. Question

Find the greatest number that will divide 445, 572 and 699 leaving remainder 4, 5 and 6 respectively.

Answer

The new numbers after subtracting remainders are:

445-4 = 441

572-5 = 567

699-6 = 693

Prime factors of 441= 3 × 3 × 7 × 7

Prime factors of 567 = 3 × 3 × 3 × 3 × 7

Prime factors of 693 = 3 × 3 × 7 ×11

Therefore HCF of 441, 567 and 693 is: 3 x 3 x7 = 63

Hence the greatest number that will divide 445, 572 and 699 leaving remainder 4, 5 and 6 respectively is 63

12. Question

Find the greatest number which divides 2011 and 2623 leaving remainder 9 and 5 respectively.

Answer

The new numbers after subtracting remainders are:

2011-9 = 2002

2623-5 = 2618

Prime factors of 2002= 2 × 7 × 11 × 13

Prime factors of 2618 = 2 × 7 × 11 × 17

Therefore HCF of 2002 and 2618 is: 2 × 7 × 11 = 154

Hence the greatest number which divides 2011 and 2623 leaving remainder 9 and 5 respectively is 154

13. Question

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer

To find maximum number of columns we should find HCF of 616 and 32

Using Euclid's algorithms:

Let a = 616 and b = 32

a = bq + r, (o ≤r<b)

616 = 32×19+8

32 = 8×4+0

∴ HCF of 616 and 32 is 8

Therefore the maximum number of columns in which army contingent to march is 8

14. Question

A merchant has 120 litres of oil of one kind, 180 litres of another kind and 240 litres of third kind. He wants to sell the oil by filling the three kinds of oil in tins of equal capacity. What should be the greatest capacity of such a tin?

Answer

To find greatest capacity of tin we should find HCF of 120 and 180 and 240

Prime factors of 120= 2 × 2 × 2 × 3 × 5

Prime factors of 180 = 2 × 2 × 3 × 3 × 5

Prime factors of 240 = 2 × 2 × 2 × 2 × 3 × 5

Therefore HCF of 120, 180 and 240 is: 2 × 2 × 3 × 5 = 60

Therefore the greatest capacity of a tin is 60 Liters

15. Question

During a sale, colour pencils were being sold in packs of 24 each and crayons in packs of 32 each. If you want full packs of both and the same number of pencils and crayons, how many of each would you need to buy?

Answer

For having the full packs of both and the same number of pencils and crayons, we need to find LCM of 24 and 32

Prime factors of 24 = 2 × 2 × 2 × 3

Prime factors of 32 = 2 × 2 × 2 × 2 × 2

Therefore LCM of 24 and 32 is: 2 × 2 × 2 × 2 × 2 × 3 = 96

Number of colour pencil packs = $\frac{96}{24}$ = 4

Number of crayons packs = $\frac{96}{32}$ = 3

Hence 4 packets of colour pencils and 3 packets of crayons would be bought.

16. Question

144 cartons of Coke Cans and 90 cartons of Pepsi Cans are to be stacked in a Canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have?

Answer

To find greatest number of cartons each stack would have, we should find HCF of 144 and 90

Prime factors of 144 = 2 × 2 × 2 × 2 × 3 × 3

Prime factors of 90 = 2 × 3 × 3 × 5

Therefore HCF of 144 and 90 is: 2 × 3 × 3 = 18

Therefore the greatest number of cartons each stack would have is: 18

17. Question

Two brands of chocolates are available in packs of 24 and 15 respectively. If I need to buy an equal number of chocolates of both kinds, what is the least number of boxes of each kind I would need to buy?

Answer

Given : Two brands of chocolates are available in packs of 24 and 15 respectively.

To find : the least number of boxes of each kind.

Solution:

To find the least number of boxes of each kind.we need to find LCM of 24 and 15

Prime factors of 24 = 2 × 2 × 2 × 3

Prime factors of 15 = 3 × 5

Therefore LCM of 24 and 15 is: 2 × 2 × 2 × 3 × 5 = 120

Number of boxes for first chocolate kind = $\frac{120}{24}$ = 5

Number of boxes for second chocolate kind = $\frac{120}{15}$ = 8

Hence 5 boxes of first kind and 8 boxes of second kind needed to buy.

18. Question

A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft. by 8 ft. What would be the size in inches of the tile required that has to be cut and how many such tiles are required?

Answer

Given: The size of the bathroom is 10 ft. by 8 ft.

To find: the size in inches of the tile required that has to be cut and how many such tiles are required?

Solution: To find the largest size of tile, we should find HCF of 10 and 8

Prime factors of 10 = 2 × 5

Prime factors of 8 = 2 × 2 × 2

Therefore HCF of 10 and 8 is: 2 ftSince i ft = 12 inches

Therefore the largest size of tile is: 2 × 12 inches = 24 inches

Area of bathroom = length of bathroom × breadth of bathroom

= 10 × 8

= 80 sq. ftArea of 1 tile = length of tile × breadth of tile

= 2 × 2

= 4 sq. ft

Number of tiles = $\frac{area\ of\ bathroom}{area\ of\ one\ tile}$

= $\frac{80}{4}$

= 20 tiles

NOTE: Always find the HCF of the given values to find their maximum.

19. Question

15 pastries and 12 biscuit packets have been donated for a school fete. These are to be packed in several smaller identical boxes with the same number of pastries and biscuit packets in each. How many biscuit packets and how many pastries will each box contain?

Answer

To find the number of biscuit packets and pastries, we should find HCF of 15 and 12

Prime factors of 15 = 3 × 5

Prime factors of 12 = 2 × 2 × 3

Therefore HCF of 15 and 12 is: 3

Number of pastries = $\frac{15}{3}$ = 5

Number of biscuit packets = $\frac{12}{3}$ = 4

20. Question

105 goats, 140 donkeys and 175 cow have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip?

Answer

To find the largest number of animals, we should find HCF of 105, 140 and 175

Prime factors of 105 = 3 × 5 × 7

Prime factors of 140 = 2 × 2 × 5 × 7

Prime factors of 175 = 5 × 5 × 7

Therefore HCF of 105, 140 and 175 is: 5 × 7 = 35

Hence, 35 animals went in each trip

21. Question

The length, breadth and height of a room are 8 m and 25 cm, 6 m 75 cm and 4 m 50 cm, respectively. Determine the longest rod which can measure the three dimensions of the room exactly.

Answer

Given: The length, breadth and height of a room are 8 m and 25 cm, 6 m 75 cm and 4 m 50 cm.

To find: the longest rod which can measure the three dimensions of the room exactly.

Solution: To find the length of largest rod, we should find HCF of 8m and 25 cm, 6 m 75 cm and 4 m 50 cmAs 1 m = 100 cm⇒ 8m and 25 cm = 8 × 100 cm + 25 cm

= 800 cm + 25 cm

= 825 cm⇒ 6 m and 75 cm = 6 × 100 cm + 75 cm

= 600 cm + 75 cm

= 675 cm

⇒ 4 m and 50 cm = 4 × 100 cm + 50 cm

= 400 cm + 50 cm

= 450 cm

Length = 825 cm; Breadth = 675 cm; Height = 450 cm

Prime factors of 825 = 3 × 5 × 5 × 11

Prime factors of 675 = 3 × 3 × 3 × 5 × 5

Prime factors of 450 = 2 × 3 × 3 × 5 × 5

Therefore HCF of 825, 675 and 450 is: 3 × 5 × 5 = 75

Hence, the length of rod is: 75 cm

NOTE: Always find the HCF of the given values to find their maximum.

Exercise 1.3

1. Question

Express each of the following integers as a product of its prime factors:

(i) 420 (ii) 468

(iii) 945 (iv) 7325

Answer

(i) Prime factor of 420 = 2 × 2 × 3 × 5 × 7

(ii) Prime factor of 468 = 2 × 2 × 3 × 3 × 13

(iii) Prime factor of 945 = 3 × 3 × 5 × 7

(v) Prime factor of 7325 = 5 × 5 × 293

2. Question

Determine the prime factorization of each of the following positive integer:

(i) 20570 (ii) 58500

(iii) 45470971

Answer

(i) Prime factors of 20570 = 2 × 5 × 11 × 11 × 17

(ii) Prime factors of 58500 = 2 × 2 × 3 x 3 × 5 x 5 × 13

(iii) Prime factors of 45470971 = 7 × 7 × 13 x 13 × 17 × 17 × 19

3. Question

Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Answer

Numbers are of two types - prime and composite. Prime numbers has only two factors namely 1 and the number itself whereas composite numbers have factors other than 1 and itself.For example: 7 is a prime number as it can be divided by 1 and 7 only whereas 14 is a composite number as it can be divided by 7, 2 and 1 From the question it can be observed that,

7 × 11 × 13 + 13 = 13 × (7 × 11 + 1) = 13 × (77 + 1)   [taking 13 common]

= 13 × 78

= 13 × 13 × 6

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 = 5 × (7 × 6 × 4 × 3 × 2 x 1 + 1)

= 5 × (1008 + 1)

= 5 ×1009

In the above we observe that 1009 cannot be factorized further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

4. Question

Check whether 6^n can end with the digit 0 for any natural numbers n.

Answer

If any number ends with the digit 0, it should be divisible by 10 or in other words its prime factorization must include primes 2 and 5 both as 10 = 2 ×5

Prime factorization of $6^n = (2 \times 3)^n$

In the above equation it is observed that 5 is not in the prime factorization of 6^n

By Fundamental Theorem of Arithmetic Prime factorization of a number is unique. So 5 is not a prime factor of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Exercise 1.4

1. Question

Find the LCM and HCF of the following pairs of integers and verify that LMC × HCF = Product of the integers:

(i) 26 and 91

(ii) 510 and 92

(iii) 336 and 54

Answer

(i) Prime factors of 26 = 2 × 13

Prime factors of 91 = 7 ×13

Hence LCM of 26 and 91 = 2 × 7 × 13 = 182

HCF of 26 and 91 is = 13

LCM × HCF = 182 × 13 = 2366

Product of two numbers = 26 × 91 = 2366

Hence from above result we got LCM × HCF = Product of the numbers

(ii) Prime factors of 510 = 2 × 3 × 5 × 17

Prime factors of 92 = 2 × 2 × 13

Hence LCM of 92 and 510 = 2 × 2 × 3 × 5 × 17 × 23 = 23460

HCF of 92 and 510 is = 2

LCM × HCF = 23460 × 2 = 46920

Product of two numbers = 92×510 = 46920

Hence from above result we got LCM × HCF = Product of the numbers

(iii) Prime factors of 336 = 2 × 2 × 2 × 2 × 3 × 7

Prime factors of 54 = 2 × 3 × 3 × 3

Hence LCM of 336 and 54 = 2 × 2 × 2 × 2 × 3 × 3 × 7 = 3024

HCF of 54 and 336 is = 2 × 3 = 6

LCM × HCF = 3024 × 6 = 18144

Product of two numbers = 336 × 54 = 18144

Hence from above result we got LCM × HCF = Product of the numbers

2. Question

Find the LCM and HCF of the following integers by applying the prime factorization method.

(i) 12, 15 and 21

(ii) 17, 23 and 29

(iii) 8, 9 and 25

(iv) 40, 36 and 126

(v) 84, 90 and 12

(vi) 24, 15 and 36

Answer

(i) Prime factors of 12 = 2 × 2 × 3

Prime factors of 15 = 3 × 5

Prime factors of 21 = 3 ×7

Hence LCM of 12, 15 and 21 = 2 × 2 × 3 × 5 × 7 = 420

HCF of 12, 15 and 21 = 3

(ii) Prime factors of 17 = 1 × 17

Prime factors of 23 = 1 ×23

Prime factors of 29 = 1 × 29

Hence LCM of 17, 23 and 29 = 11339

HCF of 17, 23 and 29 = 1

(iii) Prime factors of 8 = 1 × 2 × 2 × 2

Prime factors of 9 = 1 × 3 × 3

Prime factors of 25 = 1 × 5 × 5

Hence LCM of 8, 9 and 25 = 1800

HCF of 8, 9 and 25 = 1

(iv) Prime factors of 40 = 2 × 2 × 2 × 5

Prime factors of 36 = 2 × 2 × 3 × 3

Prime factors of 126 = 2 × 3 × 3 × 7

Hence LCM of 40, 36 and 126 = 2 × 2 × 2 × 3 × 3 × 5 × 7 = 2520

HCF of 40, 36 and 126 = 2

(v) Prime factors of 12 = 2 × 2 × 3

Prime factors of 84 = 2 × 2 × 3 × 7

Prime factors of 90 = 2 × 3 × 3 × 5

Hence LCM of 12, 84 and 90 = 2 × 2 × 3 × 3 × 5 × 7 = 1260

HCF of 12, 84 and 90 = 2 × 3 = 6

(vi) Prime factors of 15 = 3 × 5

Prime factors of 24 = 2 × 2 × 2 × 3

Prime factors of 36 = 2 × 2 × 3 × 3

Hence LCM of 15, 24 and 36 = 2 × 2 × 2 × 3 × 3 × 5 = 360

HCF of 15, 24 and 36 = 3

3. Question

Given that HCF (306, 657) = 9, find LCM (360, 657).

Answer

We know that LCM × HCF = Product of the numbers

Therefore LCM = $\frac{\text{Product of the numbers}}{\text{HCF of the numbers}} = \frac{306 \times 657}{9}$ = 22338

4. Question

Can two numbers have 16 as their HCF and 380 as their LCM? Give reason.

Answer

NO

LCM should be divisible by HCF since HCF is the common factor of both the numbers. But, in this case 380 is not divisible by 16 therefore, the two numbers are not possible which have 16 as their HCF and 380 as their LCM

5. Question

The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, find the other.

Answer

We know that LCM × HCF = Product of the numbers

Therefore Other Number = $\frac{LCM \times HCF}{\text{One number}} = \frac{2175 \times 145}{725}$ = 435

6. Question

The HCF of two numbers is 16 and their product is 3072. Find their LCM.

Answer

We know that LCM × HCF = Product of the numbers

Therefore Other Number = $\frac{\text{Product of the numbers}}{\text{HCF of the numbers}} = \frac{3072}{16}$ = 192

7. Question

The LCM and HCF of two numbers are 180 and 6 respectively. If one of the numbers is 30, find the other number.

Answer

Given: The LCM and HCF of two numbers are 180 and 6 respectively. One of the numbers is 30.

To find: The other number

Solution: We know that:LCM × HCF = Product of the numbers

LCM × HCF = a × b ...... (1)

Where a and b are numbersOne number i.e a is 30.LCM is 180.HCF is 6.

Substitute the values in eq. (1) to get the other number.

$$\Rightarrow b = \frac{LCM \times HCF}{a}$$

$$\Rightarrow b = \frac{180 \times 6}{30}$$

$$\Rightarrow b = \frac{\cancel{180}^{6} \times 6}{\cancel{30}_{1}}$$

⇒ b = 6 × 6⇒ b = 36

Therefore, the other number is 36.

8. Question

Find the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

Answer

To find the smallest number we should find LCM of 468 and 520

Prime factors of 468 = 2 × 2 × 3 × 3 × 13

Prime factors of 520 = 2 × 2 × 2 × 5 × 13

Hence LCM of 468 and 520 = 2 × 2 ×2 × 3 × 3 ×5×13 = 4680

Therefore the smallest number which when increased by 17 is exactly divisible by both 520 and 468= LCM – 17 ⇒ 4680 – 17 = 4663

9. Question

Find the greatest number of 6 digits exactly divisible by 24, 15 and 36.

Answer

Greatest number of 6 digits is 999999.

For this number to be divisible by 24, 15 and 36, Required number must be divisible by the LCM of 24, 15 and 36 i.e., by 360.Now on dividing six digit greatest number by LCM we get 279 as remainder.

Therefore the greatest number of 6 digits exactly divisible by 24, 15 and 36

= Six digit greatest number – remainder

= 999999 – 279

= 999720

10. Question

Determine the number nearest to 110000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21.

Answer

To find : The number nearest to 110000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21

Solution : To be exactly divisible by each of 8, 15 and 21,the required number must be divisible by the LCM of 8, 15 and 21 i.e.by 840.

Now on dividing 110000 by 840 we get 800 as remainder.

Therefore the number nearest to 110000 but greater than 100000 which is exactly divisible by each of 8, 15 and 21 is 110000 – Remainder ⇒ 110000 – 800 = 109200

Now 109200 gives remainder 0 when divided by 8, 15 and 21 as it is completely divisible by their LCM.

11. Question

Find the smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively.

Answer

The smallest number would be the LCM of 28 and 32

LCM of 28 and 32 = 224

Therefore the required smallest number which leaves remainders 8 and 12 when divided by 28 and 32 respectively would be:

Number = LCM – (sum of the remainders)

⇒ 224 – (12 + 8)

⇒ 224 – 20 = 204

12. Question

What is the smallest number that, when divided by 35, 56 and 91 leaves remainders of 7 in each case?

Answer

Therefore the required smallest number would be the LCM of the numbers

Prime factors of 35 = 5 × 7

Prime factors of 56 = 2 × 2 × 2×7

Prime factors of 91 = 7 × 13

LCM of 35, 56 and 91 ⇒ 2 × 2 × 2 × 5 × 7×13 = 3640

Number = LCM + Remainder

⇒ 3640 + 7 = 3647

13. Question

Find the least number that is divisible by all the numbers between 1 and 10 (both inclusive).

Answer

The least number would be the LCM of the numbers from 1 to 10

We want LCM(1,2,3,4,5,6,7,8,9,10)We can ignore 1, since any counting number is divisible by 1.We prime factor each of the counting numbers from 2 to 10 2 = 23 = 34 = 2×25 = 56 = 2×37 = 78 = 2×2×29 = 3×310 = 2×5 The LCM of all those must have as many factors ofeach prime that appears in any factorization 2 appears at most 3 times as a factor of 83 appears at most 2 times as a factor of 95 appears at most 1 time as a factor if 5 and 107 appears at most 1 time as a factor of 7

So the LCM has 3 factors of 2, 2 factors of 3, and 1 factor each of 5 and 7 LCM = 2×2×2×3×3×5×7 = 2520

Hence, the least number that is divisible by all the numbers between 1 and 10 (both inclusive) is 2520

14. Question

A rectangular courtyard is 18 m 72 cm long and 13 m 20 cm broad. It is to be paved with square tiles of the same size. Find the least possible number of such tiles.

Answer

Length of the courtyard = 18 m 72 cm= [18(100) + 72] cm [As, 1 m = 100 cm]= 1872 cm

The breadth of the courtyard = 13 m 20 cm= [13(100) + 20] cm = 1320 cm

To find the maximum edge of the tile we need to calculate HCF of length and breadth,

Using Euler’s division lemma: a = pq + r where 0≤r<p

1872 = 1320 × 1 + 552As 'r' is not equals to 0,So apply Euler's division on 1320 and 552,

1320 = 552 × 2 + 216

As 'r' is not equals to 0,So apply Euler's division on 552 and 216,

552 = 216 × 2 + 120

As 'r' is not equals to 0,So apply Euler's division on 216 and 120,

216 = 120×1 + 96

As 'r' is not equals to 0,So apply Euler's division on 120 and 96, 120 = 96 ×1 + 24

As 'r' is not equals to 0,So apply Euler's division on 96 and 24,

96 = 24 × 4 + 0

Therefore HCF of 1872 and 1320 is 24Maximum edge can be 24 cm.

Number of tile = $\frac{Area\ of\ courtyard}{Area\ of\ one\ tile}$ = $\frac{1872\times1320}{24\times24}$ = 4290 tiles

15. Question

A circular field has a circumference of 360 km. Three cyclists start together and can cycle 48, 60 and 72 km a day, round the field. When will they meet again?

Answer

Given: A circular field has a circumference of 360 km. Three cyclists start together and can cycle 48, 60 and 72 km a day.

To find: When will they meet again

Solution: Circumference of the circular field is 36 km.Use the formula:

$$speed = \frac{Distance}{Time}$$

Time taken by first cyclist = $\frac{circumference\ of\ the\ field}{speed\ of\ first\ cyclist}$ = $\frac{360}{48}$ = 7.5 days As 1 day = 24 hours⇒ 7.5×24 = 180 Hrs

Similarly,

Time taken by second cyclist = $\frac{360}{60}$ = 6 days ⇒ 6×24 = 144 Hrs

Time taken by third cyclist = $\frac{360}{72}$ = 5 days ⇒ 5×24 = 120 we need to find the minimum time at which they will meet.So we need to find LCM of the time they all take.

Now LCM of 180, 144 and 120 is 720 hours,

Therefore the time when all three will meet again = $\frac{720}{24}$ = 30 days

Note: To find the minimum of given values always find their L.C.M.

16. Question

In a morning walk three persons step off together, their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance each should walk so that he can cover the distance in complete steps?

Answer

To calculate minimum distance Minimum distance we should calculate LCM

Prime factors of 80 = 2×2×2 × 2 ×5

Prime factors of 85 = 5 ×17

Prime factors of 90 = 2×3×3×5

LCM of 80, 85 and 90 ⇒ 2 ×2 × 2 ×2 × 3×3 ×5 × 17 = 12240Minimum distance is 12240 cm,As 1 m = 100 cmSo 12240 = 120 m 40 cm

Therefore the minimum distance each should walk so that he can cover the distance in complete steps is 122 m 40 cm

Exercise 1.5

1. Question

Show that the following numbers are irrational.

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$

(iii) 6 + $\sqrt{2}$ (iv) 3 - $\sqrt{5}$

Answer

(i) Let assume that $\frac{1}{\sqrt{2}}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $\frac{1}{\sqrt{2}} = \frac{p}{q}$

$\sqrt{2} = \frac{q}{p}$

$\frac{q}{p}$ is a rational number as p and q are integers. Therefore √2 is rational which contradicts the fact that √2 is irrational.

Hence, our assumption is false and we can say that $\frac{1}{\sqrt{2}}$ is irrational.

(ii) Let assume that 7√5 is rational therefore it can be written in the form of $\frac{p}{q}$

where p and q are integers and q≠0. Moreover, let p and q have no common divisor > 1.

$7\sqrt{5} = \frac{p}{q}$ for some integers p and q

Therefore $\sqrt{5} = \frac{p}{7q}$

$\frac{p}{7q}$ is rational as p and q are integers, therefore √5 should be rational.

This contradicts the fact that √5 is irrational.

Therefore our assumption that 7√5 is rational is false. Hence 7√5 is irrational.

(iii) Let assume that 6 + √2 is rational therefore it can be written in the form of $\frac{p}{q}$

where p and q are integers and q≠0. Moreover, let p and q have no common divisor > 1.

6 + √2 = $\frac{p}{q}$ for some integers p and q

Therefore √2 = $\frac{p}{q}$ - 6

Since p and q are integers therefore $\frac{p}{q}$ - 6 is rational, hence √2 should be rational.

This contradicts the fact that √2 is irrational. Therefore our assumption is false. Hence, 6 + √2 is irrational.

(iv) Let us assume that 3 - $\sqrt{5}$ is rational

Therefore 3 - $\sqrt{5}$ can be written in the form of $\frac{p}{q}$ where p and q are integers and q≠0

3 - $\sqrt{5}$ = $\frac{p}{q}$ ⇒ $\frac{p}{q}$ -3 = $\sqrt{5}$

⇒ $\frac{p-3q}{q}$ = $\sqrt{5}$

Since p, q and 3 are integers therefore $\frac{p-3q}{q}$ is rational number

But we know $\sqrt{5}$ is irrational number, Therefore it is a contradiction.

Hence 3 - $\sqrt{5}$ is irrational

2. Question

Prove that following numbers are irrationals:

(i) $\frac{2}{\sqrt{7}}$ (ii) $\frac{3}{2\sqrt{5}}$

(iii) $4+\sqrt{2}$ (iv) $5\sqrt{2}$

Answer

(i) Let assume that $\frac{2}{\sqrt{7}}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $\frac{2}{\sqrt{7}} = \frac{p}{q}$

√7= $\frac{2q}{p}$

$\frac{2q}{p}$ is a rational number as p and q are integers.

This contradicts the fact that √7 is irrational, so our assumption is incorrect. Therefore $\frac{2}{\sqrt{7}}$ is irrational

(ii) Let assume that $\frac{3}{2\sqrt{5}}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $\frac{3}{2\sqrt{5}} = \frac{p}{q}$

$\sqrt{5} = \frac{3q}{2p}$

$\frac{3q}{2p}$ is a rational number as p and q are integers.

This contradicts the fact that √5 is irrational, so our assumption is incorrect. Therefore $\frac{3}{2\sqrt{5}}$ is irrational

(iii) Let assume that $4 + \sqrt{2}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $4 + \sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{q} - 4$

$\frac{p}{q} - 4$ is a rational number as p and q are integers.

This contradicts the fact that √2 is irrational, so our assumption is incorrect. Therefore $4 + \sqrt{2}$ is irrational.

(iv) Let assume that $5\sqrt{2}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $5\sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{5q}$

$\frac{p}{5q}$ is a rational number as p and q are integers.

This contradicts the fact that √2 is irrational, so our assumption is incorrect. Therefore $5\sqrt{2}$ is irrational.

3. Question

Show that $2 - \sqrt{3}$ is an irrational numbers.

Answer

Let assume that $2-\sqrt{3}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $2-\sqrt{3} = \frac{p}{q}$

$\sqrt{3} = 2 - \frac{p}{q}$

$\frac{2q-p}{q}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $2-\sqrt{3}$ is irrational.

4. Question

Show that $3+\sqrt{2}$ is an irrational number.

Answer

Let assume that $3+\sqrt{2}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $3+\sqrt{2} = \frac{p}{q}$

$\sqrt{2} = \frac{p}{q} - 3$

$\frac{p-3q}{q}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $3+\sqrt{2}$ is irrational.

5. Question

Prove that $4-5\sqrt{2}$ is an irrational number.

Answer

Let assume that $4-5\sqrt{2}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $4-5\sqrt{2} = \frac{p}{q}$

$5\sqrt{2} = \frac{p}{q} - 4$

$\frac{p-4q}{5q}$ is a rational number as p and q are integers.

This contradicts the fact that √2 is irrational, so our assumption is incorrect. Therefore $4-5\sqrt{2}$ is irrational.

6. Question

Show that $5-2\sqrt{3}$ is an irrational number.

Answer

To prove : $5-2\sqrt{3}$ is an irrational number.**Solution:**Let assume that $5-2\sqrt{3}$ is rational.

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $5-2\sqrt{3}=\frac{p}{q}$

$2\sqrt{3}= 5 - \frac{p}{q} \Rightarrow \sqrt{3} = \frac{5q-p}{2q}$

$\frac{5q-p}{2q}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $5-2\sqrt{3}$ is irrational.

Note: Sometimes when something needs to be proved, prove it by contradiction.Where you are asked to prove that a number is irrational prove it by assuming that it is rational numberand then contradict it.

7. Question

Prove that $2\sqrt{3}-1$ is an irrational number.

Answer

Let assume that $2\sqrt{3}-1$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $2\sqrt{3}-1 = \frac{p}{q}$

$2\sqrt{3}= \frac{p}{q} + 1$

$$\Rightarrow 2\sqrt{3} = \frac{p+q}{q}$$

$$\Rightarrow \sqrt{3} = \frac{p+q}{2q}$$

$\frac{p+q}{2q}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $2\sqrt{3}-1$ is irrational.

8. Question

Prove that $2-3\sqrt{5}$ is an irrational number.

Answer

Given: $2-3\sqrt{5}$

To prove: $2-3\sqrt{5}$ is irrational.

Proof: Let assume that $2-3\sqrt{5}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write $2-3\sqrt{5}=\frac{p}{q}$

$$2-\frac{p}{q}=3\sqrt{5}$$

$\frac{2q-p}{3q}$ = √5

$\frac{2q-p}{3q}$ is a rational number as p and q are integers. This contradicts the fact that √5 is irrational, so our assumption is incorrect. Therefore $2-3\sqrt{5}$ is irrational.

Note: Sometimes when something needs to be proved, prove it by contradiction.Where you are asked to prove that a number is irrational prove it by assuming that it is rational numberand then contradict it.

9. Question

Prove that $\sqrt{5}+\sqrt{3}$ is irrational.

Answer

Let assume that $\sqrt{5}+\sqrt{3}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and q≠0

Therefore we can write √5 = $\frac{p}{q}$ - √3

$(\sqrt{5})^2 = (\frac{p}{q} - \sqrt{3})^2$

5 = $\frac{p2}{q2}$ - $\frac{2p\sqrt{3}}{q}$ +3

5 - 3 = $\frac{p2}{q2}$ - $\frac{2p\sqrt{3}}{q}$

$\frac{p2}{q2}$ - 2 = $\frac{2p\sqrt{3}}{q}$

$\frac{p2-2q2}{qp}$ = √3

$\frac{p2-2q2}{qp}$ is a rational number as p and q are integers. This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $\sqrt{5}+\sqrt{3}$is irrational.

10. Question

Prove that $\sqrt{3}+\sqrt{4}$ is an irrational number.

Answer

Given: $\sqrt{3}+\sqrt{4}$

To prove: $\sqrt{3}+\sqrt{4}$is irrational.

Proof: Let assume that $\sqrt{3}+\sqrt{4}$ is rational

Therefore it can be expressed in the form of $\frac{\mathbf{p}}{\mathbf{q}}$, where p and q are integers and q≠0

$$\sqrt{3}+\sqrt{4}=\frac{p}{q}$$

Therefore we can write, $\Rightarrow \sqrt{4}=\frac{p}{q}-\sqrt{3}$

Squaring both sides we get, $\Rightarrow \left(\sqrt{4}\right)^2=\left(\frac{p}{q}-\sqrt{3}\right)^2$

Apply the formula (a - b)^2 = a^2 + b^2 - 2ab in $\left(\frac{p}{q}-\sqrt{3}\right)^2$

$$\Rightarrow 4=\left(\frac{p}{q}\right)^2+\left(\sqrt{3}\right)^2-2\left(\frac{p}{q}\right)\sqrt{3}$$

$$\Rightarrow 4=\frac{p^2}{q^2}+3-\frac{2p\sqrt{3}}{q}$$

$$\Rightarrow 4-3=\frac{p^2}{q^2}-\frac{2p\sqrt{3}}{q}$$

$$\Rightarrow 1=\frac{p^2}{q^2}-\frac{2p\sqrt{3}}{q}$$

$$\Rightarrow \frac{p^2}{q^2} - 1 = \frac{2p\sqrt{3}}{q}$$

$$\Rightarrow \frac{p^2 - q^2}{q^2} = \frac{2p\sqrt{3}}{q}$$

$$\Rightarrow q\left(\frac{p^2 - q^2}{q^2}\right) = 2p\sqrt{3}$$

$$\Rightarrow \frac{p^2 - q^2}{2pq} = \sqrt{3}$$

$\frac{p^2 - q^2}{2pq}$ is a rational number as p and q are integers.

This contradicts the fact that √3 is irrational, so our assumption is incorrect. Therefore $\sqrt{3} + \sqrt{4}$ is irrational.

Note: Sometimes when something needs to be proved, prove it by contradiction.Where you are asked to prove that a number is irrational prove it by assuming that it is rational numberand then contradict it.

11. Question

Prove that for any prime positive integer p, $\sqrt{p}$ is an irrational number.

Answer

Let assume that √p is rational

Therefore it can be expressed in the form of $\frac{a}{b}$, where a and b are integers and b≠0

Therefore we can write √p = $\frac{a}{b}$

$(\sqrt{p})^2 = (\frac{p}{q})^2$

P = $\frac{a2}{b2}$

$a^2 = pb^2$

Since a^2 is divided by b^2, therefore a is divisible by b.

Let a = kc

$(kc)^2 = pb^2$

$K^2c^2 = pb^2$

Here also b is divided by c, therefore b^2 is divisible by c^2. This contradicts that a and b are co-primes. Hence $\sqrt{p}$ is an irrational number.

12. Question

If p, q are prime positive integers, prove that $\sqrt{p}+\sqrt{q}$ is an irrational number.

Answer

Since it is given that p and q are prime positive integer.Let us assume that $\sqrt{p}+\sqrt{q}$ is a rational number of the form $\frac{a}{b}$,

$$\Rightarrow (\sqrt{q})^2 = \left(\frac{a}{b} - \sqrt{p}\right)^2$$

$$\Rightarrow \sqrt{p}+\sqrt{q} = \frac{a}{b}$$

$$\Rightarrow \sqrt{q} = \frac{a}{b} - \sqrt{p}$$

Squaring both sides we get,

Apply the formula $(a-b)^2 = a^2 + b^2 - 2ab$

$$\Rightarrow (\sqrt{q})^2 = \left(\frac{a}{b}\right)^2 + (\sqrt{p})^2 - 2\left(\frac{a}{b}\right)(\sqrt{p})$$

$$\Rightarrow q = \frac{a^2}{b^2} + p - 2\left(\frac{a}{b}\right)(\sqrt{p})$$

$$2\left(\frac{a}{b}\right)\sqrt{p} = \frac{a^2}{b^2} + p - q$$

$$\Rightarrow 2\left(\frac{a}{b}\right)(\sqrt{p}) = \frac{a^2 - pb^2 - qb^2}{b^2}$$

$$\Rightarrow \sqrt{p} = \frac{b\left(a^2 - pb^2 - qb^2\right)}{ab^2}$$

$$\Rightarrow \sqrt{p} = \frac{a^2 - pb^2 - qb^2}{ab}$$

Since p, q are integers therefore $\frac{a^2 - pb^2 - qb^2}{ab}$ is rational number

But we know $\sqrt{p}$ is irrational number, Therefore it is a contradiction.

Hence $\sqrt{p} + \sqrt{q}$ is irrational

Exercise 1.6

1. Question

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating

(i) $\frac{23}{8}$ (ii) $\frac{125}{441}$

(iii) $\frac{35}{50}$ (iv) $\frac{77}{210}$

(v) $\frac{129}{2^2 \times 5^7 \times 7^{17}}$

Answer

(i) $8 = 2^3$ since the denominator is of the form 2^m therefore the expression $\frac{23}{8}$ is terminating.

(ii) $441 = 3^2 \times 7^2$ since the denominator is not in the form of 2^m and 5^n Therefore the given expression is Non-terminating repeating

(iii) $\frac{7\times5}{10\times5} = \frac{7}{10}$

$10 = 2\times5$ therefore the denominator is of the form $2^m \times 5^n$

Hence, the given expression is Terminating

(iv) $\frac{77}{210} = \frac{7\times11}{30\times7} = \frac{11}{30}$

$30 = 2\times3\times5$

Since the denominator is not of the form $2^m \times 5^n$

Hence, the given expression is Non-terminating repeating Terminating

(v) $\frac{129}{2^2 \times 5^7 \times 7^{17}}$

Since the denominator is not in the form $2^m \times 5^n$

Hence, the given expression is Non-terminating repeating Terminating

2. Question

Write down the decimal expansions of the following rational numbers by writing their denominators in the form $2^m \times 5^n$, where m, n are non-negative integers.

(i) $\frac{3}{8}$ (ii) $\frac{13}{125}$

(iii) $\frac{7}{80}$ (iv) $\frac{14588}{625}$

(v) $\frac{129}{2^2 \times 5^7}$

Answer

(i) $\frac{3}{8} = \frac{3}{2^3} = \frac{3 \times 5^3}{2^3 \times 5^3}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{375}{10^3} = 0.375$

(ii) $\frac{13}{125} = \frac{13}{5^3} = \frac{13 \times 2^3}{2^3 \times 5^3}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{104}{10^3} = 0.104$

(iii) $\frac{7}{80} = \frac{7}{2^4 \times 5} = \frac{7 \times 5^3}{2^4 \times 5^4}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{875}{10^4} = 0.0875$

(iv) $\frac{14588}{625} = \frac{22 \times 7 \times 521}{5^4} = \frac{22 \times 2^4 \times 7 \times 521}{2^4 \times 5^4}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{22 \times 2^4 \times 7 \times 521}{(2 \times 5)^4} = 23.3408$

(v) $\frac{129}{2^2 \times 5^7} = \frac{2^5 \times 129}{2^7 \times 5^7} = \frac{2^5 \times 129}{(2 \times 5)^7}$ the denominator is in the form of $2^m \times 5^n$

$\Rightarrow \frac{2^5 \times 129}{(2 \times 5)^7} = 0.0004128$

3. Question

What can you say about the prime factorizations of the denominators of the following rational?

(i) 43.123456789

(ii) $43.\overline{123456789}$

(iii) $27.\overline{142857}$

(iv) 0.120120012000120000…

Answer

(i) 43.123456789 = $\frac{43123456789}{100000000} = \frac{43123456789}{2^9 \times 5^9}$

Since Prime factorization of the denominator is in the form $2^m \times 5^n$, where m, n are non-negative integers.

(ii) Since $43.\overline{123456789}$ has non-terminating repeating decimal expression, therefore Prime factorization of the denominator contains factors other than 2 or 5.

(iii) Since $27.\overline{142857}$ has non-terminating repeating decimal expression, therefore Prime factorization of the denominator contains factors other than 2 or 5.

(iv) Since $27.\overline{142857}$ has non-terminating repeating decimal expression, therefore Prime factorization of the denominator contains factors other than 2 or 5.

CCE - Formative Assessment

1. Question

State Euclid's division lemma.

Answer

Euclid's Division Lemma: Given positive integers a and b, there exist unique integers q and r satisfying a = bq + r, $0 \le r < b$. (It is a technique to compute the Highest Common Factor (HCF) of two given positive integers.)

2. Question

State Fundamental Theorem of Arithmetic.

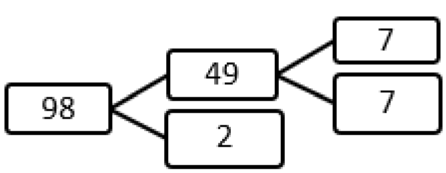
Answer

Fundamental Theorem of Arithmetic: Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

3. Question

Write 98 as product of its prime factors.

Answer



$98 = 49 \times 2 = 7^2 \times 2$

98 is factorised as a product of primes i.e. $7^2 \times 2$.

4. Question

Write the exponent of 2 in the prime factorization of 144.

Answer

The prime factorization of 144 is as follows:

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$\Rightarrow 144 = 2^4 \times 3^2$

We know that the exponent of a number a^m is m.

The exponent of 2 in the prime factorization of 144 is 4.

5. Question

Write the sum of the exponents of prime factors in the prime factorization of 98.

Answer

The prime factorization of 98 is as follows:

$98 = 2 \times 7 \times 7$

$\Rightarrow 98 = 2^1 \times 7^2$

We know that the exponent of a number a^m is m.

$\therefore$The sum of powers = 1 + 2 = 3

The sum of the exponents of prime factors in the prime factorization of 98 is 3.

6. Question

If the prime factorization of a natural number n is $2^3 \times 3^2 \times 5^2 \times 7$, write the number of consecutive zeros in n.

Answer

If any number ends with the digit 0, it should be divisible by 10,

i.e. it will be divisible by 2 and 5.

Prime factorization of n is given as $2^3 \times 3^2 \times 5^2 \times 7$.

It can be observed that there is $(2 \times 5) \times (2 \times 5)$

$\Rightarrow 10 \times 10 = 100$

Thus, there are 2 zeros in n.

The number of consecutive zeros in n is 2.

7. Question

If the product of two numbers is 1080 and their HCF is 30, find their LCM.

Answer

We know that for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

Here a × b = 1080 and HCF = 30

∴ LCM = (a × b)/ HCF

⇒ LCM = 1080/30

⇒ LCM = 36

The LCM of given product of two numbers is 36.

8. Question

Write the condition to be satisfied by q so that a rational number has a terminating 9 decimal expansion.

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

9. Question

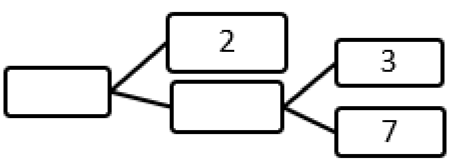
Write the condition to be satisfied by q so that a rational number has a non-terminating terminating decimal expansion.

Answer

Let x = p/q be a rational number, such that the prime factorization of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

10. Question

Complete the missing entries in the following factor tree.



Answer

Let us take the last factors of the tree 3 and 7.

Then 3 × 7 = 21

Now we get the factors in the next level as 21 and 2.

So, 21 × 2 = 42

The missing entries are 42, 21.

11. Question

The decimal expansion of the rational number $\frac{43}{2^4 \times 5^3}$ will terminate after how many places of decimals?

Answer

Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then 'x' has a decimal expansion which terminates after n or m places, whichever is maximum

The maximum power of 2 or 5 in the given rational number is 4.

So, it will terminate after 4 places of decimals.

The decimal expansion of the given rational number will terminate after 4 places of decimals.

12. Question

Has the rational number a terminating or a non terminating decimal $\frac{441}{2^2 \times 5^7 \times 7^2}$ representation?

Answer

Let $x = \frac{p}{q}$ be a rational number, then x is terminating if and only if, the denominator q has the form $2^n 5^m$

The denominator is not of the form $2^n 5^m$ as it has 7^2 as its factor.

Therefore, Given rational number has a non-terminating decimal representation.

13. Question

Write whether on simplification $\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$ gives a rational or an irrational number.

Answer

Given: number $\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$

To find: Whether the given number is rational or irrational

Solution: Factorize 45 and 20.

$\Rightarrow \frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{3\times3\times5} + 3\sqrt{2\times2\times5}}{2\sqrt{5}}$

$\Rightarrow \frac{2\sqrt{3^2\times5} + 3\sqrt{2^2\times5}}{2\sqrt{5}}$

$\Rightarrow \frac{2\times3\sqrt{5} + 3\times2\sqrt{5}}{2\sqrt{5}}$

$\Rightarrow \frac{2\sqrt{5}(3+3)}{2\sqrt{5}}$

$\Rightarrow 3 + 3 = 6$

We know that a rational number is defined as the number which can be written in the form of p/q.As 6 can be written as 6/1.

So 6 is a rational number.

The given number after simplification gives a rational number.

14. Question

What is an algorithm?

Answer

An algorithm is a series of well-defined steps which gives a procedure for solving a type of problem. The word algorithm comes from the name of the 9th-century Persian mathematician al-Khwarizmi.

15. Question

What is a lemma?

Answer

A lemma is a proven statement used for proving another statement.

16. Question

If p and q are two prime numbers, then what is their HCF?

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

For any two prime numbers, one of the common prime factors will be 1.

17. Question

If p and q are two prime numbers, then what is their LCM?

Answer

We know that LCM = Product of the greatest power of each prime factor, involved in the numbers.

If p and q are two prime numbers, then $p \times q$ is their LCM.

18. Question

What is the total number of factors of a prime number?

Answer

We know that the factors of a prime number are 1 and the number itself.

The total number of factors of a prime number is 2.

19. Question

What is a composite number?

Answer

Every composite number can be expressed (factorized) as a product of primes.

20. Question

What is the HCF of the smallest composite number and the smallest prime number?

Answer

Smallest composite number = 4

$= 2 \times 2$

Smallest prime number = 2

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

HCF (4, 2) = 2

The HCF of the smallest composite number and the smallest prime number is 2.

21. Question

HCF of two numbers is always a factor of their LCM (True/False).

Answer

True.

LCM of two or more numbers is always divisible by their HCF.

Example: Let us take two numbers 6 and 20.

The prime factorization of

$6 = 2^1 \times 3^1$ and $20 = 2^2 \times 5^1$

HCF = 2

$LCM = 2^2 \times 3 \times 5 = 60$

∴ 2 is a factor of 60.

22. Question

π is an irrational number (True/False).

Answer

True

23. Question

The sum of two prime numbers is always a prime number (True/False).

Answer

False

Prime numbers are always odd numbers and the sum of odd numbers is even.

Example: Let two prime numbers be 3 and 5.

Sum of 3 and 5 = 3 + 5 = 8 which is not a prime number.

24. Question

The product of any three consecutive natural numbers is divisible by 6 (True/False).

Answer

True

Since n(n + 1)(n + 2) is divisible by 2 and 3, then it is divisible by 6.

Example: Let us take three consecutive numbers 10, 11, 12.

Product = 10 × 11 × 12 = 1320

∴ 1320 is divisible by 6.

25. Question

Every even integer is of the form 2m, where m is an integer (True/False).

Answer

True

Let a = bq + r. b = 2, q = m

r = 0.

a = 2m

26. Question

Every odd integer is of the form 2m - 1, where m is an integer (True/False).

Answer

True

Let a = bq + r: b = 2, q = m

$-2<r<0$ i.e., r = -1

a = 2m − 1 for odd integer.

27. Question

The product of two irrational numbers is an irrational number (True/False).

Answer

False

The product of two irrational is sometimes an irrational number.

Example: If we multiply an irrational number with 0 we will get the product as 0 which is a rational number.

28. Question

The sum of two irrational numbers is an irrational number (True/False).

Answer

False

The sum of two irrational numbers is sometimes an irrational number.

Example: If the two irrational numbers have a sum zero, then the sum is a rational number.

29. Question

For what value of n, $2^n \times 5^n$ ends in 5.

Answer

Let us take n = 1.

$2^1 \times 5^1 = 10$

So, for no value of n, $2^n \times 5^n$ ends in 5.

No value of n

30. Question

If a and b are relatively prime numbers, then what is their HCF?

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

For any two relatively prime numbers, one of the common prime factors will be 1.

31. Question

If a and b are relatively prime numbers, then what is their LCM?

Answer

We know that LCM = Product of the greatest power of each prime factor, involved in the numbers.

If a and b are two prime numbers, then a × b is their LCM.

32. Question

Two numbers have 12 as their HCF and 350 as their LCM (True/False).

Answer

False

HCF of two numbers is always a factor of their LCM.

But 12 is not a factor of 350.

1. Question

The exponent of 2 in the prime factorisation of 144, is

A. 4

B. 5

C. 6

D. 3

Answer

The prime factorization of 144 is as follows:

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$\Rightarrow 144 = 2^4 \times 3^2$

We know that the exponent of a number a^m is m.

∴ The exponent of 2 in the prime factorization of 144 is 4.

2. Question

The LCM of two numbers is 1200. Which of the following cannot be their HCF?

A. 600

B. 500

C. 400

D. 200

Answer

We know that LCM of two or more numbers is always divisible by their HCF.

1200 is divisible by 600, 200 and 400 but not by 500.

3. Question

If $n = 2^3 \times 3^4 \times 5^4 \times 7$, then the number of consecutive zeros in n, where n is a natural number, is

A. 2

B. 3

C. 4

D. 7

Answer

If any number ends with the digit 0, it should be divisible by 10,

i.e. it will be divisible by 2 and 5.

Prime factorization of n is given as $2^3 \times 3^4 \times 5^4 \times 7$.

It can be observed that there is (2 × 5) × (2 × 5) × (2 × 5)

⇒ 10 × 10 × 10 = 1000

Thus, there are 3 zeros in n.

4. Question

The sum of the exponents of the prime factors in the prime factorisation of 196, is

A. 1

B. 2

C. 4

D. 6

Answer

The prime factorization of 196 is as follows:

196 = 2 × 2 × 7 × 7

⇒ 98 = $2^2 \times 7^2$

We know that the exponent of a number a^m is m.

∴The sum of powers = 2 + 2 = 4

5. Question

The number of decimal places after which the decimal expansion of the rational number $\frac{23}{2^2 \times 5}$ will terminate, is

A. 1

B. 2

C. 3

D. 4

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

The maximum power of 2 or 5 in the given rational number is 2.

So, it will terminate after 2 places of decimals.

6. Question

If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is

A. an even number

B. an odd number

C. an odd prime number

D. a prime number

Answer

Let us take $p_1 = 5$ and $p_2 = 3$

Then $p_1^2 - p_2^2 = 25 - 9 = 16$

16 is an even number.

7. Question

If two positive integers a and b are expressible in the form $a = pq^2$ and $b = p^3 q$; p, q being prime numbers, then LCM (a, b) is

A. pq

B. $p^3 q^3$

C. $p^3 q^2$

D. $p^2 q^2$

Answer

We know that LCM = Product of the greatest power of each prime factor, involved in the numbers.

So, LCM(a, b) = p^3q^2

8. Question

In Q. No. 7, HCF (a, b) is

A. pq

B. $p^3 q^3$

C. $p^3 q^2$

D. $p^2 q^2$

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

So, HCF(a, b) = pq

9. Question

If two positive integers m and n are expressible in the form $m = pq^3$ and $n = p^3 q^2$ where p, q are prime numbers, then HCF (m, n) =

A. pq

B. pq^2

C. p^3q^3

D. p^2q^3

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

So, HCF(a, b) = pq^2

10. Question

If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a =

A. 2

B. 3

C. 4

D. 1

Answer

We know that for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

Here LCM = 36, HCF = 2 and b = 18

Then, 2 × 36 = a × 18

a = (2 × 36) / 18

a = 4

11. Question

The HCF of 95 and 152, is

A. 57

B. 1

C. 19

D. 38

Answer

We know that HCF = Product of the smallest power of each common prime factor in the numbers.

The prime factorization of

95 = 5 × 19

And 152 = 2^3 × 19

∴ HCF = 19

12. Question

If HCF (26, 169) = 13, then LCM (26, 169) =

A. 26

B. 52

C. 338

D. 13

Answer

We know that for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

Here HCF = 13, a = 26 and b = 169

Then,

13 × LCM = 26 × 169

LCM = (26 × 169) / 13

LCM = 338

13. Question

If a = $2^3 \times 3$, b = 2 × 3 × 5, c = $3^n \times 5$ and LCM (a, b, c) = $2^3 \times 3^2 \times 5$, then n =

A. 1

B. 2

C. 3

D. 4

Answer

We know that LCM = Product of the greatest power of each prime factor, involved in the numbers.

Since the power of 3 in LCM is 2,

C = $3^2 \times 5$

14. Question

The decimal expansion of the rational number 14587 / 1250 will terminate after

A. One decimal place

B. two decimal place

C. three decimal place

D. four decimal place

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

The denominator 1250 can also be written as 2×5^4.

The maximum power of 2 or 5 in 1250 is 4.

So, it will terminate after 4 places of decimals.

15. Question

If p and q are co-prime numbers, then p^2 and q^2 are

A. coprime

B. not coprime

C. even

D. odd

Answer

Given: If p and q are co-prime numbers.

To find: p^2 and q^2 are

Solution: Two numbers are co-prime if their HCF is 1 i.e they have no number common other than 1.

Let us take p = 4 and q = 5.As 4 and 5 has no common factor other than 1, p and q are co-prime.

Now $p^2 = 16$ and $q^2 = 25$,

As 16 and 25 has no common factor other than 1,

So p^2 and q^2 are also co-prime.

16. Question

Which of the following rational numbers have terminating decimal?

(i)16/225 (ii)5/18

(iii)2/21 (iv)7/250

A. (i) and (ii)

B. (ii) and (iii)

C. (i) and (iii)

D. (i) and (iv)

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

(i) Here q = 225

225 can be written as $3^2 \times 5^2$

Since it is in the form of 5^m, it is a terminating decimal.

(ii) Here q = 18

18 can be written as 2×3^2

Since 3 is also there and it is not in the form of 2^n5^m, it is not a terminating decimal.

(iii) Here q = 21

21 can be written as 3×7

Since it is not in the form of 2^n5^m, it is not a terminating decimal.

(iv) Here q = 250

250 can be written as 2×5^3

Since it is in the form of 2^n5^m, it is a terminating decimal.

17. Question

If 3 is the least prime factor of number a and 7 is the least prime factor of number b, then the least prime factor of a + b, is

A. 2

B. 3

C. 5

D. 10

Answer

The prime factors of a + b

= 3 + 7 = 10 = 2 × 5

So the least prime factor is 2.

18. Question

$3.\overline{27}$ is

A. an integer

B. a rational number

C. a natural number

D. an irrational number

Answer

Since, the given number is non-terminating recurring, It is a rational number.

19. Question

The smallest number by which $\sqrt{27}$ should be multiplied so as to get a rational number is

A. $\sqrt{27}$

B. $3\sqrt{3}$

C. $\sqrt{3}$

D. 3

Answer

$\sqrt{27} = \sqrt{3 \times 3 \times 3}$

If we multiply $\sqrt{3}$ we will get a perfect square 9 which is a rational number.

20. Question

The smallest rational number by which 1/3 should be multiplied so that its decimal expansion terminates after one place of decimal, is

A. 3/10

B. 1/10

C. 3

D. 3/100

Answer

Let x = p/q be a rational number, such that the prime factorization of q is of the form $2^n\ 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

So 1/3 should be multiplied by 3/10 so that it is in the form of 2^n5^m.

21. Question

If n is a natural number, then $9^{2n} - 4^{2n}$ is always divisible by

A. 5

B. 13

C. both 5 and 13

D. None of these

Answer

$9\ 2^n - 4\ 2^n$ is of the form $a\ 2^n - b\ 2^n$.

It is divisible by both a - b and a + b.

So, 9 2^n – 4 2^n is divisible by both 9 - 4 = 5 and 9 + 4 = 13.

22. Question

If n is any natural number, then 6^n – 5^n always ends with

A. 1

B. 3

C. 5

D. 7

Answer

For any n ∈ N, e and 5^n end with 6 and 5 respectively.

Therefore, 6^n – 5^n always ends with 6 - 5 = 1.

23. Question

The LCM and HCF of two rational numbers are equal, then the numbers must be

A. prime

B. co-prime

C. composite

D. equal

Answer

When numbers are equal, LCM and HCF of two rational numbers are equal.

24. Question

If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, then the product of two numbers is

A. 203400

B. 194400

C. 198400

D. 205400

Answer

Given: The sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF.

To find: The product of two numbers.

Solution: Let LCM be x and HCF be y.

Then, by given condition,Sum of LCM and HCFx + y = 1260   (1)

Also, LCM is 900 more than HCF i.e.x = 900 + y

Substituting x = 900 + y in equation (1)

900 + y + y = 1260

900 + 2y = 1260

2y = 360

y = 180

Then, x = 900 + 180 = 108

Thus, LCM = 1080 and HCF = 180

We know that for any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b.

1080 × 180 = 194400

25. Question

The remainder when the square of any prime number greater than 3 is divided by 6, is

A. 1

B. 3

C. 2

D. 4

Answer

Any prime number greater than 3 is of the form $6k \pm 1$, where k is a natural number

and $(6k \pm 1)2 = 36k^2 \pm 12k + 1 = 6k (6k \pm 2) + 1$

Thus, the remainder is 1.