5. Trigonometric Ratios

Exercise 5.1

1. Question

In each of the following, one of the six trigonometric ratios is given. Find the values of the other trigonometric ratios.

(i) $\sin A = \frac{2}{3}$ (ii) $\cos A = \frac{4}{5}$

(iii) $\tan\theta = 11$ (iv) $\sin\theta = \frac{11}{15}$

(v) $\tan\alpha = \frac{5}{12}$ (vi) $\sin\theta = \frac{\sqrt{3}}{2}$

(vii) $\cos\theta = \frac{7}{25}$ (viii) $\tan\theta = \frac{8}{15}$

(ix) $\cot\theta = \frac{12}{5}$ (x) $\sec\theta = \frac{13}{5}$

(xi) $\cos ec\theta = \sqrt{10}$ (xii) $\cos\theta = \frac{12}{15}$

Answer

(i)

$$\sin A = \frac{2}{3} = \frac{Perpendicular}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(base)^2 = (hypotenuse)^2 - (perpendicular)^2$$

$$(base)^2 = (3)^2 - (2)^2 = 9 - 4 = 5$$

$$base = \sqrt{5}$$

$$\therefore \quad \cos A = \frac{base}{hypotenuse} = \frac{\sqrt{5}}{3}, \ \tan A = \frac{perpendicular}{base} = \frac{2}{\sqrt{5}},$$

$$\cot A = \frac{base}{perpendicular} = \frac{\sqrt{5}}{2}, \sec A = \frac{hypotenuse}{base} = \frac{3}{\sqrt{5}},$$

$$\cos ecA = \frac{hypotenuse}{Perpendicular} = \frac{3}{2}$$

(ii)

$\cos A = \frac{2}{5} = \frac{base}{hypotenuse}$

By pythagoras theorem,

$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$

$(perpendicular)^2 = (hypotenuse)^2 - (base)^2$

$(perpendicular)^2 = (5)^2 - (4)^2$

$(perpendicular)^2 = 25 - 16 = 9$

$perpendicular = 3$

$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{3}{5}, \ \tan A = \frac{perpendicular}{base} = \frac{3}{4},$

$\cot A = \frac{base}{perpendicular} = \frac{4}{3}, \sec A = \frac{hypotenuse}{base} = \frac{5}{4},$

$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{5}{3}$

(iii)

$\tan\theta = \frac{11}{1} = \frac{Perpendicular}{base}$

By pythagoras theorem,

$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$

$(hypotenuse)^2 = (1)^2 + (11)^2 = 1 + 121 = 122$

$hypotenuse = \sqrt{122}$

$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{11}{\sqrt{122}}, \ \cos A = \frac{base}{hypotenuse} = \frac{1}{\sqrt{122}},$

$\cot A = \frac{base}{perpendicular} = \frac{1}{11}, \sec A = \frac{hypotenuse}{base} = \sqrt{122},$

$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{\sqrt{122}}{11}$

(iv)

$\sin\theta = \frac{11}{15} = \frac{Perpendicular}{hypotenuse}$

By pythagoras theorem,

$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$

$(base)^2 = (hypotenuse)^2 - (perpendicular)^2$

$(base)^2 = (15)^2 - (11)^2 = 225 - 121 = 104$

$base = \sqrt{104} = 2\sqrt{26}$

$\therefore \quad \cos A = \frac{base}{hypotenuse} = \frac{2\sqrt{26}}{15}, \ \tan A = \frac{perpendicular}{base} = \frac{11}{2\sqrt{26}},$

$\cot A = \frac{base}{perpendicular} = \frac{2\sqrt{26}}{11}, \sec A = \frac{hypotenuse}{base} = \frac{15}{2\sqrt{26}},$

$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{15}{11}$

(v)

$$\tan\theta = \frac{5}{12} = \frac{Perpendicular}{base}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(hypotenuse)^2 = (12)^2 + (5)^2 = 144 + 25 = 169$$

$$hypotenuse = \sqrt{169} = 13$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{5}{13}, \ \cos A = \frac{base}{hypotenuse} = \frac{12}{13},$$

$$\cot A = \frac{base}{perpendicular} = \frac{12}{5}, \sec A = \frac{hypotenuse}{base} = \frac{13}{12},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{13}{5}$$

(vi)

$$\sin A = \frac{\sqrt{3}}{2} = \frac{Perpendicular}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(base)^2 = (hypotenuse)^2 - (perpendicular)^2$$

$$(base)^2 = (2)^2 - \left(\sqrt{3}\right)^2 = 4 - 3 = 1$$

$$base = 1$$

$$\therefore \quad \cos A = \frac{base}{hypotenuse} = \frac{1}{2}, \ \tan A = \frac{perpendicular}{base} = \frac{\sqrt{3}}{1} = \sqrt{3},$$

$$\cot A = \frac{base}{perpendicular} = \frac{1}{\sqrt{3}}, \sec A = \frac{hypotenuse}{base} = \frac{2}{1} = 2,$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{2}{\sqrt{3}}$$

(vii)

$$\cos A = \frac{7}{25} = \frac{base}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(perpendicular)^2 = (hypotenuse)^2 - (base)^2$$

$$(perpendicular)^2 = (25)^2 - (7)^2$$

$$(perpendicular)^2 = 625 - 49 = 576$$

$$perpendicular = 24$$

$$\therefore \ \sin A = \frac{perpendicular}{hypotenuse} = \frac{24}{25}, \ \tan A = \frac{perpendicular}{base} = \frac{24}{7},$$

$$\cot A = \frac{base}{perpendicular} = \frac{7}{24}, \sec A = \frac{hypotenuse}{base} = \frac{25}{7},$$

$$\operatorname{cosec} A = \frac{hypotenuse}{Perpendicular} = \frac{25}{24}$$

(viii)

$\tan\theta = \frac{8}{15} = \frac{Perpendicular}{base}$

By pythagoras theorem,

$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$

$(hypotenuse)^2 = (15)^2 + (8)^2 = 225 + 64 = 289$

$hypotenuse = \sqrt{289} = 17$

$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{8}{17}, \ \cos A = \frac{base}{hypotenuse} = \frac{15}{17},$

$\cot A = \frac{base}{perpendicular} = \frac{15}{8}, \sec A = \frac{hypotenuse}{base} = \frac{17}{15},$

$cosecA = \frac{hypotenuse}{Perpendicular} = \frac{17}{8}$

(ix)

$\cot A = \frac{base}{perpendicular} = \frac{12}{5}$

By pythagoras theorem,

$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$

$(hypotenuse)^2 = (12)^2 + (5)^2 = 144 + 25 = 169$

$hypotenuse = \sqrt{169} = 13$

$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{5}{13}, \ \cos A = \frac{base}{hypotenuse} = \frac{12}{13},$

$\tan A = \frac{perpendicular}{base} = \frac{5}{12}, \sec A = \frac{hypotenuse}{base} = \frac{13}{12},$

$cosecA = \frac{hypotenuse}{Perpendicular} = \frac{13}{5}$

(x)

$\sec A = \frac{hypotenuse}{base} = \frac{13}{5}$

By pythagoras theorem,

$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$

$(perpendicular)^2 = (hypotenuse)^2 - (base)^2$

$(perpendicular)^2 = (13)^2 - (5)^2$

$(perpendicular)^2 = 169 - 25 = 144$

$perpendicular = 12$

$\therefore \ \sin A = \frac{perpendicular}{hypotenuse} = \frac{12}{13}, \ \cos A = \frac{base}{hypotenuse} = \frac{5}{13}$

$\tan A = \frac{perpendicular}{base} = \frac{12}{5}, \cot A = \frac{base}{perpendicular} = \frac{5}{12},$

$cosecA = \frac{hypotenuse}{Perpendicular} = \frac{13}{12}$

(xi)

$$cosec\theta = \frac{hypotenuse}{Perpendicular} = \frac{\sqrt{10}}{1}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(base)^2 = (hypotenuse)^2 - (perpendicular)^2$$

$$(base)^2 = (\sqrt{10})^2 - (1)^2 = 10 - 1 = 9$$

$$base = 3$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{1}{\sqrt{10}}, \cos A = \frac{base}{hypotenuse} = \frac{3}{\sqrt{10}},$$

$$\tan A = \frac{perpendicular}{base} = \frac{1}{3}, \cot A = \frac{base}{perpendicular} = \frac{3}{1} = 3,$$

$$\sec A = \frac{hypotenuse}{base} = \frac{\sqrt{10}}{3},$$

(xii)

$$\cos A = \frac{12}{15} = \frac{base}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$

$$(perpendicular)^2 = (hypotenuse)^2 - (base)^2$$

$$(perpendicular)^2 = (15)^2 - (12)^2$$

$$(perpendicular)^2 = 225 - 144 = 81$$

$$perpendicular = 9$$

$$\therefore \quad \sin A = \frac{perpendicular}{hypotenuse} = \frac{9}{15}, \quad \tan A = \frac{perpendicular}{base} = \frac{9}{12},$$

$$\cot A = \frac{base}{perpendicular} = \frac{12}{9}, \sec A = \frac{hypotenuse}{base} = \frac{15}{12},$$

$$cosecA = \frac{hypotenuse}{Perpendicular} = \frac{15}{9}$$

2. Question

In a ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determinei-sinA,cosAii-sinC, cosC

Answer

In a ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Therefore, By Pythagoras Theorem,

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 24^2 + 7^2$$
$$AC^2 = 576 + 49$$
$$AC^2 = 625$$
$$AC = 25$$

Therefore,

$$\sin A = \frac{BC}{AC} = \frac{7}{25}, \ \cos A = \frac{AB}{AC} = \frac{24}{25}$$

$$\sin C = \frac{AB}{AC} = \frac{24}{25}, \ \cos C = \frac{BC}{AC} = \frac{7}{25}$$

3. Question

In Fig. 5.37, find tan P and cot R. Is tan P = cot R?

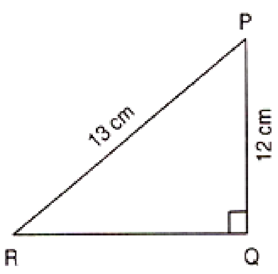


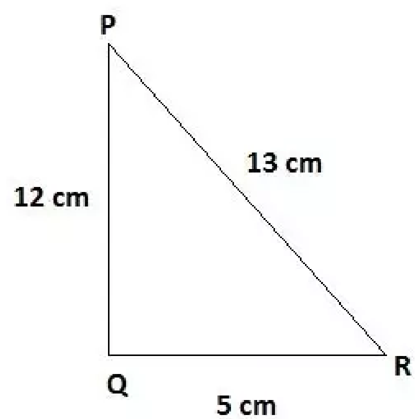
Fig. 5.37

Answer

By Pythagoras theorem we know that,(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2

$$PR^2 = PQ^2 + RQ^2$$
$$RQ^2 = PR^2 - PQ^2$$
$$RQ^2 = (13)^2 - (12)^2$$
$$RQ^2 = 169 - 144$$
$$RQ^2 = 25$$
$$RQ = 5$$

Now we have the figure as



we also know that, $\tan\,\theta = \dfrac{Perpendicular}{Base}$ $\cot\,\theta = \dfrac{Base}{Perpendicular}$

Note: When finding any trigonometric ratio, the main part is to decide perpendicular and base for that angle. Perpendicular is the side opposite to the angle for which we are calculating. For example from above figure if we are calculating sin R, then side opposite to R is PQ, So PQ will be perpendicular, PR is Hypotenuse and the side left out will be base.

$$\tan\,P = \frac{5}{12}$$

$$\cot\,R = \frac{5}{12}$$

4. Question

If $\sin A = \frac{9}{41}$, compute cos A and tan A.

Answer

$$\sin A = \frac{9}{41} = \frac{Perpendicular}{hypotenuse}$$

By pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$
$$(base)^2 = (41)^2 - (9)^2$$
$$(base)^2 = 1681 - 81 = 1600$$
$$base = 40$$
$$\therefore \quad \cos A = \frac{base}{hypotenuse} = \frac{40}{41},$$
$$\tan A = \frac{perpendicular}{base} = \frac{9}{40}$$

5. Question

Given 15 cot A =8, find sin A and sec A.

Answer

Given

$$15 \cot A = 8$$
$$\Rightarrow \quad \cot A = \frac{8}{15} = \frac{\text{base}}{\text{perpendicular}}$$

By Pythagoras theorem,

$$(hypotenuse)^2 = (base)^2 + (perpendicular)^2$$
$$(hypotenuse)^2 = (8)^2 + (15)^2$$
$$(hypotenuse)^2 = 64 + 225 = 289$$
$$hypotenuse = 17$$
$$\therefore \quad \sin A = \frac{perpendicular}{hypotenous} = \frac{15}{17}$$
$$\cos A = \frac{base}{hypotenous} = \frac{8}{17}$$

6. Question

In ΔPQR, right angled at Q, PQ = 4 cm and RQ = 3 cm. Find the values of sin P, sin R, sec P and sec R.

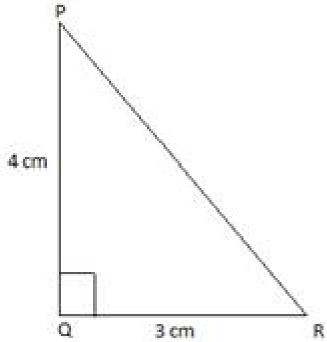
Answer

Given: In ΔPQR, right angled at Q, PQ = 4 cm and RQ = 3 cm.

To find: the values of sin P, sin R, sec P and sec R.

Solution: In triangle PQR, $\angle Q = 90^\circ$, PQ = 4 cm and RQ = 3 cm

By Pythagoras theorem,



$PR^2 = PQ^2 + RQ^2$
$RQ^2 = PR^2 - PQ^2$
$RQ^2 = (13)^2 - (12)^2$ Use the formula sinθ = perpendicular / hypotenuseSecθ = hypotenuse / base
$RQ^2 = 169 - 144$
$RQ^2 = 25$
$RQ = 5$

$\sin P = \frac{QR}{PR} = \frac{3}{5}, \sec P = \frac{PR}{PQ} = \frac{5}{4}, \sec R = \frac{PR}{PQ} = \frac{5}{3}$

$\sin R = \frac{PQ}{PR} = \frac{4}{5}$

NOTE: Always check on which angle you are asked to find any trignometricvalue and take perpendicular,base,hypotenuse accordingly.

7. Question

If $\cot\theta = \frac{7}{2}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Answer

(i)

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \cot^2\theta = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$$

(ii) $\cot^2\theta = \frac{49}{4}$

8. Question

If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer

$\cot A = \frac{4}{3} \quad \Rightarrow \tan A = \frac{3}{4}$

$$\therefore \quad \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$$= \frac{7/16}{25/16} = \frac{7}{16}\times\frac{16}{25} = \frac{7}{25}$$

And, $\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Thus, it is true.

9. Question

If $\tan\theta = \frac{a}{b}$, find the value of $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

Answer

Given: $\tan\theta = \frac{a}{b}$

To find: the value of $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

Solution: Take cosΘ common,And use the formula: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Solve,

$$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \frac{\cos\theta\left(1+\frac{\sin\theta}{\cos\theta}\right)}{\cos\theta\left(1-\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{(1+\tan\theta)}{(1-\tan\theta)}$$

$$= \frac{\left(1+\frac{a}{b}\right)}{\left(1-\frac{a}{b}\right)} = \frac{b+a}{b-a}$$

Thus, it is true.

10. Question

If 3 tanθ = 4, find the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

Answer

Given:3 tanθ = 4

To find: the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

Solution: $\tan\theta = \frac{P}{B}$

Here,3 tanθ = 4 $tan\theta = \frac{4}{3}$

Use the formula, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Solve,

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{4\cos\theta\left(1 - \frac{1}{4}\frac{\sin\theta}{\cos\theta}\right)}{2\cos\theta\left(1 + \frac{1}{2}\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{2\left(1 - \frac{1}{4}\tan\theta\right)}{\left(1 + \frac{1}{2}\tan\theta\right)}$$

$$= \frac{2\left(1 - \frac{4}{12}\right)}{\left(1 + \frac{4}{6}\right)} = 2 \times \frac{2/3}{5/3} = 2 \times \frac{2}{5} = \frac{4}{5}$$

11. Question

If 3 cot θ = 2, find the value of $\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$

Answer

$$\frac{4\sin\theta-3\cos\theta}{2\sin\theta+6\cos\theta}=\frac{4\sin\theta\left(1-\frac{3}{4}\frac{\cos\theta}{\sin\theta}\right)}{2\sin\theta\left(1+3\frac{\cos\theta}{\sin\theta}\right)}$$

$$=2\times\frac{\left(1-\frac{3}{4}\cot\theta\right)}{(1+3\cot\theta)}$$

$$=2\times\frac{\left(1-\frac{3}{4}\times\frac{2}{3}\right)}{\left(1+3\times\frac{2}{3}\right)}=2\times\frac{\left(1-\frac{1}{2}\right)}{(1+2)}=2\times\frac{\frac{1}{2}}{3}=2\times\frac{1}{6}=\frac{1}{3}$$

12. Question

If $\tan\theta=\frac{a}{b}$, prove that $\frac{a\sin\theta-b\cos\theta}{a\sin\theta+b\cos\theta}=\frac{a^2-b^2}{a^2+b^2}$

Answer

Given: $\tan\theta=\frac{a}{b}$

To prove: $\frac{a\sin\theta-b\cos\theta}{a\sin\theta+b\cos\theta}=\frac{a^2-b^2}{a^2+b^2}$ ...... (1)

Solution: Consider LHS of eq. (1)Take b cosθ common from both numerator and denominator.

$$\frac{a\ \sin\theta\ -\ b\ \cos\theta}{a\ sin\theta\ +\ b\ cos\theta}=\frac{\left(\frac{a\ sin\theta}{b\ cos\theta}-1\right)b\ cos\theta}{\left(\frac{a\ sin\theta}{b\ cos\theta}+1\right)b\ cos\theta}$$

Solve using the formula:

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\frac{a\ sin\theta\ -\ b\ cos\theta}{a\ sin\theta\ +\ b\ cos\theta}=\frac{\left(\frac{a}{b}\tan\theta-1\right)}{\left(\frac{a}{b}tan\theta+1\right)}$$

Put the value of tanθ to get,

$$\frac{a\ sin\theta\ -\ b\ cos\theta}{a\ sin\theta\ +\ b\ cos\theta}=\frac{\left(\frac{a}{b}\times\frac{a}{b}-1\right)}{\left(\frac{a}{b}\times\frac{a}{b}+1\right)}$$

$$\frac{a\ sin\theta\ -\ b\ cos\theta}{a\ sin\theta\ +\ b\ cos\theta}=\frac{\left(\frac{a^2}{b^2}-1\right)}{\left(\frac{a^2}{b^2}+1\right)}$$

$$\frac{a\ sin\theta\ -\ b\ cos\theta}{a\ sin\theta\ +\ b\ cos\theta}=\frac{\left(a^2-b^2\right)}{\left(a^2+b^2\right)}$$

hence proved

13. Question

If $\sec\theta=\frac{13}{5}$, show that $\frac{2\sin\theta-3\cos\theta}{4\sin\theta-9\cos\theta}=3$

Answer

$\sec\theta=\frac{13}{5}\qquad\Rightarrow\cos\theta=\frac{5}{13}$

$\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\left(\frac{5}{13}\right)^2}=\sqrt{1-\frac{25}{169}}=\sqrt{\frac{144}{169}}=\frac{12}{13}$

$\therefore\frac{2\sin\theta-3\cos\theta}{4\sin\theta-9\cos\theta}=\frac{2\times\frac{12}{13}-3\times\frac{5}{13}}{4\times\frac{12}{13}-9\times\frac{5}{13}}=\frac{\frac{24}{13}-\frac{15}{13}}{\frac{48}{13}-\frac{45}{13}}=\frac{9/13}{3/13}=3$

14. Question

If $\cos\theta=\frac{12}{13}$, show that $\sin\theta$ (1-$\tan\theta$) = $\frac{35}{156}$

Answer

Given: $\cos\theta=\frac{12}{13}$

To prove: $\sin\theta$ (1-$\tan\theta$) = $\frac{35}{156}$

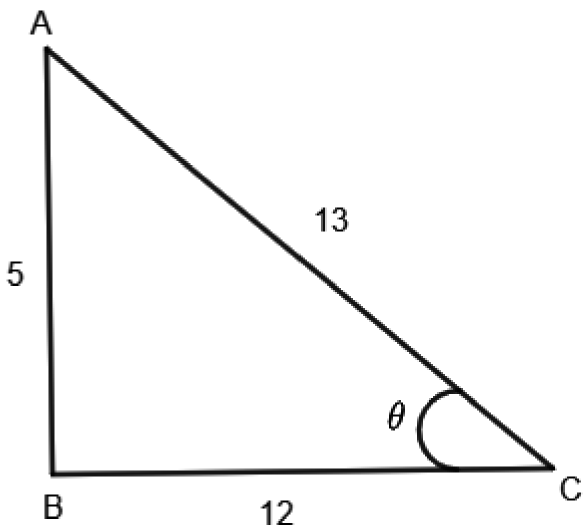
Proof: we know, $\cos\theta=\frac{B}{H}$

Where B is base and H is hypotenuse of the right angled triangle.We construct a right triangle ABC right angled at B such that $\angle ACB = \theta$

Perpendicular is AB, Base is BC = 12 and hypotenuse is AC = 13.In the triangle ABC,By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

13^2 = AB^2+$12^2$169 = AB^2+144169-144=$AB^2$25=AB^2 $AB = \sqrt{25} = 5$



$$\sin\theta = \frac{P}{H} = \frac{5}{13}$$

So,

$$\tan\theta = \frac{P}{B} = \frac{5}{12}$$

Put the values in sin θ (1-tan θ) to find its value,

$$\sin\theta(1 - \tan\theta) = \frac{5}{13}\left(1 - \frac{5}{12}\right) = \frac{5}{13} \times \frac{7}{12} = \frac{35}{156}$$ Hence Proved.

15. Question

If cot $\theta = \frac{1}{\sqrt{3}}$, show that $\frac{1 - \cos^2\theta}{2 - \sin^2\theta} = \frac{3}{5}$

Answer

$$\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{\sin^2\theta}{1+\cos^2\theta}$$

$$= \frac{1}{\frac{1+\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{1}{\operatorname{cosec}^2\theta + \cot^2\theta}$$

$$= \frac{1}{1+\cot^2\theta + \cot^2\theta}$$

$$= \frac{1}{1+2\cot^2\theta} = \frac{1}{1+2\times\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1}{1+2\times\frac{1}{3}} = \frac{1}{1+\frac{2}{3}} = \frac{3}{5}$$

Hence Proved.

16. Question

If tan $\theta = \frac{1}{\sqrt{7}}$, show that $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$

Answer

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$$

$$= \frac{1+\cot^2\theta - 1 - \tan^2\theta}{1+\cot^2\theta + 1 + \tan^2\theta}$$

$$= \frac{\cot^2\theta - \tan^2\theta}{2+\cot^2\theta + \tan^2\theta}$$

$$= \frac{(\sqrt{7})^2 - \left(\frac{1}{\sqrt{7}}\right)^2}{2+(\sqrt{7})^2 + \left(\frac{1}{\sqrt{7}}\right)^2}$$

$$= \frac{7-\frac{1}{7}}{2+7+\frac{1}{7}} = \frac{48/7}{64/7} = \frac{48}{64} = \frac{3}{4}$$

Hence Proved.

17. Question

If $\sin\theta = \frac{12}{13}$, find the value of $\frac{sin^2\theta - cos^2\theta}{2\, sin\theta\; cos\theta} \times \frac{1}{tan^2\theta}$

Answer

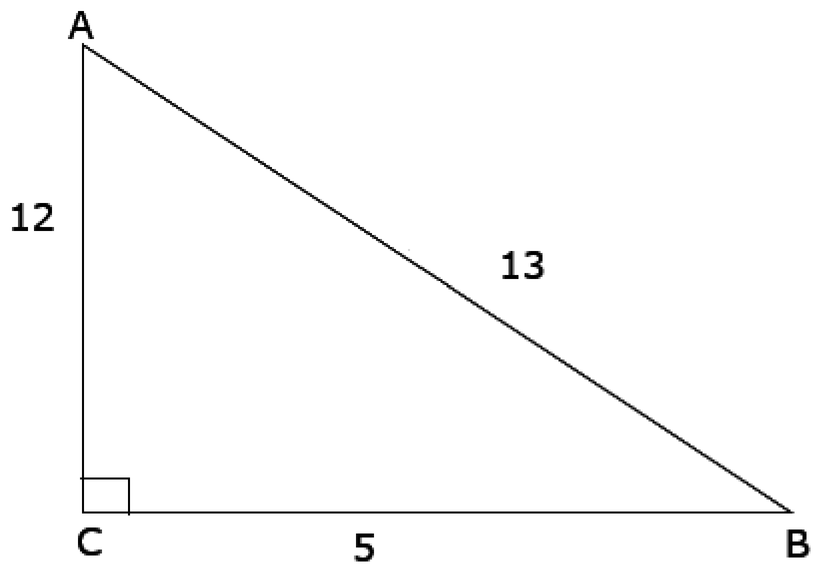
Given: $\sin\theta = \frac{12}{13}$

To find: the value of $\frac{sin^2\theta - cos^2\theta}{2\, sin\theta\; cos\theta} \times \frac{1}{tan^2\theta}$

Solution: Since $\sin\theta = \frac{perpendicular}{hypotenuse}$

So $\sin\theta = \frac{12}{13}$ implies:

Perpendicular = AC = 12, Hypotenuse = BC = 13Draw a right angled triangle at C,



By Pythagoras theorem,$AB^2 = AC^2 + BC^2$

⇒ $(13)^2 = (12)^2 + BC^2$

⇒ $BC^2 = (13)^2 - (12)^2$

⇒ $BC^2 = 169 - 144$

⇒ $BC^2 = 25$

$$\Rightarrow BC = \sqrt{25}$$

⇒ BC = 5Since cosθ = Base/Hypotenuse and tanθ = Perpendicular/Base

⇒ $\cos\theta = \frac{5}{13}\ and\ \tan\theta = \frac{12}{5}$

Substitute the known values in $\frac{\sin^2\theta - \cos^2\theta}{2\sin^2\theta\cos\theta} \times \frac{1}{\tan^2\theta}$,

$$\Rightarrow \frac{\sin^2\theta - \cos^2\theta}{2\ \sin\theta\ \cos\theta} \times \frac{1}{\tan^2\theta} = \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2 \times \frac{12}{13} \times \frac{5}{13}} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{\frac{144}{169} - \frac{25}{169}}{2 \times \frac{12}{13} \times \frac{5}{13}} \times \frac{25}{144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{\frac{144 - 25}{169}}{\frac{120}{169}} \times \frac{25}{144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{119}{120} \times \frac{25}{144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{119}{\cancel{120}_{24}} \times \frac{\cancel{25}^{5}}{144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{119 \times 5}{24 \times 144}$$

$$\Rightarrow \frac{sin^2\theta - cos^2\theta}{2\,sin\theta\,cos\theta} \times \frac{1}{tan^2\theta} = \frac{595}{3456}$$

18. Question

If $\sec\theta = \frac{5}{4}$, find the value of $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

Answer

$\sec\theta = \frac{5}{4} \quad \Rightarrow \quad \cos\theta = \frac{4}{5}$

$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$ *and* $\cot\theta = \frac{4}{3}$

$\therefore \quad \frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - 2 \times \frac{4}{5}}{\frac{3}{4} - \frac{4}{3}} = \frac{\frac{3}{5} - \frac{8}{5}}{\frac{3}{4} - \frac{4}{3}} = \frac{-1}{-\frac{7}{12}} = \frac{12}{7}$

19. Question

If $\cos\theta = \frac{5}{13}$, find the value of $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta}$

Answer

$\cos\theta = \frac{5}{13} \Rightarrow \cos^2\theta = \left(\frac{5}{13}\right)^2 = \frac{25}{169}$

$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$

$\tan^2\theta = \frac{\sin^2\theta}{\cos\theta} = \frac{144/169}{25/169} = \frac{144}{25}$

$$\frac{\sin^2\theta - \cos^2\theta}{2\sin^2\theta\cos\theta} \times \frac{1}{\tan^2\theta} = \frac{\frac{144}{169} - \frac{25}{169}}{2 \times \frac{144}{169} \times \frac{25}{169}} \times \frac{25}{144} = \frac{595}{3456}$$

20. Question

If $\tan\theta = \frac{12}{13}$, find the value of $\frac{2\sin\theta\cos\theta}{\cos^2\theta\sin^2\theta}$

Answer

$\tan\theta = \frac{12}{13} \Rightarrow \sin\theta = \frac{12}{\sqrt{313}}$ *and* $\cos\theta = \frac{13}{\sqrt{313}}$

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta\sin^2\theta} = \frac{2}{\cos\theta\sin\theta} = \frac{2}{\frac{13}{\sqrt{313}} \times \frac{12}{\sqrt{313}}} = \frac{2 \times 313}{60} = \frac{313}{30}$$

21. Question

If $\cos\theta = \frac{3}{5}$, find the value of $\frac{\sin\theta - \frac{1}{\tan\theta}}{2tan\theta}$

Answer

Given: $\cos\theta = \frac{3}{5}$

To find: the value of $\frac{\sin\theta - \frac{1}{\tan\theta}}{2tan\theta}$

Solution: We know:

$$\cos\theta = \frac{Base}{Hypotenuse}$$

By applying Pythagoras theorem,we have$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$

$\Rightarrow BC^2 = AB^2 + AC^2$

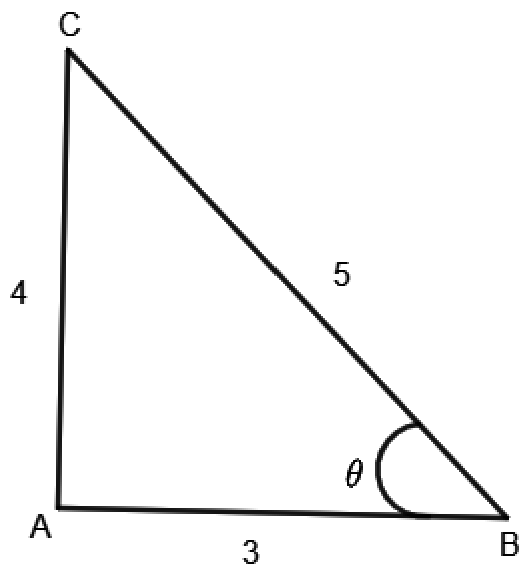
$\Rightarrow BC^2 = 3^2 + 4^2$

$\Rightarrow BC^2 = 9 + 16$

$\Rightarrow BC^2 = 25$

⇒ BC = $\sqrt{25}$

⇒ BC = 5



Use: $Sin\theta = \frac{Perpendicular}{Hypotenuse}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\Rightarrow \sin\theta = \frac{4}{5}$$

$$\Rightarrow \frac{\sin\theta - \frac{1}{\tan\theta}}{2tan\theta} = \frac{\frac{4}{5} - \frac{\cos\theta}{\sin\theta}}{2\frac{sin\theta}{cos\theta}}$$

Substitute the known values,

$$\Rightarrow \frac{sin\theta - \frac{1}{tan\theta}}{2tan\theta} = \frac{\frac{4}{5} - \frac{\frac{3}{5}}{\frac{4}{5}}}{2 \times \frac{\frac{4}{5}}{\frac{3}{5}}}$$

$$\Rightarrow \frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta} = \frac{\frac{4}{5} - \frac{3}{4}}{2 \times \frac{4}{3}}$$

$$\Rightarrow \frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta} = \frac{\frac{16-15}{20}}{\frac{8}{3}}$$

$$\Rightarrow \frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta} = \frac{\frac{1}{20}}{\frac{8}{3}}$$

$$\Rightarrow \frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta} = \frac{3}{20 \times 8}$$

$$\Rightarrow \frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta} = \frac{3}{160}$$

22. Question

If $\sin\theta = \frac{3}{5}$, find the value of $\frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta}$

Answer

$\sin\theta = \frac{3}{5}$

$$\frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta} = \frac{\cos\theta - \cot\theta}{2\cot\theta}$$

$$= \frac{1}{2}\left(\frac{\cos\theta}{\cot\theta} - 1\right)$$

$$= \frac{1}{2}(\sin\theta - 1)$$

$$= \frac{1}{2}\left(\frac{3}{5} - 1\right) = \frac{1}{2}\left(-\frac{2}{5}\right) = -\frac{1}{5}$$

23. Question

If $\sec A = \frac{5}{4}$, verify that $\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Answer

$\sec A = \frac{5}{4} \quad \Rightarrow \cos A = \frac{4}{5} \quad and \ \sin A = \frac{3}{5} \ \tan A = \frac{3}{4}$

$$\therefore \ \frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\times\frac{3}{5} - 4\times\frac{27}{125}}{4\times\frac{64}{125} - 3\times\frac{4}{5}}$$

$$= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{216}{125} - \frac{12}{5}} = \frac{\frac{225-108}{125}}{\frac{216-300}{125}} = -\frac{117}{84}$$

$$and \quad \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} = \frac{3\times\frac{3}{4} - \frac{27}{64}}{1 - 3\times\frac{9}{16}} = \frac{\frac{9}{4} - \frac{27}{64}}{1 - \frac{27}{16}} = \frac{\frac{144-27}{64}}{\frac{16-27}{64}} - \frac{117}{84}$$

24. Question

If $\sin\theta = \frac{3}{4}$, prove that $\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$

Answer

$\sin\theta = \frac{3}{4} \quad \Rightarrow \quad \cos\theta = \frac{\sqrt{7}}{4}$

$$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \sqrt{\frac{1+\cot^2\theta - \cot^2\theta}{1+\tan^2\theta - 1}}$$

$$= \sqrt{\frac{1}{\tan^2\theta}} = \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{7}}{4}\times\frac{4}{3} = \frac{\sqrt{7}}{3}$$

25. Question

If sec A = $\frac{17}{8}$, verify that $\frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A}$

Answer

$\sec A = \frac{17}{8} \quad \Rightarrow \cos A = \frac{8}{17} \quad \Rightarrow \sin A = \frac{15}{17} \ and \ \tan A = \frac{15}{8}$

$$\therefore \ \frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - 4\times\frac{225}{289}}{4\times\frac{64}{289} - 3} = \frac{3 - \frac{900}{289}}{\frac{216}{289} - 3} = \frac{-\frac{33}{289}}{-\frac{651}{289}} = \frac{33}{651} = \frac{11}{217}$$

$$\frac{3 - \tan^2 A}{1 - 3\tan^2 A} = \frac{3 - \frac{225}{64}}{1 - 3\times\frac{225}{64}} = \frac{11}{217}$$

26. Question

If $\cot\theta = \frac{3}{4}$, prove that $\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \frac{1}{\sqrt{7}}$

Answer

$\cot\theta = \frac{3}{4} \quad \Rightarrow \sin\theta = \frac{4}{5} \ \text{or} \ \operatorname{cosec}\theta = \frac{5}{4}$

$\text{and} \ \cos\theta = \frac{3}{5} \ \text{or} \ \sec\theta = \frac{5}{3}$

$$\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{\frac{5}{3} - \frac{5}{4}}{\frac{5}{3} + \frac{5}{4}}} = \sqrt{\frac{\frac{5}{12}}{\frac{35}{12}}} = \frac{1}{\sqrt{7}}$$

27. Question

If $\tan\theta = \frac{24}{7}$, find that $\sin\theta + \cos\theta$.

Answer

$\tan\theta = \frac{24}{7} \quad \Rightarrow \sin\theta = \frac{24}{25} \ \text{and} \ \cos\theta = \frac{7}{25}$

$\therefore \quad \sin\theta + \cos\theta = \frac{24}{25} + \frac{7}{25} = \frac{31}{25}$

28. Question

If $\sin\theta = \frac{a}{b}$, find $\sec\theta + \tan\theta$ in terms of a and b.

Answer

Given: $\sin\theta = \frac{a}{b}$

To find: $\sec\theta + \tan\theta$ in terms of a and b.

Solution:

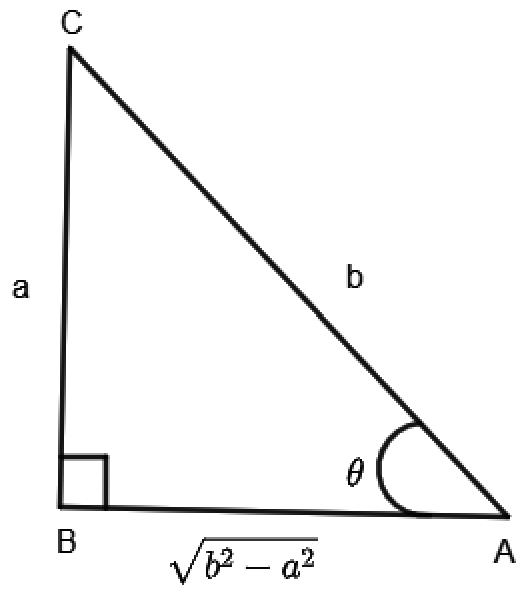
$\sin\theta = \frac{a}{b} \sin\theta = \frac{P}{H}$

We will construct a right angled triangle, right angled at B such that, $\angle BAC = \theta$
Perpendicular=BC=a and hypotenuse=AC=b$AC^2 = AB^2 + BC^2$

$b^2 = AB^2 + a^2$

$AB^2 = b^2 - a^2$

$$AB = \sqrt{b^2 - a^2}$$



Use the formula: $\cos\theta = \frac{B}{H},\ \tan\theta = \frac{P}{B}, \sec\theta = \frac{H}{B}$

Solve,

$$\sin\theta = \frac{a}{b} \quad \Rightarrow \cos\theta = \frac{\sqrt{b^2-a^2}}{b} \quad or \quad \sec\theta = \frac{b}{\sqrt{b^2-a^2}}$$

$$and\ \tan\theta = \frac{a}{\sqrt{b^2-a^2}}$$

$$\therefore \quad \sec\theta + \tan\theta = \frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}} = \sqrt{\frac{b+a}{b-a}}$$

29. Question

If 8 tan A = 15, find $\sin A - \cos A$

Answer

$$\tan A = \frac{15}{8} \qquad \Rightarrow \sin A = \frac{15}{17} \ \ and \ \ \cos A = \frac{8}{17}$$

$$\therefore \sin A - \cos A = \frac{15}{17} - \frac{8}{17} = \frac{7}{17}$$

30. Question

If $3\cos\theta - 4\sin\theta = 2\cos\theta + \sin\theta$, find $\tan\theta$

Answer

$$3\cos\theta - 4\sin\theta = 2\cos\theta + \sin\theta$$

$$3\cos\theta - 2\cos\theta = 4\sin\theta + \sin\theta$$

$$\cos\theta = 5\ \sin\theta$$

$$\therefore \qquad \tan\theta = \frac{1}{5}$$

31. Question

If $\tan\theta = \frac{20}{21}$, show that $\frac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta} = \frac{3}{7}$

Answer

Given: $\tan\theta = \frac{20}{21}$

To show: $\frac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta} = \frac{3}{7}$

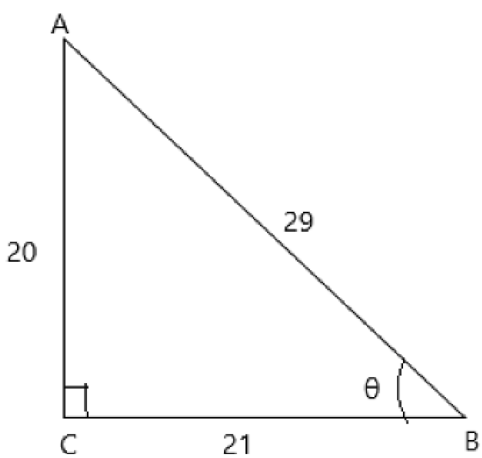
Solution: Since tanθ = perpendicular / baseSo we construct a right triangle ABC right angled at C such that∠ABC=θ and AC = Perpendicular = 20BC = base = 21By Pythagoras theorem,$AB^2 = AC^2 + BC^2$

⇒ $AB^2 = (20)^2 + (21)^2$

⇒ $AB^2 = 400 + 441$

⇒ $AB^2 = 841$

⇒ AB = √841⇒ AB = 29



As sinθ = perpendicular / hypotenusecosθ = base / hypotenuseSo,

$$\tan\theta = \frac{20}{21} \Rightarrow \sin\theta = \frac{20}{29} \ \textit{and} \ \cos\theta = \frac{21}{29}$$

$$\therefore \frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{1-\frac{20}{29}+\frac{21}{29}}{1+\frac{20}{29}+\frac{21}{29}} = \frac{\frac{30}{29}}{\frac{70}{29}} = \frac{3}{7}$$

Hence proved

32. Question

If $\cos ecA = 2$, find the value of $\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A}$

Answer

$$\operatorname{cosec} A = \frac{2}{1} \Rightarrow \sin A = \frac{1}{2}, \cos A = \frac{\sqrt{3}}{2}, \tan A = \sqrt{3}$$

$$\therefore \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{1}{\sqrt{3}} + \frac{1/2}{1+\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}} + \frac{1}{2} \times \frac{2}{2+\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} + \frac{1}{2+\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3}+1\right)}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3}+1\right)\left(2-\sqrt{3}\right)}{4-3} = 2$$

33. Question

If $\angle A$ *and* $\angle B$ are acute angles such that cos A = cos B, then show that $\angle A = \angle B$.

Answer

In a right angled triangle ABC,

$$\cos A = \frac{AC}{AB} \text{ and } \cos B = \frac{BC}{AB}$$

$$\because \cos A = \cos B$$

$$\frac{AC}{AB} = \frac{BC}{AB}$$

$$\therefore \quad AC = BC$$

We have, opposite sides of equal angles are equal. Therefore, In a right angled triangle ABC

$$\angle A = \angle B = 45^\circ$$

34. Question

If $\angle A$ *and* $\angle P$ are acute angles such that tan A = tan P, then show that $\angle A = \angle P$.

Answer

In a right angled triangle APQ,

$$\tan A = \frac{PQ}{AQ} \text{ and } \tan P = \frac{AQ}{PQ}$$

$$\because \tan A = \tan P$$

$$\frac{PQ}{AQ} = \frac{AQ}{PQ}$$

$$\therefore \quad PQ = AQ$$

$$\angle P = \angle A = 45^\circ$$

35. Question

In a ΔABC, right angled at A, if tan C = √3, find the value of sin B cos C + cos B sin C.

Answer

In a ΔABC, right angled at A,

tan C = √3

i.e ∠C = 60° and ∠B = 90 - 60 = 30°

Sin C = Sin 60° = √3/2

Cos C = Cos 60° = 1/2

Sin B = Sin 30° = 1/2

Cos B = Cos 30° = √3/2

According to the question,

sin B cos C + cos B sin C

= (1/2) (1/2) + (√3/2) (√3/2)

= 1/4 + 3/4

= 1

36. Question

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than (ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Answer

(i) The value of tan 90° is greater than 1. Therefore, given statement is false.

(ii) $\sec A = \frac{12}{5} \Rightarrow \cos A = \frac{5}{12}$ as 12 is the hypotenuse largest side. Therefore, given statement is true.

(iii) cos A is the abbreviation used for cosine of angle A Therefore, given statement is true.

(iv) cot A is not the product of cot and A. Therefore, given statement is false.

(v) Since, the hypotenuse is the longest side whereas in sin A = $\frac{4}{3}$, 3 which is the denominator and cannot be hypotenous.

Exercise 5.2

1. Question

Evaluate each of the following:

$\sin 45° \sin 30° + \cos 45° \cos 30°$

Answer

Given: $\sin 45° \sin 30° + \cos 45° \cos 30°$

To find: The value of above equation.

Solution: Use the values:

$$\sin 30^{\circ} = \frac{1}{2}, \cos 30^{\circ} = \frac{\sqrt{3}}{2}, sin 45^{\circ} = \frac{1}{\sqrt{2}}, \cos 45^{\circ} = \frac{1}{\sqrt{2}}$$

Solve, $\sin 45^{\circ} sin 30^{\circ} + \cos 45^{\circ} \cos 30^{\circ} = \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right)$

$$\Rightarrow sin 45^{\circ} sin 30^{\circ} + cos 45^{\circ} cos 30^{\circ} = \left(\frac{1}{2\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2\sqrt{2}}\right)$$

$$\Rightarrow sin 45^{\circ} sin 30^{\circ} + cos 45^{\circ} cos 30^{\circ} = \left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)$$

2. Question

Evaluate each of the following:

$\sin 60° \cos 30° + \cos 60° \sin 30°$

Answer

Given : $\sin 60° \cos 30° + \cos 60° \sin 30°$

To find : The value of $\sin 60° \cos 30° + \cos 60° \sin 30°$

Solution : Use the values:

$$sin 30^{\circ} = \frac{1}{2}, \sin 60^{\circ} = \frac{\sqrt{3}}{2}, \cos 30^{\circ} = \frac{\sqrt{3}}{2} and\ cos 60^{\circ} = \frac{1}{2}$$

Solve,
$$\begin{aligned} &\sin 60^{o} \cos 30^{o} + \cos 60^{o} \sin 30^{o} \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{4} + \frac{1}{4} = 1 \end{aligned}$$

+Hence the value of $\sin 60° \cos 30° + \cos 60° \sin 30°$ is 1.

3. Question

Evaluate each of the following:

cos60° cos45° - sin60° sin 45°

Answer

$\cos 60^{o} \cos 45^{o} - \sin 60^{o} \sin 45^{o}$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}$

4. Question

Evaluate each of the following:

$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

Answer

$\sin^2 30^{o} + \sin^2 45^{o} + \sin^2 60^{o} + \sin^2 190^{o}$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 = \frac{5}{2}$

5. Question

Evaluate each of the following:

$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

Answer

$\cos^2 30^{o} + \cos^2 45^{o} + \cos^2 60^{o} + \cos^2 90^{o}$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + (0)^2$

$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2}$

6. Question

Evaluate each of the following:

$\tan^2 30^\circ + \tan^2 60^\circ + \tan^2 45^\circ$

Answer

$\tan^2 30^{o} + \tan^2 60^{o} + \tan^2 45^{o}$

$= \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 + (1)^2$

$= \frac{1}{3} + 3 + 1 = \frac{13}{3}$

7. Question

Evaluate each of the following:

$2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$

Answer

$$2\sin^2 30^o - 3\cos^2 45^o + \tan^2 60^o$$
$$= 2\times\left(\frac{1}{2}\right)^2 - 3\times\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2 3$$
$$= 2\times\frac{1}{4} - 3\times\frac{1}{2} + 3$$
$$= \frac{1}{2} - \frac{3}{2} + 3 = 2$$

8. Question

Evaluate each of the following:

$$\sin^2 30^\circ \cos^2 45^\circ + 4\tan^2 30^\circ + \frac{1}{2}\sin^2 90^\circ \ \ -2\cos^2 90^\circ + \frac{1}{24}\cos^2 0^\circ$$

Answer

$$\sin^2 30^o \cos^2 45^o + 4\tan^2 30^o + \frac{1}{2}\sin^2 90^o - 2\cos^2 90^o + \frac{1}{24}\cos^2 0^o$$
$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\times\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}\times(1)^2 - 2\times(0)^2 + \frac{1}{24}\times(1)^2$$
$$= \frac{1}{4}\times\frac{1}{2} + \frac{4}{3} + \frac{1}{2} - 0 + \frac{1}{24}$$
$$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = 2$$

9. Question

Evaluate each of the following:

$$4(\sin^4 60^\circ + \cos^4 30^\circ) - 3(\tan^2 60^\circ - tan^2 45^\circ) \ \ +5\cos^2 45^\circ$$

Answer

Given: 4 ($\sin^4$ 60° + $\cos^4$ 30°)-3 ($\tan^2$ 60° - $\tan^2$ 45°) + 5$\cos^2$ 45°

To find: The value of 4 ($\sin^4$ 60° + $\cos^4$ 30°)-3 ($\tan^2$ 60° - $\tan^2$ 45°) + 5$\cos^2$ 45°.

Solution:

We know,

$$\sin\ 60^\circ\ = \frac{\sqrt{3}}{2}, \cos\ 30^\circ\ = \frac{\sqrt{3}}{2}, \cos\ 45^\circ\ = \frac{1}{\sqrt{2}}, \tan\ 45^\circ\ = 1, tan\ 60^\circ = \sqrt{3}$$

Substitute the above values in 4 ($\sin^4$ 60° + $\cos^4$ 30°)-3 ($\tan^2$ 60° - $\tan^2$ 45°) + 5$\cos^2$ 45°,

Solve,

$$4(\sin^4 60^o + \cos^4 30^o) - 3(\tan^2 60^o - tan^2 45^o) + 5\cos^2 45^o$$
$$= \left[4\left\{\left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{\sqrt{3}}{2}\right)^4\right\} - 3\left\{\left(\sqrt{3}\right)^2 - (1)^2\right\} + 5\times\left(\frac{1}{\sqrt{2}}\right)^2\right]$$
$$= \left[4\left\{\frac{9}{16} + \frac{9}{16}\right\} - 3\{3-1\} + 5\times\frac{1}{2}\right]$$
$$= \frac{18}{4} - 6 + \frac{5}{2} = 1$$

10. Question

Evaluate each of the following:

$(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)\ (\sin^2 30^\circ + 4\cot^2 45^\circ\ \ - \sec^2 60^\circ)$

Answer

$$\left(\operatorname{cosec}^2 45^o \sec^2 30^o\right)\left(\sin^2 30^o + 4\cot^2 45^o - \sec^2 60^o\right)$$
$$= \left(2\times\frac{2}{3}\right)\left(\frac{1}{4} + 4\times 1 - 4\right)$$
$$= \frac{4}{3}\left(\frac{1}{4} + 4\times 1 - 4\right) = \frac{1}{3}$$

11. Question

Evaluate each of the following:

$\operatorname{cosec}^3 30^\circ \cos 60^\circ \tan^3 45^\circ\ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$

Answer

$$\operatorname{cosec}^3 30^o \cos 60^o \tan^3 45^o \sin^2 90^o \sec^2 45^o \cot 30^o$$
$$= (2)^3 \times \frac{1}{2} \times (1)^3 \times (1)^2 \times \left(\sqrt{2}\right)^2 - \sqrt{3}$$
$$= 8\times\frac{1}{2}\times 1\times 1\times 2 - \sqrt{3}$$
$$= 8\sqrt{3}$$

12. Question

Evaluate each of the following:

$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$

Answer

$$\cot^2 30^o - 2\cos^2 60^o - \frac{3}{4}\sec^2 45^o - 4\sec^2 30^o$$
$$= \left(\sqrt{3}\right)^2 - 2\left(\frac{1}{2}\right)^2 - \frac{3}{4}\times\left(\sqrt{2}\right)^2 - 4\left(\frac{2}{\sqrt{3}}\right)^2$$
$$= 3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3} = -\frac{13}{3}$$

13. Question

Evaluate each of the following:(cos 0°+sin 45°+sin 30°) (sin 90° - cos 45° + cos 60°)

Answer

Given: (cos 0°+sin 45°+sin 30°) (sin 90° - cos 45° + cos 60°)

To find: The value of the above.

Solution: Use the formulas:

$$\sin 30^{\circ} = \frac{1}{2}, sin45^{\circ} = \frac{1}{\sqrt{2}}, sin90^{\circ} = 1$$

$$cos0^{\circ} = 1, cos45^{\circ} = \frac{1}{\sqrt{2}}, cos60^{\circ} = \frac{1}{2}$$

Solve,(cos 0°+sin 45°+sin 30°) (sin 90° - cos 45° + cos 60°)

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

$$= \left(\frac{2\sqrt{2} + 2 + \sqrt{2}}{2\sqrt{2}}\right)\left(\frac{2\sqrt{2} - 2 + \sqrt{2}}{2\sqrt{2}}\right)$$

$$= \left(\frac{3\sqrt{2} + 2}{2\sqrt{2}}\right)\left(\frac{3\sqrt{2} - 2}{2\sqrt{2}}\right)$$

$$= \frac{(3\sqrt{2} + 2)(3\sqrt{2} - 2)}{(2\sqrt{2})^2}$$

Use the identity: $(a-b)(a+b)=a^2-b^2$

$$= \frac{(3\sqrt{2})^2 - (2)^2}{(2\sqrt{2})^2}$$

$$= \frac{(9 \times 2) - 4}{(2\sqrt{2})^2}$$

$$= \frac{18 - 4}{8}$$

$$= \frac{14}{8}$$

$$= \frac{7}{2}$$

14. Question

Evaluate each of the following:

$$\frac{\sin 30^{\circ} - \sin 90^{\circ} + 2\cos 0^{\circ}}{\tan 30^{\circ} \tan 60^{\circ}}$$

Answer

$$\frac{\sin 30^{o} - \sin 90^{o} + 2\cos 0^{o}}{\tan 30^{o} \tan 60^{o}} = \frac{\frac{1}{2} - 1 + 2 \times 1}{\frac{1}{\sqrt{3}} \times \sqrt{3}} = \frac{3}{2}$$

15. Question

Evaluate each of the following:

$$\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cos^2 45^\circ$$

Answer

$$\frac{4}{\cot^2 30^o} + \frac{1}{\sin^2 60^o} - \cos^2 45^o = \frac{4}{\left(\sqrt{3}\right)^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{4}{3} + \frac{4}{3} - \frac{1}{2} = \frac{13}{6}$$

16. Question

Evaluate each of the following:

$$4(\sin^4 30^\circ + \cos^2 60^\circ) \quad -3(\cos^2 45^\circ \quad -\sin^2 90^\circ) \quad -\sin^2 60^\circ$$

Answer

$$4(\sin^4 30^o + \cos^2 60^o) - 3(\cos^2 45^o - \sin^2 90^o) - \sin^2 60^o$$
$$= 4\left(\frac{1}{16} + \frac{1}{4}\right) - 3\left(\frac{1}{2} - 1\right) - \frac{3}{4}$$
$$= \frac{20}{16} + \frac{3}{2} - \frac{3}{4}$$
$$= 2$$

17. Question

Evaluate each of the following:

$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\cos ec30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

Answer

$$\frac{\tan^2 60^o + 4\cos^2 45^o + 3\sec^2 30^o + 5\cos^2 90^o}{\cos ec30^o + \sec 60^o - \cot^2 30^o}$$
$$= \frac{\left(\sqrt{3}\right)^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 + 5\times(0)^2}{2 + 2 - \left(\sqrt{3}\right)^2}$$
$$= \frac{3+2+4}{2+2-3} = 9$$

18. Question

Evaluate each of the following:

$$\frac{\sin 30^\circ}{\sin 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} \quad - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

Answer

$$\frac{\sin 30^o}{\sin 45^o}+\frac{\tan 45^o}{\sec 60^o}-\frac{\sin 60^o}{\cot 45^o}-\frac{\cos 30^o}{\sin 90^o}$$
$$=\frac{1/2}{1/\sqrt{2}}+\frac{1}{2}-\frac{\sqrt{3}/2}{1}-\frac{\sqrt{3}/2}{1}$$
$$=\frac{1}{\sqrt{2}}+\frac{1}{2}-\frac{2\sqrt{3}}{2}=\frac{\sqrt{2}+1-2\sqrt{3}}{2}$$

19. Question

Evaluate each of the following:

$$\frac{\tan 45^\circ}{\operatorname{cosec} 30^\circ}+\frac{\sec 60^\circ}{\cot 45^\circ}-\frac{5\sin 90^\circ}{2\cos 0^\circ}$$

Answer

$$\frac{\tan 45^o}{\operatorname{cosec} 30^o}+\frac{\sec 60^o}{\cot 45^o}-\frac{5\sin 90^o}{2\cos 0^o}$$
$$=\frac{1}{2}+\frac{2}{1}-\frac{5\times 1}{2\times 1}$$
$$=\frac{1}{2}+2-\frac{5}{2}=0$$

20. Question

Find the value of x in each of the following:

$$2\sin 3x=\sqrt{3}$$

Answer

$$2\sin 3x=\sqrt{3}$$
$$\Rightarrow \sin 3x=\frac{\sqrt{3}}{2}$$
$$\Rightarrow \sin 3x=\sin 60^\circ$$
$$\Rightarrow 3x=60^\circ$$
$$\Rightarrow x=20^\circ$$

21. Question

Find the value of x in each of the following:

$$2\sin\frac{x}{2}=1$$

Answer

$$2\sin\frac{x}{2}=1$$
$$\Rightarrow \sin\frac{x}{2}=\frac{1}{2}$$
$$\Rightarrow \sin\frac{x}{2}=\sin 30^\circ$$
$$\Rightarrow \frac{x}{2}=30^\circ$$
$$\Rightarrow x=60^\circ$$

22. Question

Find the value of x in each of the following:

$\sqrt{3}\sin x = \cos x$

Answer

Given: $\sqrt{3}\sin x = \cos x$

To find: The value of $\sqrt{3}\sin x = \cos x$

Solution: Apply cross multiplication in the given expression,

$$\sqrt{3}\sin x = \cos x$$
$$\Rightarrow \quad \frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad \tan x = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad \tan x = \tan 30^\circ$$
$$\Rightarrow \quad x = 30^\circ$$

23. Question

Find the value of x in each of the following:

$\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

Answer

Given : $\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

To find : The value of x.

Solution : Put the values:

$$sin30^\circ = \frac{1}{2}, \sin45^\circ = \frac{1}{\sqrt{2}} \ and \ \cos45^\circ = \frac{1}{\sqrt{2}}$$

solve,

$$\tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$$
$$\Rightarrow \quad \tan x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$$
$$\Rightarrow \quad \tan x = \frac{1}{2} + \frac{1}{2}$$
$$\Rightarrow \quad \tan x = 1$$
$$\Rightarrow \quad \tan x = \tan 45^\circ$$
$$\Rightarrow \quad x = 45^\circ$$

24. Question

Find the value of x in each of the following:

$\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$

Answer

$$\sqrt{3}\tan 2x = \cos 60^o + \sin 45^o \cos 45^o$$

$$\Rightarrow \quad \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \quad \sqrt{3}\tan 2x = 1$$

$$\Rightarrow \quad \tan 2x = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan 2x = \tan 30^\circ$$

$$\Rightarrow \quad 2x = 30^\circ$$

$$\Rightarrow \quad x = 15^\circ$$

25. Question

Find the value of x in each of the following:

$\cos 2x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

Answer

$$\cos 2x = \cos 60^o \cos 30^o + \sin 60^o \sin 30^o$$

$$\Rightarrow \quad \cos 2x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$\Rightarrow \quad \cos 2x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$\Rightarrow \quad \cos 2x = \frac{2\sqrt{3}}{4}$$

$$\Rightarrow \quad \cos 2x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \cos 2x = \cos 30^\circ$$

$$\Rightarrow \quad 2x = 30^\circ$$

$$\Rightarrow \quad x = 15^\circ$$

26. Question

If $\theta = 30^\circ$, verify that:

(i) $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

(ii) $\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta}$

(iii) $\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

(iv) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Answer

(i) Use the values:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, tan 60^\circ = \sqrt{3}$$

L.H.S.

$$\tan 2\theta = \tan(2\times 30^\circ) = \tan 60^\circ = \sqrt{3}$$

R.H.S

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}}\times\frac{3}{2} = \sqrt{3}$$

Hence proved

(ii) Use the formula:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, tan30^\circ = \frac{1}{\sqrt{3}}$$

L.H.S.

$$\sin 2\theta = \sin(2\times 30^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

R.H.S

$$\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2/\sqrt{3}}{4/3} = \frac{2}{\sqrt{3}}\times\frac{3}{4} = \frac{\sqrt{3}}{2}$$

Hence proved

(iii) Use the formula,

$$\cos 60^\circ = \frac{1}{2}, tan30^\circ = \frac{1}{\sqrt{3}}$$

L.H.S.

$$\cos 2\theta = \cos(2\times 30^\circ) = \cos 60^\circ = \frac{1}{2}$$

R.H.S

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ} = \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2/3}{4/3} = \frac{2}{3}\times\frac{3}{4} = \frac{1}{2}$$

Hence proved

(iv) Use the formula, $cos30^\circ = \frac{\sqrt{3}}{2}, \cos 90^\circ = 1$

L.H.S.

$$\cos 3\theta = \cos(3\times 30^\circ) = \cos 90^\circ = 0$$

R.H.S

$$4\cos^3\theta - 3\cos\theta = 4\cos^3 30^\circ - 3\cos 30^\circ$$
$$= 4\times\left(\frac{\sqrt{3}}{2}\right)^3 - 3\times\frac{\sqrt{3}}{2}$$
$$= 4\times\frac{3\sqrt{3}}{8} - 3\times\frac{\sqrt{3}}{2} = 0$$

Hence proved

27. Question

If $A = B = 60^\circ$, verify that

(i) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

(ii) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

(iii) $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A\tan B}$

Answer

(i)

$$\cos(A-B) = \cos(60^\circ - 60^\circ) = \cos 0^\circ = 1$$
$$\cos A\cos B + \sin A\sin B = \cos 60^\circ\cos 60^\circ + \sin 60^\circ\sin 60^\circ$$
$$= \frac{1}{2}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$$
$$= \frac{1}{4} + \frac{3}{4} = 1$$

(ii)

$$\sin(A-B) = \sin(60^\circ - 60^\circ) = \sin 0^\circ = 0$$
$$\sin A\cos B - \cos A\sin B = \sin 60^\circ\cos 60^\circ - \cos 60^\circ\sin 60^\circ$$
$$= \frac{\sqrt{3}}{2}\times\frac{1}{2} - \frac{1}{2}\times\frac{\sqrt{3}}{2}$$
$$= 0$$

(iii)

$$\tan(A-B) = \tan(60^\circ - 60^\circ) = \tan 0^\circ = 0$$
$$\frac{\tan A - \tan B}{1+\tan A\tan B} = \frac{\tan 60^\circ - \tan 60^\circ}{1+\tan 60^\circ\tan 60^\circ} = \frac{\sqrt{3}-\sqrt{3}}{1-\sqrt{3}\sqrt{3}} = \frac{0}{1-3} = 0$$

28. Question

If A = 30° and B = 60°, verify that

(i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

(ii) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

Answer

(i) Use the formula,

$$\sin 30^\circ = \frac{1}{2}, sin60^\circ = \frac{\sqrt{3}}{2}, sin90^\circ = 1, \cos 30^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ = \frac{1}{2}$$

$$\sin(A+B) = \sin(30^\circ + 60^\circ) = \sin 90^\circ = 1$$

$$\sin A \cos B + \cos A \sin B = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = 1$$

Hence proved

(ii) Use the formula,

$$sin30^\circ = \frac{1}{2}, sin60^\circ = \frac{\sqrt{3}}{2}, \cos 90^\circ = 0, cos30^\circ = \frac{\sqrt{3}}{2}, cos60^\circ = \frac{1}{2}$$

$$\cos(A+B) = \cos(30^\circ + 60^\circ) = \cos 90^\circ = 0$$

$$\cos A \cos B - \sin A \sin B = \cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= 0$$

29. Question

If sin (A - B) = sin A cos B – cos A sin B and cos (A - B) = cos A cos B + sin A sin B, find the values of sin 15° and cos 15°.

Answer

For finding the values of sin 15º and cos 15°, we can split 15° into two angles such that,15° = 45° - 30° or we also can split 15° as 15° = 60° - 45°You can use either way, answer won't change.Formula to use for calculating this value is already given in the question,

sin(A - B) = sin A cos B - cos A sin B (1)

cos(A - B) = cos A cos B + sin A sin B (2)

Put A = 60° and B = 45°A - B = 15°Now put the values in formula 1 and 2,

$$\sin 15^\circ = \sin(60^\circ - 45^\circ)$$

$$= \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

And,

$$\cos 15^\circ = \cos(60^\circ - 45^\circ)$$
$$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ$$
$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Thus we have,

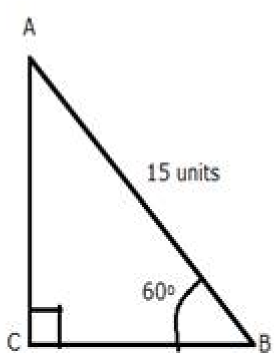
$$\sin 15^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$\cos 15^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

30. Question

In a right triangle ABC, right angled at C, if $\angle B$ = 60° and AB = 15 units. Find the remaining angles and sides.

Answer



In a right triangle ABC, right angled at C, if $\angle B$ = 60° and AB = 15 units. Therefore,

$$\sin 60^\circ = \frac{AC}{AB}$$
$$\frac{\sqrt{3}}{2} = \frac{AC}{15}$$
$$AC = \frac{15}{2}\sqrt{3} \text{ units}$$

And,

$$\cos 60^\circ = \frac{BC}{AB}$$
$$\frac{1}{2} = \frac{BC}{15}$$
$$BC = \frac{15}{2} = 7.5 \text{units}$$

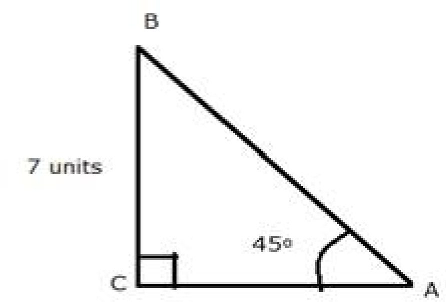
$$\angle A = 180^\circ - (90^\circ + 60^\circ)$$
$$= 180^\circ - 150^\circ$$
$$= 30^\circ$$

31. Question

If ΔABC is a right triangle such that $\angle C = 90°$, $\angle A = 45°$ and BC = 7 units. Find $\angle B$, AB and AC.

Answer

If ΔABC is a right triangle such that $\angle C = 90°$, $\angle A = 45°$ and BC = 7 units. Therefore,



$$\sin A = \frac{BC}{AB}$$
$$\sin 45^\circ = \frac{AC}{AB}$$
$$\frac{1}{\sqrt{2}} = \frac{7}{AB}$$
$$AB = 7\sqrt{2} \text{ units}$$

And,

$$\cos A^\circ = \frac{AC}{AB}$$
$$\cos 45^\circ = \frac{AC}{AB}$$
$$\frac{1}{\sqrt{2}} = \frac{AC}{7\sqrt{2}}$$
$$AC = 7 \text{units}$$

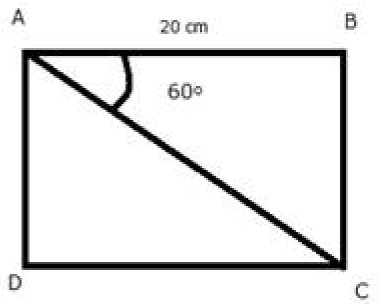
$$\angle B = 180^\circ - (90^\circ + 45^\circ)$$
$$= 180^\circ - 135^\circ$$
$$= 45^\circ$$

32. Question

In a rectangle ABCD, AB = 20 cm, $\angle BAC$ = 60°, calculate side BC and diagonals AC and BD.

Answer

If rectangle ABCD AB = 20 cm, $\angle BAC$ = 60°. Therefore,



$$\tan A = \frac{BC}{AB}$$
$$\tan 60^\circ = \frac{BC}{20}$$
$$\sqrt{3} = \frac{BC}{20}$$
$$BC = 20\sqrt{3}\ \text{cm}$$

And,

$$\cos 60^\circ = \frac{AB}{AC}$$
$$\frac{1}{2} = \frac{20}{AC}$$
$$AC = 40\ \text{cm}$$

Therefore, AC = BD = 40 cm

33. Question

If sin (A + B) = 1 and cos (A – B) = 1, 0° < A + B ≤90°, A ≥ B find A and B.

Answer

Given: sin (A + B) = 1 and cos (A – B) = 1

sin(A+B) = 1

Also, we know, sin 90^0 = 1

⇒ sin(A+B) = sin 90^0

or (A+B) = 90^0 ....(1)

Now, cos (A – B) = 1

And, we know, cos 0^0 = 1

⇒ (A - B) = 0^0 ....(2)

On solving both equations (1) and (2) , we get

2A = 90^0

or A = 90^0/2

or **A = 45°**

Similarly, **B = 45°**

34. Question

If tan (A – B) = $\frac{1}{\sqrt{3}}$ and tan (A + B) = $\sqrt{3}$, 0° < A + B ≤90°, A > B find A and B.

Answer

$$\tan(A-B)=\frac{1}{\sqrt{3}}$$
$$\tan(A-B)=\tan 30^\circ$$
$$A-B=30^\circ \ldots\ldots (1)$$
$$\tan(A+B)=\sqrt{3}$$
$$\tan(A+B)=\tan 60^\circ$$
$$A+B=60^\circ \ldots\ldots (2)$$

On solving both equations, we get

A = 45° and B = 15°

35. Question

If sin (A – B) = $\frac{1}{2}$ and cos (A + B) = $\frac{1}{2}$, 0° < A + B ≤90°, A < B find A and B.

Answer

$$\sin(A-B)=\frac{1}{2}$$
$$\sin(A-B)=\sin 30^\circ$$
$$A-B=30^\circ \ldots\ldots (1)$$
$$\cos(A+B)=\frac{1}{2}$$
$$\cos(A+B)=\cos 60^\circ$$
$$A+B=60^\circ \ldots\ldots (2)$$

On solving both equations, we get

A = 45°, B = 15°

36. Question

In a ΔABC right angled at B, $\angle A=\angle C$. Find the values of

(i) $\sin A\cos C+\cos A\sin C$

(ii) $\sin A\sin B+\cos A\cos B$

Answer

In a ΔABC right angled at B, $\angle A=\angle C$, therefore,

$$\angle A=\angle C=45^\circ$$

(i)

$$\sin A\ \cos C + \cos A\ \sin C = \sin(45^\circ + 45^\circ)$$
$$= \sin 90^\circ = 1$$

(ii)

$$\sin A\ \sin B + \cos A\ \cos B = \cos(A + B)$$
$$= \cos(90^\circ + 45^\circ)$$
$$= \sin(45^\circ)$$
$$= \frac{1}{\sqrt{2}}$$

37. Question

Find acute angles A and B, if sin (A + 2B) = $\frac{\sqrt{3}}{2}$ and cos (A + 4B) = 0, A > B.

Answer

Given: sin (A + 2B) = $\frac{\sqrt{3}}{2}$ and cos (A + 4B) = 0

To find: The values of acute angles A and B.

Solution: We know,

$\sin 60^\circ = \frac{\sqrt{3}}{2}\ and\ \cos 90^\circ = 0$ So,

$$\sin(A + 2B) = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \quad \sin(A + 2B) = \sin 60^\circ$$
$$\Rightarrow \quad A + 2B = 60^\circ \quad \ldots.(1)$$
$$\text{and} \quad \cos(A + 4B) = \cos 90^\circ$$
$$\Rightarrow \quad A + 4B = 90^\circ \quad \ldots.(2)$$

Solve eq. (1) and eq. (2) to get the values of a and b.

Subtract eq. (1) from eq. (2),⇒A+4B-(A+2B)=90°-60°

⇒A+4B-A-2B=90°-60°

⇒2B=30° ⇒B=15° Substitute the value of B in eq. (1) to get,⇒A + 2B =60° ⇒A + 30°=60°⇒A =60°-30°⇒A =30°

Hence the values of A and B are A = 30°, B = 15°

38. Question

If A and B ae acute angles such that tan A = $\frac{1}{2}$, tan B = $\frac{1}{3}$ and tan (A + B) = $\frac{\tan A + \tan B}{1 - \tan A \tan B}$, find A + B.

Answer

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan(A+B)=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$$

$$\tan(A+B)=\frac{\frac{5}{6}}{1-\frac{1}{6}}=\frac{\frac{5}{6}}{\frac{5}{6}}=1$$

$$\therefore \quad \tan(A+B)=\tan 45^\circ$$

$$\Rightarrow \quad A+B=45^\circ$$

39. Question

In ΔPQR, right-angled at Q, PQ = 3 cm and PR = 6 cm. Determine $\angle P$ *and* $\angle R$.

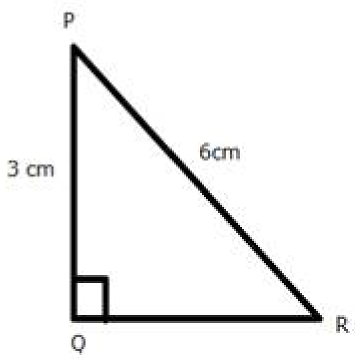
Answer

Given: In ΔPQR, right-angled at Q, PQ = 3 cm and PR = 6 cm.

To find: $\angle P$ *and* $\angle R$

Solution:

Take right angle ΔPQR right angled at Q.



Since,

$$\sin\theta=\frac{P}{H}$$

Here,

$$\sin R=\frac{PQ}{PR}$$

$$sin R=\frac{3}{6}=\frac{1}{2}$$

since, $\sin 30^\circ=\frac{1}{2}$

So, sin R = sin 30°

⇒ R = 30°

Now, in ΔPQRAs sum of all angles of a triangle is 180°.∠P = 180° - (90° + 30°)∠P = 180° - 120° ∠P = 60°

Hence the value of ∠R is 30° and ∠P is 60°.

Exercise 5.3

1. Question

Evaluate the following:

(i) $\frac{\sin 20°}{\cos 70°}$ (ii) $\frac{\cos 19°}{\sin 71°}$

(iii) $\frac{\sin 21°}{\cos 69°}$ (iv) $\frac{\tan 10°}{\cot 80°}$

(v) $\frac{\sec 11°}{\cos ec 79°}$

Answer

Use:

$$\sin\left(90^{\circ} - \theta\right) = \cos\theta$$

$$cos\left(90^{\circ} - \theta\right) = \sin\theta$$

$$\tan\left(90^{\circ} - \theta\right) = \cot\theta$$

$$cot\left(90^{\circ} - \theta\right) = tan\theta$$

$$\sec\left(90^{\circ} - \theta\right) = \mathrm{cosec}\theta$$

$$cosec\left(90^{\circ} - \theta\right) = sec\theta$$

(i) $\frac{\sin 20^{o}}{\cos 70^{o}} = \frac{\sin(90^{o} - 70^{o})}{\cos 70^{o}} = \frac{\cos 70^{o}}{\cos 70^{o}} = 1$

(ii) $\frac{\cos 19^{o}}{\sin 71^{o}} = \frac{\cos(90^{o} - 71^{o})}{\sin 71^{o}} = \frac{\sin 71^{o}}{\sin 71^{o}} = 1$

(iii) $\frac{\sin 21^{o}}{\cos 69^{o}}=\frac{\sin(90^{o}-69^{o})}{\cos 69^{o}}=\frac{\cos 69^{o}}{\cos 69^{o}}=1$

(iv) $\frac{\tan 10^{o}}{\cot 80^{o}}=\frac{\tan(90^{\circ}-80^{o})}{\cot 80^{o}}=\frac{\cot 80^{\circ}}{\cot 80^{o}}=1$

(v) $\frac{\sec 11^{o}}{\operatorname{cosec} 79^{o}}=\frac{\sec(90^{\circ}-79^{o})}{\operatorname{cosec} 79^{o}}=\frac{\operatorname{cosec} 79^{o}}{\operatorname{cosec} 79^{o}}=1$

2. Question

Evaluate the following:

(i) $\left(\frac{\sin 49^{\circ}}{\cos 41^{\circ}}\right)^{2}+\left(\frac{\cos 41^{\circ}}{\sin 49^{\circ}}\right)^{2}$

(ii) $\cos 48^{\circ}-\sin 42^{\circ}$

(iii) $\frac{\cot 40^{\circ}}{\tan 50^{\circ}}-\frac{1}{2}\left(\frac{\cos 35^{\circ}}{\sin 55^{\circ}}\right)$

(iv) $\left(\frac{\sin 27^{\circ}}{\cos 63^{\circ}}\right)^{2}-\left(\frac{\cos 63^{\circ}}{\sin 27^{\circ}}\right)^{2}$

(v) $\frac{\tan 35^{\circ}}{\cot 55^{\circ}}+\frac{\cot 78^{\circ}}{\tan 12^{\circ}}-1$

(vi) $\frac{\sec 70^{\circ}}{\operatorname{cosec} 20^{\circ}}+\frac{\sin 59^{\circ}}{\cos 31^{\circ}}$

(vii) cosec 31° - sec 59°

(viii) $(\sin 72^{\circ}+\cos 18^{\circ})(\sin 72^{\circ}-\cos 18^{\circ})$

(ix) $\sin 35^{\circ}\sin 55^{\circ}-\cos 35^{\circ}\cos 55^{\circ}$

(x) $\tan 48^{\circ}\tan 23^{\circ}\tan 42^{\circ}\tan 67^{\circ}$

(xi) $\sec 50^{\circ}\sin 40^{\circ}+\cos 40^{\circ}\operatorname{cosec} 50^{\circ}$

Answer

(i)

$$\left(\frac{\sin 49^{o}}{\cos 41^{o}}\right)^{2}+\left(\frac{\cos 41^{o}}{\sin 49^{o}}\right)^{2}$$
$$=\left(\frac{\sin(90^{o}-41^{o})}{\cos 41^{o}}\right)^{2}+\left(\frac{\cos(90^{o}-49^{o})}{\sin 49^{o}}\right)^{2}$$
$$=\left(\frac{\cos 41^{\circ}}{\cos 41^{o}}\right)^{2}+\left(\frac{\sin 49^{\circ}}{\sin 49^{o}}\right)^{2}$$
$$=(1)^{2}+(1)^{2}=2$$

(ii)

$$\cos 48^\circ - \sin 42^\circ = \cos 48^\circ - \sin(90^\circ - 48^\circ)$$
$$= \cos 48^\circ - \cos 48^\circ$$
$$= 0$$

(iii)

$$\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$$
$$= \frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$$
$$= \frac{\cot(90^\circ - 40^\circ)}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos(90^\circ - 35^\circ)}{\sin 55^\circ}\right)$$
$$= \frac{\tan 50^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\sin 55^\circ}{\sin 55^\circ}\right)$$
$$= 1 - \frac{1}{2} \times 1 = \frac{1}{2}$$

(iv)

$$\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2 = \left(\frac{\sin(90^\circ - 63^\circ)}{\cos 63^\circ}\right)^2 - \left(\frac{\cos(90^\circ - 27^\circ)}{\sin 27^\circ}\right)^2$$
$$= \left(\frac{\cos 63^\circ}{\cos 63^\circ}\right)^2 - \left(\frac{\sin 27^\circ}{\sin 27^\circ}\right)^2$$
$$= (1)^2 - (1)^2 = 0$$

(v)

$$\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$$
$$= \frac{\tan(90^\circ - 55^\circ)}{\cot 55^\circ} + \frac{\cot(90^\circ - 12^\circ)}{\tan 12^\circ} - 1$$
$$= \frac{\cot 55^\circ}{\cot 55^\circ} + \frac{\tan 12^\circ}{\tan 12^\circ} - 1$$
$$= 1 + 1 - 1 = 0$$

(vi)

$$\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ} = \frac{\sec(90^\circ - 20^\circ)}{\operatorname{cosec} 20^\circ} + \frac{\sin(90^\circ - 31^\circ)}{\cos 31^\circ}$$
$$= \frac{\operatorname{cosec} 20^\circ}{\operatorname{cosec} 20^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ}$$
$$= 1 + 1 = 2$$

(vii)

$$\operatorname{cosec} 31^\circ - \sec 59^\circ = \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ$$
$$= \sec 59^\circ - \sec 59^\circ$$
$$= 0$$

(viii)

$$
\begin{aligned}
&\left(\sin 72^\circ+\cos 18^\circ\right)\left(\sin 72^\circ-\cos 18^\circ\right)\\
&=\left\{\sin\left(90^\circ-72^\circ\right)+\cos 18^\circ\right\}\left\{\sin\left(90^\circ-72^\circ\right)-\cos 18^\circ\right\}\\
&=\left\{\cos 18^\circ+\cos 18^\circ\right\}\left\{\cos 18^\circ-\cos 18^\circ\right\}\\
&=2\times 0\\
&=0
\end{aligned}
$$

(ix)

$$
\begin{aligned}
&\sin 35^\circ \sin\left(90^\circ-35^\circ\right)-\cos 35^\circ \cos\left(90^\circ-55^\circ\right)\\
&=\sin 35^\circ \cos 35^\circ-\cos 35^\circ \sin 35^\circ\\
&=\sin\left(35^\circ-35^\circ\right)\\
&=\sin 0^\circ=0
\end{aligned}
$$

(x)

$$
\begin{aligned}
&\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ\\
&=\tan 48^\circ \tan 23^\circ \tan\left(90^\circ-48^\circ\right)\tan\left(90^\circ-23^\circ\right)\\
&=\tan 48^\circ \tan 23^\circ \cot 48^\circ \cot 23^\circ\\
&=\left(\tan 48^\circ \cot 48^\circ\right)\left(\tan 23^\circ \cot 23^\circ\right)\\
&=1\times 1=1
\end{aligned}
$$

(xi)

$$
\begin{aligned}
&\sec 50^\circ \sin 40^\circ+\cos 40^\circ \operatorname{cosec} 50^\circ\\
&=\sec 50^\circ \sin\left(90^\circ-50^\circ\right)+\cos\left(90^\circ-50^\circ\right)\operatorname{cosec} 50^\circ\\
&=\frac{1}{\cos 50^\circ}\cos 50^\circ+\sin 50^\circ\frac{1}{\sin 50^\circ}\\
&=1+1=2
\end{aligned}
$$

3. Question

Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°

(i) $\sin 59^\circ+\cos 56^\circ$

(ii) $\tan 65^\circ+\cot 49^\circ$

(iii) $\sec 76^\circ+\operatorname{cosec} 52^\circ$

(iv) $\cos 78^\circ+\sec 78^\circ$

(v) $\operatorname{cosec} 54^\circ+\sin 72^\circ$

(vi) $\cot 85^\circ+\cos 75^\circ$

(vii) $\sin 67^\circ+\cos 75^\circ$

Answer

Use the formula:

$$\sin\left(90^\circ-\theta\right)=\cos\theta$$

$$cos\left(90^{\circ} - \theta\right) = \sin\theta$$

$$\tan\left(90^{\circ} - \theta\right) = \cot\theta$$

$$cot\left(90^{\circ} - \theta\right) = \tan\theta$$

$$\sec\left(90^{\circ} - \theta\right) = \text{cosec}\theta$$

$$cosec\left(90^{\circ} - \theta\right) = sec\theta$$

(i)

$$\begin{aligned} \sin 59^{o} + \cos 56^{o} &= \sin\left(90^{\circ} - 31^{o}\right) + \cos\left(90^{\circ} - 34^{o}\right) \\ &= \cos 31^{o} + \sin 34^{o} \end{aligned}$$

(ii)

$$\begin{aligned} &\tan 65^{o} + \cot 49^{o} \\ &= \tan\left(90^{\circ} - 25^{o}\right) + \cot\left(90^{\circ} - 41^{o}\right) \\ &= \cot 25^{o} + \tan 41^{o} \end{aligned}$$

(iii)

$$\begin{aligned} &\sec 76^{\circ} + \text{cosec} 52^{\circ} \\ &= \sec\left(90^{\circ} - 14^{\circ}\right) + \text{cosec}\left(90^{\circ} - 52^{\circ}\right) \\ &= \text{cosec} 14^{o} + \sec 38^{o} \end{aligned}$$

(iv)

$$\begin{aligned} &\cos 78^{\circ} + \sec 78^{\circ} \\ &= \cos\left(90^{\circ} - 78^{\circ}\right) + \sec\left(90^{\circ} - 78^{\circ}\right) \\ &= \sin 12^{o} + \text{cosec} 12^{o} \end{aligned}$$

(v)

$$\begin{aligned} &\text{cosec} 54^{o} + \sin 54^{o} \\ &= \text{cosec}\left(90^{\circ} - 54^{o}\right) + \sin\left(90^{\circ} - 72^{o}\right) \\ &= \sec 36^{o} + \cos 18^{o} \end{aligned}$$

(vi)

$$\begin{aligned} &\cot 85^{\circ} + \cos 75^{\circ} \\ &= \cot\left(90^{\circ} - 85^{\circ}\right) + \cos\left(90^{\circ} - 75^{\circ}\right) \\ &= \tan 5^{o} + \sin 15^{o} \end{aligned}$$

(vii)

$$\sin 77^\circ + \cos 75^\circ$$
$$= \sin(90^\circ - 77^\circ) + \cos(90^\circ - 75)$$
$$= \cos 23^o + \sin 15^o$$

4. Question

Express $\cos 75^\circ + \cot 75^\circ$ in terms of angles between 0° and 30°.

Answer

Given : $\cos 75^\circ + \cot 75^\circ$

To find : Expression in terms of angles between 0° and 30°.

Solution : Use the values:

$$\cos\left(90^\circ - \theta\right) = \sin\theta$$

$$\cot\left(90^\circ - \theta\right) = \tan\theta$$

Solve,

$$\cos 75^o + \cot 75^o = \cos(90^\circ - 15^\circ) + \cot(90^\circ - 15^\circ)$$
$$= \sin 15^\circ + \tan 15^\circ$$

5. Question

If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A.

Answer

$$\sin 3A = \cos(A - 26^\circ)$$
$$\cos(90^\circ - 3A) = \cos(A - 26^\circ)$$
$$90^\circ - 3A = A - 26^\circ$$
$$4A = 116^\circ$$
$$A = \frac{116^\circ}{4} = 29^\circ$$

6. Question

If A, B, C, are the interior angles of a triangle ABC, prove that

(i) $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

(ii) $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

Answer

(i) Since, A, B, C, are the interior angles of a triangle ABC.

Therefore,

$$A + B + C = 180^\circ$$
$$\Rightarrow \quad A + C = 180^\circ - B$$
$$\Rightarrow \quad \frac{A + C}{2} = \frac{180^\circ - B}{2}$$
$$\Rightarrow \quad \tan\left(\frac{A + C}{2}\right) = \tan\left(90^\circ - \frac{B}{2}\right)$$
$$\Rightarrow \quad \tan\left(\frac{A + C}{2}\right) = \cot\left(\frac{B}{2}\right)$$

Hence proved.

(ii)

$$A + B + C = 180^\circ$$
$$\Rightarrow \quad A + C = 180^\circ - B$$
$$\Rightarrow \quad \frac{A + C}{2} = \frac{180^\circ - B}{2}$$
$$\Rightarrow \quad \sin\left(\frac{A + C}{2}\right) = \sin\left(90^\circ - \frac{B}{2}\right)$$
$$\Rightarrow \quad \sin\left(\frac{A + C}{2}\right) = \cos\left(\frac{B}{2}\right)$$

Hence proved.

7. Question

Prove that:

(i) $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1$

(ii) $\sin 48^\circ \sec 42^\circ + \cos 48^\circ \cos ec 42^\circ = 2$

(iii) $\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\cos ec 20^\circ}{\sec 70^\circ} - 2\cos 70^\circ \cos ec 20^\circ = 0$

(iv) $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \cos ec 31^\circ = 2$

Answer

(i)

we know,

tan(90 - A) = cot Aand cotA tanA = 1By using above concepts, we can solve the question as:Consider

LHS,$\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = \tan(90^\circ - 70^\circ)\tan(90^\circ - 55^\circ)\tan 45^\circ \tan 55^\circ \tan 70^\circ$

$\Rightarrow \tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = \cot 70^\circ \cot 55^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ$

$\Rightarrow \tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = (\cot 70^\circ \tan 70^\circ)(\cot 55^\circ \tan 55^\circ)\tan 45^\circ$Since $\tan 45^\circ = 1$,

$\Rightarrow \tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1 \times 1 \times 1$

Which is equal to RHS.

Hence Proved

(ii)

we know, sec(90 - A) = cosec Aand sinA.cosecA = 1By using above concepts, we can solve the question as

$$\begin{aligned} & \sin 48^o \sec 42^o + \cos 48^o \operatorname{cosec} 42^o \\ & = \sin 48^o \sec\left(90^\circ - 48^o\right) + \cos 48^o \operatorname{cosec}\left(90^\circ - 48^o\right) \\ & = \sin 48^o \operatorname{cosec} 48^\circ + \cos 48^o \sec 48^\circ \\ & = \sin 48^o \frac{1}{\sin 48^\circ} + \cos 48^o \frac{1}{\cos 48^\circ} \\ & = 1 + 1 = 2 \end{aligned}$$

Proved

(iii)

we know, sin(90 - A) = cos A andcosec(90 - A) = sec A, By using this information we can solve our question as

$$\begin{aligned} & \frac{\sin 70^o}{\cos 20^o} + \frac{\operatorname{cosec} 20^o}{\sec 70^o} - 2\cos 70^o \operatorname{cosec} 20^o \\ & = \frac{\sin\left(90^\circ - 20^o\right)}{\cos 20^o} + \frac{\operatorname{cosec}\left(90^\circ - 70^o\right)}{\sec 70^o} - 2\cos 70^o \operatorname{cosec}\left(90^\circ - 70^o\right) \\ & = \frac{\cos 20^\circ}{\cos 20^o} + \frac{\sec 70^\circ}{\sec 70^o} - 2\cos 70^o \sec 70^\circ \\ & = 1 + 1 - 2 \times 1 \\ & = 0 \end{aligned}$$

Proved

(iv)

we know, sin(90 - A) = cos A andcosec(90 - A) = sec A, By using this information we can solve our question as

$$\begin{aligned} & \frac{\cos 80^o}{\sin 10^o} + \cos 59^o \operatorname{cosec} 31^o \\ & = \frac{\cos\left(90^\circ - 10^o\right)}{\sin 10^o} + \cos 59^o \operatorname{cosec}\left(90^\circ - 59^o\right) \\ & = \frac{\sin 10^\circ}{\sin 10^o} + \cos 59^o \sec 59^\circ \\ & = 1 + 1 = 2 \end{aligned}$$

Proved

8. Question

Prove the following:

(i) $\sin\theta \sin(90^\circ - \theta) - \cos\theta \cos(90^\circ - \theta) = 0$

(ii) $\dfrac{\cos(90^\circ - \theta)\sec(90^\circ - \theta)\tan\theta}{\operatorname{cosec}(90^\circ - \theta)\sin(90^\circ - \theta)\cot(90^\circ - \theta)} + \dfrac{\tan(90^\circ - \theta)}{\cot\theta} = 2$

(iii) $\dfrac{\tan(90^\circ - A)\cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$

(iv) $\frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$

(v) $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ = 1$

Answer

In the given parts use:

$$\sin\left(90^\circ - \theta\right) = \cos\theta,\ cos\left(90^\circ - \theta\right) = \sin\theta,$$

$$\sec\left(90^\circ - \theta\right) = \text{cosec}\theta,\ cosec\left(90^\circ - \theta\right) = \sec\theta$$

$$\tan\left(90^\circ - \theta\right) = \cot\theta,\ cot\left(90^\circ - \theta\right) = tan\theta$$

(i)

$\sin\theta\sin(90^\circ - \theta) - \cos\theta\cos(90^\circ - \theta) = 0$

solve LHS,

$$\begin{aligned} & \sin\theta\sin(90^\circ - \theta) - \cos\theta\cos(90^\circ - \theta) \\ &= \sin\theta\cos\theta - \cos\theta\sin\theta \\ &= 0 \end{aligned}$$

Which is equal to RHS.

(ii)

$$\frac{\cos(90^\circ - \theta)\sec(90^\circ - \theta)\tan\theta}{\cos ec(90^\circ - \theta)\sin(90^\circ - \theta)\cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot\theta} = 2$$

solve LHS,Use: $\sin\theta = \frac{1}{\text{cosec}\theta},\ \sec\theta = \frac{1}{\cos\theta}$

Solve,

$$\begin{aligned} & \frac{\cos(90^\circ - \theta)\sec(90^\circ - \theta)\tan\theta}{\cos ec(90^\circ - \theta)\sin(90^\circ - \theta)\cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot\theta} \\ &= \frac{\sin\theta\ \cos\sec\theta\tan\theta}{\sec\theta\ \cos\theta\ \tan\theta} + \frac{\cot\theta}{\cot\theta} \\ &= 1 + 1 = 2 \end{aligned}$$

Which is equal to RHS.

(iii) $\frac{\tan(90^\circ - A)\cot A}{\cos ec^2 A} - \cos^2 A = 0$

solve LHS,Use:

$$\tan\theta = \frac{sin\theta}{cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin\theta = \frac{1}{\text{cosec}\theta},\ \sec\theta = \frac{1}{\cos\theta}$$

Solve,

$$\frac{\tan(90^\circ - A)\cot A}{\text{cosec}^2 A} - \cos^2 A$$
$$= \frac{\cot A \cot A}{\text{cosec}^2 A} - \cos^2 A$$
$$= \frac{\cot^2 A}{\text{cosec}^2 A} - \cos^2 A$$
$$= \frac{\cos^2 A}{\sin^2 A \times \text{cosec}^2 A} - \cos^2 A$$
$$= \frac{\cos^2 A}{1} - \cos^2 A$$
$$= \cos^2 A - \cos^2 A = 0$$

Which is equal to RHS.

(iv) $\frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$

solve LHS,Use: $\tan\theta = \frac{sin\theta}{cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$

$$\frac{\cos(90^\circ - A)\sin(90^\circ - A)}{\tan(90^\circ - A)}$$
$$= \frac{\sin A\ \cos A}{\cot A}$$
$$= \frac{\sin A\ \cos A}{\frac{\cos A}{\sin A}}$$
$$= \frac{\sin^2 A\ \cos A}{\cos A} = \sin^2 A$$

Which is equal to RHS.

(v) $\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ\ \tan 20^\circ\ \tan 70^\circ \tan 80^\circ\ \tan 89^\circ = 1$

solve LHS,

$$\sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ$$
$$= \sin(50^\circ + \theta) - \sin\{90^\circ - (40^\circ - \theta)\} + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan(90^\circ - 20^\circ)\tan(90^\circ - 10^\circ)\tan(90^\circ - 1^\circ)$$
$$= \sin(50^\circ + \theta) - \sin(50^\circ + \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \cot 20^\circ \cot 10^\circ \cot 1^\circ$$
$$= (\tan 1^\circ \cot 1^\circ)(\tan 10^\circ \cot 10^\circ)(\tan 20^\circ \cot 20^\circ)$$
$$= 1 \times 1 \times 1 = 1$$

Which is equal to RHS.

9. Question

Evaluate:

(i) $\frac{2}{3}(\cos^4 30^\circ - \sin^4 45^\circ) - 3(\sin^2 60^\circ - \sec^2 45^\circ) + \frac{1}{4}\cot^2 30^\circ$

(ii) $4(\sin^4 30^\circ + \cos^4 60^\circ) - \frac{2}{3}(sin^2 60^\circ - cos^2 45^\circ) + \frac{1}{2}\tan^2 60^\circ$

(iii) $\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\cos ec 40^\circ}{\sec 50^\circ} - 4\cos 50^\circ \cos ec 40^\circ$

(iv) $\tan 35^\circ \tan 40^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ$

(v) $\cos ec(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cos(35^\circ + \theta)$

(vi) $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

(vii) $\frac{2\sin 68^\circ}{\cos 22^\circ} - \frac{2\cot 15^\circ}{5\tan 75^\circ} - \frac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$

(viii) $\frac{3\cos 55^\circ}{7\sin 35^\circ} - \frac{4(\cos 70^\circ \cos ec 20^\circ)}{7(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$

(ix) $\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$

(x) $\frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ \cos ec 52^\circ)}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ)}$

Answer

(i)

$$\frac{2}{3}\left(\cos^4 30^o - \sin^4 45^o\right) - 3\left(\sin^2 60^o - \sec^2 45^o\right) + \frac{1}{4}\cot^2 30^o$$

$$= \frac{2}{3}\left\{\left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^4\right\} - 3\left\{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\sqrt{2}\right)^2\right\} + \frac{1}{4}\left(\sqrt{3}\right)^2$$

$$= \frac{2}{3}\left\{\frac{9}{16} - \frac{1}{4}\right\} - 3\left\{\frac{3}{4} - 2\right\} + \frac{3}{4}$$

$$= \frac{2}{3} \times \frac{5}{16} + 3 \times \frac{5}{4} + \frac{3}{4}$$

$$= \frac{5}{24} + \frac{15}{4} + \frac{3}{4} = \frac{113}{24}$$

(ii) $4(\sin^4 30^\circ + \cos^4 60^\circ) - \frac{2}{3}(sin^2 60^\circ - cos^2 45^\circ) + \frac{1}{2}\tan^2 60^\circ$

$$= 4\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right) - \frac{2}{3}\left(\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right) + \frac{1}{2}(\sqrt{3})^2$$

$$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - \frac{2}{3}\left(\frac{3}{4} - \frac{1}{2}\right) + \frac{1}{2}(3)$$

$$= 4\left(\frac{1+1}{16}\right) - \frac{2}{3}\left(\frac{3-2}{4}\right) + \frac{1}{2}(3)$$

$$= 4\left(\frac{2}{16}\right) - \frac{2}{3}\left(\frac{1}{4}\right) + \frac{1}{2}(3)$$

$$= \frac{1}{2} - \frac{1}{6} + \frac{3}{2}$$

$$= \frac{3-1+9}{6}$$

$$= \frac{11}{6}$$

(iii)

$$= \frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ} - 4\cos 50^\circ \operatorname{cosec} 40^\circ$$

$$= \frac{\sin(90^\circ - 40^\circ)}{\cos 40^\circ} + \frac{\operatorname{cosec}(90^\circ - 50^\circ)}{\sec 50^\circ} - 4\cos 50^\circ \operatorname{cosec}(90^\circ - 50^\circ)$$

$$= \frac{\cos 40^\circ}{\cos 40^\circ} + \frac{\sec 50^\circ}{\sec 50^\circ} - 4\cos 50^\circ \sec 50^\circ$$

$$= 1 + 1 - 4 = -2$$

(iv)

$$\tan 35^\circ \tan 40^\circ \tan 45^\circ \tan 50^\circ \tan 55^\circ$$

$$= \tan 35^\circ \tan 40^\circ \tan 45^\circ \tan(90^\circ - 40^\circ) \tan(90^\circ - 35^\circ)$$

$$= \tan 35^\circ \tan 40^\circ \tan 45^\circ \cot 40^\circ \cot 35^\circ$$

$$= (\tan 35^\circ \cot 35^\circ)(\tan 40^\circ \cot 40^\circ) \tan 45^\circ$$

$$= 1 \times 1 \times 1 = 1$$

(v)

$$\operatorname{cosec}(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta)$$

$$= \sec\{90^\circ - (65^\circ + \theta)\} - \sec(25^\circ - \theta) - \cot\{90^\circ - (55^\circ - \theta)\} + \cot(35^\circ + \theta)$$

$$= \sec(25^\circ - \theta) - \sec(25^\circ - \theta) - \cot(35^\circ + \theta) + \cot(35^\circ + \theta)$$

$$= 0$$

(vi)

$$= \tan 7^\circ \tan 23^\circ \tan 60^\circ \tan(90^\circ - 23^\circ) \tan(90^\circ - 7^\circ)$$

$$= \tan 7^\circ \tan 23^\circ \tan 60^\circ \cot 23^\circ \cot 7^\circ$$

$$= (\tan 7^\circ \cot 7^\circ)(\tan 23^\circ \cot 23^\circ) \tan 60^\circ$$

$$= 1 \times 1 \times \sqrt{3} = \sqrt{3}$$

(vii)

$$\frac{2\sin 68^\circ}{\cos 22^\circ}-\frac{2\cot 15^\circ}{5\tan 75^\circ}-\frac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$$
$$=\frac{2\sin(90^\circ-22^\circ)}{\cos 22^\circ}-\frac{2\cot(90^\circ-75^\circ)}{5\tan 75^\circ}-\frac{3\tan 45^\circ \tan 20^\circ \tan 40^\circ \tan(90^\circ-40^\circ)\tan(90^\circ-20^\circ)}{5}$$
$$=\frac{2\cos 22^\circ}{\cos 22^\circ}-\frac{2\tan 75^\circ}{5\tan 75^\circ}-\frac{3\tan 45^\circ(\tan 20^\circ \cot 20^\circ)(\tan 40^\circ \cot 40^\circ)}{5}$$
$$=2-\frac{2}{5}-\frac{3}{5}=1$$

(viii)

$$\frac{3\cos 55^\circ}{7\sin 35^\circ}-\frac{4(\cos 70^\circ \operatorname{cosec} 20^\circ)}{7(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$$
$$=\frac{3\cos(90^\circ-35^\circ)}{7\sin 35^\circ}-\frac{4\cos 70^\circ \operatorname{cosec}(90^\circ-70^\circ)}{7\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan(90^\circ-25^\circ)\tan(90^\circ-5^\circ)}$$
$$=\frac{3\sin 35^\circ}{7\sin 35^\circ}-\frac{4\cos 70^\circ \sec 70^\circ}{7(\tan 5^\circ \cot 5^\circ)(\tan 25^\circ \cot 25^\circ)\tan 45^\circ}$$
$$=\frac{3}{7}-\frac{4}{7}=-\frac{1}{7}$$

(ix)

$$\frac{\sin 18^\circ}{\cos 72^\circ}+\sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$$
$$=\frac{\sin(90^\circ-72^\circ)}{\cos 72^\circ}+\sqrt{3}\{\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan(90^\circ-40^\circ)\tan(90^\circ-10^\circ)\}$$
$$=\frac{\cos 72^\circ}{\cos 72^\circ}+\sqrt{3}\{(\tan 10^\circ \cot 10^\circ)\tan 30^\circ(\tan 40^\circ \cot 40^\circ)\}$$
$$=1+\sqrt{3}\left\{1\times\frac{1}{\sqrt{3}}\times 1\right\}=1+1=2$$

(x)

$$\frac{\cos 58^\circ}{\sin 32^\circ}+\frac{\sin 22^\circ}{\cos 68^\circ}-\frac{\cos 38^\circ \operatorname{cosec} 52^\circ)}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ)}$$
$$=\frac{\cos(90^\circ-32^\circ)}{\sin 32^\circ}+\frac{\sin(90^\circ-68^\circ)}{\cos 68^\circ}-\frac{\cos 38^\circ \operatorname{cosec}(90^\circ-38^\circ)}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan(90^\circ-18^\circ)\tan(90^\circ-35^\circ)}$$
$$=\frac{\sin 32^\circ}{\sin 32^\circ}+\frac{\cos 68^\circ}{\cos 68^\circ}-\frac{\cos 38^\circ \sec 38^\circ}{(\tan 18^\circ \cot 18^\circ)(\tan 35^\circ \cot 35^\circ)\tan 60^\circ}$$
$$=1+1-\frac{1}{1\times 1\times\sqrt{3}}$$
$$=2-\frac{1}{\sqrt{3}}=\frac{6-\sqrt{3}}{3}$$

10. Question

In $\sin\theta=\cos(\theta-45^\circ)$, where θ *and* $\theta-45^\circ$ are acute angles, find the degree measure of θ.

Answer

$$\sin\theta=\cos(\theta-45^\circ)$$
$$\Rightarrow \cos(90^\circ-\theta)=\cos(\theta-45^\circ)$$
$$\Rightarrow 90^\circ-\theta=\theta-45^\circ$$
$$\Rightarrow 2\theta=135^\circ$$
$$\Rightarrow \theta=\frac{135^\circ}{2}=67\frac{1}{2}^\circ$$

11. Question

If A, B, C are the interior angles of a ΔABC, show that:

(i) $\sin\frac{B+C}{2}=\cos\frac{A}{2}$ (ii) $\cos\frac{B+C}{2}=\sin\frac{A}{2}$

Answer

(i) Since, A, B, C are the interior angles of a ΔABC

$$A+B+C=180^\circ$$
$$\Rightarrow \quad B+C=180^\circ-A$$
$$\Rightarrow \quad \frac{B+C}{2}=\frac{180^\circ-A}{2}$$
$$\Rightarrow \quad \sin\left(\frac{B+C}{2}\right)=\sin\left(90^\circ-\frac{A}{2}\right)$$
$$\Rightarrow \quad \sin\left(\frac{B+C}{2}\right)=\cos\left(\frac{A}{2}\right)$$

(ii)

$$A+B+C=180^\circ$$
$$\Rightarrow \quad B+C=180^\circ-A$$
$$\Rightarrow \quad \frac{B+C}{2}=\frac{180^\circ-A}{2}$$
$$\Rightarrow \quad \cos\left(\frac{B+C}{2}\right)=\cos\left(90^\circ-\frac{A}{2}\right)$$
$$\Rightarrow \quad \cos\left(\frac{B+C}{2}\right)=\sin\left(\frac{A}{2}\right)$$

12. Question

If $2\theta+45^\circ$ *and* $30^\circ-\theta$ are acute angles, find the degree measure of θ satisfying

Answer

$$\sin(2\theta+45^o)=\cos(30^o-\theta)$$
$$\Rightarrow \quad \sin(2\theta+45^o)=\sin\{90^\circ-(30^o-\theta)\}$$
$$\Rightarrow \quad 2\theta+45^o=90^\circ-30^o+\theta$$
$$\Rightarrow \quad \theta=15^\circ$$

13. Question

If θ is a positive acute angle such that $\sec\theta=\operatorname{cosec}60^\circ$, find the value of $2\cos^2\theta-1$.

Answer

$$\sec\theta=\operatorname{cosec}60^o$$
$$\Rightarrow \quad \operatorname{cosec}(90^\circ-\theta)=\operatorname{cosec}60^o$$
$$\Rightarrow \quad 90^\circ-\theta=60^o$$
$$\Rightarrow \quad \theta=30^\circ$$
$$Now, \quad 2\cos^2\theta-1=2\cos^2 30^\circ-1$$
$$=2\times\left(\frac{\sqrt{3}}{2}\right)^2-1$$
$$=2\times\frac{3}{4}-1=\frac{1}{2}$$

14. Question

If $\cos 2\theta = \sin 4\theta$, where 2θ and 4θ are acute angles, find the value of θ.

Answer

Given: $\cos 2\theta = \sin 4\theta$, where 2θ and 4θ are acute angles.

To find: The value of θ.

Solution:since, $\sin(90^\circ - \theta) = \cos\theta$ So,

$$\begin{aligned} &\cos 2\theta = \sin 4\theta \\ \Rightarrow &\sin(90^\circ - 2\theta) = \sin 4\theta \\ \Rightarrow &90^\circ - 2\theta = 4\theta \\ \Rightarrow &6\theta = 90^\circ \\ \Rightarrow &\theta = 15^\circ \end{aligned}$$

15. Question

If $\sin 3\theta = \cos(\theta - 6^\circ)$, where 3θ and $\theta - 6^\circ$ are acute angles, find the value of θ.

Answer

$$\begin{aligned} &\sin 3\theta = \cos(\theta - 6^\circ) \\ \Rightarrow &\cos(90^\circ - 3\theta) = \cos(\theta - 6^\circ) \\ \Rightarrow &90^\circ - 3\theta = \theta - 6^\circ \\ \Rightarrow &4\theta = 96^\circ \\ \Rightarrow &\theta = \frac{96^\circ}{4} = 24^\circ \end{aligned}$$

16. Question

If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, where $4A$ is an acute angle, find the value of A.

Answer

$$\begin{aligned} &\sec 4A = \operatorname{cosec}(A - 20^\circ) \\ \Rightarrow &\operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ) \\ \Rightarrow &90^\circ - 4A = A - 20^\circ \\ \Rightarrow &5A = 110^\circ \\ \Rightarrow &A = \frac{110^\circ}{5} = 22^\circ \end{aligned}$$

17. Question

If $\sec 2A = \operatorname{cosec}(A - 42^\circ)$, where $2A$ is an acute angle, find the value of A.

Answer

$$\begin{aligned} &\sec 2A = \operatorname{cosec}(A - 42^\circ) \\ \Rightarrow &\operatorname{cosec}(90^\circ - 2A) = \operatorname{cosec}(A - 42^\circ) \\ \Rightarrow &90^\circ - 2A = A - 42^\circ \\ \Rightarrow &3A = 132^\circ \\ \Rightarrow &A = \frac{132^\circ}{3} = 44^\circ \end{aligned}$$

CCE - Formative Assessment

1. Question

Write the maximum and minimum values of sin θ.

Answer

With the help of Minimum-Maximum Value Table we can find the Value of sin θ

Therefore,

Minimum Value of sin θ = - 1 and

Maximum Value of sin θ = 1

2. Question

Write the maximum and minimum values of cos θ.

Answer

With the help of Minimum-Maximum Value Table we can find the Value of cos θ

Therefore,

Minimum Value of cos θ = - 1 and

Maximum Value of cos θ = 1

3. Question

What is the maximum value of $\frac{1}{\sec\theta}$?

Answer

As we know,

$$\frac{1}{\sec\theta} = \frac{1}{\frac{1}{\cos\theta}} = \cos\theta$$

And,

Maximum value of cos θ = 1

So,

The maximum value of 1/sec θ is 1

4. Question

What is the maximum value of $\frac{1}{\text{cosec}\theta}$?

Answer

As we know,

$$\frac{1}{\operatorname{cosec}\theta} = \sin\theta$$

And,

Maximum value of sin θ = 1

So,

The maximum value of 1/cosec θ is 1.

5. Question

If $\tan\theta = \frac{4}{5}$, find the value of $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

Answer

Given,

tan θ = 4/5

As we know,

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{4}{5}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{16}{25}}} = \frac{5}{\sqrt{41}}$$

And,

$$\sin\theta = \sqrt{1 - \left(\frac{5}{\sqrt{41}}\right)^2}$$

$$= \sqrt{1 - \frac{25}{41}}$$

$$= \sqrt{\frac{41 - 25}{41}} = \frac{4}{\sqrt{41}}$$

Now,

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\frac{5}{\sqrt{41}} - \frac{4}{\sqrt{41}}}{\frac{5}{\sqrt{41}} + \frac{4}{\sqrt{41}}}$$

$$= \frac{\frac{1}{\sqrt{41}}}{\frac{9}{\sqrt{41}}} = \frac{1}{9}$$

Therefore,

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1}{9}$$

6. Question

If $\cos\theta = \frac{2}{3}$, find the value of $\frac{\sec\theta - 1}{\sec\theta + 1}$

Answer

Given,

cos θ = 2/3

We know,

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

So,

$$\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{\frac{3}{2} - 1}{\frac{3}{2} + 1}$$

$$= \frac{\frac{1}{2}}{\frac{5}{2}}$$

$$= \frac{2}{10} = \frac{1}{5}$$

Therefore,

$$\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{1}{5}$$

7. Question

If 3cot θ = 4, find the value of $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

Answer

Given,

3 cot θ = 4

So,

cot θ = 4/3

As we know,

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}}$$

$$= \frac{1}{\sqrt{1 + \left(\frac{4}{5}\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{16}{25}}} = \frac{5}{\sqrt{41}}$$

And,

$$\sin\theta = \sqrt{1 - \left(\frac{5}{\sqrt{41}}\right)^2}$$

$$= \sqrt{1 - \frac{25}{41}}$$

$$= \sqrt{\frac{41 - 25}{41}} = \frac{4}{\sqrt{41}}$$

Now,

$$\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \frac{(4)\frac{5}{\sqrt{41}} - \frac{4}{\sqrt{41}}}{(2)\frac{5}{\sqrt{41}} + \frac{4}{\sqrt{41}}} =$$

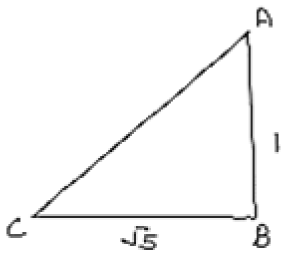
8. Question

Given $\tan\theta = \frac{1}{\sqrt{5}}$, what is the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$?

Answer

Given,

tan θ = 1/√5



In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = 1^2 + (\sqrt{5})^2$

$AC^2 = 1 + 5 = 6$

AC = √6

We have,

cosec θ = AC/AB = √6/1

sec θ = AC/BC = √6/√5

Now,

$$\frac{\text{cosec}^2\,\theta - \sec^2\theta}{\text{cosec}^2\,\theta\ +\ \sec^2\theta} = \frac{(\sqrt{6})^2 - \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}{(\sqrt{6})^2\ +\ \left(\frac{\sqrt{6}}{\sqrt{6}}\right)^2}$$

$$= \frac{6 - \frac{6}{5}}{6\ +\ \frac{6}{5}}$$

$$= \frac{\frac{30 - 6}{5}}{\frac{30\ +\ 6}{5}}$$

$$= \frac{24}{36} = \frac{2}{3}$$

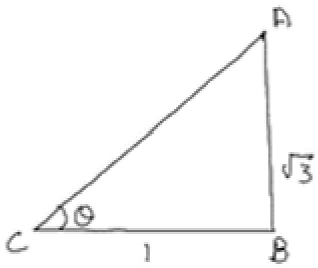
9. Question

If $\cot\theta = \frac{1}{\sqrt{3}}$, rite the value of $\frac{1-\cos^2\theta}{2-\sin^2\theta}$

Answer

Given,

cot θ = 1/√3



In ΔABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (\sqrt{3})^2 + 1^2$

$AC^2 = 3 + 1 = 4$

AC= 2

We get;

cos θ = BC/AC = 1/2

sin θ = AB/AC = √3/2

Now,

$$\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$$

$$= \frac{\frac{4-1}{4}}{\frac{8-3}{4}} = \frac{3}{5}$$

10. Question

If tan A = 3/4 and A + B = 90°, then what is the value of cot B ?

Answer

Given,

tan A = 3/4 and

A + b = 90°

So we have,

A = 90° - B

So,

tan A = tan (90° - B) = 3/4

∵ cot B = tan A

Therefore,

cot B = 3/4

11. Question

If A + B = 90° and cos B = 3/5, what is the value of sin A?

Answer

Given,

cos B = 3/5 and

A + B = 90°

So we have,

B = 90° - A

So,

cos B = cos (90° - A) = 3/5

∵ cos B = sin A

Therefore,

sin A = 3/5

12. Question

Write the acute angle θ satisfying √3 sin θ = cos θ

Answer

Given,

√3 sin θ = cos θ

sin θ / cos θ = 1/√3tan θ = 1/ √3 …(the value of tan 30° is 1/√3)

Therefore,tan θ = tan 30°θ = 30°

13. Question

Write the value of cos 1° cos 2° cos 3°…. cos 179° cos 180°.

Answer

cos 1° cos 2° cos 3°…. cos 179° cos 180°= cos 1° × cos 2° × cos 3° × .....× cos 90° × ..... × cos 179°

As we know,

cos 90° = 0

So,= cos 1° × cos 2° × cos 3° × .....× 0 × ..... × cos 179°= 0

As, cos 90° has the value 0 that's why the whole answer will be zero.

14. Question

Write the value of tan 10° tan 15° tan 75° tan 80°.

Answer

tan 10° × tan 15° × tan 75° × tan 80°

= tan 10 × tan 15 × tan(90-75) × tan (90-80)= tan10 × tan15 × cot 15 × cot10As we know,

tan θ × cot θ = 1So,tan 10 × cot 10 × tan 15 × cot 15= 1

15. Question

If A + B = 90° and tan A = 3/4, what is cot B?

Answer

Given,

tan A = 3/4

A + B = 90°B = 90 – A

So,cot B = cot (90-A)cot B = tan A

Therefore,cot B = 3/4

16. Question

If tan A = 5/12, find the value of (sin A + cos A) sec A.

Answer

Given,

tan A = 5/12

We get,

$$\cot A = \frac{1}{\tan A} = \frac{1}{\frac{5}{12}} = \frac{5}{12}$$

Now,

(sin A + cos A) sec A = (sin A + cos A)× 1/cos A

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\cos A}$$

= tan A + 1

$$= \frac{5}{12} + 1 = \frac{5 + 12}{12} = \frac{17}{12}$$

Therefore,

cot A = 17/12

1. Question

If θ is an acute angle such that cos θ = 3/5, then $\frac{\sin\theta\tan\theta - 1}{2\tan^2\theta}$ =

A. $\frac{160}{3}$

B. $\frac{1}{36}$

C. $\frac{3}{160}$

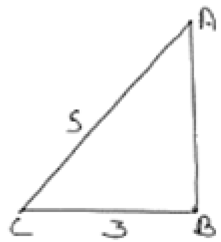
D. $\frac{160}{3}$

Answer

Given,

cos θ = 3/5

In Δ ABC,



$AC^2 = AB^2 + BC^2$

$(5)^2 = AB^2 + (3)^2$

$25 = AB^2 + 9$

$25 - 9 = AB^2$

$16 = AB^2$

$4 = AB$

Therefore,

AB = 4

As,

sin θ = AB/AC = 4/5

tan θ = AB/BC = 4/3

Now,

$$\frac{\sin\theta\tan\theta - 1}{2\tan^2\theta} = \frac{\frac{4}{5}\times\frac{4}{3} - 1}{2\times\left(\frac{4}{3}\right)^2}$$

$$= \frac{\frac{16-15}{15}}{2\times\frac{16}{9}}$$

$$= \frac{\frac{1}{15}}{\frac{32}{9}}$$

$$= \frac{1}{15}\times\frac{9}{32}$$

$$= \frac{9}{480} = \frac{3}{160}$$

2. Question

If tan θ = a/b then $\frac{a\sin\theta + b\cos\theta}{a\sin\theta - b\cos\theta}$ is equal to

A. $\frac{a+b}{a-b}$

B. $\frac{a^2 - b^2}{a^2 + b^2}$

C. $\frac{a + b}{a - b}$

D. $\frac{a - b}{a + b}$

Answer

Given,

tan θ = a/b

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = a^2 + b^2$

$$AC = \sqrt{a^2 + b^2}$$

Therefore,

$$\sin\theta = \frac{AB}{AC} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\cos\theta = \frac{BC}{AC} = \frac{b}{\sqrt{a^2 + b^2}}$$

Now,

$$\frac{a\sin\theta + b\cos\theta}{a\sin\theta - b\cos\theta} = \frac{a \times \frac{a}{\sqrt{a^2 + b^2}} + b \times \frac{b}{\sqrt{a^2 + b^2}}}{a \times \frac{a}{\sqrt{a^2 + b^2}} - b \times \frac{b}{\sqrt{a^2 + b^2}}}$$

$$= \frac{\frac{a^2 + b^2}{\sqrt{a^2 + b^2}}}{\frac{a^2 - b^2}{\sqrt{a^2 + b^2}}} = \frac{a^2 + b^2}{a^2 - b^2}$$

3. Question

If 5 tan θ – 4 = 0, then the value of $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$ is

A. 5/3

B. 5/6

C. 0

D. 1/6

Answer

Given,

5 tan θ – 4 = 0

So,

5 tan θ = 4

tan θ = 4/5

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (4)^2 + (5)^2$

$AC^2 = 16 + 25$

$AC^2 = 41$

AC = √41

Therefore,

sin θ = AB/AC = 4/√41

cos θ = BC/AC = 5/√41

Now,

$$\frac{5\sin\theta - 4\cos\theta}{5\sin\theta\ +\ 4\cos\theta} = \frac{5\times\frac{4}{\sqrt{41}} - 4\times\frac{5}{\sqrt{41}}}{5\times\frac{4}{\sqrt{41}}\ +\ 4\times\frac{5}{\sqrt{41}}} = 0$$

4. Question

If 16 cot x = 12, then $\frac{\sin x - \cos x}{\sin x + \cos x}$ equals

A. 1/7

B. 3/7

C. 2/7

D. 0

Answer

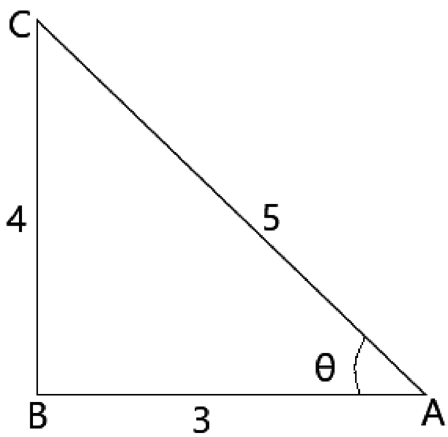
Given: 16 cot x = 12

To find: The value of $\frac{\sin x - \cos x}{\sin x + \cos x}$.

Solution:

cot x = 12/16

cot x = 3/4Also cotθ = base/perpendicularSo we now construct a right triangle ABC, right angled at B such that∠BAC = θ, Base = 3 and perpendicular = 4

In Δ ABC, 

$AC^2 = AB^2 + BC^2$

$AC^2 = (3)^2 + (4)^2$

$AC^2 = 9 + 16 = 25$

AC = 5

As we know sinθ = perpendicular / hypotenuse cosθ = base / hypotenuse

Therefore,

sin x = AB/AC = 4/5

cos x = BC/AC = 3/5

Now,

$$\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{\frac{4}{5} - \frac{3}{5}}{\frac{4}{5} + \frac{3}{5}}$$

$$= \frac{\frac{4-3}{5}}{\frac{4+3}{5}} = \frac{1}{7}$$

5. Question

If 8 tan x = 15, then sin x - cos x is equal to

A. $\frac{7}{17}$

B. $\frac{17}{7}$

C. $\frac{1}{17}$

D. $\frac{7}{17}$

Answer

Given,

8 tan x = 15

So,

tan x = 15/8

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (15)^2 + (8)^2$

$AC^2 = 225 + 64 = 289$

AC = 17

Therefore,

sin x = AB/AC = 15/17

cos x = BC/AC = 8/17

Now,

$$\sin x - \cos x = \frac{15}{17} - \frac{8}{17}$$

$$= \frac{15-8}{17} = \frac{7}{17}$$

6. Question

If $\tan\theta = \frac{1}{\sqrt{7}}$, then $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2 + \sec^2\theta}$ =

A. 5/7

B. 3/7

C. 1/12

D. 3/4

Answer

Given,

tan θ = 1/√7

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (1)^2 + (\sqrt{7})^2$

$AC^2 = 8$

AC = 2√2

Therefore,

$$\text{cosec}\,\theta = \frac{2\sqrt{2}}{1} \text{ and } \sec\theta = \frac{2\sqrt{2}}{\sqrt{7}}$$

Now,

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{\frac{56 - 8}{7}}{\frac{56 + 8}{7}}$$

$$= \frac{48}{64} = \frac{3}{4}$$

7. Question

If tan θ = 3/4, then $\cos^2\theta - \sin^2\theta =$

A. $\frac{7}{25}$

B. 1

C. $\frac{4}{25}$

D. $\frac{4}{25}$

Answer

Given,

tan θ = 3/4

In Δ ABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (3)^2 + (4)^2$

$AC^2 = 9 + 16 = 25$

AC = 5

Therefore,

sin θ = p/h = 3/5 and

cos θ = b/h = 4/5

Now putting these values in the given equation we get,

$\cos^2 \theta - \sin^2 \theta = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{16-9}{25}$

$= \frac{7}{25}$

8. Question

If θ is an acute angle such that $\tan^2 \theta = 8/7$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is

A. 7/8

B. 8/7

C. 7/4

D. 64/49

Answer

Given,

$\tan^2 \theta = 8/7$

Now,

$$\frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)} = \frac{1 - \sin^2\theta}{1 - \cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta$$

$$= \frac{1}{\tan^2\theta}$$

$$= \frac{1}{\frac{8}{7}} = \frac{7}{8}$$

9. Question

If 3cos θ = 5 sin θ, then the value of $\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta}$ is

A. $\frac{542}{2937}$

B. $\frac{316}{2937}$

C. $\frac{542}{2937}$

D. None of these

Answer

Given,

3 cos θ = 5 sin θ

$$\frac{\cos\theta}{\sin\theta} = \frac{5}{3}$$

$$\cot\theta = \frac{5}{3}$$

In ΔABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = (3)^2 + (5)^2$

$AC^2 = 9 + 25 = 34$

$AC = \sqrt{34}$

Therefore,

$\sin\theta = 3/\sqrt{34}$

$\cos\theta = 5/\sqrt{34}$

$\sec\theta = \sqrt{34}/5$

Now,

$$\frac{5\sin\theta - 2\sec^3\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta} = \frac{5 \times \frac{3}{\sqrt{34}} - 2\left(\frac{\sqrt{34}}{5}\right)^3 + 2 \times \frac{5}{\sqrt{34}}}{5 \times \frac{3}{\sqrt{34}} + 2\left(\frac{\sqrt{34}}{5}\right)^3 - 2 \times \frac{5}{\sqrt{34}}}$$

$$= \frac{\frac{125 \times 15 - 2 \times 34 \times 34 + 10 \times 125}{125\sqrt{34}}}{\frac{125 \times 15 + 2 \times 34 \times 34 - 10 \times 125}{125\sqrt{34}}}$$

$$= \frac{1875 - 2312 + 1250}{1875 + 2312 - 1250}$$

$$= \frac{813}{2937}$$

$$= \frac{271}{979}$$

10. Question

If $\tan^2 45° - \cos^2 30° = x \sin 45° \cos 45°$, then x =

A. 2

B. - 2

C. −1/2

D. 1/2

Answer

Given,

$\tan^2 45° - \cos^2 30° = x \sin 45° \cos 45°$ ....(i)

put the values in equation (i),

$$(1)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = x \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$1 - \frac{3}{4} = x \times \frac{1}{2}$$

$$\frac{1}{4} = \frac{x}{2}$$

$$x = \frac{1}{2}$$

11. Question

The value of $\cos^2 17° - \sin^2 73°$ is

A. 1

B. $\frac{1}{3}$

C. 0

D. -1

Answer

$\cos^2 17° - \sin^2 73° = \cos^2 17° - \sin^2(90° - 17°)$

$= \cos^2 17° - \cos^2 17° = 0$

12. Question

The value of $\frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°}$ is

A. 1/2

B. $1/\sqrt{2}$

C. 1

D. 2

Answer

$$\frac{\cos^3 20° - \cos^3 70°}{\sin^3 70° - \sin^3 20°} = \frac{\cos^3 20° - \cos^3(90° - 20°)}{\sin^3(90° - 20°) - \sin^3 20°}$$

$$= \frac{\cos^3 20° - \sin^3 20°}{\cos^3 20° - \sin^3 20°} = 1$$

13. Question

If $\dfrac{x \text{cosec}^2 30° \sec^2 45°}{8\cos^2 45° \sin^2 60°}$ = $\tan^2 60°$ – $\tan^2 30°$ then

A. 1

B. - 1

C. 2

D. 0

Answer

Given,

$$\frac{x \text{ cosec}^2 30° \sec^2 45°}{8 \cos^2 45° \sin^2 60°} = \tan^2 60° - \tan^2 30°$$

$$\frac{x \times (2)^2}{8\left(\frac{1}{\sqrt{2}}\right)^2} = \left(\sqrt{3}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\frac{4x \times 2}{\frac{8}{2} \times \frac{3}{4}} = 3 - \frac{1}{3}$$

$$\frac{8x}{3} = \frac{9-1}{3}$$

8x = 8

x = 8/8 = 1

So, the value of x = 1

14. Question

If A and B are complementary angles, then

A. sin A = sin B

B. cos A = cos B

C. tan A = tan B

D. sec A = cosec B

Answer

Given,

A + B = 90°

B = 90° - A

sin B = sin (90° - A)

sin B = cos A

Taking the reciprocal,

cosec B = sec A

Or

sec A = cosec B

15. Question

If x sin (90° - 0) cot (90° - θ) = cos (90° - θ), then x =?

A. 0

B. 1

C. - 1

D. 2

Answer

Given: x sin (90° - 0) cot (90° - θ) = cos (90° - θ)

To find: The value of x.

Solution:

x sin (90° - 0) cot (90° - θ) = cos (90° - θ)

Since, sin (90° - θ)= cos θcot (90° - θ) = tan θ

⇒ x cos θ.tan θ = sin θ

We know $\tan\,\theta\;=\frac{\sin\,\theta}{\cos\,\theta}$ ...... (1)Put this value in (1)

$\Rightarrow x\,\cos\,\theta\;.\frac{\sin\theta}{cos\,\theta}=sin\,\theta$

$\Rightarrow x=\frac{\sin\,\theta.\;\cos\,\theta}{\cos\,\theta.\sin\,\theta}$

⇒ x = 1

Hence, the value is x = 1

16. Question

If x tan 45° cos 60° = sin 60° cot 60°, then x is equal to

A. 1

B. $\sqrt{3}$

C. 1/2

D. $1/\sqrt{2}$

Answer

Given,

x tan 45° cos 60° = sin 60° cot 60°

$(x)\times(1)\times 1/2 = \sqrt{3}/2 \times 1/\sqrt{3}$

x/2 = 1/2

x = 2/2 = 1

So the value of x is 1.

17. Question

If angles A, B, C of a ΔABC form an increasing AP, then sin B =

A. 1/2

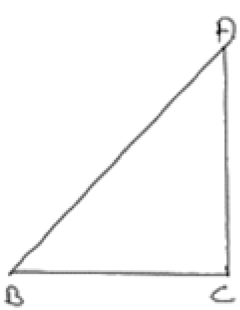
B. $\sqrt{3}/2$

C. 1

D. $1/\sqrt{2}$

Answer

Let suppose A, B and C are the angles of a triangle ABC,



In ΔABC,

∠ A = (a − d)

$\angle B = a$

$\angle C = a + d$

Now, form an increasing A.P

As we know Sum of all the angle of a triangle is 180°,

Therefore,

$\angle A + \angle B + \angle C = 180°$

$(a - d) + a + (a + d) = 180°$

$3a = 180°$

$a = 180/3 = 60°$

From the table,

$\sin B = \sin A = \sin 60°$

$= \sqrt{3}/2$

18. Question

If θ is an acute angle such that $\sec^2\theta = 3$, then the value of $\frac{\tan^2\theta - \text{cosec}^2\theta}{\tan^2\theta + \text{cosec}^2\theta}$ is

A. 4/7

B. 3/7

C. 2/7

D. 1/7

Answer

Given,

$\text{Sec}^2\theta = 3$

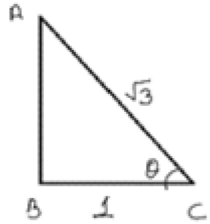
So,

$\text{Sec}\,\theta = \sqrt{3} = h/b = k$

Therefore,

$h = \sqrt{3}k,\ b = k$

In ΔABC,



$h^2 = p^2 + b^2$

$(\sqrt{3}k)^2 = p^2 + (k)^2$

$3k^2 = p^2 + k^2$

$3k^2 - k^2 = p^2$

$2k^2 = p^2$

$\sqrt{2}\ K = p$

We know,

$\tan\theta = p/b = \sqrt{2}k/k = \sqrt{2}$

$\text{cosec}\,\theta = h/p = \sqrt{3}k/\sqrt{2}k = \sqrt{3}/\sqrt{2}$

Put these value in,

$$\frac{\tan^2 - \text{cosec}^2}{\tan^2 + \text{cosec}^2} = \frac{(\sqrt{2})^2 - \left(\sqrt{\frac{3}{2}}\right)^2}{(\sqrt{2})^2 + \left(\sqrt{\frac{3}{2}}\right)^2}$$

$$= \frac{2 - \frac{3}{2}}{2 + \frac{3}{2}}$$

$$= \frac{\frac{4-3}{2}}{\frac{4+3}{2}} = \frac{1}{7}$$

19. Question

The value of tan 1° tan 2° tan 3° ……tan 89° is

A. 1

B. -1

C. 0

D. None of these

Answer

tan 1° × tan 2° × tan 3° × ………. × tan 89°

As we know tan (90 – θ) = cot θ

So here we get,

tan (90 – 89) × tan (90 - 88) × tan (90 - 87) × ……….× tan 87 × tan 88 × tan 89°

cot 89° × cot 88° × cot 87°…………….. tan 45°× tan 46° …………….× tan 87°× tan 88°× tan 89°

∵ cot θ = 1/tan θ

Therefore,

(cot 89° × tan89°)(cot 88° × tan88°)(cot 87° × tan87°)…… (cot 46° × tan46°)(tan 45°)

$$\left(\frac{1}{\tan 89^\circ} \times \tan 89^\circ\right)\left(\frac{1}{\tan 88^\circ} \times \tan 88^\circ\right)\left(\frac{1}{\tan 87^\circ} \times \tan 87^\circ\right) \ldots . . \left(\frac{1}{\tan 46^\circ} \times \tan 46^\circ\right)(\tan 45^\circ$$

= tan 45°

= 1

20. Question

The value of cos 1° cos 2° cos 3°…… cos 180° is

A. 1

B. 0

C. -1

D. None of these

Answer

Given,

cos 1° cos 2° cos 3° ………………..cos 180°

cos 1° × cos 2° × cos 3° …… cos 89° × cos 90° × cos 91° ……….cos 180°

As we know from the table,

cos 90° = 0

Therefore,

cos 1° × cos 2° × cos 3° …… cos 89° × 0 × cos 91° ……….cos 180°

= 0

21. Question

The value of tan 10° tan 15° tan 75° tan 80. is

A. - 1

B. 0

C. 1

D. None of these

Answer

Given,

tan 10° tan 15° tan 75° tan 80°

As we know,

tan (90 – θ) = than θ

Therefore,

⇒ tan (90°-80°) tan (90°-75°) tan 75° tan 80°

⇒ cot 80° cot 75° tan 75° tan 80°

⇒ (cot 80° tan 80°) (cot 75° tan 75°)……… [cot θ = 1/tanθ]

⇒ $\left(\frac{1}{\tan 80^\circ} \times \tan 80^\circ\right)\left(\frac{1}{\tan 75^\circ} \times \tan 75^\circ\right)$

⇒ (1)(1) = 1

22. Question

The value of $\frac{\cos\ (90^\circ-\theta)\sec(90^\circ-\theta)\ \tan\theta}{\operatorname{cosec}(90^\circ-\theta)\sin(90^\circ-\theta)\cot(90^\circ-\theta)}+\frac{\tan(90^\circ-\theta)}{\cot\theta}$ is

A. 1

B. - 1

C. 2

D. - 2

Answer

Given,

$\frac{\cos\ (90-\theta)\sec(90-\theta)\tan\theta}{\operatorname{cosec}(90-\theta)\sin(90-\theta)\cot(90-\theta)}+\frac{\tan(90-0)}{\cot\theta}$…….. (i)

∵ cos (90 – θ) = sin θ cos (90 – θ) = cos θ

sec (90 – θ) = cosec θ cot (90 – θ) = tan θ

cosec (90 – θ) = sec θ tan (90 – θ) = cot θ

Putting these values in (i),

We get,

$$\frac{\sin\theta\ \operatorname{cosec}\theta\tan\theta}{\sec\theta\ \cos\theta\tan\theta}+\frac{\cot\theta}{\cot\theta}$$

∵ cosec θ = 1/sin θ and

sec θ = 1/cos θ

$$\Rightarrow \frac{\sin\theta\times\frac{1}{\sin\theta}\times\tan\theta}{\frac{1}{\cos\theta}\times\cos\theta\times\tan\theta}+\frac{\cot\theta}{\cot\theta}$$

⇒ 1 + 1 = 2

23. Question

If θ and 2θ – 45° are acute angles such that sin θ = cos (2θ – 45°), then tan θ is equal to

A. 1

B. - 1

C. √3

D. 1/√3

Answer

Given,

θ and 2θ – 45° are acute angle,

sin θ = cos (2θ – 45)………….. (i)

[∵ cos (90 – θ) = sin θ]

Putting these value in equation (i),

Cos (90 – θ) = cos (2θ – 45°)

90 - θ = 2 θ – 45°

90 + 45 = 3 θ

3 θ = 135

θ = 135/3 = 45

tan θ = tan 45° = 1

24. Question

If 5θ and 4θ are acute angles satisfying sin 5θ = cos 4θ, then 2 sin 3θ – √3 tan 4θ is equal to

A. 1

B. 0

C. - 1

D. $1 + \sqrt{3}$

Answer

Given,

5θ and 4 θ are acute angles,

Therefore,

5θ + 4θ = 90°

9θ = 90°

θ = 90/9 = 10°

Then value of-

2 sin 3θ - √3 tan 3 θ

Putting value of θ = 10°,

We get,

⇒ 2 sin 3(10) - √3 tan 3(10)

⇒ 2 sin 30° - √3 tan 30° [∵ sin 30° = 1/2 and tan 30° = 1/√3]

⇒ 2× 1/2 - √3 x

⇒ 1 − 1 = 0

25. Question

If A + B = 90°, then $\frac{\tan A\ \tan B + \tan A\ \cot B}{\sin A \sec B} - \frac{\sin^2 B}{\cos^2 A}$ is equal to

A. $\cot^2 A$

B. $\cot^2 B$

C. $-\tan^2 A$

D. $-\cot^2 A$

Answer

Given,

A + B = 90°

B = 90° - A

Putting this Value in the given equation we get,

$$\Rightarrow \frac{\tan A \tan B + \tan A . \cot B}{\sin A . \sec B} - \frac{\sin^2 B}{\cos^2 A}$$

$$\Rightarrow \frac{\tan A \tan(90 - A) + \tan A . \cot(90 - A)}{\sin A . \sec(90 - A)} - \frac{\sin^2 (90 - A)}{\cos^2 A}$$

∵ tan (90 – A) = Cot A sin(90 – A) = cos A

cot (90 – A) = tan A

sec (90 – A) = cosec A

$$\Rightarrow \frac{\tan A . \cot A + \tan A . \tan A}{\sin A . \cos B} - \frac{\cos^2 A}{\cos^2 A}$$

[∵ cot A = 1/tan A and cosec A = 1/sin A]

$$\Rightarrow \frac{\tan A . \frac{1}{\tan A} + \tan A . \tan A}{\sin A . \frac{1}{\sin A}} - 1$$

⇒ 1 + $\tan^2$ A – 1

∵ A + B = 90°

A = 90 – B

So,

⇒ $\tan^2(90 – B)$

⇒ $\cot^2 B$

26. Question

$\frac{2 \tan 30°}{1 + \tan^2 30°}$ is equal to

A. sin 60°

B. cos 60°

C. tan 60°

D. sin 30°

Answer

Given,

$$\frac{2 \tan 30°}{1 + \tan^2 30°}$$

$$\frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$\Rightarrow \frac{\frac{2}{\sqrt{3}}}{\frac{3 + 1}{3}}$$

$$\Rightarrow \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$\Rightarrow \frac{3}{2\sqrt{3}}$$

[∵ sin 60° = √3/2]

$$\Rightarrow \frac{\sqrt{3}}{2} = \sin 60°$$

27. Question

$\frac{1 - \tan^2 45°}{1 + \tan^2 45°}$ is equal to

A. tan 90°

B. 1

C. sin 45°

D. sin 0°

Answer

Given,

$$\frac{1 - \tan^2 45}{1 + \tan^2 45}$$

∵ tan 45 = 1

Put this value,

We get;

$$\frac{1-(1)^2}{1+(1)^2}=0$$

sin 0°

Since sin 0° = 0

28. Question

sin 2A = 2 sin A is true when A =

A. 0°

B. 30°

C. 45°

D. 60°

Answer

Sin 2 A = 2 sin A

[∵ 2A = 2 sin A. Cos A]

⇒ 2 sin A . cos A = 2 sin A

⇒ cos A = 1 = cos 0°

⇒ A = 0°

29. Question

$\frac{2\tan 30°}{1-\tan^2 30°}$ is equal to

A. cos 60°

B. sin 60°

C. tan 60°

D. sin 30°

Answer

$$\frac{2\tan 30°}{1-\tan^2 30°}$$

$$\frac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$$

$$\Rightarrow \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}}$$

$$\Rightarrow \frac{2}{\sqrt{3}} \times \frac{3}{2}$$

$$\Rightarrow \sqrt{3} = \tan 60°$$

tan 60° [∵ tan 60° = √3]

30. Question

If A, B and C are interior angles of a triangle ABC, then $\sin\left(\frac{B+C}{2}\right)$ =.

A. sin A/2

B. cos A/2

C. −sin A/2

D. -cos A/2

Answer

Given,

A, B and C are the interior angles of Δ ABC,

Therefore,

A + B + C = 180°

B + C = 180° - A

$$\frac{B\ +\ C}{2} = \frac{180 - A}{2}$$

$$\frac{B\ +\ C}{2} = 90 - \frac{A}{2}$$

Now put this value in the given equation we get,

$$\sin\left(\frac{B\ +\ C}{2}\right) = \sin\left(90 - \frac{A}{2}\right)$$

$\Rightarrow \cos\frac{A}{2}$ [∵ sin (90 – θ) = cos θ]

31. Question

If cos θ = 2/3 then 2 $\sec^2\theta$ + 2$\tan^2\theta$ – 4 is equal to

A. 1

B. 0

C. 3

D. 4

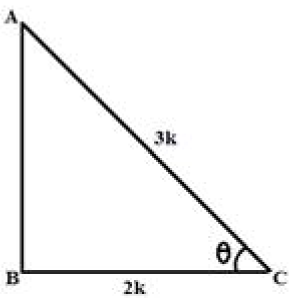
Answer

Given,

cos θ = 2/3 = b/h = k

2$\sec^2$ θ + 2 $\tan^2\theta$ – 7

b = 2k, h = 3k



In ΔABC,

$h^2 = p^2 + b^2$

⇒ $(3k)^2 = p^2 + (2k)^2$

⇒ $9k^2 = p^2 + 4k^2$

⇒ $p^2 = 9k^2 – 4k^2$

⇒ $p^2 = 5k^2$

⇒ p = √5k

Then,

Sec θ = h/b = 3k/2k = 3/2 and

Tan θ = p/b = √5k/2k = √5/2

⇒ 2 $\sec^2$ θ + 2 $\tan^2$ θ – 7

$\Rightarrow 2\left(\frac{3}{2}\right)^2 + 2\left(\frac{\sqrt{5}}{2}\right)^2 - 7$

$\Rightarrow 2 \times \frac{9}{4} + 2 \times \frac{5}{4} - 7$

$\Rightarrow \frac{9}{2} + \frac{5}{4} - 7$

$\Rightarrow \frac{9+5-17}{2} = 0$

32. Question

tan 5° × tan 30° × 4 tan 85° is equal to

A. 4/√3

B. 4√3

C. 1

D. 4

Answer

⇒ tan 5° x tan 30° x 4 tan 85°

As,

tan (90 – θ) = cot θ

Therefore,

⇒ tan (90 – 85) x tan 30° x 4 tan 85°

⇒ cot 85° × tan 85° × 4 × tan 30°

⇒ 1/tan 85 × tan 85° × 4 × tan 30°

As we know,

tan 30° = 1/√3

⇒ 4 × tan 30°

⇒ 4 × (1/√3) = 4/√3

33. Question

The value of $\frac{\tan 55^\circ}{\cot 35^\circ}$ + cot1° cot 2° cot 3°.... cot 90°, is

A. - 2

B. 2

C. 1

D. 0

Answer

Given,

$$\frac{\tan 55°}{\cot 35°} + \cot 1°.\cot 2°.\cot 3°....\cot 90°$$

$$\frac{\tan(90 - 35)}{\cot 35°} + \cot(90-89).\cot(90-88).\cot(90-87)....\cot 90°$$

As we know,

Cot 90° = 0

Therefore,

$$\frac{\cot 35°}{\cot 35°} + \tan 89°.\tan 88°.\tan 87°....\cot 89°.\cot 90°$$

1 + 0 = 1

34. Question

In Fig. 5.47, the value of cos ϕ is

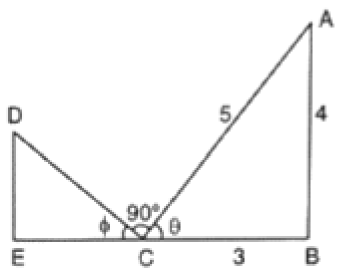


Fig. 5.47

A. 5/4

B. 5/3

C. 3/5

D. 4/5

Answer

As we know that sum of the angles of the straight line is 180°,

Therefore,

$\angle\theta + \angle 90 + \angle\phi = 180$

$\angle\theta + \angle\phi = 90$

In ΔABC,

$\sin\theta = 4/5 = p/h$

Putting $\theta = 90 - \phi$

We get,

sin (90 - ϕ) = 4/5

As,

Sin (90 - ϕ) = cos ϕ

cos ϕ = 4/5

35. Question

In Fig. 5.48, AD = 4 cm BD = 3 cm and CB = 12 cm, find cot θ.

A. $\frac{12}{13}$

B. $\frac{5}{12}$

C. $\frac{13}{12}$

D. $\frac{12}{13}$

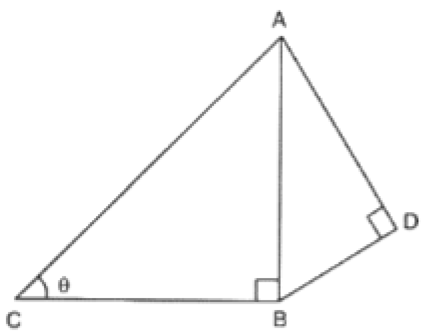


Fig. 5.48

Answer

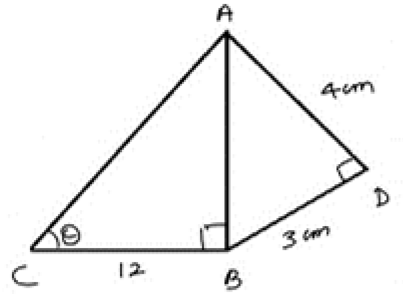
Given,

AD = 4cm

BD = 3 cm

CB = 12 cm

In ΔABC,



$AB^2 = AD^2 + BD^2$

$AB^2 = 4^2 + 3^2$

AB = √16 + 9 = 5 cm

Then,

cot θ = CB/AB = 12/5