12. Some Application of Trigonometry

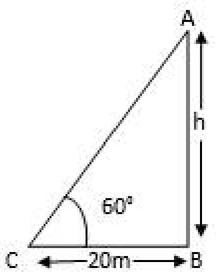
Exercise 12.1

1. Question

A tower stands vertically on the ground. From a point on the ground, 20 m away from the foot of the tower, the angle of elevation of the top the tower is 60°. What is the height of the tower?

Answer

Let the height of the tower = h (m)



In ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{h}{20}$

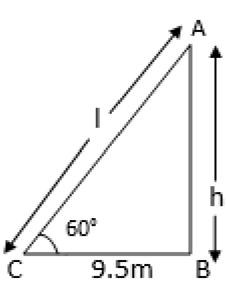
$h = 20\sqrt{3}$ m

Therefore height of the tower is $20\sqrt{3}$ m

2. Question

The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.

Answer



Let the length of the ladder=l (m)

In ΔABC,

$\cos 60° = \frac{BC}{AC}$

$\frac{1}{2} = \frac{9.5}{l}$

l =9.5×2 ⇒ 19 m

Therefore length of Ladder is 19 m.

3. Question

A ladder is placed along a wall of a house such that its upper end is touching the top of the wall. The foot of the ladder is 2 m away from the wall and the ladder is making an angle of 60° with the level of the ground. Determine the height of the wall.

Answer

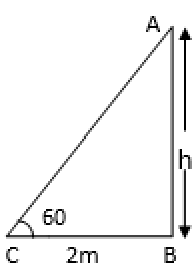
Let the length of the wall = h (m)

In ΔABC,

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{h}{2}$

$h = 2\sqrt{3}$ m

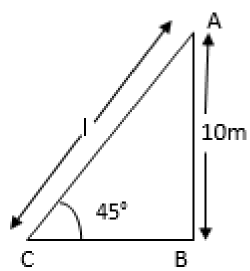


Therefore length of the wall is $2\sqrt{3}$ m

4. Question

An electric pole is 10 m high. A steel wire tied to top of the pole is affixed at a point on the ground to keep the pole up right. If the wire makes an angle of 45° with the horizontal through the foot of the pole, find the length of the wire.

Answer



Let the length of the wire = l (m)

In ΔABC,

Sin 45° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{2}} = \frac{10}{l}$

l ⇒ 10√2 m

⇒ 10x1.41

⇒14.1 m

Therefore Length of wire is 14.1 m

5. Question

A kite is flying at a height of 75 metres from the ground level, attached to a string inclined at 60° to the horizontal. Find the length of the string to the nearest metre.

Answer

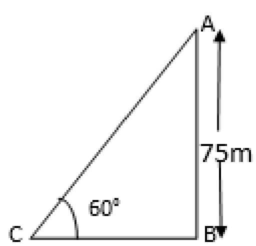
Let the length of the wire = l (m)

In ΔABC,

Sin 60° = $\frac{AB}{AC}$

$\frac{\sqrt{3}}{2} = \frac{75}{l}$

$\sqrt{3}l$ = 2×75



l = $\frac{150}{\sqrt{3}}$

l = $\frac{150\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$ ⇒ $\frac{150\sqrt{3}}{3}$ ⇒ 50√3 ⇒ 86.6 m.

Therefore length of string is 86.6 m.

6. Question

The length of a string between a kite and a point on the ground is 90 metres. If the string makes an angle θ with the ground level such that tanθ =15/8, how high is the kite? Assume that there is no slack in the string.

Answer

Given: The length of a string between a kite and a point on the ground is 90 metres. If the string makes an angle θ with the ground level such that tanθ =15/8.

To find: how high is the kite.

Solution: Let the height of string = h (m)

tan θ = $\frac{15}{8}$ (given)

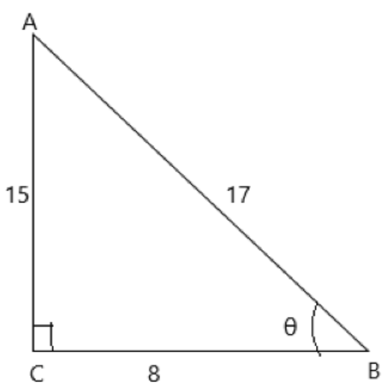
Since tanθ = perpendicular/baseSo perpendicular= 15 and base=8So we construct a right triangle ABC right angled at C such that∠ABC=θ and AC = Perpendicular = 15BC = base = 8By Pythagoras theorem,$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = (15)^2 + (8)^2$

$\Rightarrow AB^2 = 225 + 64$

$\Rightarrow AB^2 = 289$

$\Rightarrow AB = \sqrt{289} \Rightarrow AB = 17$

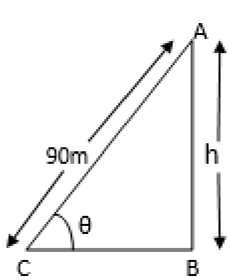


Since Sin θ = perpendicular/hypotenuse

⇒ Sin θ = $\frac{15}{17}$ ..... (1)

In ΔABC,

Sin θ = $\frac{AB}{AC}$ Sin θ = $\frac{h}{90}$ ...... (2)



Equating (1) and (2) we get,

$\frac{15}{17} = \frac{h}{90}$ 17h = 90×15

$\Rightarrow h = \frac{90 \times 15}{17} \Rightarrow h = 79.41$ m.

Therefore length of string is 79.41 m.

7. Question

A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff. At a point on the plane 70 metres away from the tower, an observer notices that the angles of elevation of the top and the bottom of the flag-staff are respectively 60° and 45°. Find the height of the flag-staff and that of the tower.

Answer

Let the height of tower = h (m)

Let the height of the flag-staff = t (m)

In ΔDBC,

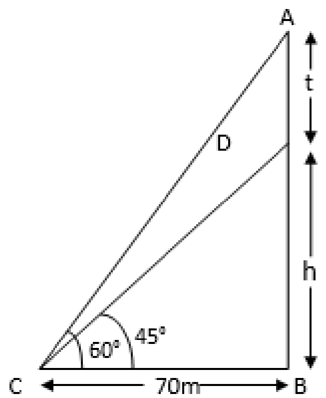
$\tan 45° = \frac{DB}{BC}$

$1 = \frac{h}{70}$

h = 70 m

Therefore height of tower = 70m.

Now in ΔABC,



$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{h+t}{70} \Rightarrow \sqrt{3} = \frac{70+t}{70}$ (on substituting value of h =70)

$70+t = 70\sqrt{3}$

$t = 70\sqrt{3}-70$

$t = 70 (\sqrt{3} -1)$

$t = 70 \times (1.732-1)$

$t = 70 \times 0.732 \Rightarrow 51.24$ m.

Therefore height of the flag- staff is 51.24 m.

8. Question

A vertically straight tree, 15m height, is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground. At what height from the ground did the tree break?

Answer

Total height of the tree is 15 m.

I.e AB= 15m.

Let height at which tree is broken is h (m)

Therefore BC = h (m)

CD = AB-BC

= 15-h.

In ΔDBC,

sin 60° = $\frac{BC}{CD}$

$\frac{\sqrt{3}}{2} = \frac{h}{15-h}$

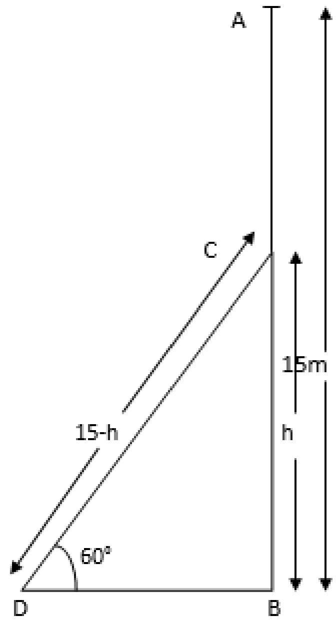
On cross-multiplication

$\sqrt{3}(15-h) = 2h$

$\Rightarrow 15\sqrt{3}-\sqrt{3}h = 2h$

$\Rightarrow h(2+\sqrt{3}) = 15\sqrt{3}$

$h = \frac{15\sqrt{3}}{2+\sqrt{3}}$



on multiplying and dividing by 2-√3

h = $\frac{15\sqrt{3}(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

h = $\frac{30\sqrt{3}-45}{4-3} \Rightarrow \frac{30\sqrt{3}-45}{1} \Rightarrow$ 6.96 m.

Therefore the tree is broken at 6.96 m from the ground.

9. Question

A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 5 metres. At a point on the plane, the angles of elevation of the bottom and the top of the flag-staff are respectively 30° and 60°. Find the height of the tower.

Answer

Let the height of the tower = h (m)

Let the point of elevation on the ground is x (m) away from the foot of the tower.

In ΔDBC,

tan 30° = $\frac{BD}{BC}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

On the cross multiplication

x = h√3 -------(1)

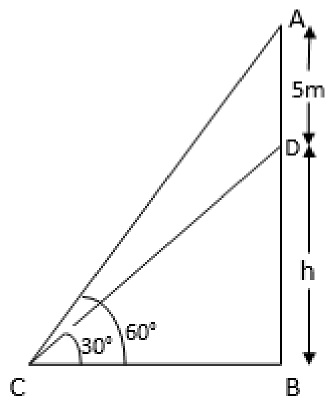
In ΔABC,

tan 60° = $\frac{AB}{BC}$

$\sqrt{3} = \frac{AD+BD}{BC}$

$\sqrt{3} = 5 + \frac{h}{x}$ ---------(2)

On substituting value of x from equn. (1) in eqn. (2)



$\sqrt{3} = \frac{5+h}{h\sqrt{3}}$

$h\sqrt{3}\times\sqrt{3} = 5+h$

3h = 5+h

3h-h = 5

$2h = 5 \Rightarrow h = \frac{5}{2}$

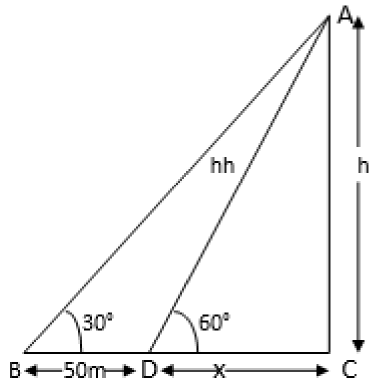
h = 2.5 m.

Therefore height of the tower is 2.5 m.

10. Question

A person observed the angle of elevation of the top of a tower as 30°. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60°. Find the height of the tower.

Answer



Let the height of the tower = h (m)

In ΔABD,

tan 60° = $\frac{AB}{BD}$

√3 = $\frac{h}{x}$

h = $x\sqrt{3}$ ---------- (1)

In ΔABC,

tan 30° = $\frac{AB}{BC}$

tan 30° = $\frac{AB}{BD+CD}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+50}$ ------(2)

on substituting value of h from equn. (1) In equn. (2)

$$\frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{x+50}$$

On cross multiplication

√3×√3 x = $x + 50$

3 x = $x + 50$

3 $x - x = 50$

2 $x = 50$

$x = \frac{50}{2}$ ⇒ 25 m.

Now substituting value of $x = 25$ in eqn. (1)

$h = 25\sqrt{3} \Rightarrow 43.3m.$

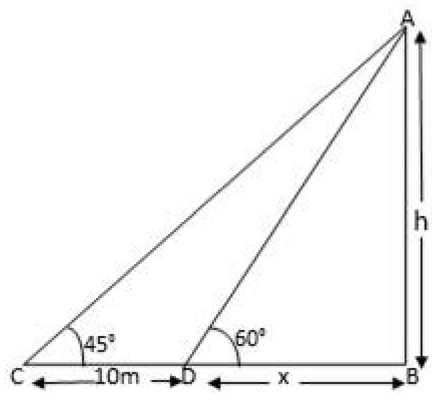
Therefore height of tower is 43.3 m.

11. Question

The shadow of a tower, when the angle of elevation of the sun is 45°, is found to be 10 m longer than when it was 60°. Find the height of the tower.

Answer

Let the height of the tower = h (m)



Let the point of 60° elevation is x (m) away from the foot of the tower.

In ΔABC,

$\tan 45° = \frac{AB}{BC}$

$1 = \frac{AB}{CD+BD}$

$1 = \frac{h}{10+x}$

$h = 10+x$ ----(1)

In ΔABD,

$\tan 60° = \frac{AB}{BD}$

$\sqrt{3} = \frac{h}{x}$

$h = \sqrt{3}\, x$

$x = \frac{h}{\sqrt{3}}$ ---------(2)

$substituting value\ of\ x$ From eqn. (2) in eqn. (1)

$h = 10 + \frac{h}{\sqrt{3}}$

$h - \frac{h}{\sqrt{3}} = 10$

$\frac{\sqrt{3}h-h}{\sqrt{3}} = 10$

$h\left(\sqrt{3}-1\right) = 10\sqrt{3}$

$h = \frac{10\sqrt{3}}{\sqrt{3}-1} \Rightarrow \frac{(10\sqrt{3})\,(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$h= \frac{30+10\sqrt{3}}{3-1} \Rightarrow h= \frac{30+10\sqrt{3}}{2}$

$\Rightarrow 15+5\sqrt{3} \Rightarrow 23.66$ m.

Therefore height of the tower is 23.66 m.

12. Question

A parachutist is descending vertically and makes angles of elevation of 45° and 60° at two observing points 100 m apart from each other on the left side of himself. Find the maximum height from which he falls and the distance of the point where he falls on the ground from the just observation point.

Answer

Let the height of the parachutist = h (m)

Let the distance of falling point from observation point = x (m)

In ΔABC,

$\tan 45° = \frac{AB}{BC}$

$1 = \frac{AB}{CD+BD} \Rightarrow \frac{h}{100+x}$

$h = 100+x$ ----------(1)

In ΔABD,

$\tan 60° = \frac{AB}{BD} \Rightarrow \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

$h = \sqrt{3}\,x$ -----------(2)

From eqn. (1) and eqn. (2) we get,

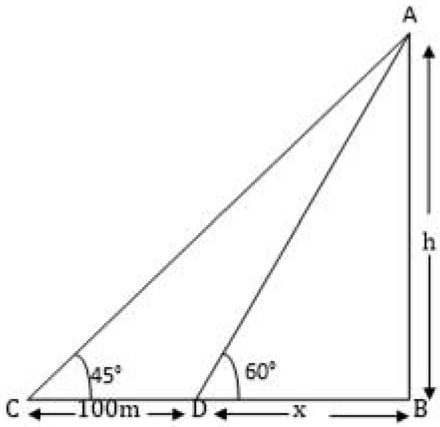
$100+x = \sqrt{3}\,x$

$100 = x\,(\sqrt{3}-1)$

$x = \frac{100}{\sqrt{3}-1} \Rightarrow \frac{100(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \Rightarrow \frac{100\times\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$x = \frac{100\times2.732}{2}$

$x = 136.6$ m.



Now using value of x in eqn. (1)

$h = 100 + x$

h = 100+136.6

h = 236.6 m.

Therefore height of parachutist is 236.6 m. and distance of point where he falls

is 136.6 m.

13. Question

On the same side of a tower, two objects are located. When observed from the top of the tower, their angles of depression are 45° and 60°. If the height of the tower is 150 m, find the distance between the objects.

Answer

Let the distance between the objects = x (m.)

In ΔABC,

tan 45° = $\frac{AB}{BC}$

1 = $\frac{AB}{CD+BD}$ ⇒

1 = $\frac{150}{x+y}$

$x + y$ = 150 ----------(1)

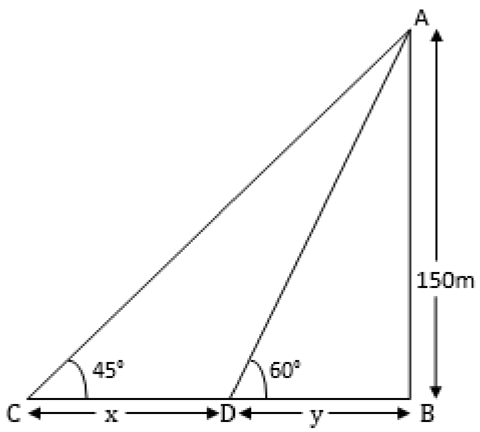
In ΔABD,

$\tan 60° = \frac{AB}{BD}$

$\sqrt{3} = \frac{150}{y}$

$y\sqrt{3} = 150$

$y = 150/\sqrt{3}$ -------(2)



substituting value of y in eqn.(1)

$$\frac{150}{\sqrt{3}} + x = 150$$

$x = 150 - \frac{150}{\sqrt{3}}$

$x = \frac{150\sqrt{3}-150}{\sqrt{3}} \Rightarrow \frac{150(\sqrt{3}-1)}{\sqrt{3}} \Rightarrow \frac{(150\sqrt{3})(\sqrt{3}-1)}{\sqrt{3}\times\sqrt{3}}$

$x = \frac{(150\sqrt{3})(\sqrt{3}-1)}{3}$

$x = 63.4$ m.

Therefore the distance between the points is 63.4 m.

14. Question

The angle of elevation of a tower from a point on the same level as the foot of the tower is 30°. On advancing 150 meters towards the foot of the tower, the angle of elevation of the tower becomes 60°. Show that the height of the tower is 129.9 metres (Use $\sqrt{3} = 1.732$).

Answer

Let the height of the tower = h (m)

In ΔABC,

tan 30° = $\frac{AB}{BC}$

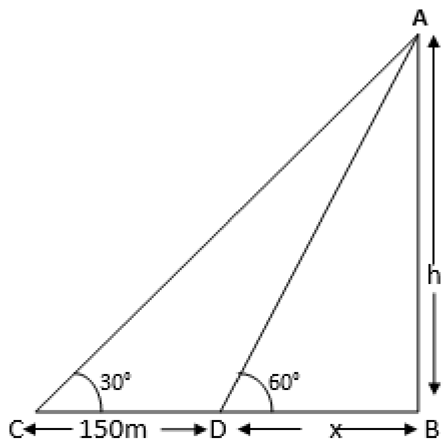
$\frac{1}{\sqrt{3}}$ = $\frac{AB}{CD+BD}$

$\frac{1}{\sqrt{3}}$ = $\frac{h}{150+x}$

$\sqrt{3}$h = 150+ x

$x = \sqrt{3\,h} - 150$ -------- (1)

In ΔABD,



tan 60° = $\frac{AB}{BD}$

$\sqrt{3}$ = $\frac{h}{x}$ ⇒ h = $\sqrt{3}$ x

$x = \frac{h}{\sqrt{3}}$ -------(2)

on substituting value of *x* from eqn.(2)*i* eqn.(1)

$$\frac{h}{\sqrt{3}} = \sqrt{3}\,h - 150$$

$$h = 3h - 150\sqrt{3}$$

h-3h = -150$\sqrt{3}$

2h = 150$\sqrt{3}$

h = $\frac{150\sqrt{3}}{2}$ ⇒ 75$\sqrt{3}$

h= 129.9 m.

Hence height of tower is 129.9 m.

15. Question

The angle of elevation of the top of a tower as observed from a point in a horizontal plane through the foot of the tower is 32°. When the observer moves towards the tower a distance of 100 m, he finds the angle of elevation of the top to be 63°. Find the height of the tower and the distance of the first position from the tower. [Take tan 32° = 0.6248 and tan 63° = 1.9626]

Answer

Let the height of the tower = h (m)

Let the distance of point from the foot of the tower = x (m)

In ΔABC,

tan 32° = $\frac{AB}{BC}$

tan 32° = $\frac{AB}{CD+DB}$

0.6248 = $\frac{h}{100+x}$

h = 0.6248(100+ x) -----------(1)

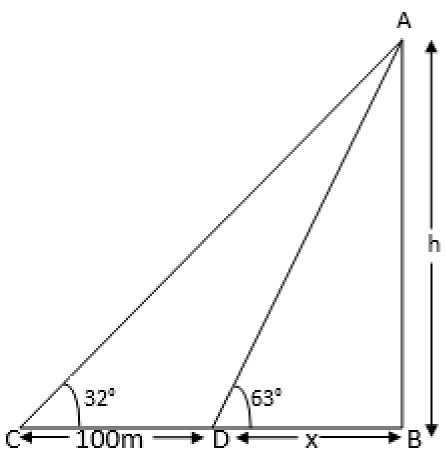
In ΔABD,

tan 63° = $\frac{AB}{BD}$

1.9626 = $\frac{h}{x}$

h = 1.9626 x -----------(2)

Substituting value of h from eqn. (2) in eqn. (1)



1.9626 x= 0.6248(100+ x)

1.9626 x= 62.48+0.6248 x

$1.9626\,x - 0.6248\,x = 62.48$

$1.3378\,x = 62.48$

$$x = \frac{62.48}{1.3378}$$

$x = 46.70$ m.

on substituting value of x in eqn.(2)

$h = 1.9626 \times 46.7$

$h = 91.66$ m.

Distance of the first position from tower=

BC = CD+DB

$BC = 100+46.7$

$BC = 146.7\ m.$

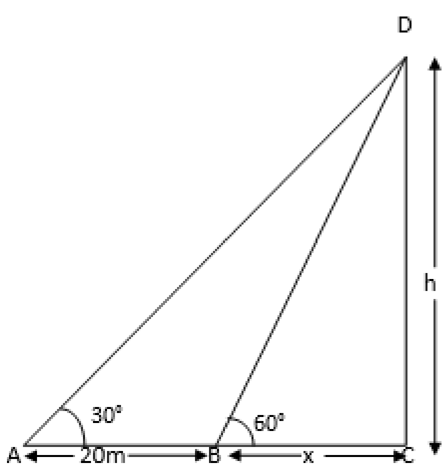
height of tower is 91.66 m.

16. Question

The angle of elevation of the top of a tower from a point A on the ground is 30°. On moving a distance of 20 metres towards the foot of the tower to a point B the angle of elevation increases to 60°. Find the height of the tower and the distance of the tower from the point A.

Answer

Let the height of the tower is = h (m)



Distance of point B from foot of the tower is = x (m)

In ΔADC,

$\tan 30° = \frac{DC}{AC}$

$\frac{1}{\sqrt{3}} = \frac{h}{20+x}$

$\sqrt{3}$ h = 20+ x -----------(1)

In ΔDCB,

$\tan 60° = \frac{DC}{BC}$

$\sqrt{3} = \frac{h}{x}$

h = $\sqrt{3}\,x$ ----------(2)

On substituting value of h from eqn. (2) in eqn. (1)

$\sqrt{3}\times \sqrt{3}\,x = 20 + x$

$3\,x = 20+ x$

$3\,x- x= 20$

$x = 10$

Therefore distance of point A from tower is

AC = AB+BC

AC = 20+10 ⇒ 30

Ac = 30 m.

Now substituting value of x in eqn. (1)

$\sqrt{3}$ h = 20+10 ⇒ 30

$h = \frac{30}{\sqrt{3}}$ ⇒ 17.32 m.

Therefore height of tower is 17.32 m.

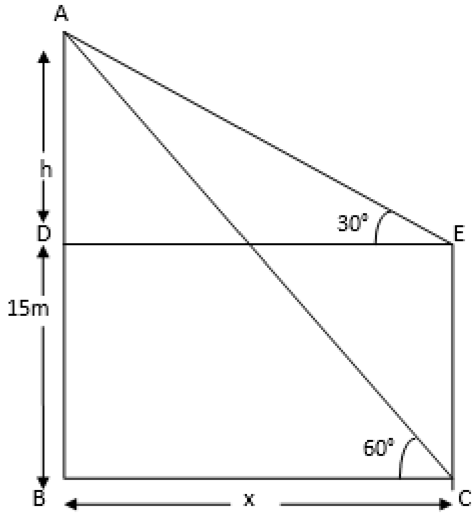
17. Question

From the top of a building 15 m high the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower and the distance between the tower and building.

Answer

Let the distance between tower and building = x (m)

In ΔABC,



tan 60° = $\frac{AB}{BC}$

√3 = $\frac{AD+BD}{BC}$

$\frac{\sqrt{3}}{1} = \frac{h+15}{x}$

√3 x = h+15

h = √3 x-15 ------(1)

In ΔABE,

tan 30° = $\frac{AD}{DE}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$x = \sqrt{3}\,h$ ---------(2)

$Substiuting\ value\ of\, x\ in\ eqn.(1)$

$$h = \sqrt{3}\times\sqrt{3}h - 15$$

h = 3h-15 ⇒ 2h = 15

h = $\frac{15}{2}$ ⇒ 7.5 m.

Height of tower = 15+7.5 ⇒ 22.5 m.

$x = \sqrt{3}\times 7.5$ ⇒ 12.99 m.

Therefore distance between tower and building is 12.99 m.

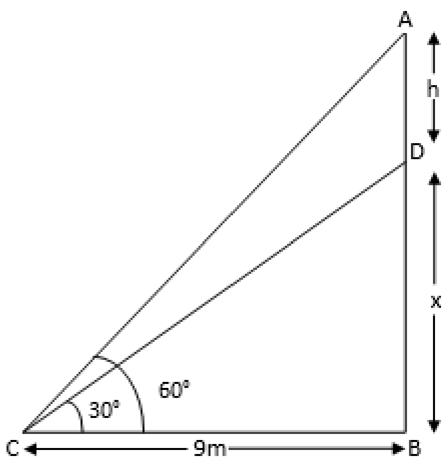
18. Question

On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. At a point 9 metres away from the foot of the tower the angle of elevation of the top and bottom of the flag pole are 60° and 30° respectively. Find the height of the tower and the flag pole mounted on it.

Answer

Let the height of the Flag-pole = h(m)

And height of tower = x (m)



In ΔABC,

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{h+x}{9}$

h+ x = 9√3 -------(1)

In ΔDBC,

tan 30° = $\frac{DB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{x}{9}$

√3 x = 9

$x = \frac{9}{\sqrt{3}} \Rightarrow \frac{9\sqrt{3}}{3}$

$x = 3\sqrt{3}$ -------(2)

Now substituting value of x in eqn. (1)

h+3√3 = 9√3

h = 9√3-3√3

h = 6√3 m.

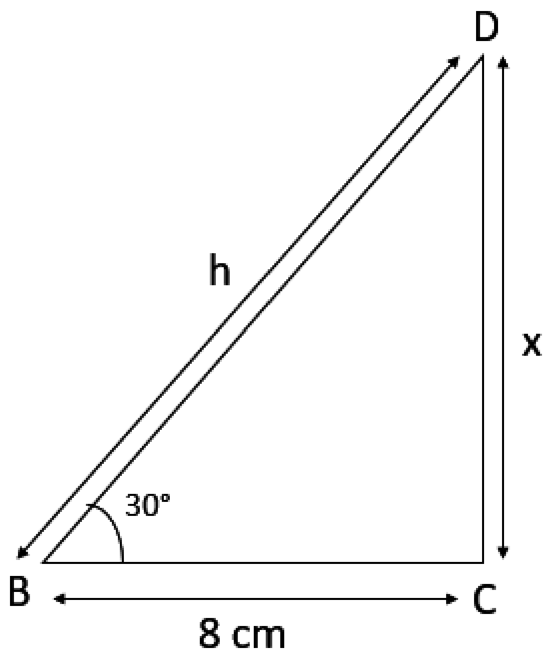
Therefore height of tower is 3√3 m. and height of flag pole is 6√3 m.

19. Question

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Answer

let the broken part be DB.



Distance from the foot of the tree and point C is 8 cm.

BC = 8 cm

Height of tree = Height of broken part + height of the remaining tree= DC + DB

In ΔDBC,

$\tan\theta = \frac{perpendicular}{base}$ $\cos\theta = \frac{base}{hypotenuse}$

tan 30° = $\frac{DB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{x}{8}$

$\sqrt{3}\,x = 8$

$x = \frac{8}{\sqrt{3}}$ ---------(1)

Cos 30° = $\frac{BC}{DC}$

$\frac{\sqrt{3}}{2} = \frac{8}{h}$

$\sqrt{3h} = 16$

h $= \frac{16}{\sqrt{3}}$

Height of tree = $\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = \frac{24}{\sqrt{3}}$

$= \frac{24\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{24\sqrt{3}}{3}$

Height of tree = 8√3 m

20. Question

From a point P on the ground the angle of elevation of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flag-staff from P is 45°. Find the length of the flag-staff and the distance of the building from the point P. (Take $\sqrt{3} = 1.732$).

Answer

Let the height of the flag-staff = h (m)

And the distance of point P from foot of building = x (m)

In ΔAPB,

tan 45° = $\frac{AB}{BP}$

tan 45° = $\frac{h+10}{x}$

1 = = $\frac{h+10}{x}$

h+10 = x -------(1)

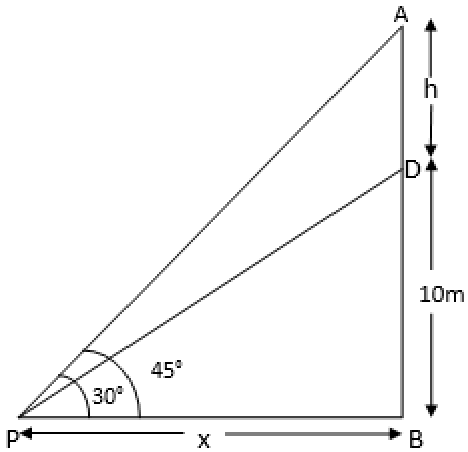
In ΔDPB,

tan 30° = $\frac{DB}{BP}$

$\frac{1}{\sqrt{3}} = \frac{10}{x}$

$x = 10\sqrt{3}$ ⇒17.32 m. -----------(2)

On substituting value of x in eqn. (1)



h = $x - 10$

h = 17.32-10 ⇒ 7.32 m.

Therefore height of flag-staff is 7.32 m. and distance of point P from tower is 17.32 m.

21. Question

A 1.6 m tall girl stands at a distance of 3.2 m from a lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using (i) trigonometric ratios (ii) property of similar triangles.

Answer

Let the height of the lamp post = h (m)

And height of girl is CD = 1.6 m.

Length of shadow is OD = 4.8 m.

In ΔCDO

$\tan\theta = \frac{CD}{OD} \Rightarrow \frac{1.6}{4.8} \Rightarrow \frac{1}{3}$ ------(1)

In ΔABO

$\tan\theta = \frac{AB}{BO}$

$\frac{1}{3} = \frac{x}{(3.2+4.8)}$

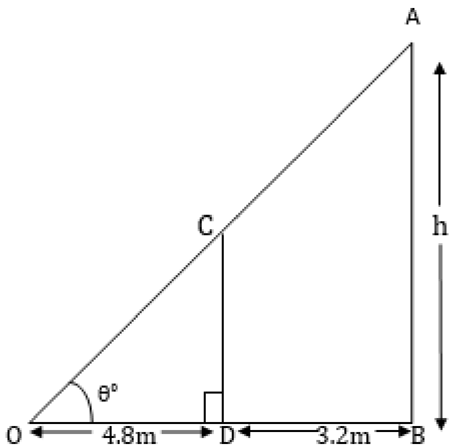
$x = \frac{8}{3}$

$Therefore\ height\ of\ lamp\ post\ is\ \frac{8}{3}\ m.$

By Smilariting

In ΔABO and in ΔCDO

$\angle ABO = \angle CDO = 90°$



$\angle AOB = \angle COD$

$\Delta ABO \sim \Delta CDO$ (AA similarity)

For similar Δs sides are in ratio

Hance,

$$\frac{AB}{CD} = \frac{BO}{DO}$$

$$\frac{x}{1.6} = \frac{8}{4.8}$$

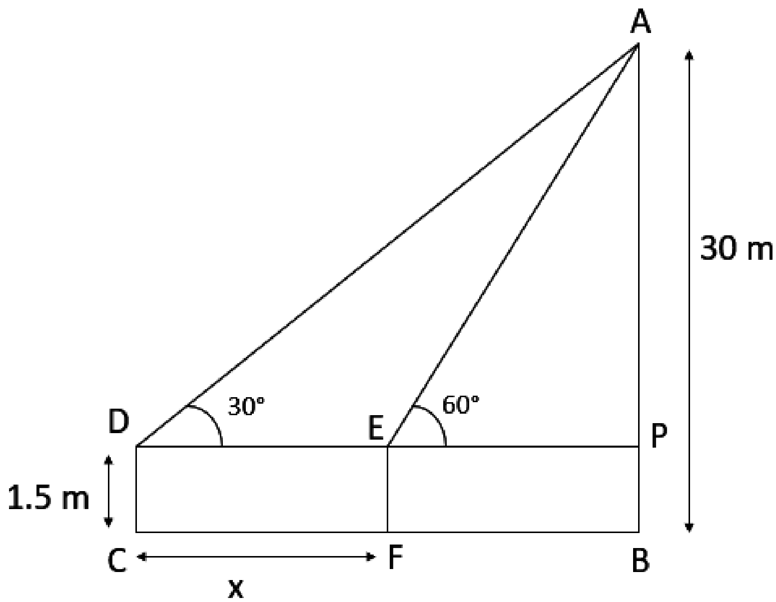
$$x = \frac{8}{3} m.$$

$$Therefore\ height\ of\ lamp\ post\ is \frac{8}{3} m.$$

22. Question

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer



The height of the boy is DC = 1.5 mLet he was at point C initially and then moved to point F.Let CF = x Now DC is parallel to PB.DP is parallel to CB.⇒ DE = CF EP = FBNow,

AP = AB - BP

= 30-1.5 = 28.5 m.

Since the tower is vertical,∠APE = 90°We know, in a right-angled triangle, $\tan\ \theta = \frac{perpendicular}{base}$

In ΔADP,

$$tan\ 30^{\circ} = \frac{AP}{DP} \quad \frac{1}{\sqrt{3}} = \frac{28.5}{DP}$$

DP = 28.5√3 ------------(1)

In ΔAEB,

$$tan\ 60^{\circ} = \frac{AP}{PE} \quad \Rightarrow \sqrt{3} = \frac{AP}{PE}$$

$$\Rightarrow \sqrt{3} = \frac{28.5}{PE}$$

$$\Rightarrow PE = \frac{28.5}{\sqrt{3}} \quad PE = \frac{28.5\sqrt{3}}{3}$$

DE = DP - EP $= 28.5\sqrt{3} - \frac{28.5\sqrt{3}}{3}$

$$= 28.5\sqrt{3}\left(1 - \frac{1}{3}\right)$$

$$= 28.5\sqrt{3}\left(\frac{2}{3}\right) \quad = \frac{57\sqrt{3}}{3}$$

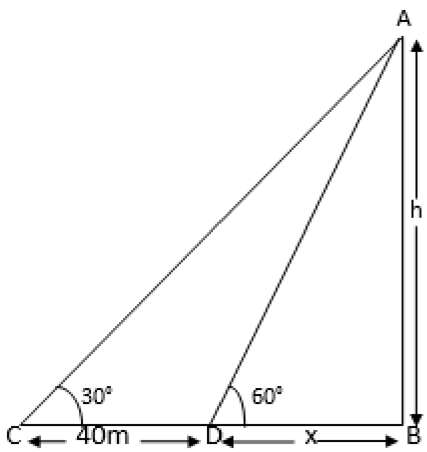
= 19√3 m.

Therefore the walking distance of boy is 19√3 m.

23. Question

The shadow of a tower standing on level ground is found to be 40 m longer when Sun's altitude is 30° than when it was 60°. Find the height of the tower.

Answer



Let the height of tower = h (m)Since the tower is vertical to the ground.∠ABC = 90°We know, in a right-angle triangle, $\tan\,\theta = \frac{perpendicular}{base}$

In ΔABC,

$$\tan\,30^\circ = \frac{AB}{BC}$$

$\frac{1}{\sqrt{3}} = \frac{AB}{CD+BD}$

$\frac{1}{\sqrt{3}} = \frac{h}{40+x}$

√3h = 40+ x ---------------(1)

In ΔABD,

$$tan\,60^\circ = \frac{AB}{BD}$$

√3 = $\frac{h}{x}$

h =√3 x -------(2)

on substituting the value of h from eqn. (2) in eqn. (1)

$h = \sqrt{3} \times (\sqrt{3}h - 40)$

$h = 3h - 40\sqrt{3}$

$2h = 40\sqrt{3}$

$h = 20\sqrt{3}$ m.

Therefore the height of the tower is 20√3 m.

24. Question

From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of 20 m high building are 45° and 60° respectively. Find the height of the transmission tower.

Answer

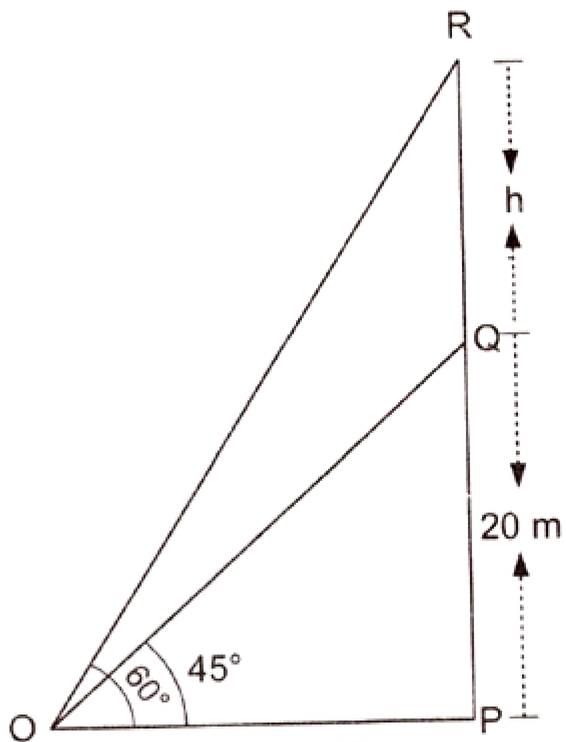


Fig. 12.48

Since the building is vertical. $\angle QPO = 90°$

In a right-angled triangle, we know, $\tan\theta = \frac{perpendicular}{base}$

In ΔOPQ,

$\tan 45° = \frac{QP}{OP}$

$1 = \frac{20}{OP}$

OP = 20 -------(1)

Now in ΔOPR

tan 60° = $\frac{PR}{OP}$

$\sqrt{3} = \frac{RQ+QP}{OP}$

$\sqrt{3} = \frac{h+20}{20}$

20√3 = h+20

h = 20√3-20

h = 20(√3-1) m.

Therefore the height of transmission tower is 20(√3-1) m.

25. Question

The angles of depression of the top and bottom of 8 m tall building from the top of a multistoried building are 30° and 45° respectively. Find the height of the multistoried building and the distance between the two buildings.

Answer

Let DC is tall building and AB is multistoried building.

AB = AE+EB

AB = h+8 ----------(1)

In ΔABC,

tan 45° = $\frac{AB}{BC}$

1 = $\frac{h+8}{x}$

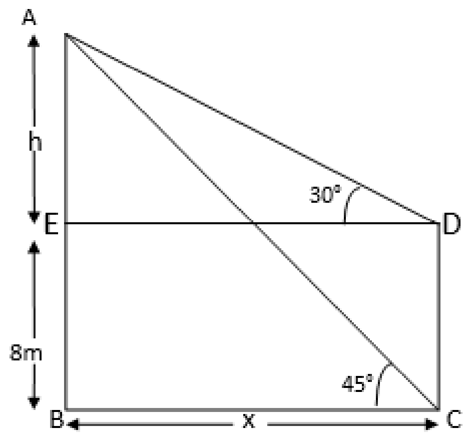
$x = h + 8$ ----------(2)

In ΔAED,

tan 30° = $\frac{AE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

√3h = x ---------(3)



Substituting value of x from eqn. (2) in eqn. (1)

$$\sqrt{3}h = h + 8$$

$\sqrt{3}h - h = 8$

h = $\frac{8}{\sqrt{3}-1}$

h = $\frac{8\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} \Rightarrow \frac{8(\sqrt{3}+1)}{2}$

h = 4(√3+1)m. --------(4)

Substituting value of h from eqn. (4) in eqn. (3)

√3h = x

$x =$ √3× 4(√3+1)

$x =$ √3 (4√3+4)

$$x = 12 + 4\sqrt{3}$$

$$x = 4(3 + \sqrt{3})m.$$

Therefore height of multistoried building is

= 8+4(√3+1)

= 8+4√3+4

= 12+4√3

= 4(3+√3) m.

Distance between two building is 4(3+√3) m.

26. Question

A statue 1.6 m tall stands on the top of pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Answer

Let AD is statue of height 1.6 m. and BD is pedestal of height h (m).

Let the distance between point of elevation and foot of pedestal is x (m).

In ΔDBC,

tan 45° = $\frac{BD}{BC}$

1 = $\frac{h}{x}$

h = x ----------(1)

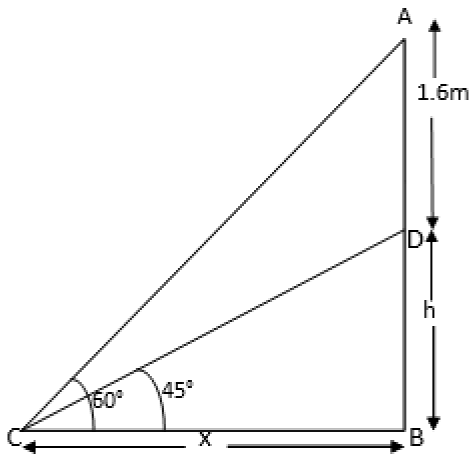
In ΔABC,

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{AD+BD}{BC}$

√3 = $\frac{1.6+h}{x}$

√3 x = 1.6+h -----------(2)



On substituting value of x from eqn. (1) in eqn. (2)

√3h = 1.6+h

√3h – h = 1.6

h = $\frac{1.6}{\sqrt{3}-1}$

on rationalizing we get.

$h = \frac{1.6(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}) \Rightarrow \frac{1.6(\sqrt{3}+1)}{3-1} \Rightarrow \frac{1.6(\sqrt{3}+1)}{2}$

$0.8(\sqrt{3}+1) = \frac{8(\sqrt{3}+1)}{10} = \frac{4(\sqrt{3}+1)}{5}$ m.

Therefore height of pedestal is $\frac{4(\sqrt{3}+1)}{5}$ m.

27. Question

A T.V. tower stands vertically on a bank of a river. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From a point 20 m away this point on the same bank, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the river.

Answer

Let BC be the height of the T.V tower and AB be the width of the river.

In ΔABC,

$\tan 60° = \frac{BC}{AB}$

$\sqrt{3} = \frac{h}{x}$ ------(1)

In ΔABC,

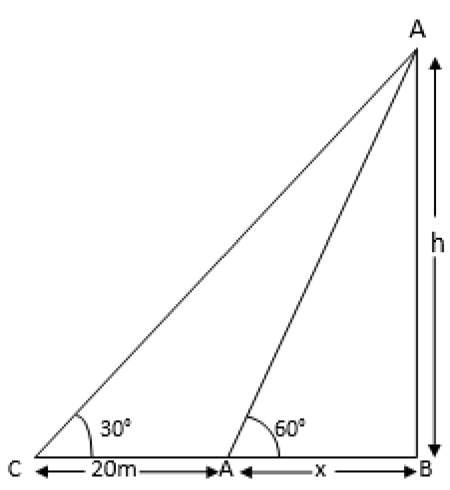
$\tan 30° = \frac{BC}{BD}$

$\tan 30° = \frac{BC}{DA+AB}$

$\frac{1}{\sqrt{3}} = \frac{h}{20+x}$

$\sqrt{3}h = 20+ x$ ---------(2)

On substituting value of h in eqn.(2)



$\sqrt{3}\times \sqrt{3}\,x$ = 20+ x

$3\,x = 20 + x$

$3\,x - x = 20$

$2\,x = 20$

$x = 10\,m.$

On substituting value of x in eqn. (1)

h = $\sqrt{3}\,x$

h = $10\sqrt{3}$ m.

Therefore height of T.V tower is $10\sqrt{3}$ m. and width of river is 10 m.

28. Question

From the top a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Answer

Let the distance between the foots of building and cable tower is x (m).

The height of cable tower = AB = AE+EB ⇒ (h+7)m.

In ΔAED,

tan 60° = $\frac{AE}{DE}$

$\sqrt{3}$ = $\frac{h}{x}$

h = $\sqrt{3}\,x$ ---------(1)

The height of cable tower = AB = AE+EB ⇒ (h+7)m.

In ΔDEB,

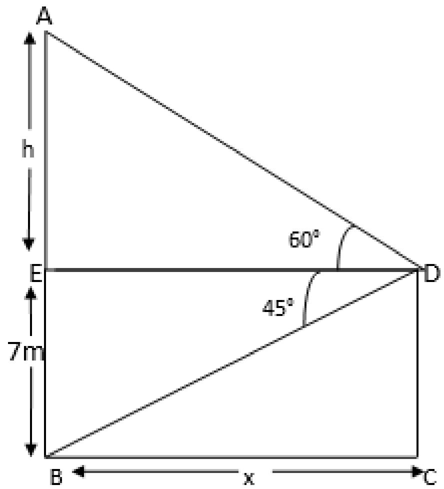
tan 45° = $\frac{BE}{DE}$

1 = $\frac{7}{x}$

$x = 7$ ----------(2)

On substituting value of x in eqn. (1)

h = $7\sqrt{3}$



Height of cable tower is (h+7)m.

⇒ $7\sqrt{3}+7$

⇒ $7(\sqrt{3}+1)$m.

Therefore height of cable tower is $7(\sqrt{3}+1)$m.

29. Question

As observed from the top of a 75 m tall lighthouse, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer

Let the distance between the two ships be x (m.)

And distance between the ship and foot of light house is y (m.)

In ΔABD

tan 45° = $\frac{AB}{BD}$

1 = $\frac{AB}{BD}$

AB = BD

75 = y

$y = 75\ m.$ --------(1)

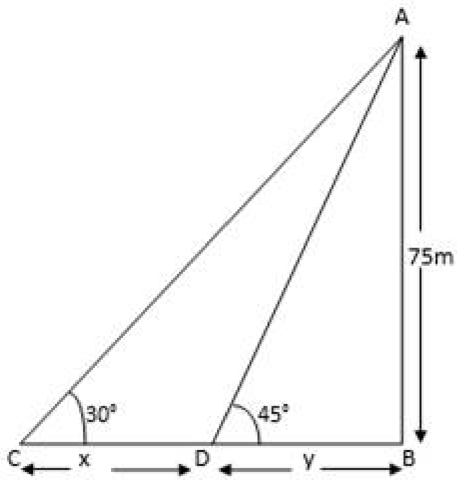
In ΔABC

tan 30° = $\frac{AB}{BC}$

tan 30° = $\frac{AB}{BD+DC}$

$$\frac{1}{\sqrt{3}} = \frac{75}{y + x}$$

$y + x$ = 75√3 --------(2)



On substituting value of y in eqn. (2)

$$x = 75\sqrt{3} - 75$$

$$x = 75\left(\sqrt{3} - 1\right)m.$$

Therefore distance between the two ships is $75\left(\sqrt{3} - 1\right)m.$

30. Question

The angle of elevation of the top of the building from the foot of the tower is 30° and the angle of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Answer

Let AB be the building of height 50 m. and tower of height h (m.)

In ΔABC

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{50}{x}$

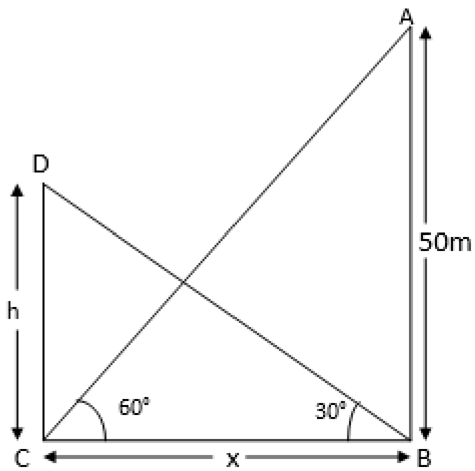
$x = \frac{50}{\sqrt{3}}$ ------(1)

Now in ΔDCB

tan 30° = $\frac{DC}{BC}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$x = \sqrt{3}h$ -----(2)



On substituting value of x in eqn. (1)

$$x = \frac{50}{\sqrt{3}}$$

$\sqrt{3}h = \frac{50}{\sqrt{3}}$

$h = \frac{50}{\sqrt{3} \times \sqrt{3}}$

$h = \frac{50}{3}$

Therefore height of tower is $\frac{50}{3}$ m.

31. Question

From a point on a bridge across a river the angles of depression of the banks on opposite side of the river are 30° and 45° respectively. If bridge is at the height of 30m from the banks, find the width of the river.

Answer

Let A and B be the points on the bank n opposite sides f the river and BC be the width of the river.

BC = BD+DC ⇒ $(x + y)$m.

AD be the height of the bridge.

In ΔADC

tan 45° = $\frac{AD}{DC}$

$1 = \frac{30}{y}$

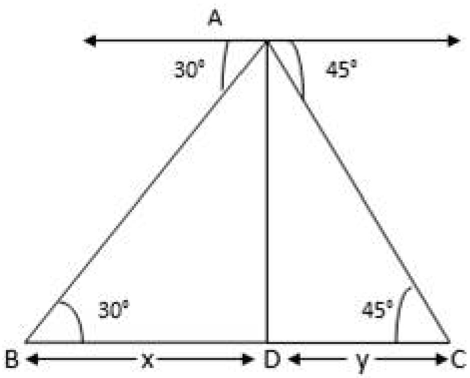
$y = 30\ m.$ ---------(1)

In ΔADB

tan 30° = $\frac{AD}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{30}{x}$$

$x = 30\sqrt{3}$ ------------(2)



Width of the river = ($x + y$)

⇒ 30√3+30

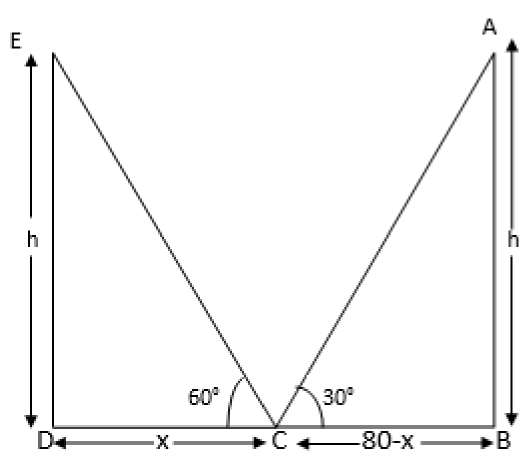
⇒ 30(√3+1)m.

32. Question

Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point between them on the road the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Answer

Let AB and ED are two poles of equal height.



Let C be the point between the poles on the ground.

Since poles are vertical to the ground.∠ADC = ∠ABC = 90°In a right-angled triangle, we know,

$$\tan\theta = \frac{perpendicular}{base}$$

In ΔEDC

tan 60° = $\frac{ED}{DC}$

$$\sqrt{3} = \frac{h}{x}$$

h = √3 x --------(1)

In ΔABC

tan 30° = $\frac{AB}{BC}$

$$\frac{1}{\sqrt{3}} = \frac{h}{80 - x}$$

√3h = 80-x----------(2)

On substituting value of h from eqn.(1) in eqn. (2)

√3 × √3x = 80 - x

⇒ 3x = 80 - x

⇒ 4x = 80

⇒ x = 20 m

On substituting value of x in eqn. (1)

h = 20√3Distance of C from pole ED = 20 mDistance of C from pole AB = 80 - 20= 60 m

Therefore the height of the poles is 20√3 m. and distances of the points from one pole is 20 m and from other pole is 60 m.

33. Question

A man sitting at a height of 20 m on a tall tree on a small island in the middle of a river observes two poles directly opposite to each other on the two banks of the river and in line with the foot of tree. If the angles of depression of the feet of the poles from a point at which the man is sitting on the tree on either side of the river are 60° and 30° respectively. Find the width of the river.

Answer

Let width of the river be DC= DB+BC ⇒ $(x + y)$ m.

In ΔABC

$\tan 60° = \frac{AB}{BC}$

$$\sqrt{3} = \frac{20}{y}$$

$20 = \sqrt{3}\, y$

$y = \frac{20}{\sqrt{3}}$ -----(1)

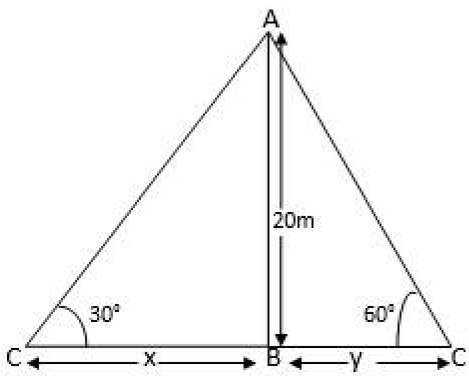
Now in ΔABD

$\tan 30° = \frac{AB}{DB}$

$$\frac{1}{\sqrt{3}} = \frac{20}{x}$$

$$x = \frac{20}{x}$$

Therefore width of river = $(x + y)$ m.



$\Rightarrow 20\sqrt{3} + \frac{20}{\sqrt{3}}$

$\Rightarrow \frac{20\sqrt{3}+\sqrt{3}+20}{\sqrt{3}} \Rightarrow \frac{60+20}{\sqrt{3}} \Rightarrow \frac{80}{\sqrt{3}}$ m.

Width of river is $\frac{80}{\sqrt{3}}$ m.

34. Question

A vertical tower stands on a horizontal plane and is surmounted by a flag-staff of height 7 m. From a point on the plane, the angle of elevation of the bottom of the flag-staff is 30° and that of the top of the flag-staff is 45°. Find the height of the tower.

Answer

Let the height of tower is BD = h (m.)

Now in ΔABC

$\tan 45° = \frac{AB}{BC} \Rightarrow \frac{AD+BD}{BC}$

$1 = \frac{7+h}{x}$

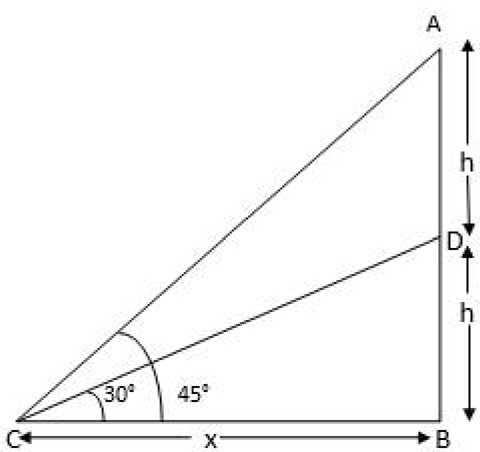
$7+h = x$ ------(1)

Now in ΔDBC

$\tan 30° = \frac{BD}{BC}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$x = \sqrt{3}h$ ----------(2)



On substituting value of x in eqn.(1)

$7+h = \sqrt{3}h$

$\sqrt{3}h - h = 7$

$h(\sqrt{3}-1) = 7$

$h = \frac{7}{\sqrt{3}-1} \Rightarrow \frac{7}{1.732-1}$

$\frac{7}{0.732} = 9.56m.$

Therefore height of tower is 9.56m.

35. Question

The length of the shadow of a tower standing on level plane is found to be 2x metres longer when the sun’s altitude is 30° than when it was 45°. Prove that the height of tower is $x\left(\sqrt{3}+1\right)$ metres.

Answer

Let the height of tower is AB = h (m.)

Now in ΔABD

tan 45° = $\frac{AB}{BD}$

1 = $\frac{h}{y}$

h = y ------(1)

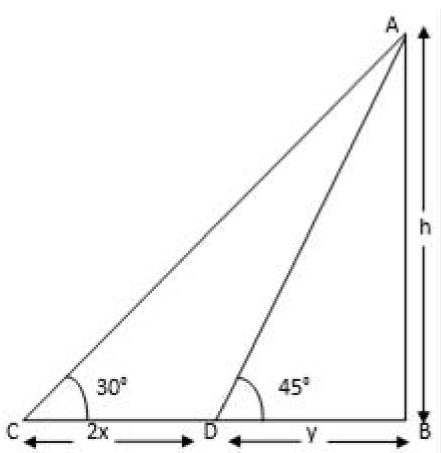
Now in ΔABC

tan 30° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BD+DC}$

$\frac{1}{\sqrt{3}} = \frac{h}{y+2x}$

√3h = 2 $x + y$ -------(2)



On substituting value of y from eqn. (1) in eqn. (2)

√3h = 2 $x + y$

√3h = 2 $x + h$

√3h-h = 2 x

h (√3-1) = 2 x

h = $\frac{2x}{\sqrt{3}-1}$

on rationalsing above fraction we get,

h = $\frac{2x(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

h = $\frac{2x(\sqrt{3}+1)}{2}$

$x = (\sqrt{3}+1)$

Therefore height of tower is $(\sqrt{3}+1)$ m.

36. Question

A tree breaks due to the storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 metres. Find the height of the tree.

Answer

In fig.BD is the height of the tree. Let the broken part touches the ground at point C

BD = AB+AD

in ΔABC

tan 30° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{h}{10}$

√3h = 10

h = $\frac{10}{\sqrt{3}}$

h = $\frac{10\sqrt{3}}{3}$ m. -------(1)

Again ΔABC.

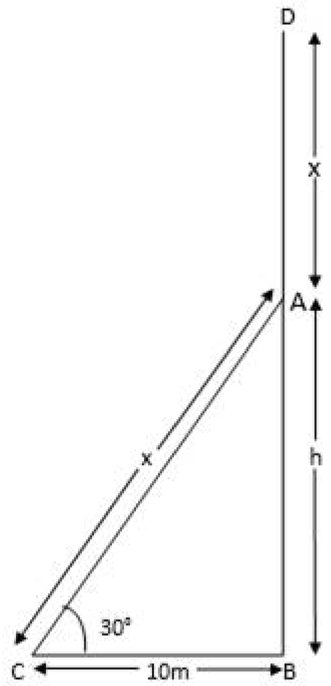
COS 30° = $\frac{BC}{AC}$

$$\frac{\sqrt{3}}{2} = \frac{10}{x}$$

√3 $x = 20$

$x = \frac{20}{\sqrt{3}}$

$x = \frac{20\sqrt{3}}{3}$ ------(2)



Therefore height of tree = $(x + h)$ m.

Adding eqn. (1) and (2) we get,

$\frac{10\sqrt{3}}{3} + \frac{20\sqrt{3}}{3}$

$\frac{10\sqrt{3}+20\sqrt{3}}{3} = \frac{30\sqrt{3}}{3} \Rightarrow 10\sqrt{3}$

$10\times 1.732 \Rightarrow 17.32$ m.

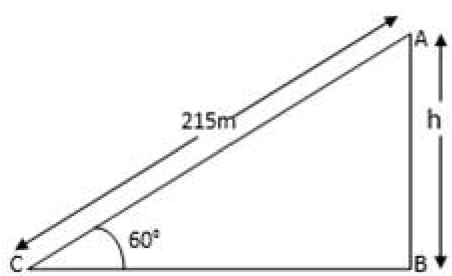
Therefore height of tree is 17.32 m.

37. Question

A balloon is connected to a meteorological ground station by a cable of length 215 m inclined at 60° to the horizontal. Determine the height of the balloon from the ground. Assume that there is no slack in the cable.

Answer

In the fig. Let AB be the height of the balloon.



In ΔABC

$\sin 60° = \frac{AB}{AC}$

$\frac{\sqrt{3}}{2} = \frac{h}{215}$

2h = 215√3

h = $\frac{215\sqrt{3}}{2}$ ⇒ 186.19 m.

Therefore height of balloon from the ground is 186.19 m.

38. Question

Two men on either side of the cliff 80 m high observes the angles of elevation of the top of the cliff to be 30° and 60° respectively. Find the distance between the two men.

Answer

In fig. AB be the height of the cliff.

Let the distance between the two men is DC.

DC = $(x + y)$ m.

In ΔABC

$\tan 60° = \frac{AB}{BC}$

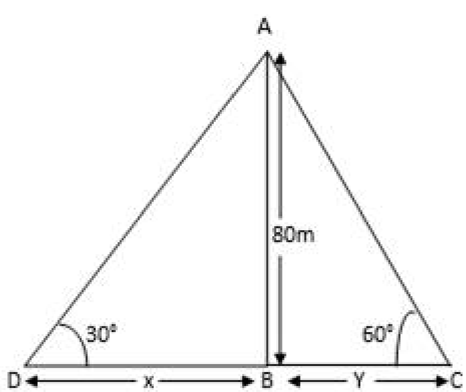
$\sqrt{3} = \frac{80}{y}$

$\sqrt{3}\,y = 80$

$$y = \frac{80}{\sqrt{3}}$$

$y = \frac{80\sqrt{3}}{3}$ ------(1)

In ΔABD



tan 30° = $\frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{80}{x}$$

$x = 80\sqrt{3}$ --------(2)

Adding eqn. (1) and (2) we get,

$(x + y)$ = $80\sqrt{3} + \frac{80\sqrt{3}}{3}$

$\frac{3\times80\sqrt{3}+80\sqrt{3}}{3} \Rightarrow \frac{240\sqrt{3}+80\sqrt{3}}{3} \Rightarrow \frac{320\sqrt{3}}{3} \Rightarrow$ 184.75 m.

Therefore distance between two men is 184.75 m.

39. Question

Find the angle of elevation of the sun (sun’s altitude) when the length of the shadow of a vertical pole is equal to its height.

Answer

Let the position of the sun be at O and AB be the height of the pole.

Now BC is the shadow cast by the pole. From point C, the angle of elevation of the top of the pole (point A) and the sun would be the same.

Since length of shadow is equal to the height of the vertical pole.

Therefore AB = BC

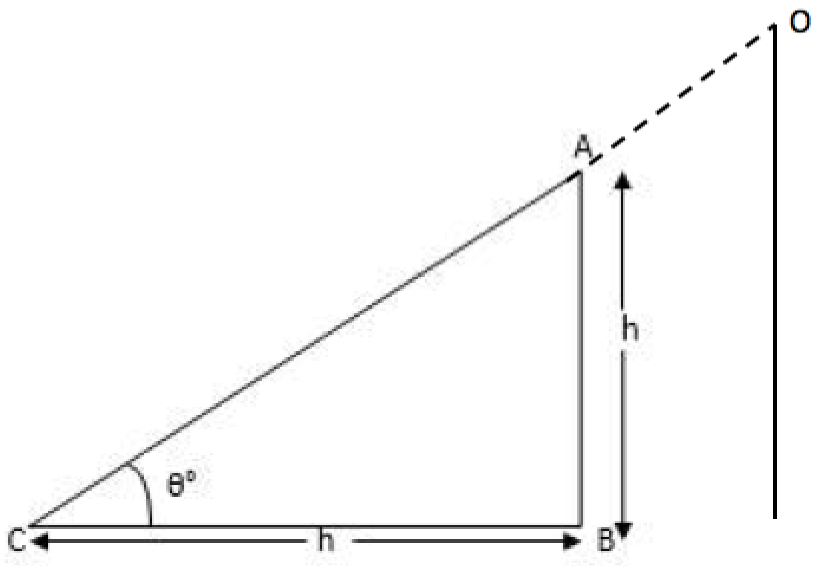
In ΔABD

tan θ = $\frac{AB}{BC}$

tan θ = $\frac{h}{h} \Rightarrow 1$

θ = tan 1 (since tan 45°= 1)

θ = 45°



Therefore angle of elevation is 45°

40. Question

A fire in a building B is reported on teleported on telephone to two fire stations P and Q, 20 km apart from each other on a straight road. P observes that the fire is at an angle of 60° to the road and Q observes that it is at an angle of 45° to the road. Which station should send its team and how much will this team have to travel?

Answer

Let the height of building be 'h'.Let the distance between P and foot of building is 'x' metres.

In ΔPRS

tan 60° = $\frac{PS}{PR}$

√3 = $\frac{h}{x}$

h = √3x ---------(1)

In ΔQRS

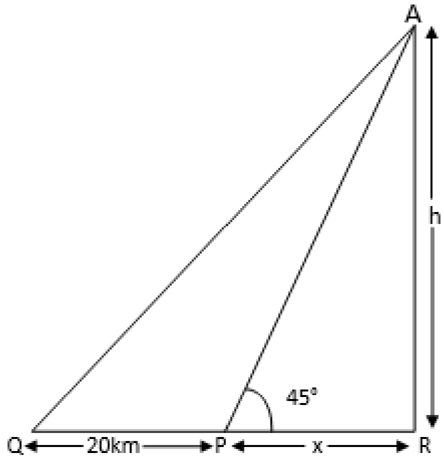
tan 45° = $\frac{RS}{QR}$

tan 45° = $\frac{h}{20+x}$

1 = $\frac{h}{20+x}$

h = 20+ x ---------(2)

on substituting the value of h from eqn. (2) in eqn. (1)



$20 + x = \sqrt{3}\,x$

$\sqrt{3}\,x - x = 20$

$$x = \frac{20}{\sqrt{3} - 1}$$

On rationalising above fraction we get,

$x = \frac{20 \times \sqrt{3} + 1}{(\sqrt{3} - 1)(\sqrt{3} + 1)} \Rightarrow \frac{20 \times \sqrt{3} + 1}{2} \Rightarrow 10(\sqrt{3}+1)$

x = 10(1.732+1)

$x = 10 \times 2.732$

$x = 27.32\ m.$

Therefore Station P has to send the team. And the distance between station P and the building is 27.32 m.

41. Question

A man on the deck of a ship is 10 m above the water level. He observes that the angle of elevation of the top of a cliff is 45° and the angle of depression of the base is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

Answer

In the fig. AB is the height of the cliff.

AB = AE+EB ⇒ h+10

In ΔAED

tan 45° = $\frac{AE}{DE}$

1 = $\frac{h}{DE}$

DE = h -----(1)

In ΔDEB

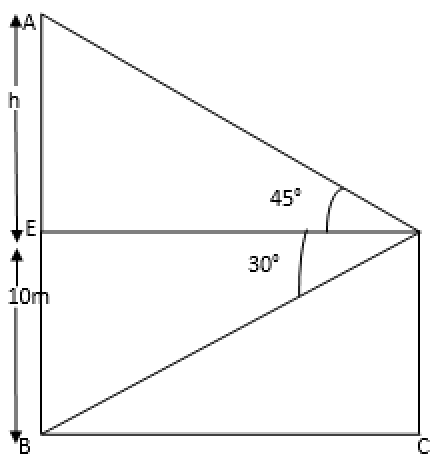
tan 30° = $\frac{EB}{DE}$

$$\frac{1}{\sqrt{3}} = \frac{10}{DE}$$

DE = 10√3 -----(2)

From eqn. (1) and eqn. (2) we get,

h = 10√3



Height of cliff = AE+EB

= 10√3+10

= 27.32 m.

Therefore distance of the cliff from the ship is 10√3 and Height of cliff is 27.32m.

42. Question

A man standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.

Answer

In the fig. AB is the height of hill AB = AE+EB ⇒ h+8

In ΔAEB

tan 60° = $\frac{AE}{DE}$

√3 = $\frac{h}{DE}$

h = √3 DE ------- (1)

In ΔDEB

$\tan 30° = \frac{EB}{DE}$

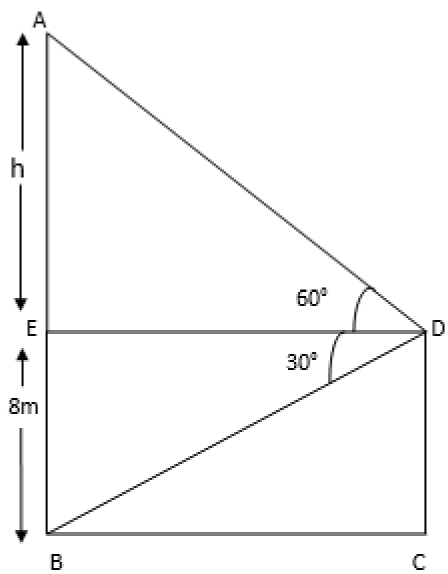
$$\frac{1}{\sqrt{3}} = \frac{8}{DE}$$

DE = $8\sqrt{3}$ ------(2)

From eqn. (1) and eqn. (2) we get,

h = $\sqrt{3} \times 8\sqrt{3} \Rightarrow 24$ m.

Height of hill = $24+8 \Rightarrow 32$ m.



On substitution value of h in eqn. (1)

$24 = \sqrt{3}$ DE

DE = $\frac{24}{\sqrt{3}}$

DE = $\frac{24\sqrt{3}}{\sqrt{3}\times\sqrt{3}} \Rightarrow \frac{24\sqrt{3}}{3}$

DE = $8\sqrt{3}$ m.

Therefore distance between ship and hill is $8\sqrt{3}$ m and height of the hill is 32 m.

43. Question

There are two temples, one on each bank of a river, just opposite to each other. One temple is 50 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.

Answer

In fig let the height of the other temple is h (m.) and distance between two temple is x (m.)

In ΔABC

$\tan 60° = \frac{AB}{BC}$

$\tan 60° = \frac{AB+EB}{BC}$

$\sqrt{3} = \frac{50}{x}$

$x = \frac{50}{\sqrt{3}} \Rightarrow \frac{50\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

$x = \frac{50\sqrt{}}{3}$ (1)

In ΔAED

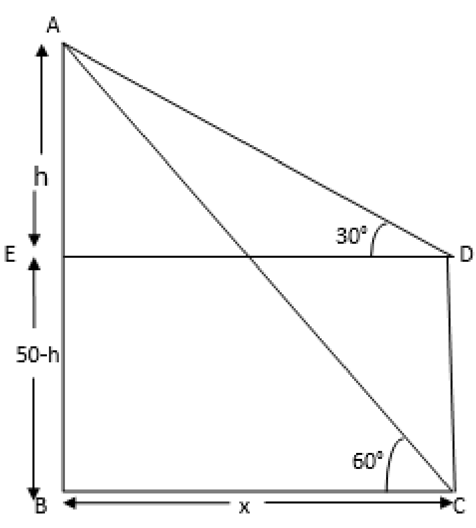
$\tan 30° = \frac{AE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$

$X = h\sqrt{3}$ (2)

From eqn. (1) and eqn. (2) we get,

$\sqrt{3}h = \frac{50\sqrt{3}}{3}$



$h = \frac{50}{3} = 16.67$m

On substituting the value of ‘h’ in eqn (2)

$X = \frac{50\sqrt{3}}{3} \Rightarrow 28.87$m

Therefore height of the temple is 50-16.67 = 33.33m

And the distance between the two temples is 28.87m

44. Question

The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 seconds, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.

Answer

In the fig let C be the initial position of the aeroplane. After 15 seconds the position of the aeroplane becomes E.

In ΔABC

$\tan 45° = \frac{BC}{AB}$

$1 = \frac{3000}{y}$

Y = 3000m ------(2)

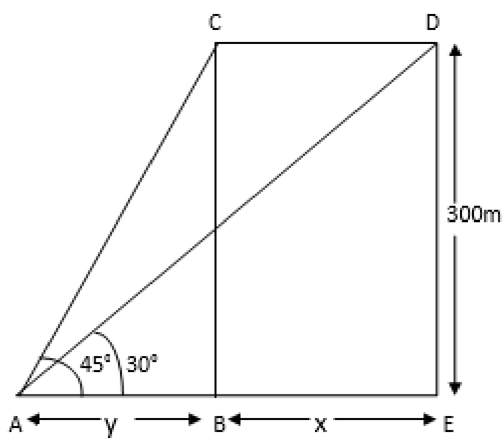
In ΔADE

$\tan 30° = \frac{DE}{AD}$

$\frac{1}{\sqrt{3}} = \frac{3000}{x+y}$

$x + y = 3000\sqrt{3}$

Using equation (1) to replace value of y, we get



$x = 3000\sqrt{3} - 3000$

$\Rightarrow 3000(\sqrt{3}-1)$

⇒ 2196m

Since the distance travelled by aeroplane in 15 seconds is 2196m. Therefore distance travelled by aeroplane in 1 hour =

$\frac{2196 \times 3600}{15 \times 1000} \Rightarrow 527.04$ km/hr

Therefore speed of aeroplane is 527.04 km/hr

45. Question

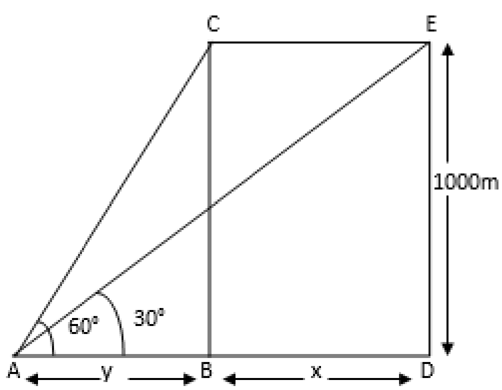
An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the aeroplane in km / hr.

Answer

Given: An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°.

To find: the speed of the aeroplane in km / hr.

Solution: Draw the figure according to given information



In the fig let C be the initial position of the aeroplane. After 10 seconds the position of the aeroplane becomes E.

As $\tan\theta = \frac{perpendicular}{base}$

In ΔABC

tan 60° = $\frac{BC}{AB}$

we know tan 60° = √3

⇒ √3 = $\frac{1000}{y}$

It is given it is flying horizontally 1 km above the ground.

⇒ √3y = 1000 m

⇒ y = $\frac{1000}{\sqrt{3}}$ -------(1)

In ΔADE

tan 30° = $\frac{DE}{AD}$

we know tan 30° = $\frac{1}{\sqrt{3}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1000}{x+y}$

$\Rightarrow$ x + y = 1000√3

On substituting value of y from eqn (1)

$x + \frac{1000}{\sqrt{3}} = 1000\sqrt{3}$

$x = 1000\sqrt{3} - \frac{1000}{\sqrt{3}}$

$\Rightarrow x = \frac{1000\sqrt{3}\times\sqrt{3}-1000}{\sqrt{3}}$

$\Rightarrow x = \frac{3000-1000}{\sqrt{3}}$

$\Rightarrow x = \frac{2000}{\sqrt{3}} \Rightarrow x = 1154.7\text{m}$

Since the distance travelled by aeroplane in 10 seconds is 1154.7 m. As 1 hr = 3600 sec and 1 km = 1000 mTherefore distance travelled by aeroplane in 1 hour $= \frac{1154.7\times3600}{10\times1000} \Rightarrow$ 415.69 km/hr

Therefore speed of aeroplane is 415.69 km/hr

46. Question

From the top of a 50 m high tower, the angles of depression of the top and bottom of a pole are observed to be 45° and 60° respectively. Find the height of the pole.

Answer

In the fig CE is the height of the pole and x be the distance between tower and pole.

In ΔADE

$\tan 45° = \frac{AD}{DE}$

$1 = \frac{50-h}{x}$

x = 50-h ----- (1)

In ΔABC

$\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{50}{x}$

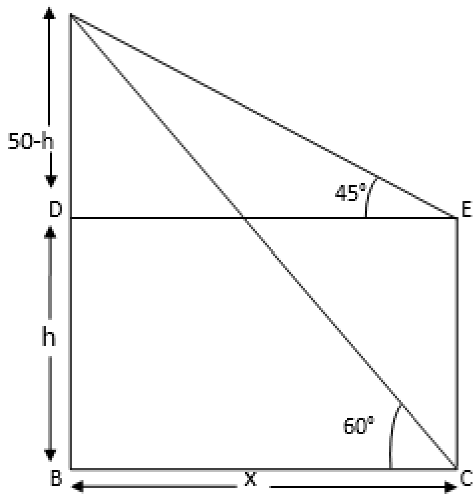
$x = \frac{50}{\sqrt{3}}$ -----(2)

On substituting value of x in eqn (1), we get

$50 - h = \frac{50}{\sqrt{3}}$

$h = 50 - \frac{50}{\sqrt{3}} \Rightarrow \frac{50(\sqrt{3}-1)}{\sqrt{3}}$

$\Rightarrow$ 21.13m



Therefore the height of the pole is 21.13m

47. Question

The horizontal distance between two trees of different heights is 60 m. The angle of depression of the top of the first tree when seen from the top of the second tree is 45°. If the height of the second tree is 80 m, find the height of the first tree.

Answer

In the fig let CE be the height of the first tree

and AB is the height of the second tree.

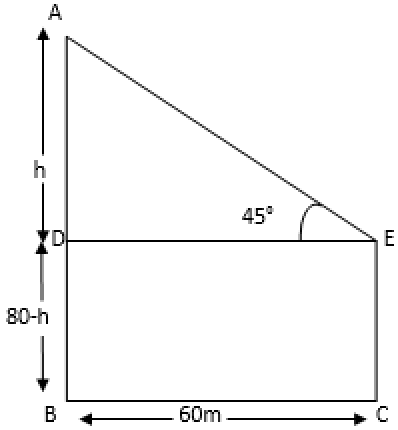
In ΔADE

$\tan 45° = \frac{AD}{DE}$

$1 = \frac{h}{60}$

h = 60m

Therefore height of the first tree is 80-60 = 20m

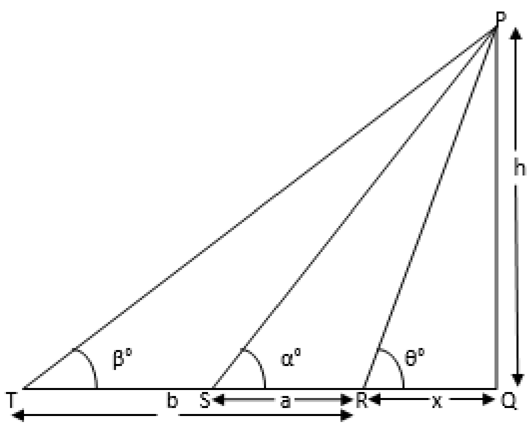


48. Question

A tree standing on a horizontal plane is leaning towards east. At two points situated at distances a and b exactly due west on it, the angles of elevation of the top are respectively α and β. Prove that the height of the top from the ground is $\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

Answer

In the fig let RP be the leaning tree, R & S be the two points at distance 'a' and 'b' from point Q.



In ΔPQR

$\tan\theta° = \frac{PQ}{QR}$

$\tan\theta° = \frac{h}{x}$

$x = \frac{h}{tan\theta}$ ....(1)

In ΔPQS

$\tan\alpha = \frac{PQ}{QS} \Rightarrow \frac{PQ}{QR+RS}$

$\tan\alpha = \frac{h}{x+a}$ ....(2)

In ΔPQT

tan α = $\frac{PQ}{QS} \Rightarrow \frac{PQ}{QR+RS}$

tan β = $\frac{PQ}{PT} \Rightarrow \frac{PQ}{QR+RS+ST}$

tan β = $\frac{h}{x+b}$ ....(3)

On substituting value of x from eqn (1) in eqn (2) we get,

tan α = $\frac{h \tan\theta}{h+a \tan\theta}$

h tan α + a tan θ tan α = h tan θ

h tan α = tan θ(h-a tan α)

tan θ = $\frac{h \tan\alpha}{h- a \tan\alpha}$ ....(4)

Now on substituting value of x in eqn (3)

tan β = $\frac{h \tan\theta}{h+b \tan\theta}$

Now on substituting value of tan θ in eqn (4)

tan β = $\frac{h^2 + \tan\theta}{h^2 - a\, h \tan\alpha + b\, h \tan\alpha}$

h^2tan β$-\ a\ h \tan\alpha \tan\beta + b\ h \tan\alpha \tan\beta - h^2 \tan\alpha$ = 0

h(h tan β$-\ h \tan\alpha + b \tan\alpha \tan\beta - a \tan\alpha \tan\beta)$ = 0

h$(\tan\beta - \tan\alpha)$+tan αtan β(b-a) = 0

h = $\frac{(b-a)\tan\alpha \tan\beta}{\tan\alpha - \tan\beta}$ Proved

49. Question

The angle of elevation of the top of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Calculate the height of the tower

Answer

In the fig PQ be the height of the tower.

In ΔQRT

tan 45° = $\frac{QR}{RT}$

1 = $\frac{h}{x}$

h = x ……….(1)

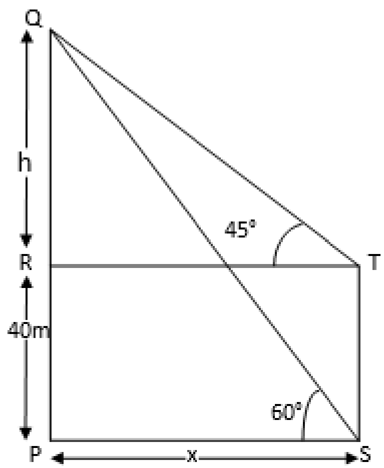
In ΔQRT

tan 60° = $\frac{QR}{PS}$

√3 = $\frac{h+40}{x}$

√3x = h+40 ………….(2)

On substituting the value of x from eqn (1) in eqn (2)



√3h = h+40

h (√3-1) = 40

h = $\frac{40}{\sqrt{3}-1}$ ⇒ 54.64m

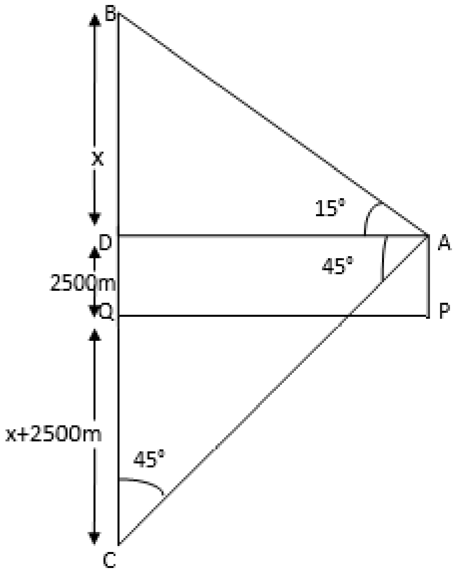
Therefore height of the tower is = h+40 = 54.64+40 ⇒ 94.64m

50. Question

The angle of elevation of a stationery cloud from a point 2500 m above a lake is 15° and the angle of depression of its reflection in the lake is 45°. What is the height of the cloud above the lake level? (Use tan 15° = 0.268)

Answer

In the fig B is the position of the cloud and C is the point of reflection of the cloud in the lake.



In the fig BD = x

DQ = AP= 2500m

QC = BQ = BD + DQ = (2500 + x)m

DC = DQ +QC = 2500 + 2500 + x = (5000 + x)m

In ΔQRT

tan 45° = $\frac{DC}{AD}$

1 = $\frac{5000+x}{AD}$

AD = 5000 + x ....(1)

Tan15° = tan (45° - 30°) = $\frac{tan45^\circ - tan30^\circ}{1+tan45^\circ tan30^\circ}$ [Using formula tan(α-β) = $\frac{tan\alpha - tan\beta}{1+tan\alpha tan\beta}$]

Tan15° = $\frac{1-1/\sqrt{3}}{1+1/\sqrt{3}}$ = $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

In ΔABD

tan 15° = $\frac{BD}{AD}$

$\frac{\sqrt{3}-1}{\sqrt{3}+1}$ = $\frac{x}{5000+x}$

(√3+1)x = (√3+1)(5000+x)

√3x + x = 5000√3 + √3x – 5000 - x

2x = 5000(√3-1)

x = 2500(√3-1)m

Now BQ = BD + DQ = x + 2500

(On substituting value of x)

BQ = 2500(√3-1) + 2500

⇒ 2500√3 – 2500 + 2500

2500√3m

Therefore height of cloud is 2500√3m

51. Question

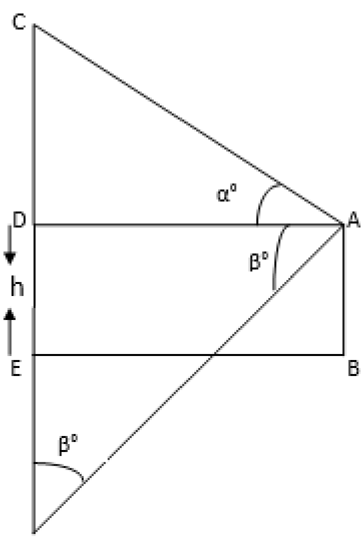
If the angle of elevation of a cloud from a point h metres above a lake is a and the angle of depression of its reflection in the lake be b, prove that the distance of the cloud from the point of observation is $\frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$

Answer

In the fig A is the point of observation and C is the position of the cloud.

Let the distance between the cloud and point of observation is x.

In ΔACD



Sin α = $\frac{CD}{AC}$

CD = AC Sin α ⇒ x Sin α

Cos α = $\frac{AD}{AC}$

AD = x Cos α ……………………(1)

CE = CD + DE = (h + x Sin α)

EF = CE = (h + x Sin α)

DF = DE + EF = (h + h + x Sin α) = (2h + x Sin α) …………….(2)

In ΔADF

$\tan \beta = \frac{DF}{AD}$

On substituting value of DF & AD

from above eqns (1) and (2)

$\tan \beta = \frac{2h + x \, Sin \, \alpha}{x \, Cos \, \alpha}$

$\frac{Sin \, \beta}{Cos \, \beta} = \frac{2h + x \, Sin \, \alpha}{x \, Cos \, \alpha}$

2h Cos β + x Sin α Cos β = x Sin β Cos α

x (Cos α Sin β - Sin α Cos β) = 2h Cos β

$x = \frac{2h \, Cos \, \beta}{Cos \, \alpha \, Sin \, \beta - Sin \, \alpha \, Cos \, \beta}$

On dividing numerator and denominator by Cos α Cos β, we get

$x = \frac{2h \, Sec \, \alpha}{\tan \beta - \tan \alpha}$

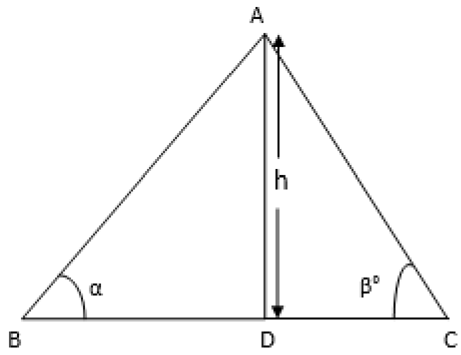
Therefore the distance between cloud and point of observation is $\frac{2h \, Sec \, \alpha}{\tan \beta - \tan \alpha}$ m

52. Question

From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive mile stones on opposite sides of the aeroplane are observed to be α and β. Show that the height in miles of aeroplane above the road is given by $\frac{\tan \alpha \tan \beta}{\tan \alpha + \tan \beta}$

Answer

In the fig let B & C be two mile stones. And height of the aeroplane is AD



In ΔABD

$\tan \alpha = \frac{AD}{BD}$

$\tan \alpha = \frac{h}{BD}$

h = BD tan α ....(1)

In ΔACD

tan β = $\frac{AC}{CD}$

tan β = $\frac{h}{CD}$

CD = $\frac{h}{\tan\beta}$ ....(2)

On adding eqn (1) and (2)

BD + CD = $\frac{h\,(\tan\alpha+\tan\beta)}{\tan\alpha\,\tan\beta}$

Now as B and C are milestones, the distance between them = 1

Therefore, BD + CD = 1

1 = $\frac{h\,(\tan\alpha+\tan\beta)}{\tan\alpha\,\tan\beta}$

h = $\frac{(\tan\alpha\tan\beta)}{\tan\alpha+\tan\beta}$ Proved

53. Question

PQ is a post of given height a, and AB is a tower at some distance. If α and β are the angles of elevation of B, the top of the tower, at P and Q respectively. Find the height of the tower and its distance from the post.

Answer

In the fig PQ is the post off height ‘a’ and AB is the tower of height ‘h’

In ΔBAP

tan α = $\frac{AB}{AP}$

tan α = $\frac{h}{AP}$

AP = h tan α ....(1)

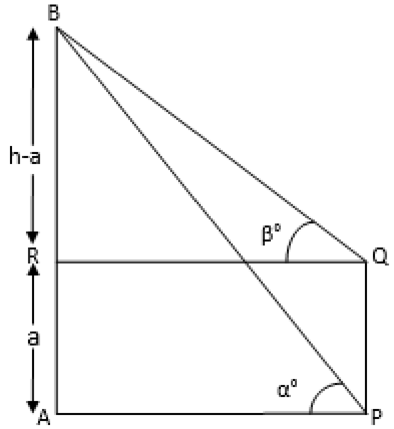
In ΔBRQ

tan β = $\frac{BR}{QR}$

tan β = $\frac{h-a}{AP}$

AP = $\frac{h-a}{\tan\beta}$ ....(2)

From eqn (1) and (2) we get,



$\frac{h-a}{\tan\beta} = \frac{h}{\tan\alpha}$

h tan β = (h-a) tan α

h tan β = h tan α − a tan α

h (tan α - tan β) = a tan α

h = $\frac{a\tan\alpha}{\tan\alpha - \tan\beta}$

Therefore height of the tower is $\frac{a\tan\alpha}{\tan\alpha - \tan\beta}$

Now AP = $\frac{h}{\tan\alpha}$ = $\frac{\frac{a\tan\alpha}{\tan\alpha - \tan\beta}}{\tan\alpha}$

⇒ $\frac{a\tan\alpha}{\tan\alpha\,(\tan\alpha - \tan\beta)}$

⇒ $\frac{a\tan\alpha}{\tan\alpha\,(\tan\alpha - \tan\beta)}$

Therefore distance is ⇒ $\frac{a}{\tan\alpha - \tan\beta}$

54. Question

A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle β with the horizontal. Show that $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

Answer

In the fig let the length of ladder is h (m)

In ΔAEB

Sin α = $\frac{AE}{AB}$

Sin α = $\frac{AE}{h}$

AE = h Sin α

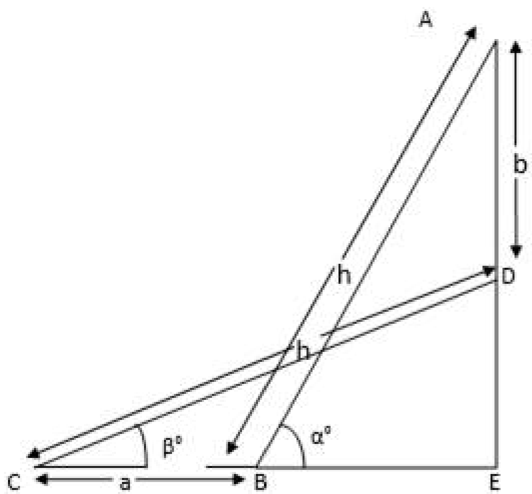
Cos α = $\frac{BE}{AB}$

Cos α = $\frac{BE}{h}$

BE = h Cos α

In ΔDEC

Sin β = $\frac{DE}{CD}$



Sin β = $\frac{DE}{h}$

DE = h Sin β

Cos β = $\frac{CE}{CD}$

Cos β = $\frac{CE}{h}$

CE = h Cos β

Now

$\frac{BC}{AD} = \frac{a}{b}$

$\frac{CE-BE}{AE-DE} = \frac{a}{b}$

$\frac{a}{b} = \frac{h\ Cos\beta - h\ Cos\ \alpha}{h\ Sin\ \alpha - h\ Sin\ \beta}$

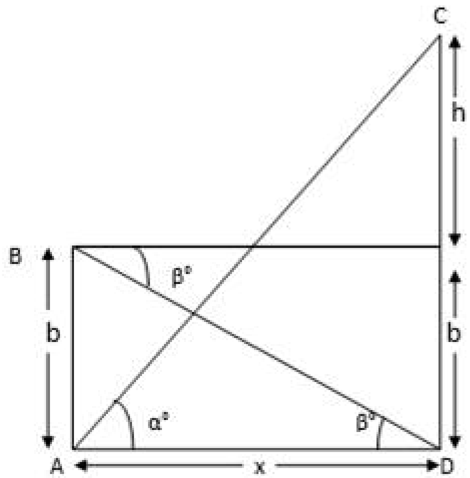
$\frac{a}{b} = \frac{Cos\beta - Cos\ \alpha}{Sin\ \alpha - Sin\ \beta}$ Proved

55. Question

A tower subtends an angle α at a point A in the plane of its base and the angle of depression of the foot of the tower at a point b metres just above A is β. Prove that the height of the tower is b tan α cot β.

Answer

In the fig let CD is the height of tower



CD = a + b

In ΔABD

Cot β = $\frac{AD}{AB}$

Cot β = $\frac{x}{b}$

x = b Cot β ……………..(1)

In ΔADC

tan α = $\frac{CD}{AD}$

tan α = $\frac{CD}{b\ Cot\ \beta}$

⇒ CD = b Cot β tan α

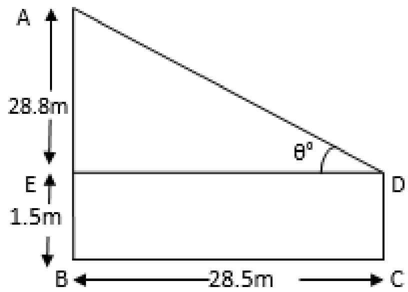
Therefore height of the tower is b Cot β tan α

56. Question

An observer, 1.5 m tall, is 28.5 m away from a tower 30 m high. Determine the angle of elevation of the tower from his eye.

Answer

In the fig let DC is the observer of the height 1.5m.



In ΔAED

$\tan\theta = \frac{AE}{DE}$

$\tan\theta = \frac{28.5}{28.5}$

$\tan\theta = 1$

$\theta = \tan^{-1} 1$

$\theta = 45°$

Hence the angle of the observation of

the tower from observer's eye is 45°

57. Question

A carpenter makes stools for electricians with a square top of side 0.5 m and at a height of 1.5 m above the ground. Also, each leg is inclined at an angle of 60° to the ground. Find the length of each leg and also the lengths of two steps to be put at equal distances.

Answer

In the fig let the height of the stool from the ground is 1.5m i.e. AL = BM = 1.5m

In ΔACL

$\tan 60° = \frac{AL}{CL}$

$\sqrt{3} = \frac{1.5}{CL}$

$CL = \frac{1.5}{\sqrt{3}} \Rightarrow \frac{1.5\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$

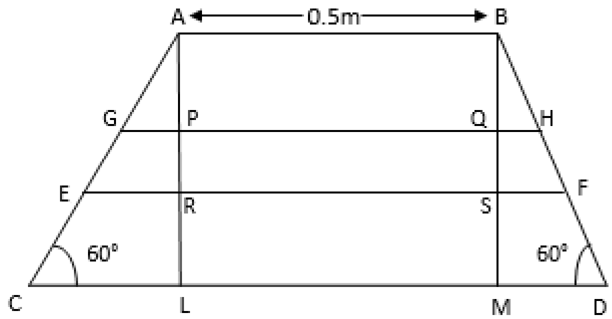
$\Rightarrow \frac{1.5\sqrt{3}}{3} = 0.866m$

Again in ΔACL

$\sin 60° = \frac{AL}{AC}$

$\frac{\sqrt{3}}{2} = \frac{1.5}{AC}$

AC = $\frac{3}{\sqrt{3}}$ = √3m ⇒ 1.732m



Lengths of steps are GH and EF and steps are put at equal distance such that AP =PR = RL

Now consider ΔAGP and ΔACL

$\frac{AP}{AL} = \frac{GP}{CL}$

$\frac{AP}{3AP} = \frac{GP}{0.866}$

GP = 0.2887m

Length of step = GH = GP + PQ + QH = GP + AB + GP (Since QH = GP)

GH = 2GP + AB = 2 × 0.2887 + 0.5 = 1.0774m

Similarly we can say that,

ER = $\frac{2CL}{3}$ ⇒ $\frac{2\times 0.866}{3}$ = 0.577m

Length of the step is EF = ER + RS + SF

⇒ ER + AB + ER = 2ER + AB

EF = 2× 0.5773 + 0.5 ⇒ 1.6546m

58. Question

A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30°. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of 45°. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

Answer

In the fig 'C' is the position of the kites. Let the length of second kite string is h.

In ΔABC

Sin 30° = $\frac{BC}{AC}$

$\frac{1}{2} = \frac{10+x}{100}$

20 + 2x = 100

x = 40m …………(1)

In ΔCFD

Sin 45° = $\frac{CF}{CD}$

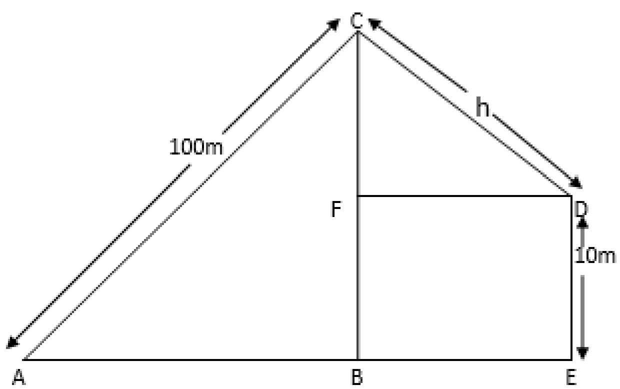
$\frac{1}{\sqrt{2}} = \frac{x}{h}$

h = √2 x ……………(2)

On substituting value

of x from eqn (1) in eqn (2)

h = √2 × 40 ⇒ 40√2 m



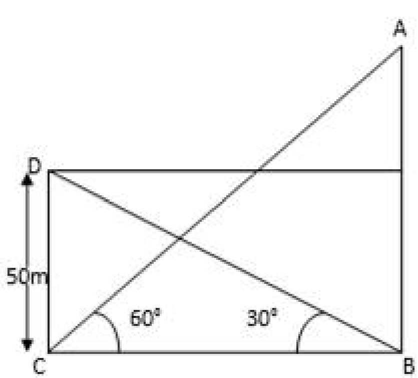
Therefore length of string of second kite is 40√2 m

59. Question

The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If the tower is 50 m high, what is the height of the hill?

Answer

In the fig DC is the tower and AB is the hill.



In ΔDCB

tan 30° = $\frac{DC}{BC}$

$\frac{1}{\sqrt{3}} = \frac{50}{BC}$

BC = 50√3 m

In ΔABC

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{AB}{50\sqrt{3}}$

AB = 50√3× √3 m ⇒ 150m

Therefore the distance between tower and hill is 50√3m and height of hill is 150m

60. Question

Two boats approach a light house in mid-sea from opposite directions. The angles of elevation of the top of the light house from two boats are 30° and 45° respectively. If the distance between two boats is 100 m, find the height of the light house.

Answer

In the fig AB is the light house of height h (m)

In ΔABC

tan 30° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{h}{100-x}$

x = 100 - √3 h ....(1)

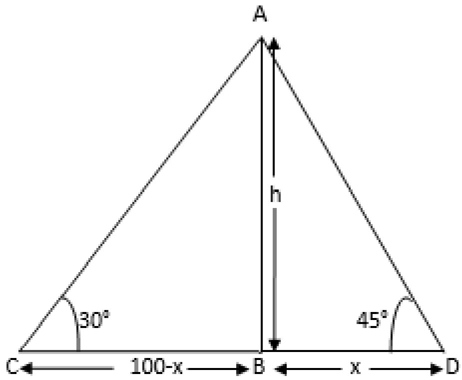
In ΔABD

tan 45° = $\frac{AB}{BD}$

1= $\frac{h}{x}$

x = h ....(2)

On substituting value of x from eqn (2) in eqn (1)



h = 100 - √3 h

h + √3 h = 100

h (1 + √3) = 100

h = $\frac{100}{1+\sqrt{3}}$ = $\frac{100(\sqrt{3}-1)}{2}$ ⇒ 50 (√3 - 1)

Therefore height of the light house is 50 (√3 - 1)m

61. Question

From the top of a building AB, 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find

(i) the horizontal distance between AB and CD.

(ii) the height of the lamp post.

(iii) the difference between the heights of the building and the lamp post.

Answer

In the fig AB is the building of height 60m and CD is the lamp post of height h (m)

(i) In ΔABC

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{60}{BC}$

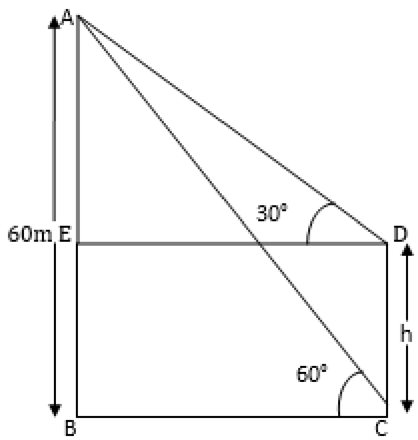
BC = $\frac{60}{\sqrt{3}}$

On multiplying and dividing by √3, we get

BC = $\frac{60\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$ ⇒ 20√3

Therefore distance between building

and lamp post is 20√3 m



(ii) In ΔAED

tan 30° = $\frac{AE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$

$\sqrt{3}AE$ = 20√3

AE = $\frac{20\sqrt{3}}{\sqrt{3}}$ = 20

Therefore height of lamp post is CD = AB-AE ⇒ 60-20 = 40 m

Therefore height of lamp post is 40 m

(iii) The difference between the height of the building and the lamp post is 60-40 = 20m

62. Question

From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the light house be h metres and the line joining the ships passes through the foot of the light house, show that the distance between the ship is $\frac{h(\tan\alpha + \tan\beta)}{\tan\alpha\tan\beta} metres$

Answer

In the fig AB is the light house of height h (m)

In ΔADC

tan β = $\frac{AD}{DC}$

tan β = $\frac{h}{y}$

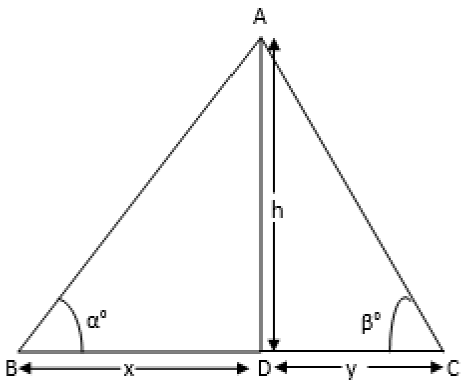
h = y tan β or y = $\frac{h}{\tan\beta}$……………….(1)

In ΔADB

$\tan \alpha = \frac{AD}{BD}$

$\tan \alpha = \frac{h}{x}$

h = x tan α or x = $\frac{h}{\tan \alpha}$ ....(2)



The distance between the two ships is BC = x + y

On adding eqn (1) & (2) we get,

x + y = $\frac{h}{\tan \alpha} + \frac{h}{\tan \beta}$

$\Rightarrow \frac{h\ tan\ \beta + h\ tan\ \alpha}{\tan \alpha \tan \beta}$

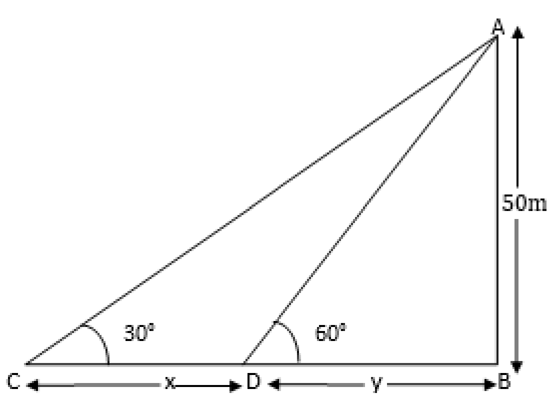
$\frac{h\ (tan\ \beta + tan\ \alpha)}{\tan \alpha \tan \beta}$ meters *PROVED*

63. Question

A straight highway leads to the foot of a tower of height 50 m. From the top of the tower, the angles of depression of two cars standing on the highway are 30° and 60° respectively. What is the distance between the two cars and how far is each car from the tower?

Answer

In the fig AB is the tower on the highway.



In ΔABC

tan 30° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{50}{x+y}$

x + y = 50√3 ………………..(1)

In ΔABD

tan 60° = $\frac{AB}{BD}$

√3 = $\frac{50}{y}$

y = $\frac{50}{\sqrt{3}}$

On multiplying and dividing by √3, we get

y = $\frac{50\sqrt{3}}{\sqrt{3}\times\sqrt{3}} \Rightarrow \frac{50\sqrt{3}}{3}$ ……………..(2)

Therefore the distance between

the first car and tower is 28.87m

On substituting value of y

from eqn (2) in eqn (1)

x + $\frac{50\sqrt{3}}{3}$ = 50√3

x = 50√3 - $\frac{50\sqrt{3}}{3}$

$\Rightarrow \frac{3\times50\sqrt{3}-50\sqrt{3}}{3}$

$\Rightarrow \frac{150\sqrt{3}-50\sqrt{3}}{3} = \frac{100\sqrt{3}}{3}$ = 57.73m

The distance between two cars is 57.73m

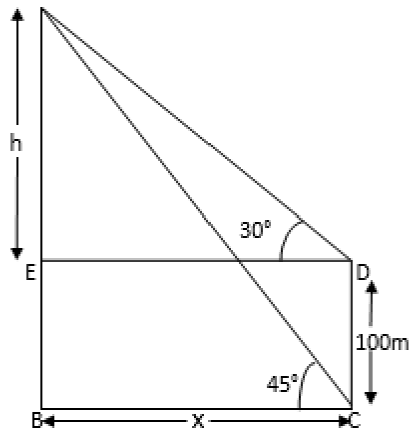
The distance between second car and tower is (x + y) = 57.73 + 28.87 = 86.60m

64. Question

The angles of elevation of the top of a rock from the top and foot of a 100 m high tower are respectively 30° and 45°. Find the height of the rock.

Answer

In the fig AB is the Rock and CD is the tower.



AB = AE + EB ⇒ h + 100

In ΔABC

tan 45° = $\frac{AB}{BC}$

1 = $\frac{100+h}{x}$

x = 100 + h ………………(1)

In ΔAEB

tan 30° = $\frac{AE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{h}{100+h}$

√3h= 100 + h

h(√3-1) = 100

h = $\frac{100}{\sqrt{3}-1}$

On multiplying and dividing by √3 + 1, we get

h = $\frac{100(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} \Rightarrow \frac{100(\sqrt{3}+1)}{2}$

⇒ 50 (√3 + 1) = 136.6m

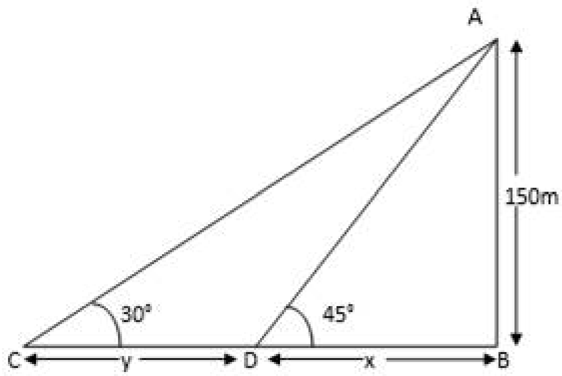
Therefore height of the rock = h +100 ⇒ 136.6 + 100 = 236.6m

65. Question

As observed from the top of a 150 m tall light house, the angles of depression of two ships approaching it are 30° and 45°. If one ship is directly behind the other, find the distance between the two ships.

Answer

In ΔABD



tan 45° = $\frac{AB}{BD}$

1 = $\frac{150}{x}$

x = 150m ……………….(1)

In ΔABC

tan 30° = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{150}{x+y}$

x + y = 150√3 ……………….(2)

On substituting value of x from eqn (1) in eqn (2), we get

150 + y = 150√3

y = 150 (√3 - 1) ⇒ 109.8m

Therefore the distance between two ships is 109.8m

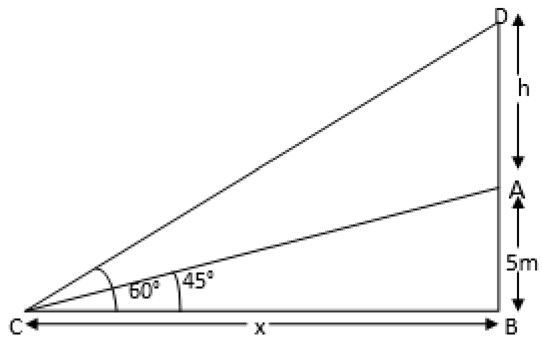
66. Question

A flag-staff stands on the top of a 5 m high tower. From a point on the ground, the angle of elevation of the top of the flag-staff is 60° and from the same point, the angle of elevation of the top of the tower is 45°. Find the height of the flag-staff.

Answer

In the fig let AD is the Flag-Staff of height h (m)

In ΔABC



$\tan 45° = \frac{AB}{BC}$

$1 = \frac{5}{x}$

x = 5m

In ΔDBC

$\tan 60° = \frac{DB}{BC}$

$\sqrt{3} = \frac{h+5}{5}$

$h + 5 = 5\sqrt{3}$

$h = 5\sqrt{3}-5$

$h = 5(\sqrt{3}-1)$

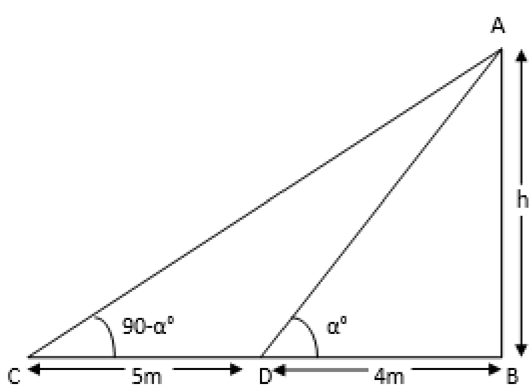
h = 3.66 m

Therefore height of flag-staff is 3.66m

67. Question

The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Answer



In the fig let AB is the Tower of height h (m).Since the tower is vertical to the ground.

∠ABC = 90°In a right-angled triangle, we know,

$$\tan\ \theta = \frac{perpendicular}{base}$$

In ΔABD

tan α = $\frac{AB}{BD}$

tan α = $\frac{h}{4}$

h = 4 tan α …………(1)

In ΔABC

tan (90°-α)= $\frac{AB}{BC}$

We know tan(90°- θ) = cot θ

⇒ cot α = $\frac{h}{9}$

⇒ h = 9 cot β ……….(2)

On multiplying eqn (1) and eqn (2), we get

h × h = 4 tan α × 9 cot α

h^2 = 36 tan α × cot αWe know, $tan\ \theta = \frac{1}{cot\ \theta}$

$$h^2 = 36 \times\ \tan\ \alpha \times \frac{1}{tan\ \alpha}$$

h^2 = 36h = ± 36

As the height cannot be negative.

⇒ h = 6m

Therefore the height of the tower is 6m.

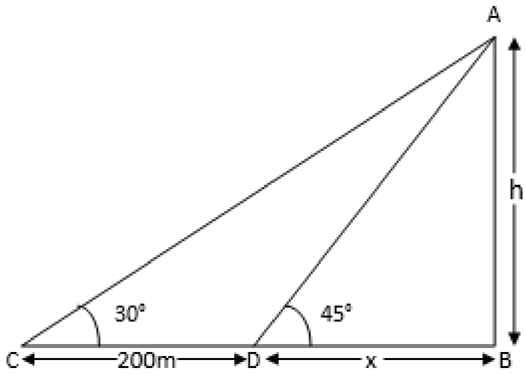
68. Question

The angles of depression of two ships from the top of a light house and on the same side of it are found to be 45° and 30° respectively. If the ships are 200 m apart, find the height of the light house.

Answer

In the fig let AB is the light house of height h (m)

In ΔABC



$\tan 30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{h}{200+x}$

$\sqrt{3}h = 200 + x$ ....(1)

In ΔABD

$\tan 45° = \frac{AB}{BD}$

$1 = \frac{h}{x}$

h = x ....(2)

From eqn (1) and (2) we get

$\sqrt{3}h = 200 + h$

$h(\sqrt{3} + 1) = 200$

$h = \frac{200}{\sqrt{3}+1}$

On multiplying and dividing by √3-1, we get

$h = \frac{200(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$h = \frac{200(\sqrt{3}-1)}{2} \Rightarrow 273.2\text{m}$

Therefore height of the light house is 273.2m

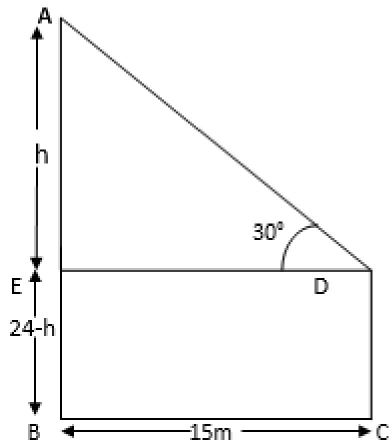
39. Question

The horizontal distance between two poles is 15 m. The angle of depression of the top of the first pole as seen from the top of the second pole is 30°. If the height of the second pole is 24m, find the height of the first pole. $(\sqrt{3} = 1.732)$

Answer

In the fig let DC is the first pole

of height h (m)



In ΔAED

tan 30° = $\frac{AE}{DE}$

$\frac{1}{\sqrt{3}} = \frac{h}{15}$

√3h = 15

h = $\frac{15}{\sqrt{3}}$ ⇒ 8.66m

Therefore the height of first

pole is 24 – 8.66 = 15.34m

70. Question

Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships are observed from the top of the light house are 60° and 45° respectively. If the height of the light house is 200 m, find the distance between the two ships. $\left(Use\ \sqrt{3} = 1.73\right)$

Answer

In the fig AB is the light house of height of 200m.

In ΔAED

tan 45° = $\frac{AB}{BD}$

1 = $\frac{200}{x}$

x = 200m ………………….(1)

In ΔABC

tan 60° = $\frac{AB}{BC}$

$\sqrt{3} = \frac{200}{y}$

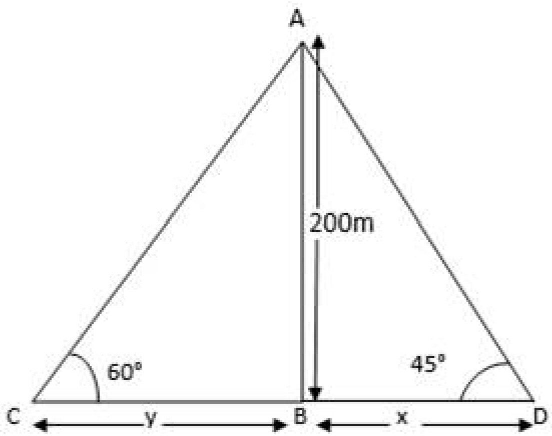
$\sqrt{3}y = 200$

$y = \frac{200}{\sqrt{3}}$

On multiplying and dividing by √3, we get

$y = \frac{200\sqrt{3}}{\sqrt{3}\times\sqrt{3}} \Rightarrow \frac{200\sqrt{3}}{3}$ m

Therefore the distance between two ships is:



DC = x + y

$x + y = 200 + \frac{200\sqrt{3}}{3}$

$\Rightarrow \frac{3\times200+200\sqrt{3}}{3}$

$\Rightarrow \frac{600+200\sqrt{3}}{3} = 315.47\text{m}$

Therefore the distance between two ships is 315.47m

71. Question

The angle of elevation of the top of a chimney from the top of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40 m, find the height of the chimney. According to pollution control norms, the minimum height of a smoke emitting chimney should be 100 m. State if the height of the above mentioned chimney meets the pollution norms. What value is discussed in this question?

Answer

In the fig AB is the Chimney and CD is the tower of 40m

In ΔAED

$\tan 60° = \frac{AB}{DE}$

$\sqrt{3} = \frac{h}{DE}$

h = √3DE ……………..(1)

In ΔDEB

tan 30° = $\frac{BE}{DE}$

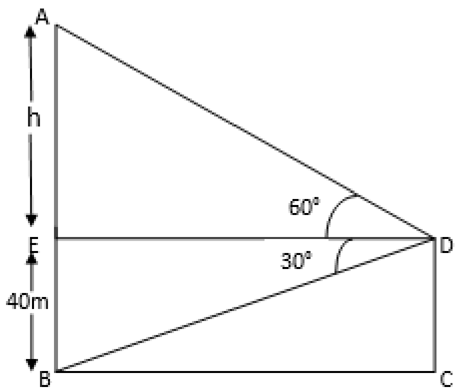
$\frac{1}{\sqrt{3}} = \frac{40}{DE}$

DE = 40√3 ………………..(2)

On substituting value of DE from

eqn (2) in eqn (2), we get

h = √3 × 40√3 = 120



Therefore height of Chimney is 40 + 120 = 160m

Yes, the height of the chimney meets the pollution norms.

Chimneys are made tall so that smoke should go high in the atmosphere in order to minimize air pollution.

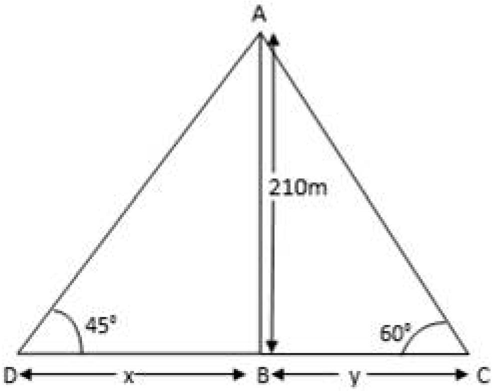
72. Question

An aeroplane is flying at a height of 210 m. Flying at this height at some instant the angles of depression of two points in a line in opposite directions on both the banks of the river are 45° and 60°. Find the width of the river. $\left(Use\ \sqrt{3} = 1.73\right)$

Answer

In the fig AD is the position of the aeroplane. Let the width of the river is DC = DB + BC

In ΔABD



tan 45° = $\frac{AB}{DB}$

1 = $\frac{210}{x}$

x = 210m …………………(1)

In ΔABC

tan 60° = $\frac{AB}{BC}$

√3 = $\frac{210}{y}$

√3y = 210

y = $\frac{210}{\sqrt{3}}$

On multiplying and dividing by √3, we get

y = $\frac{210\sqrt{3}}{\sqrt{3}\times\sqrt{3}} \Rightarrow \frac{210\sqrt{3}}{3}$ = 70√3 ……………………(2)

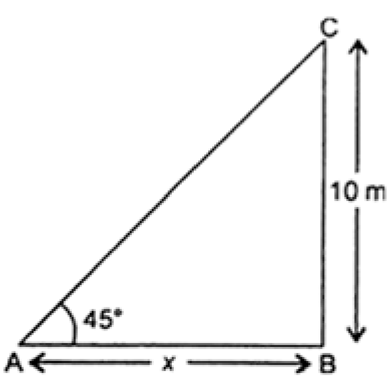
Therefore width of the river is = 210 +70√3 = 331.24m

CCE - Formative Assessment

1. Question

The height of a tower is 10 m. What is the length of its shadow when Sun's altitude is 45°?

Answer



Let BC be the height of tower which is 10m

Sun`s altitude is 45°

Let AB would be the shadow of the tower x meters

∴ $\tan 45° = BC/AB$ ($\tan\theta$ = perdendicular/base)

⇒ $1 = 10/x$

$x = 10$m

2. Question

If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, what is the angle of elevation of the Sun?

Answer

The ratio of height of a tower and the length of its shadow= $\sqrt{3}:1$

Angel of elevation = θ

$\tan\theta = \sqrt{3}:1$

$\tan\theta = \sqrt{3}$ ($\because \tan 60° = \sqrt{3}$)

∴ $\theta = 60°$

3. Question

What is the angle of elevation of the Sun when the length of the shadow of a vertical pole is equal to its height?

Answer

Here length of the shadow of a vertical pole is equal to its height

Let them both be x m

Angle of elevation = $\tan\theta = P/B$ (p = perpendicular, b= base)

Here $p = b = x$

∴ $\tan\theta = x/x$

$\tan\theta = 1$

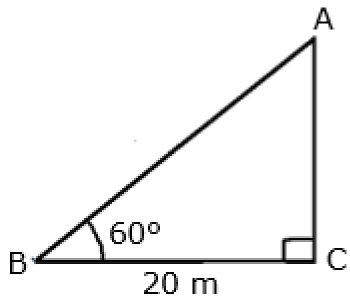
∴ $\theta = 45°$ ($\because \tan 45° = 1$)

4. Question

From a point on the ground, 20 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is 60°, what is the height of the tower?

Answer

Let the height of the tower be AC = x m



Distance from the foot of the vertical tower = 20m

tan 60° = AC/ BC

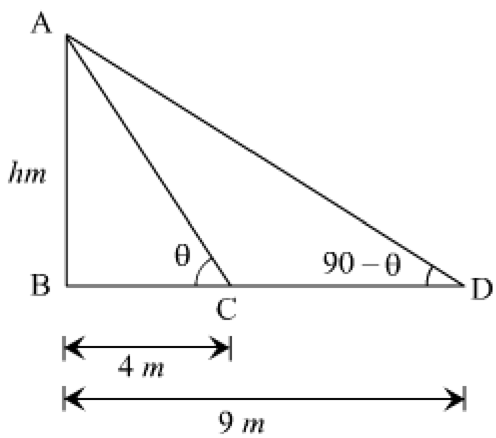
√3 = x / 20 (tan 60° = √3)

x = 20√3

5. Question

If the angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complimentary, find the height of the tower.

Answer



Let the height of the tower be h meters

Given, the angles of elevation of the top of a tower from two points are complimentary

∴ ∠ACB = θ and ∠ADB = 90° - θ

In Δ ABC

tan θ = 4 / h

h = 4tan θ………..1

In ΔABD

tan (90° - θ) = h / 9

h = 9 (cot θ) ………………..(tan (90° - θ) = cot θ) 2

cot θ = h/9

$\cot\theta = \frac{4\tan\theta}{9}$

$1/\tan\theta = \frac{4\tan\theta}{9}$

$9 = 4\tan^2\theta$

$\tan\theta = 3/2$

Height of tower (h)= 4 × 3/2………..putting value of tan θ in 1

= 6m

6. Question

In Fig. 12.58, what are the angles of depression from the observing positions O_1 and O_2 of the object at A?

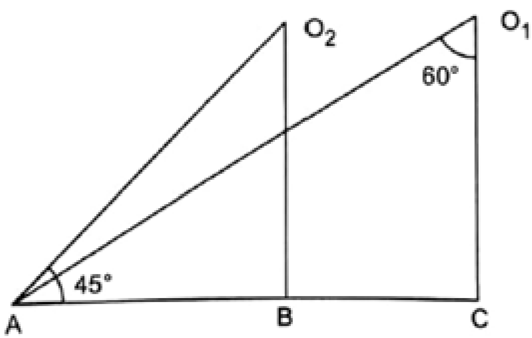
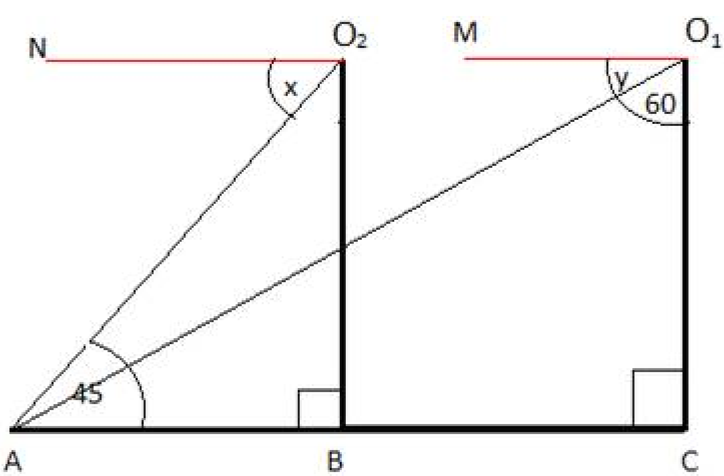


Fig. 12.58

Answer



Let x be the angle of depression of object A from the point O_2

And y be the angle of depression of object A from the point O_1

In Δ AO_1C

$\angle O_1AC + \angle ACO_1 + \angle ACO_1 = 180°$ (angle sum property)

$\angle O_1AC + 90° + 60° = 180°$

$\angle O_1AC = 30°$

Through O_1 , draw $O_1M \parallel AC$

And through O_2 draw $O_2N \parallel AC$

Now $O_1M \parallel AC$ and AO_1 is transversal

∴ $\angle O_1AC = y = 30° = \angle MO_1A$ (vertically opposite ∠s)

Similarly, $O_2N \parallel AC$ and AO_2 is transversal

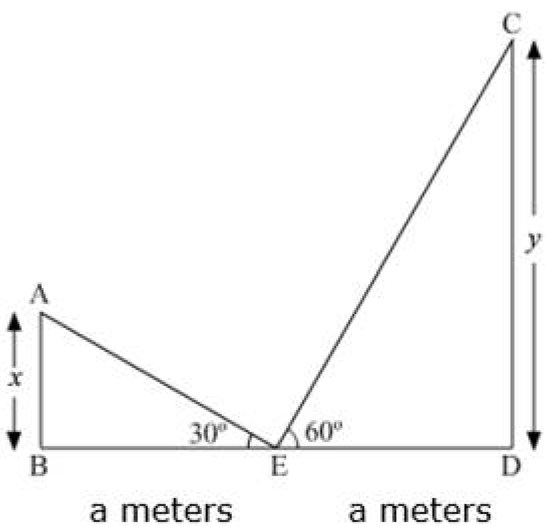
$\angle NO_2 A = x = 45° = \angle O_2 AB$ (vertically opposite angles)

∴ Angles of depression are 30° and 45°

7. Question

The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find x: y.

Answer



In ΔABE

tan 30° = x/a

⇒ x = a tan 30°

Now, in Δ ECD

tan 60° = y/a

⇒ y = a tan 60°

$$\frac{x}{y} = \frac{\tan 30°}{\tan 60°}$$

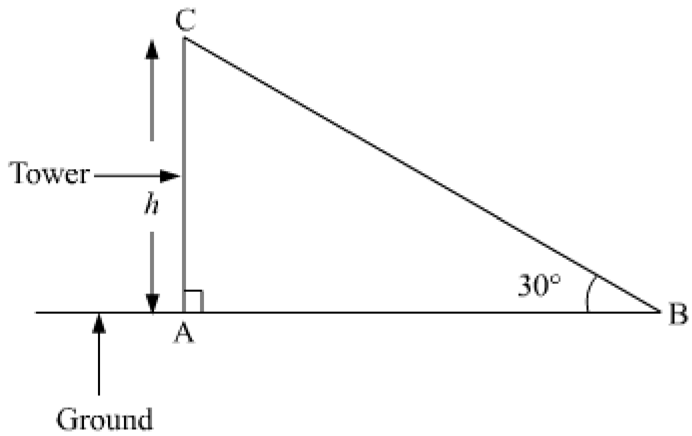
$$\frac{x}{y} = \frac{1}{3}$$

⇒ x :y = 1:3

8. Question

The angle of elevation of the top of a tower at a point on the ground is 30°. What will be the angle of elevation, if the height of the tower is tripled?

Answer



Let h be the height of the tower

∴ tan 30°= h/ AB (tan θ = perpendicular / base)

$h = \frac{1}{\sqrt{3}} AB$ 1

When the height is tripled h becomes 3h

tan θ = 3h / AB

tan θ = $3 \times \frac{\frac{1}{\sqrt{3}}AB}{AB}$ (from 1)

tan θ = $\frac{3\times1}{\sqrt{3}}$

tan θ = √3 (by rationalizing the denominator)

θ = 60° (tan 60° = √3)

1. Question

The ratio of the length of a rod and its shadow is 1 : √3. The angle of elevation of the sum is

A. 30°

B. 45°

C. 60°

D. 90°

Answer

The ratio of the length of rod and its shadow = 1: √3

Let the angle of elevation of sun be θ

tan θ = P / B (P = perpendicular, B = base)

Here tan θ = 1: √3 = $\frac{1}{\sqrt{3}}$

tan θ = $\frac{\sqrt{3}}{3}$ (by rationalizing the denominator)

θ = 30° (∵ tan 30° = $\frac{\sqrt{3}}{3}$)

2. Question

If the angle of elevation of a tower from a distance of 100 meters from its foot is 60°, then the height of the tower is

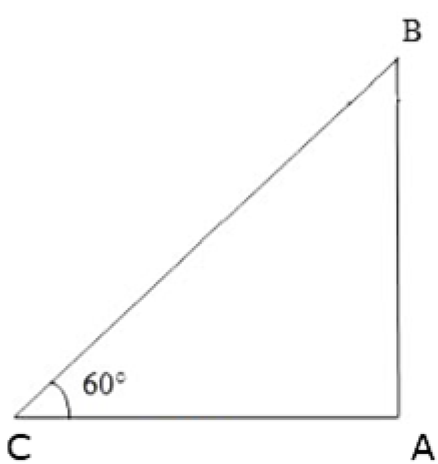
A. $100\sqrt{3}\text{m}$

B. $100\sqrt{3}\text{m}$

C. $50\sqrt{3}\text{m}$

D. $\frac{200}{\sqrt{3}}\text{m}$

Answer



Here, angle of elevation = 60°

Distance between the foot of tower and the shadow = 100m

Let height of the tower be h meters

Angle of elevation = tan θ = $\frac{\text{height of tower}}{\text{distance of shadow from its foot}}$

tan 60° = $\frac{h}{100}$

$\sqrt{3} \times 100 = h$

Height of tower = $100\sqrt{3}$ m

3. Question

If the altitude of the sun is at 60°, then the height of the vertical tower that will cast a shadow of length 30 m is

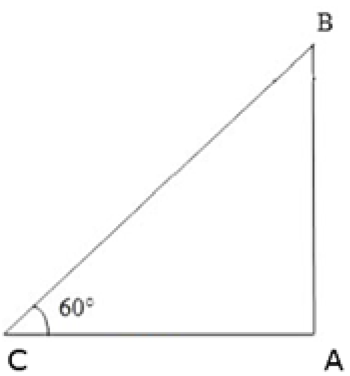
A. $30\sqrt{3}\text{m}$

B. 15 m

C. $\frac{30}{\sqrt{3}}\text{m}$

D. $15\sqrt{2}\text{m}$

Answer



Altitude of the sun = 60°

Length of the vertical tower = 30 m

Height of tower be h meters

tan θ = H/ B

$\tan 60° = \frac{h}{30}$

$h = 30\sqrt{3}$ m

4. Question

If the angles of elevation of a tower from two points distant a and (a > b) from its foot and in the same straight line from it are 30° and 60°, then the height of the tower is

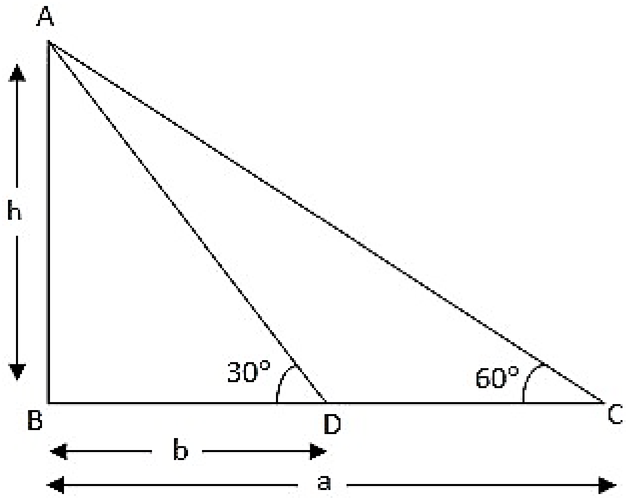
A. $\sqrt{a+b}$

B. $\sqrt{ab}$

C. $\sqrt{a-b}$

D. $\sqrt{\frac{a}{b}}$

Answer



Given BC = a and BD = bLet the height AB = hIn right ΔABD,

tan θ = $\frac{AB}{BD}$

tan 30° = $\frac{h}{b}$

$\frac{\sqrt{3}}{3} = \frac{h}{b}$

$h = \frac{\sqrt{3}}{3}$ b...............1

In Δ ABC

tan 60° = $\frac{h}{a}$

√3 a = h.................2

Multiplying 1 and 2

$h^2 = \frac{\sqrt{3}}{3} b \times \sqrt{3}\, a$

h = √ab

5. Question

If the angles of elevation of the top of a tower from two points distant a and b from the base and in the same straight line with it are complementary, then the height of the tower is

A. ab

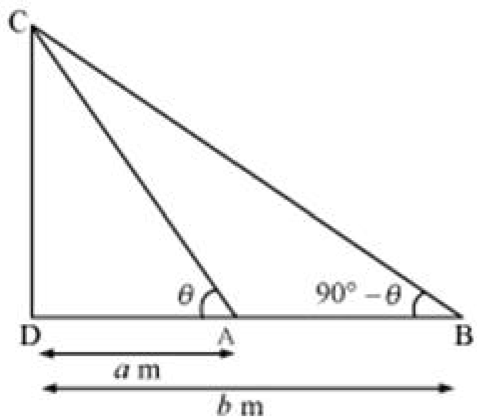
B. $\sqrt{ab}$

C. $\frac{a}{b}$

D. $\sqrt{\frac{a}{b}}$

Answer

Since the angles of elevation are complementary then if one angle is θ other would be 90° - θ



Here CD is the height of tower which forms two complementary angles θ and 90° - θ from its top to the distance a meters and b meters respectively.

In Δ CAD

$\tan\theta = \frac{CD}{a}$.1

In Δ CBD

$\tan 90° - \theta = \frac{CD}{b}$

$\cot\theta = \frac{CD}{b}$

$\frac{1}{\tan\theta} = \frac{CD}{b}$

Putting value of tan θ From 1

$\frac{a}{CD} = \frac{CD}{b}$

$(CD)^2 = ab$

$CD = \sqrt{ab}$

6. Question

From a light house the angles of depression of two ships on opposite sides of the light house are observed to be 30° and 45°. If the height of the light house is h metres, the distance between the ships is

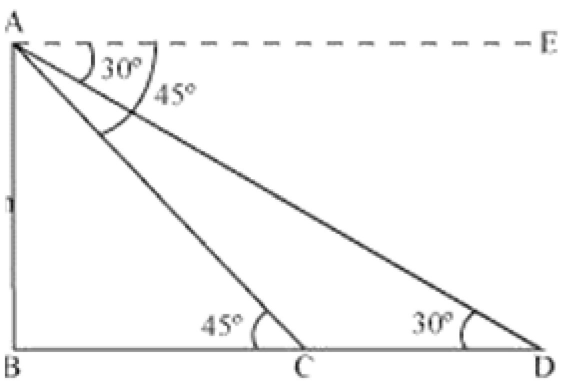
A. $(\sqrt{3}+1)\mathrm{h\,metres}$

B. $(\sqrt{3}-1)\mathrm{h\,metres}$

C. $\sqrt{3}\,\mathrm{h\,metres}$

D. $1+\left(1+\frac{1}{\sqrt{3}}\right)\mathrm{h\,metres}$

Answer



Here AB is the light house of height h meters

The angles of depression from the light house are 30° and 45°

The distance between the two ships is CD

In Δ ABC

tan 45° = $\frac{AB}{BC}$

⇒ BC = AB (∵ tan 45° = 1)

⇒ BC = h

In Δ ABD

tan 30° = $\frac{AB}{BD}$

$$\frac{1}{\sqrt{3}}=\frac{AB}{BD}$$

⇒ (h + CD) = √3 h (AB = BC = h)

$\Rightarrow$ CD = $(\sqrt{3} - 1)$ h

7. Question

The angle of elevation of the top of a tower standing on a horizontal plane from a point C is α. After walking a distance d towards the foot of the tower the angle of elevation is found to be β. The height of the tower is

A. $\dfrac{d}{\cot\alpha + \cot\beta}$

B. $\dfrac{d}{\cot\alpha - \cot\beta}$

C. $\dfrac{d}{\tan\beta - \tan\alpha}$

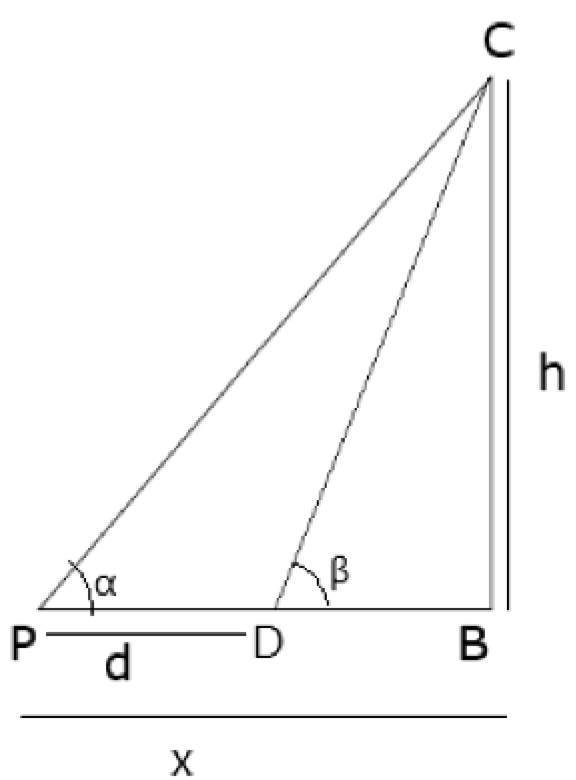
D. $\dfrac{d}{\tan\beta + \tan\alpha}$

Answer

Given: The angle of elevation of the top of a tower standing on a horizontal plane from a point C is α. After walking a distance d towards the foot of the tower the angle of elevation is found to be β.

To find: The height of the tower

Solution:



Let h be the height of the tower on horizontal plane.

Let α be the angle of elevation from point C and β be the angle of elevation from point B

Given CB = d

In Δ PCB

tan (α) = $\frac{h}{x}$

x = $\frac{h}{\tan\alpha}$

In Δ CDB

tan(β) = $\frac{h}{x-d}$

$$\Rightarrow \tan\beta = \frac{h}{\frac{h}{\tan\alpha} - d}$$

$$\Rightarrow \tan\beta = \frac{h\ \tan\alpha}{h - d\ \tan\alpha}$$

⇒ tanβ (h - d tanα) = h tanα

⇒ h tanβ - d tan a tanβ = h tan a

⇒ h (tanβ − tan a) = d tan a tanβ

$$\Rightarrow h = \frac{d\ tan\alpha\ tan\beta}{tan\beta - tan\alpha}$$

Use the formula:

$$\tan\theta = \frac{1}{\cot\theta}$$

$$\Rightarrow h = \frac{d \frac{1}{\cot\alpha}\ \frac{1}{cot\beta}}{\frac{1}{cot\beta} - \frac{1}{cot\alpha}}$$

$$\Rightarrow h = \frac{d \frac{1}{cot\alpha}\ \frac{1}{cot\beta}}{\frac{cot\alpha - cot\beta}{cot\beta\ cot\alpha}}$$

$$\Rightarrow h = \frac{d}{cot\alpha - cot\beta}$$

Hence (b) is the answer.

8. Question

The tops of two poles of height of 20 m and 14 m are connected by a wire. If the wire makes an angle of 30° with horizontal, then the length of the wire is

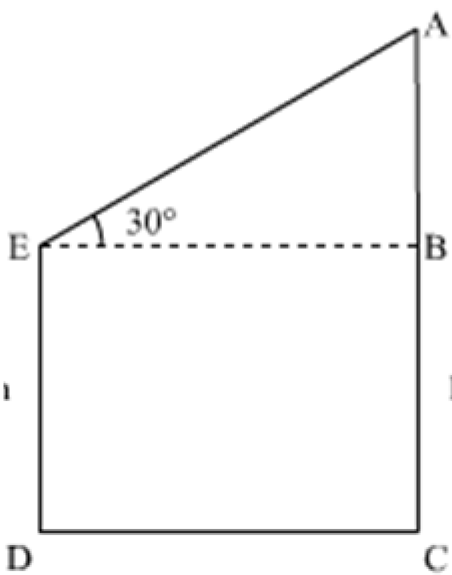
A. 12 m

B. 10 m

C. 8 m

D. 6 m

Answer



Here ED = 14m and AC = 20 m are two poles

⇒ AB = 20 – 14 = 6 m

Wire AE connects them making angle with horizontal of 30°

We have to find AE which is length of wire

Sin 30° = $\frac{6}{AE}$

$\frac{1}{2} = \frac{6}{AE}$

AE = 12 m

9. Question

From the top of a cliff 25 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is

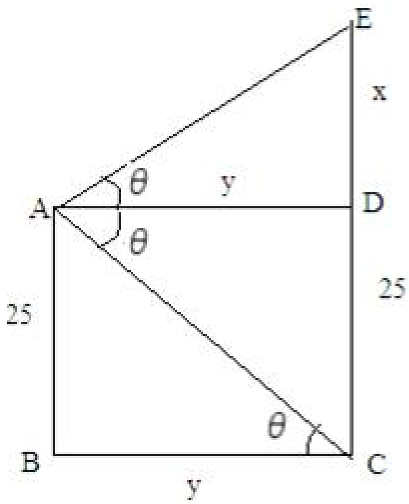
A. 25 m

B. 50 m

C. 75 m

D. 100 m

Answer



Here AB = 25 m is cliff and CE = (25 + x) m is tower

In Δ ADE

tan θ = $\frac{x}{y}$...............1

In Δ ABC

tan θ = $\frac{25}{y}$ 2

From 1 and 2

$$\frac{25}{y} = \frac{x}{y}$$

x =25 m

Total height of tower is 25 + x = 25 +25 = 50 m

10. Question

The angles of depression of two ships from the top of a light house are 45° and 30° towards east. If the ships are 100 m apart, the height of the light house is

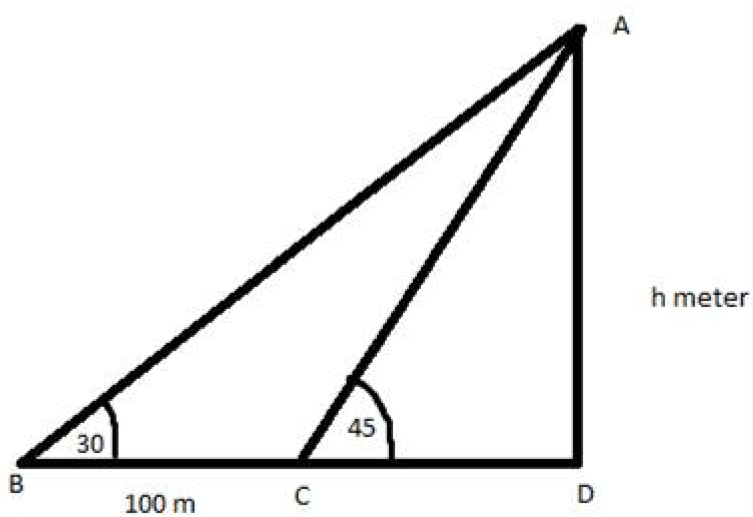
A. $\frac{50}{\sqrt{3}+1}$m

B. $\dfrac{50}{\sqrt{3}-1}\,\mathrm{m}$

C. $50(\sqrt{3}-1)\mathrm{m}$

D. $50(\sqrt{3}+1)\mathrm{m}$

Answer



Here AD = h meter is the tower

The ships B and C are 100 m apart so BC = 100 m

In Δ ACD

tan 45° = $\frac{h}{CD}$

CD = h (∵ tan 45° = 1)

In Δ ABD

tan 30° = $\frac{h}{100+CD}$

$\frac{1}{\sqrt{3}} = \frac{h}{100+CD}$

h = $\frac{100+CD}{\sqrt{3}}$

$\sqrt{3}$ h= 100 + h

h = $\frac{100}{\sqrt{3}-1}$ m

On rationalizing the denominator

h = $\frac{100(\sqrt{3}+1)}{2}$

$h = 50\ (\sqrt{3}+1)$m

11. Question

If the angle of elevation of a cloud from a point 200 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°, then the height of the cloud above the lake is

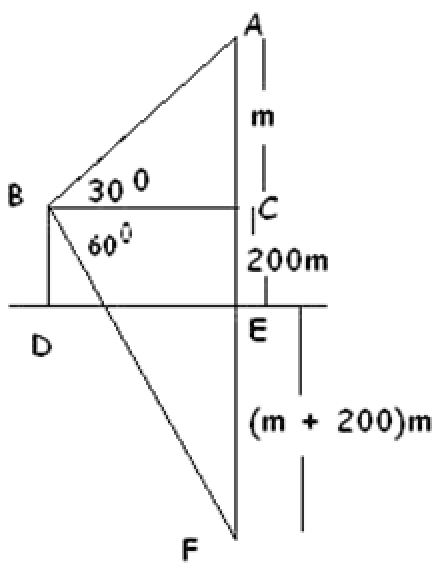
A. 200 m

B. 500 m

C. 30 m

D. 400 m

Answer



A is the position of the cloud, B is the point 200 m above the lake and F is the reflection in the lake.

Here AE = EF

EF = (m + 200) m

In ΔABC

$$cot\ 30^{\circ} = \frac{BC}{AC} \quad \Rightarrow cot\ 30^{\circ} = \frac{BC}{m}$$

BC = m cot 30° ……………….1

In Δ BCF

CF = 200 + m +200

= (400 + m) meters

$$\cot 60^{\circ} = \frac{BC}{FC} \quad \cot 60^{\circ} = \frac{BC}{400+m}$$

BC = (400 + m) cot 60° ………………….2

From 1 and 2

m cot 30° = (400 + m) cot 60°

m $\sqrt{3} = \frac{(400 + m)}{\sqrt{3}}$

3 m = (400 + m)

2m = 400

m = 200 m

The height above the cloud above the lake is (200 + 200)m = 400 m.

12. Question

The height of a tower is 100 m. When the angle of elevation of the sun changes from 30° to 45°, the shadow of the tower becomes x metres less. The value of x is

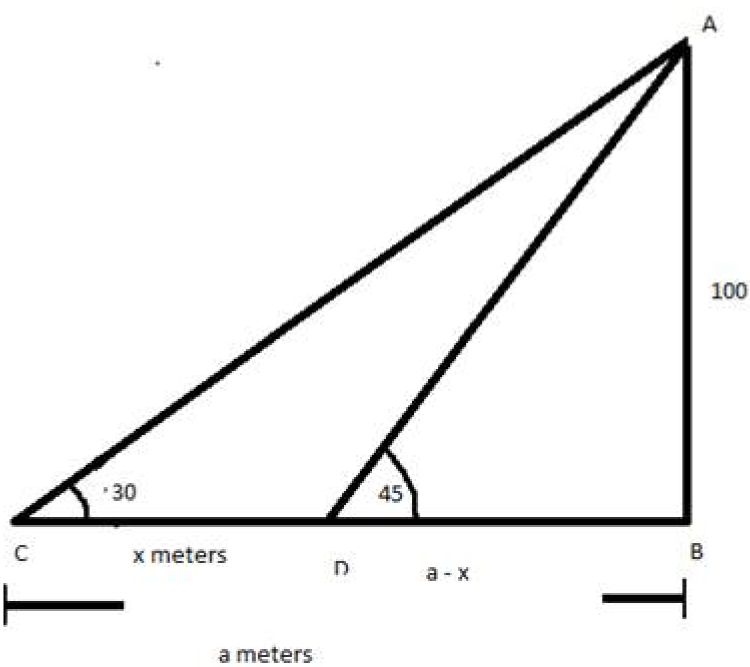
A. 100 m

B. $100\sqrt{3}\text{m}$

C. $100(\sqrt{3}-1)\text{m}$

D. $\frac{100}{\sqrt{3}}\text{m}$

Answer



Let AB be the tower of height 100 m

BC is the total distance of shadow formed at two different angles namely ACB and ADB 30° and 45° respectively

In Δ ACB

tan 30° = $\frac{100}{a}$

$\frac{1a}{\sqrt{3}}$ = 100

a = $100\sqrt{3}$

In Δ ADB

tan 45° = $\frac{100}{100\sqrt{3}-x}$

100√3-x = 100

x = 100(√3-1) m

13. Question

Two persons are a metres apart and the height of one is double that of the other. If from the middle point of the line joining their feet, an observer finds the angular elevation of their tops to be complementary, then the height of the shorter person is

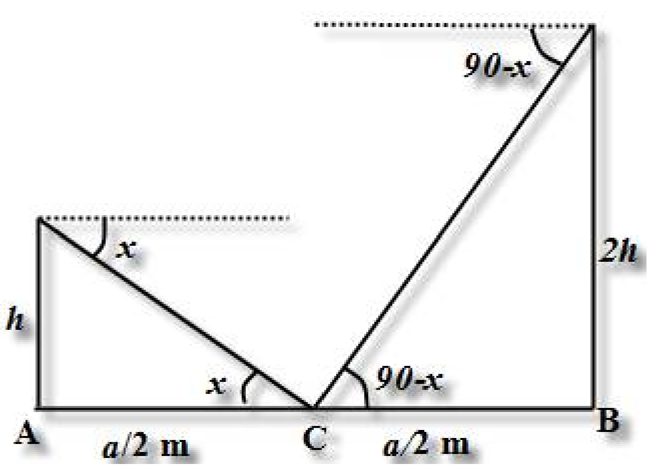
A. $\frac{a}{4}$

B. $\frac{a}{\sqrt{2}}$

C. $a\sqrt{2}$

D. $\frac{a}{2\sqrt{2}}$

Answer



Let AB be the line joining the distance between two persons. Given C is the midpoint of AB so AC = a/2 and CB = a/2

Height of taller person is double the height of shorter person which is 2h and h respectively forming complementary angles

tan x = $\frac{2h}{a}$...............1

And tan 90- x = $\frac{4h}{a}$

cot x= $\frac{4h}{a}$ 2

Multiplying 1 and 2

tan x cot x = $\frac{2h}{a} \times \frac{4h}{a}$

$a^2 = 8h^2$

a = 2√2 h

$$h = \frac{a}{2\sqrt{2}} \text{ m}$$

14. Question

The angle of elevation of a cloud from a point h metre above a lake is θ. The angle of depression of its reflection in the lake is 45°. The height of the cloud is

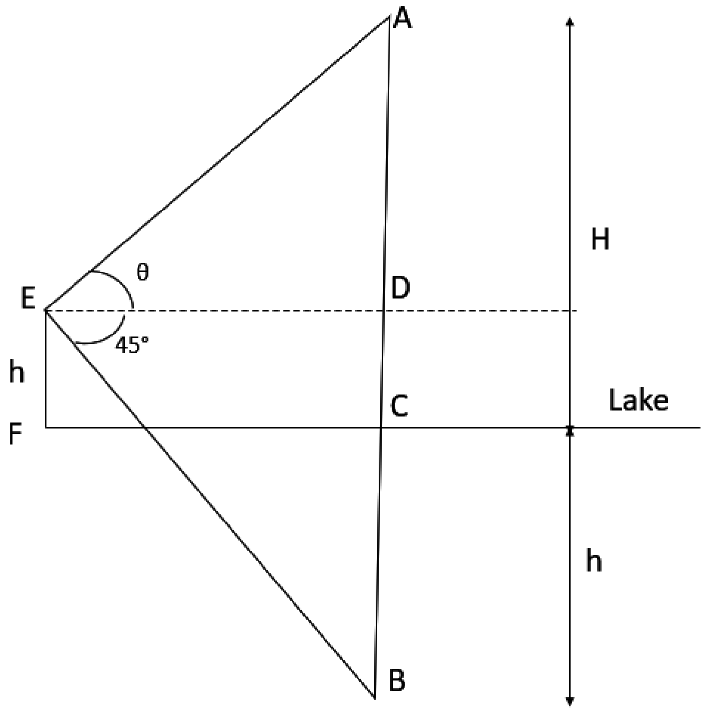
A. h tan (45° + θ)

B. h cot (45° – θ)

C. h tan (45° – θ)

D. h cot (45° + θ)

Answer



Let A and B be the position of the cloud and its reflection in the lake.

Let the height of the cloud be H m.

Given EF = h m, ∠AED = θ and ∠DEB = 45°

As EF || CD

CD = h m

By law of reflection,

AC = BC = H

AD = AC – DC

= H – h

And,

BD = BC + DC

= H + hIn ΔDEB, $\tan 45^{\circ} = \frac{BD}{DE}$

$\Rightarrow 1 = \frac{BD}{DE}$

⇒ BD = DE⇒ DE = H + h ..... (1)In ΔAED, $tan\ \theta = \frac{AD}{DE}$

$\Rightarrow \tan\ \theta = \frac{H-h}{DE}$

$$\Rightarrow DE = \frac{H-h}{\tan\ \theta} \quad \text{...... (2)}$$

From (1) and (2), $\dfrac{H-h}{tan\ \theta} = H + h$

⇒ H - h = (H + h) tan θ⇒ H - h = H tan θ + h tan θ⇒ H - H tan θ = h + h tan θ⇒ H (1 - tan θ) = h (1 + tan θ)

$$\Rightarrow H = \frac{h(1+tan\ \theta)}{\left(1-tan\ \theta\right)}$$

$$\Rightarrow H = \frac{h\left(\tan\ 45^{\circ} + tan\ \theta\right)}{\left(1-\tan\ 45^{\circ}\ tan\ \theta\right)}$$

As we know,

$\tan(A+B) = \dfrac{\tan\ A\ + \tan\ B}{1-\ \tan\ A\ \tan\ B}$ ⇒ H = h (tan 45° + θ)

15. Question

A tower subtends an angle of 30° at a point on the same level as its foot. At a second point h metres above the first, the depression of the foot of the tower is 60°. The height of the tower is

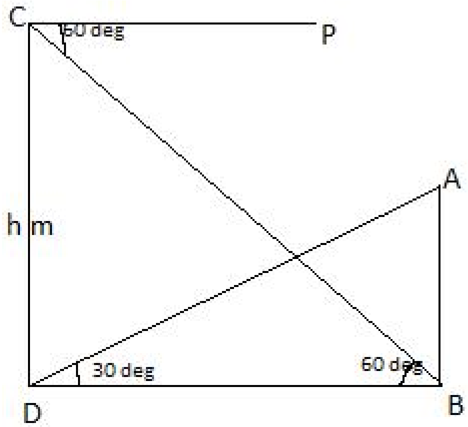
A. $\frac{\text{h}}{2}\text{m}$

B. $\sqrt{3}\text{h m}$

C. $\frac{\text{h}}{3}\text{m}$

D. $\frac{\text{h}}{\sqrt{3}}\text{m}$

Answer



Let the AB be the height be the tower

Let D be the point where the tower subtends angle of 30°

Let C be the point where such that CD = h meters. From C the angle of depression subtended at the foot of the tower is 60°

In Δ CDB

tan 60° = $\frac{h}{BD}$

BD = h cot 60°

BD =$\frac{h}{\sqrt{3}}$ 1

In Δ ADB

tan 30° = AB/ BD

$\frac{1}{\sqrt{3}}$ = AB / BD

AB = $\frac{1}{\sqrt{3}}\times\frac{h}{\sqrt{3}}$

AB = $\frac{h}{3}$ m

16. Question

It is found that on walking x meters towards a chimney in a horizontal line through its base, the elevation of its top changes from 30° to 60°. The height of the chimney is

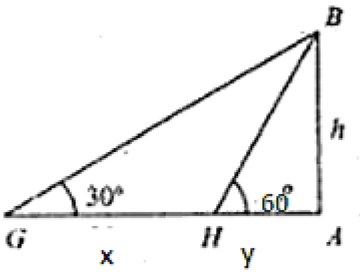
A. $3\sqrt{2}x$

B. $2\sqrt{3}x$

C. $\frac{\sqrt{3}}{2}x$

D. $\frac{2}{\sqrt{3}}x$

Answer



Here AB is the chimney of height h

By walking x meters toward chimney the angle of elevation changes from 30° to 60°

In Δ ABH

tan 60° = $\frac{h}{y}$

h = √3 y

$\frac{h}{\sqrt{3}}$ = y 1

In Δ ABG

tan 30° = $\frac{h}{x+y}$

$\frac{1}{\sqrt{3}} = \frac{h}{x+y}$

√3 h = x+y

√3 h - $\frac{h}{\sqrt{3}}$ = x (from 1)

$\frac{2h}{\sqrt{3}}$ = x

h = $\frac{\sqrt{3}x}{2}$

17. Question

The length of the shadow of a tower standing on level ground is found to be 2x metres longer when the sun's elevation is 30° than when it was 45°. The height of the tower in metres is

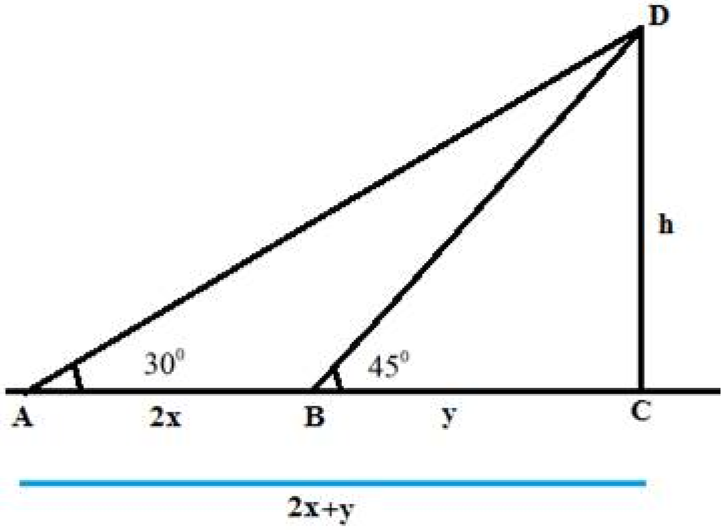
A. $(\sqrt{3}+1)x$

B. $(\sqrt{3}-1)x$

C. $2\sqrt{3}x$

D. $3\sqrt{2}x$

Answer



In Δ DBC

tan 45° = h /y

h = y (∵ tan 45° = 1)……….1

In ΔACD

tan 30° = $\frac{h}{2x+y}$

$\frac{1}{\sqrt{3}} = \frac{h}{2x+y}$

√3 h = 2x + h (from 1)

2x= (√3- 1) hh = $\frac{2x}{(\sqrt{3}-1)}$

h = (√3 +1)x meter

18. Question

Two poles are ‘a’ metres apart and the height of one is double of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the smaller is

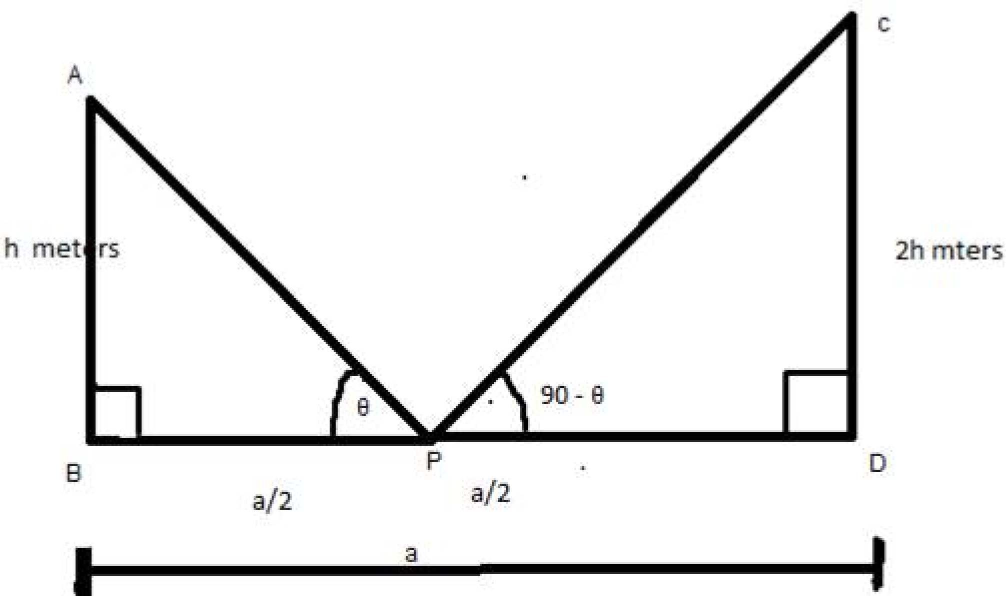
A. √2 ametres

B. $\frac{a}{2\sqrt{2}}$ metres

C. $\frac{a}{\sqrt{2}}$ metres

D. 2 ametres

Answer



Let AB and CD be the two poles of height h meters and 2h meters respectively such that BD be a km i.e.; the distance between the two poles and P be the midpoint of BD .

Given ∠APB = θ and ∠CPD = 90 − θ

In Δ ABP

tan θ = $\frac{h}{a/2}$

tan θ = $\frac{2h}{a}$.....................1

In Δ CDP

cot (90° − θ) = PD / CD = $\frac{a}{2\times 2h}$

cot (90° − θ)= $\frac{a}{4h}$

tan θ = $\frac{a}{4h}$.............2

Equating 1 and 2

$\frac{2h}{a} = \frac{a}{4h}$

8 h ^{2} = a^2

h = $\frac{a}{2\sqrt{2}}$ m

19. Question

The tops of two poles of height 16 m and 10 m are connected by a wire of length l metres. If the wire makes an angle of 30° with the horizontal, then l =

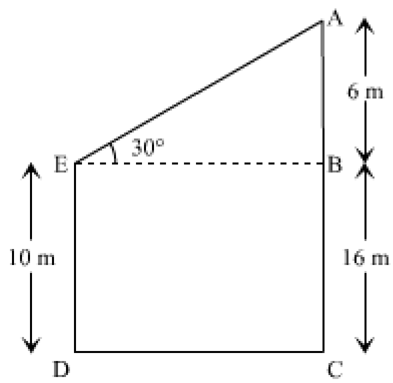
A. 26

B. 16

C. 12

D. 10

Answer



Let AE be the wire to connect the two poles ED and AC of height 10m and 16m forming angle of 30° with horizontal

In Δ AEB

Sin 30° = $\frac{AB}{AE}$

$\frac{1}{2} = \frac{6}{AE}$

AE = 12 m

20. Question

If a 1.5 m tall girl stands at a distance of 3 m from a lamp-post and casts a shadow of length 4.5 m on the ground, then the height of the lamp-post is

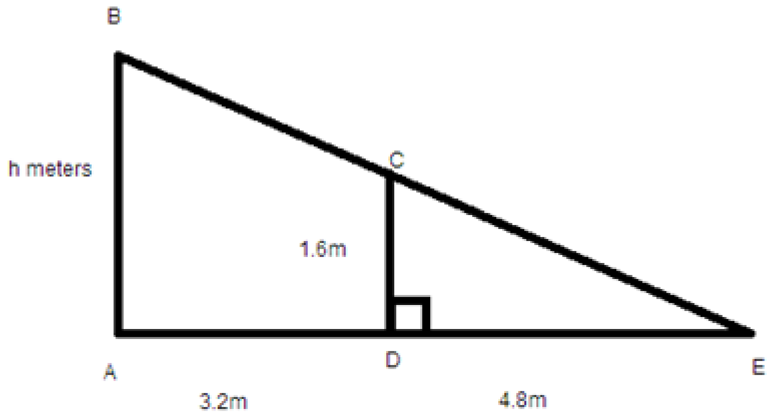
A. 1.5 m

B. 2 m

C. 2.5 m

D. 2.8 m

Answer



Let AB be the tower of h meters and CD be the girl of 1.6m height casting shadow = DE of 4.8 m, standing at the distance of 3.2m from the tower.

In Δ CDE

tan ∠CED = CD/ DE = $\frac{1.6}{4.8}$ 1

tan ∠CED = 1/3

In Δ ABE

tan ∠CED = $\frac{AB}{AE} = \frac{h}{8}$.....................2

From 1 and 2

$\frac{1}{3} = \frac{h}{8}$

h = $\frac{8}{3}$ m

h = 2.5m

21. Question

The length of shadow of a tower on the plane ground is √3 times the height of the tower. The angle of elevation of sun is

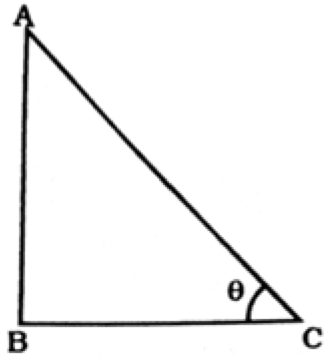
A. 45°

B. 30°

C. 60°

D. 90°

Answer



AB is the height of the tower and BC is the length of the shadow forming angle of elevation

tan θ = $\frac{AB}{BC}$

Given AB= h and BC = √3 h

tan θ = $\frac{h}{\sqrt{3}h}$

tan θ = $\frac{1}{\sqrt{3}}$

θ = 30°

22. Question

The angle of depression of a car, standing on the ground, from the top of a 75 m tower, is 30°. The distance of the car from the base of the tower (in metres) is

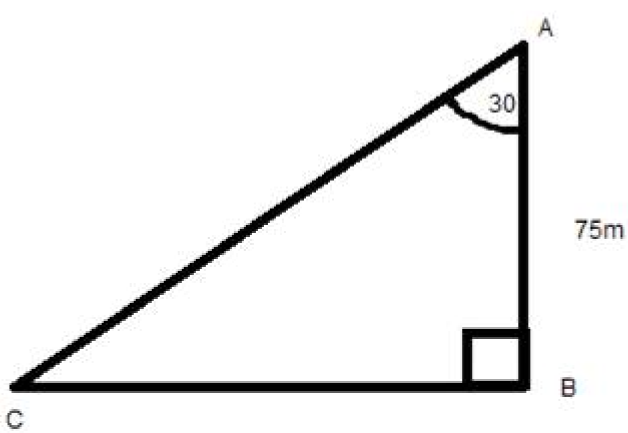
A. 25√3

B. 50√3

C. 75√3

D. 150

Answer



Here AB is the tower of height 75m forming angle of depression = 30°

∴ Angle of elevation is 90° - 30° = 60°

$\tan 60° = \frac{75}{x}$

$\sqrt{3} = \frac{75}{x}$

$x = \frac{75}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$x = 25\sqrt{3}$

23. Question

A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is

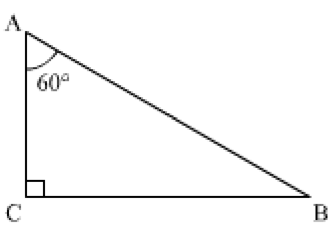
A. $15\sqrt{3}\text{m}$

B. $\frac{15\sqrt{3}}{2}\text{m}$

C. $\frac{15}{2}\text{m}$

D. 15 m

Answer



Given AB be the ladder of 15 m length and AC be the wall of h meters making angle with wall as 60°.

$\text{Cos } 60° = \frac{p}{h} = \frac{AC}{AB}$

$\frac{1}{2} = \frac{AC}{15}$

$AC = \frac{15}{2}\text{m}$

24. Question

The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. The distance of the car from the tower (in metres) is

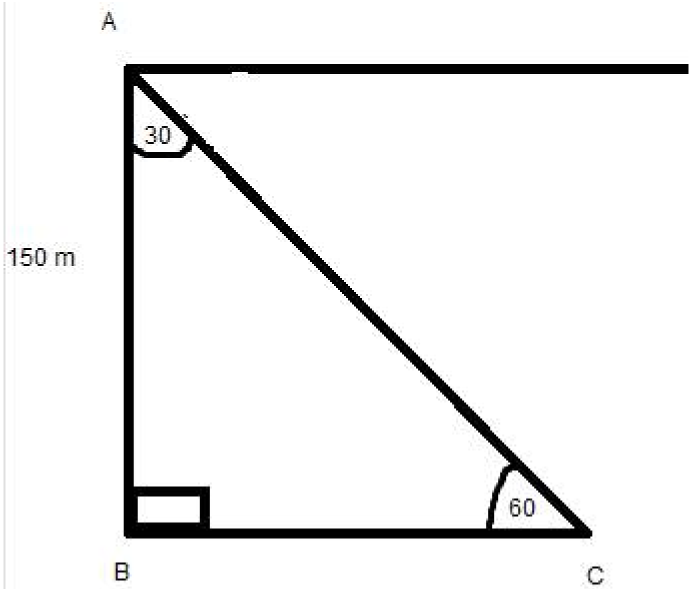
A. $50\sqrt{3}$

B. $150\sqrt{3}$

C. $150\sqrt{2}$

D. 75

Answer



Here AB is the tower of 150m which forms the angel of depression = 30° from the top

Hence the angle of elevation from the car is 90° - 30° = 60°

In Δ ABC

tan 60° = $\frac{150}{BC}$

BC = $\frac{150}{\sqrt{3}}$

BC = 50 √3 m

25. Question

If the height of a vertical pole is √3 times the length of its shadow on the ground, then the angle of elevation of the sun at that times is

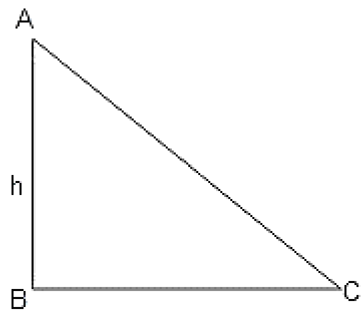
A. 30°

B. 60°

C. 45°

D. 75°

Answer



Consider the height of tower be √3 h∴ height of shadow =h .In a triangle ABC,

tan ∠ACB = $\frac{\sqrt{3}h}{h}$

tan ∠ACB = √3

∠ACB = 60°.Therefore, angle of elevation is 60° (∵ tan 60° = √3)

26. Question

The angle of elevation of the top of a tower at a point on the ground 50 m away from the foot of the tower is 45°. Then the height of the tower (in metres) is

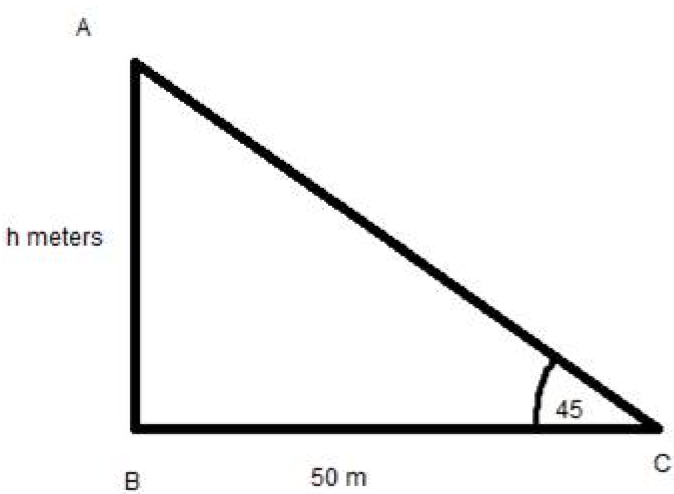
A. 50√3

B. 50

C. $\frac{50}{\sqrt{2}}$

D. $\frac{50}{\sqrt{3}}$

Answer



Let AB be the tower of h meters height forming angle of elevation= 45° from 50m distance from the tower

In Δ ABC

$\tan 45° = \frac{h}{50}$

h = 50 m

27. Question

A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length of the ladder (in metres) is

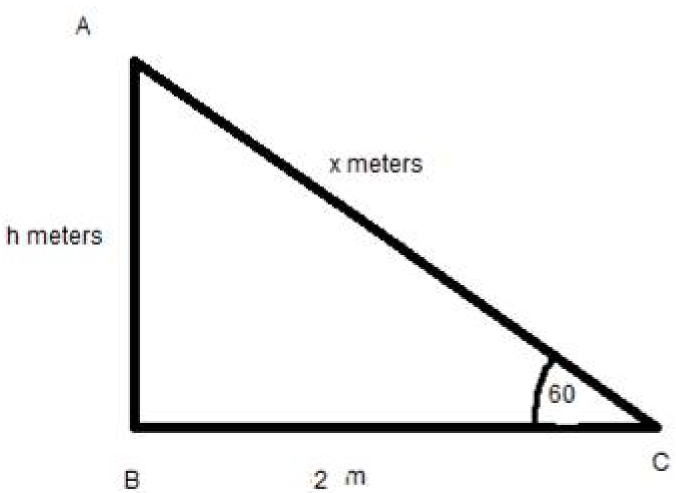
A. $\frac{4}{\sqrt{3}}$

B. $4\sqrt{3}$

C. $2\sqrt{2}$

D. 4

Answer



Here AC is the ladder of x meters placed against the wall of h meters at the distance of 2m from the wall.

It forms the angle of elevation to the top of wall as 60°

$\text{Cos } 60° = \frac{\text{base}}{\text{hypotenuse}}$

$\frac{1}{2} = \frac{2}{\text{hypotenuse}}$

Hypotenuse = 4 (length of the ladder)