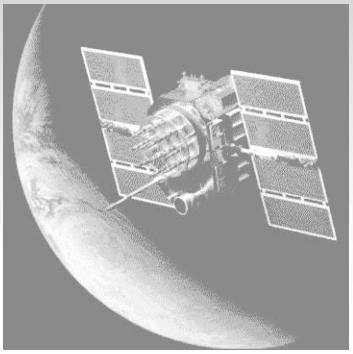


Chapter

8

Gravitation

Contents

Canadian Space Agency astronaut Chris Hadfield experiences the physiological effects and the freedom of weightlessness.

Weightlessness poses many serious problems to the astronauts. It becomes quite difficult for them to control their movements. Everything in the satellite

Gravitation

8.1 Introduction

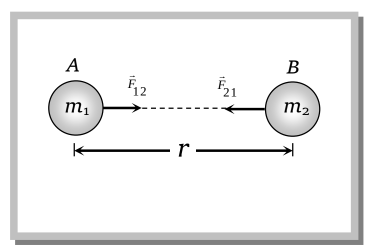
Newton at the age of twenty-three is said to have seen an apple falling down from tree in his orchid. This was the year 1665. He started thinking about the role of earth's attraction in the motion of moon and other heavenly bodies.

By comparing the acceleration due to gravity due to earth with the acceleration required to keep the moon in its orbit around the earth, he was able to arrive the Basic Law of Gravitation.

8.2 Newton's law of Gravitation

Newton's law of gravitation states that every body in this universe attracts every other body with a force, which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres. The direction of the force is along the line joining the particles.

Thus the magnitude of the gravitational force F that two particles of masses m_1 and m_2 separated by a distance r exert on each other is given by $F \propto \frac{m_1 m_2}{r^2}$



or $$F = G\frac{m_1 m_2}{r^2}$$

Vector form : According to Newton's law of gravitation

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r^2}\hat{r}_{21} = \frac{-Gm_1m_2}{r^3}\vec{r}_{21} = \frac{-Gm_1m_2}{|\vec{r}_{21}|^3}\vec{r}_{21}$$

$\hat{r}_{12}$ = unit vector from A to B

$\hat{r}_{21}$ = unit vector from B to A,

$\vec{F}_{12}$ = gravitational force exerted on body A by body B

$\vec{F}_{21}$

Here negative sign indicates that the direction of $\vec{F}_{12}$ is opposite to that of $\hat{r}_{21}$.

Similarly $$\vec{F}_{21} = \frac{-Gm_1m_2}{r^2}\hat{r}_{12} = \frac{-Gm_1m_2}{r^3}\vec{r}_{12} = \frac{-Gm_1m_2}{|\vec{r}_{12}|^3}\vec{r}_{12}$$

$$= \frac{Gm_1m_2}{r^2}\hat{r}_{21} \quad [\because \hat{r}_{12} = -\hat{r}_{21}]$$

$\therefore$ It is clear that $\vec{F}_{12} = -\vec{F}_{21}$. Which is Newton's third law of motion.

Here G is constant of proportionality which is called 'Universal gravitational constant'.

If $m_1 = m_2$ and $r = 1$ then $G = F$

i.e. universal gravitational constant is equal to the force of attraction between two bodies each of unit mass whose centres are placed unit distance apart.

Important points

(i) The value of G in the laboratory was first determined by Cavendish using the torsional balance.

(ii) The value of G is 6.67×10^{-11} $N\text{–}m^2\ kg^{-2}$ in S.I. and 6.67×10^{-8} $dyne\text{-}\ cm^2\text{-}g^{-2}$ in C.G.S. system.

(iii) Dimensional formula $[M^{-1}L^3T^{-2}]$.

(iv) The value of G does not depend upon the nature and size of the bodies.

(v) It also does not depend upon the nature of the medium between the two bodies.

(vi) As G is very small hence gravitational forces are very small, unless one (or both) of the masses is huge.

8.3 Properties of Gravitational Force

(1) It is always attractive in nature while electric and magnetic force can be attractive or repulsive.

(2) It is independent of the medium between the particles while electric and magnetic force depend on the nature of the medium between the particles.

(3) It holds good over a wide range of distances. It is found true for interplanetary to inter atomic distances.

(4) It is a central force *i.e.* acts along the line joining the centres of two interacting bodies.

(5) It is a two-body interaction *i.e.* gravitational force between two particles is independent of the presence or absence of other particles; so the principle of superposition is valid *i.e.* force on a particle due to number of particles is the resultant of forces due to individual particles *i.e.* $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots..$

While nuclear force is many body interaction

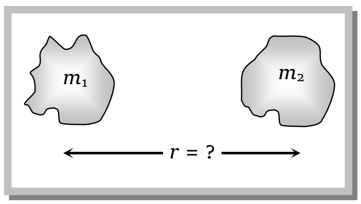
(6) It is the weakest force in nature : As $F_{nuclear} > F_{electromagnetic} > F_{gravitational}$.

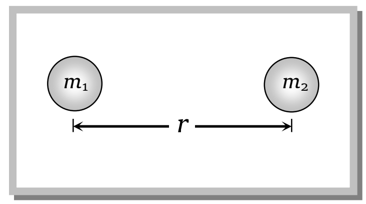
(7) The ratio of gravitational force to electrostatic force between two electrons is of the order of 10^{-43}.

(8) It is a conservative force *i.e.* work done by it is path independent or work done in moving a particle round a closed path under the action of gravitational force is zero.

(9) It is an action reaction pair *i.e.* the force with which one body (say earth) attracts the second body (say moon) is equal to the force with which moon attracts the earth. This is in accordance with Newton's third law of motion.

Note : ❑ The law of gravitation is stated for two point masses, therefore for any two arbitrary finite size bodies, as shown in the figure, It can not be applied as there is not unique value for the separation.





But if the two bodies are uniform spheres then the separation r may be taken as the distance between their centres because a sphere of uniform mass behave as a point mass for any point lying outside it.

Sample problems based on Newton's law of gravitation

Problem 1. The gravitational force between two objects does not depend on **[RPET 2003]**

(a) Sum of the masses (b) Product of the masses

(c) Gravitational constant (d) Distance between the masses

Solution : (a) $F = \dfrac{\text{Gravitational constant} \times \text{product of the masses}}{(\text{Distance between the masses})^2}$.

Problem 2. Mass M is divided into two parts xM and $(1 - x)M$. For a given separation, the value of x for which the gravitational attraction between the two pieces becomes maximum is

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) 1 (d) 2

Solution : (a) Gravitational force $F = \dfrac{Gm_1m_2}{r^2} = \dfrac{GxM(1-x)M}{r^2} = \dfrac{GM^2}{r^2}x(1-x)$

For maximum value of force $\dfrac{dF}{dx} = 0$ $\therefore$ $\dfrac{d}{dx}\left[\dfrac{GM^2x}{r^2}(1-x)\right] = 0$

$\Rightarrow \dfrac{d}{dx}(x - x^2) = 0 \Rightarrow 1 - 2x = 0 \Rightarrow x = 1/2$

Problem 3. The mass of the moon is about 1.2% of the mass of the earth. Compared to the gravitational force the earth exerts on the moon, the gravitational force the moon exerts on earth

(a) Is the same (b) Is smaller (c) Is greater (d) Varies with its phase

Solution : (a) Earth and moon both exerts same force on each other.

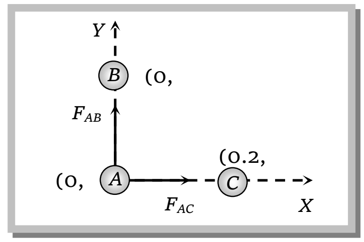
***Problem* 4.** Three identical point masses, each of mass 1*kg* lie in the *x*-*y* plane at points (0, 0), (0, 0.2*m*) and (0.2*m*, 0). The net gravitational force on the mass at the origin is

(a) $1.67\times10^{-9}(\hat{j}+\hat{j})N$ (b) $3.34\times10^{-10}(\hat{i}+\hat{j})N$

(c) $1.67\times10^{-9}(\hat{i}-\hat{j})N$ (d) $3.34\times10^{-10}(\hat{i}+\hat{j})N$

Solution : (a) Let particle A lies at origin, particle B and C on y and x -axis respectively

$$\overrightarrow{F_{AC}}=\frac{G\,m_A m_B}{r_{AB}^2}\hat{i}=\frac{6.67\times10^{-11}\times1\times1}{(0.2)^2}\hat{i}=1.67\times10^{-9}\,\hat{i}\,N$$



Similarly $\overrightarrow{F_{AB}}=1.67\times10^{-9}\,\hat{j}\,N$

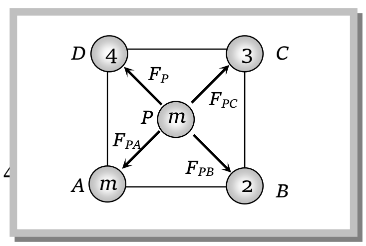
$\therefore$ Net force on particle A $\vec{F}=\vec{F}_{AC}+\vec{F}_{AB}=1.67\times10^{-9}(\hat{i}+\hat{j})\,N$

***Problem* 5.** Four particles of masses *m*, 2*m*, 3*m* and 4*m* are kept in sequence at the corners of a square of side *a*. The magnitude of gravitational force acting on a particle of mass *m* placed at the centre of the square will be

(a) $\frac{24m^2G}{a^2}$ (b) $\frac{6m^2G}{a^2}$ (c) $\frac{4\sqrt{2}Gm^2}{a^2}$ (d) Zero

Solution : (c) If two particles of mass m are placed x distance apart then force of attraction $\frac{Gmm}{x^2}=F$ (Let)

Now according to problem particle of mass m is placed at the centre (P) of square. Then it will experience four forces



$F_{PA}=$ force at point P due to particle $A=\frac{Gmm}{x^2}=F$

Similarly $F_{PB}=\frac{G2mm}{x^2}=2F$, $F_{PC}=\frac{G3mm}{x^2}=3F$ and $F_{PD}=\frac{G4mm}{x^2}=4F$

Hence the net force on P $\vec{F}_{net}=\vec{F}_{PA}+\vec{F}_{PB}+\vec{F}_{PC}+\vec{F}_{PD}=2\sqrt{2}\,F$

$\therefore$ $\vec{F}_{net}=2\sqrt{2}\frac{Gmm}{x^2}=2\sqrt{2}\frac{Gm^2}{(a/\sqrt{2})^2}$ [$x=\frac{a}{\sqrt{2}}=$ half of the diagonal of the square]

$$=\frac{4\sqrt{2}\,Gm^2}{a^2}.$$

8.4 Acceleration Due to Gravity

The force of attraction exerted by the earth on a body is called gravitational pull or gravity.

We know that when force acts on a body, it produces acceleration. Therefore, a body under the effect of gravitational pull must accelerate.

The acceleration produced in the motion of a body under the effect of gravity is called acceleration due to gravity, it is denoted by g.

Consider a body of mass m is lying on the surface of earth then gravitational force on the body is given by

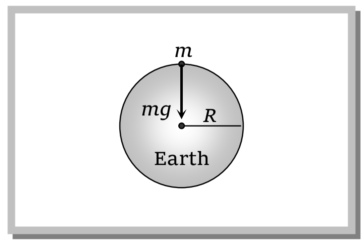
$$F = \frac{GMm}{R^2} \qquad .....(i)$$

Where M = mass of the earth and R = radius of the earth.

If g is the acceleration due to gravity, then the force on the body due to earth is given by

Force = mass × acceleration

or $$F = mg \qquad .....(ii)$$



From (i) and (ii) we have $mg = \frac{GMm}{R^2}$

$$\therefore \quad g = \frac{GM}{R^2} \qquad .....(iii)$$

$$\Rightarrow \quad g = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right)$$ [As mass (M) = volume ($\frac{4}{3}\pi R^3$) × density (ρ)]

$$\therefore \quad g = \frac{4}{3}\pi\rho G R \qquad .....(iv)$$

Important points

(i) From the expression $g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR$ it is clear that its value depends upon the mass radius and density of planet and it is independent of mass, shape and density of the body placed on the surface of the planet. *i.e.* a given planet (reference body) produces same acceleration in a light as well as heavy body.

(ii) The greater the value of (M/R^2) or ρR, greater will be value of g for that planet.

(iii) Acceleration due to gravity is a vector quantity and its direction is always towards the centre of the planet.

(iv) Dimension $[g] = [LT^{-2}]$

(v) it's average value is taken to be 9.8 m/s^2 or 981 cm/sec^2 or 32 $feet/sec^2$, on the surface of the earth at mean sea level.

(vi) The value of acceleration due to gravity vary due to the following factors : (a) Shape of the earth, (b) Height above the earth surface, (c) Depth below the earth surface and (d) Axial rotation of the earth.

Sample problems based on acceleration due to gravity

***Problem* 6.** Acceleration due to gravity on moon is 1/6 of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_m) and moon (ρ_e) is $\left(\frac{\rho_e}{\rho_m}\right) = \frac{5}{3}$ then radius of moon R_m in terms of R_e will be **[MP PMT 2003]**

(a) $\frac{5}{18}R_e$ (b) $\frac{1}{6}R_e$ (c) $\frac{3}{18}R_e$ (d) $\frac{1}{2\sqrt{3}}R_e$

Solution : (a) Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR$ $\therefore$ $g \propto \rho R$ or $\frac{g_m}{g_e} = \frac{\rho_m}{\rho_e}.\frac{R_m}{R_e}$ [As $\frac{g_m}{g_e} = \frac{1}{6}$ and $\frac{\rho_e}{\rho_m} = \frac{5}{3}$ (given)]

$\therefore$ $\frac{R_m}{R_e} = \left(\frac{g_m}{g_e}\right)\left(\frac{\rho_e}{\rho_m}\right) = \frac{1}{6}\times\frac{5}{3}$ $\therefore$ $R_m = \frac{5}{18}R_e$

***Problem* 7.** A spherical planet far out in space has a mass M_0 and diameter D_0. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity which is equal to

[MP PMT 1987; DPMT 2002]

(a) GM_0/D_0^2 (b) $4mGM_0/D_0^2$ (c) $4GM_0/D_0^2$ (d) GmM_0/D_0^2

Solution : (c) We know $g = \frac{GM}{R^2} = \frac{GM}{(D/2)^2} = \frac{4GM}{D^2}$

If mass of the planet $= M_0$ and diameter of the planet $= D_0$. Then $g = \frac{4GM_0}{D_0^2}$.

***Problem* 8.** The moon's radius is 1/4 that of the earth and its mass is 1/80 times that of the earth. If g represents the acceleration due to gravity on the surface of the earth, that on the surface of the moon is

[MP PMT 1997; RPET 2000; MP PET 2000, 2001]

(a) $\frac{g}{4}$ (b) $\frac{g}{5}$ (c) $\frac{g}{6}$ (d) $\frac{g}{8}$

Solution : (b) Acceleration due to gravity $g = \frac{GM}{R^2}$ $\therefore$ $\frac{g_{moon}}{g_{earth}} = \frac{M_{moon}}{M_{earth}}.\frac{R_{earth}^2}{R_{moon}^2} = \left(\frac{1}{80}\right)\left(\frac{4}{1}\right)^2$

$g_{moon} = g_{earth}\times\frac{16}{80} = \frac{g}{5}$.

***Problem* 9.** If the radius of the earth were to shrink by 1% its mass remaining the same, the acceleration due to gravity on the earth's surface would**[IIT-JEE 1981; CPMT 1981; MP PMT 1996, 97; Roo**

(a) Decrease by 2% (b) Remain unchanged (c) Increase by 2% (d)

Solution : (c) We know $g \propto \frac{1}{R^2}$ [As R decreases, g increases]

So % change in $g = 2$ (% change in R) $= 2\times 1\% = 2\%$

$\therefore$ acceleration due to gravity increases by 2%.

***Problem* 10.** Mass of moon is $7.34\times10^{22}kg$. If the acceleration due to gravity on the moon is $1.4 m/s^2$, the radius of the moon is $(G = 6.667\times10^{-11}Nm^2/kg^2)$

(a) $0.56\times10^4 m$ (b) $1.87\times10^6 m$ (c) $1.92\times10^6 m$ (d) $1.01\times10^8 m$

Solution : (b) We know $g = \frac{GM}{R^2}$ $\therefore R = \sqrt{\frac{GM}{g}} = \sqrt{\frac{6.67\times10^{-11}\times7.34\times10^{22}}{1.4}} = 1.87\times10^{6}\,m$.

Problem **11.** A planet has mass 1/10 of that of earth, while radius is 1/3 that of earth. If a person can throw a stone on earth surface to a height of 90*m*, then he will be able to throw the stone on that planet to a height **[RPMT 1994]**

(a) 90*m* (b) 40*m* (c) 100*m* (d) 45*m*

Solution : (c) Acceleration due to gravity $g = \frac{GM}{R^2}$ $\therefore \frac{g_{planet}}{g_{earth}} = \frac{M_{planet}}{M_{earth}}\left(\frac{R_{earth}}{R_{planet}}\right)^2 = \frac{1}{10}\times\left(\frac{3}{1}\right)^2 = \frac{9}{10}$

If a stone is thrown with velocity u from the surface of the planet then maximum height $H = \frac{u^2}{2g}$

$\frac{H_{planet}}{H_{earth}} = \frac{g_{earth}}{g_{planet}}$ $\Rightarrow H_{planet} = \frac{10}{9}\times H_{earth} = \frac{10}{9}\times 90$ = 100 *metre.*

Problem **12.** The radii of two planets are respectively R_1 and R_2 and their densities are respectively ρ_1 and ρ_2. The ratio of the accelerations due to gravity at their surfaces is

(a) $g_1 : g_2 = \frac{\rho_1}{R_1^2} : \frac{\rho_2}{R_2^2}$ (b) $g_1 : g_2 = R_1R_2 : \rho_1\rho_2$

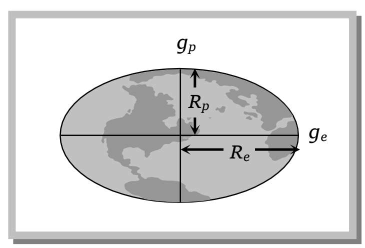
(c) $g_1 : g_2 = R_1\rho_2 : R_2\rho_1$ (d) $g_1 : g_2 = R_1\rho_1 : R_2\rho_2$

Solution : (d) Acceleration due to gravity $g = \frac{4}{3}\pi\rho GR$ $\therefore g_1 : g_2 = R_1\rho_1 : R_2\rho_2$.

8.5 Variation in *g* Due to Shape of Earth

Earth is elliptical in shape. It is flattened at the poles and bulged out at the equator. The equatorial radius is about 21 *km* longer than polar radius, from

$g = \frac{GM}{R^2}$



At equator $g_e = \frac{GM}{R_e^2}$ ......(i)

At poles $g_p = \frac{GM}{R_p^2}$ .....(ii)

From (i) and (ii) $\frac{g_e}{g_p} = \frac{R_p^2}{R_e^2}$

Since $R_{equator} > R_{pole}$ $\therefore g_{pole} > g_{equator}$ and $g_p = g_e + 0.018\ ms^{-2}$

Therefore the weight of body increases as it is taken from equator to the pole.

Sample problems based on variation in g due to shape of the earth

Problem **13.** Where will it be profitable to purchase 1 *kg* sugar (by spring balance)

(a) At poles (b) At equator (c) At 45° latitude (d) At 40°latitude

Solution : (b) At equator the value of g is minimum so it is profitable to purchase sugar at this position.

***Problem* 14.** Force of gravity is least at

(a) The equator (b) The poles

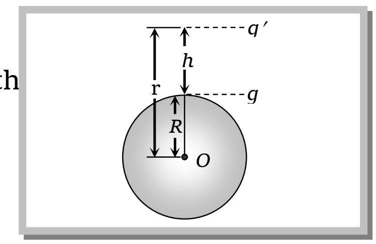
(c) A point in between equator and any pole (d) None of these

Solution : (a)

8.6 Variation in *g* With Height

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} \quad .....(i)$$



Acceleration due to gravity at height h from the surface of th

$$g' = \frac{GM}{(R+h)^2} \quad .....(ii)$$

From (i) and (ii) $g' = g\left(\frac{R}{R+h}\right)^2$ .....(iii)

$$= g\frac{R^2}{r^2} \quad .....(iv) \quad [\text{As } r = R + h]$$

Important points

(i) As we go above the surface of the earth, the value of g decreases because $g' \propto \frac{1}{r^2}$.

(ii) If $r = \infty$ then $g' = 0$, *i.e.*, at infinite distance from the earth, the value of g becomes zero.

(iii) If $h << R$ *i.e.*, height is negligible in comparison to the radius then from equation (iii) we get

$$g' = g\left(\frac{R}{R+h}\right)^2 = g\left(1+\frac{h}{R}\right)^{-2} = g\left[1-\frac{2h}{R}\right] \quad [\text{As } h << R]$$

(iv) If $h << R$ then decrease in the value of g with height :

Absolute decrease $\Delta g = g - g' = \frac{2hg}{R}$

Fractional decrease $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{2h}{R}$

Percentage decrease $\frac{\Delta g}{g} \times 100\,\% = \frac{2h}{R} \times 100\,\%$

Sample problems based on variation in g with height

***Problem* 15.** The acceleration of a body due to the attraction of the earth (radius R) at a distance $2R$ from the surface of the earth is (g = acceleration due to gravity at the surface of the earth)

(a) $\frac{g}{9}$ (b) $\frac{g}{3}$ (c) $\frac{g}{4}$ (d) g

Solution : (a) $\frac{g'}{g} = \left(\frac{R}{R+h}\right)^2 = \left(\frac{R}{R+2R}\right)^2 = \frac{1}{9}$ $\therefore\ g' = \frac{g}{9}$.

Problem **16.** The height of the point vertically above the earth's surface, at which acceleration due to gravity becomes 1% of its value at the surface is (Radius of the earth = R)

(a) $8R$ (b) $9R$ (c) $10\ R$ (d) $20R$

Solution : (b) Acceleration due to gravity at height h is given by $g' = g\left(\frac{R}{R+h}\right)^2$

$$\Rightarrow \frac{g}{100} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{10} \Rightarrow h = 9R .$$

Problem **17.** At surface of earth weight of a person is 72 N then his weight at height $R/2$ from surface of earth is (R = radius of earth)

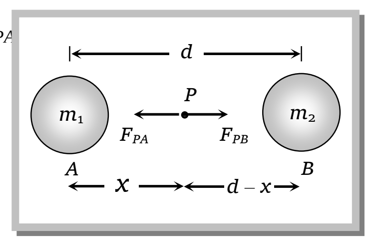
(a) $28N$ (b) $16N$ (c) $32N$ (d) $72N$

Solution : (c) Weight of the body at height R, $W' = W\left(\frac{R}{R+h}\right)^2 = W\left(\frac{R}{R+\frac{R}{2}}\right)^2 = W\left(\frac{2}{3}\right)^2 = \frac{4}{9}W = \frac{4}{9}\times 72 = 32\ N$.

Problem **18.** If the distance between centres of earth and moon is D and the mass of earth is 81 times the mass of moon, then at what distance from centre of earth the gravitational force will be zero **[RPET 1996]**

(a) $D/2$ (b) $2D/3$ (c) $4D/3$ (d) $9D/10$

Solution : (d) If P is the point where net gravitational force is zero then F_{PA}



$$\frac{Gm_1m}{x^2} = \frac{Gm_2m}{(d-x)^2}$$

By solving $x = \frac{\sqrt{m_1}\,d}{\sqrt{m_1}+\sqrt{m_2}}$

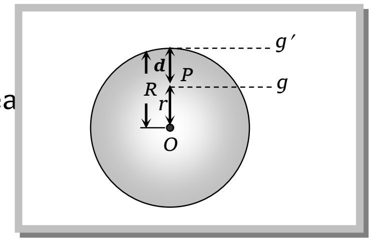
For the given problem $d = D$, m_1 = earth, m_2 = moon and $m_1 = 81m_2$ $\therefore m_2 = \frac{m_1}{81}$

So $x = \frac{\sqrt{m_1}\,D}{\sqrt{m_1}+\sqrt{m_2}} = \frac{\sqrt{m_1}\,D}{\sqrt{m_1}+\sqrt{\frac{m_1}{81}}} = \frac{D}{1+\frac{1}{9}} = \frac{9D}{10}$

8.7 Variation in *g* With Depth

Acceleration due to gravity at the surface of the earth

$$g = \frac{GM}{R^2} = \frac{4}{3}\pi\rho GR \quad .....(i)$$



Acceleration due to gravity at depth d from the surface of the ea

$$g' = \frac{4}{3}\pi\rho G(R-d) \quad .....(ii)$$

From (i) and (ii) $\quad g' = g\left[1 - \frac{d}{R}\right]$

Important points

(i) The value of g decreases on going below the surface of the earth. From equation (ii) we get $g' \propto (R-d)$.

So it is clear that if d increase, the value of g decreases.

(ii) At the centre of earth $d = R \;\therefore\; g' = 0$, *i.e.,* the acceleration due to gravity at the centre of earth becomes zero.

(iii) Decrease in the value of g with depth

Absolute decrease $\Delta g = g - g' = \frac{dg}{R}$

Fractional decrease $\frac{\Delta g}{g} = \frac{g - g'}{g} = \frac{d}{R}$

Percentage decrease $\frac{\Delta g}{g} \times 100\,\% = \frac{d}{R} \times 100\,\%$

(iv) The rate of decrease of gravity outside the earth (if $h << R$) is double to that of inside the earth.

Sample problems based on variation in g with depth

***Problem* 19.** Weight of a body of mass m decreases by 1% when it is raised to height h above the earth's surface. If the body is taken to a depth h in a mine, change in its weight is **[KCET 2003; MP PMT 2003]**

(a) 2% decrease (b) 0.5% decrease (c) 1% increase (d) 0.5% increase

Solution : (b) Percentage change in g when the body is raised to height h, $\frac{\Delta g}{g} \times 100\,\% = \frac{2h \times 100}{R} = 1\%$

Percentage change in g when the body is taken into depth d, $\frac{\Delta g}{g} \times 100\,\% = \frac{d}{R} \times 100\,\% = \frac{h}{R} \times 100\,\%$

[As $d = h$]

$\therefore$ Percentage decrease in weight $= \frac{1}{2}\left(\frac{2h}{R} \times 100\right) = \frac{1}{2}(1\%) = 0.5\%$.

Problem **20.** The depth at which the effective value of acceleration due to gravity is $\frac{g}{4}$ is (R = radius of the earth)

[MP PET 2003]

(a) R (b) $\frac{3R}{4}$ (c) $\frac{R}{2}$ (d) $\frac{R}{4}$

Solution : (b) $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{4} = g\left(1 - \frac{d}{R}\right) \Rightarrow d = \frac{3R}{4}$

Problem **21.** Assuming earth to be a sphere of a uniform density, what is the value of gravitational acceleration in a mine 100 *km* below the earth's surface (Given R = 6400*km*)

(a) $9.66\, m/s^2$ (b) $7.64\, m/s^2$ (c) $5.06\, m/s^2$ (d) $3.10\, m/s^2$

Solution : (a) Acceleration due to gravity at depth *d*, $g' = g\left[1 - \frac{d}{R}\right] = g\left[1 - \frac{100}{6400}\right] = 9.8\left[1 - \frac{1}{64}\right]$

$= 9.8 \times \frac{63}{64} = 9.66\, m/s^2$.

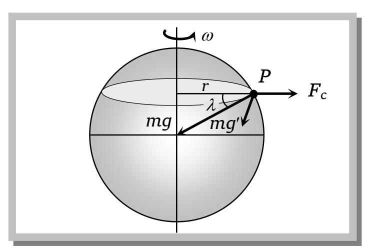
Problem **22.** The depth *d* at which the value of acceleration due to gravity becomes $\frac{1}{n}$ times the value at the surface, is [R = radius of the earth]

(a) $\frac{R}{n}$ (b) $R\left(\frac{n-1}{n}\right)$ (c) $\frac{R}{n^2}$ (d) $R\left(\frac{n}{n+1}\right)$

Solution : (b) $g' = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{g}{n} = g\left(1 - \frac{d}{R}\right) \Rightarrow \frac{d}{R} = 1 - \frac{1}{n} \Rightarrow d = \left(\frac{n-1}{n}\right)R$

8.8 Variation in *g* Due to Rotation of Earth

As the earth rotates, a body placed on its surface moves along the circular path and hence experiences centrifugal force, due to it, the apparent weight of the body decreases.



Since the magnitude of centrifugal force varies with the latitude of the place, therefore the apparent weight of the body varies with latitude due to variation in the magnitude of centrifugal force on the body.

If the body of mass m lying at point P, whose latitude is λ, then due to rotation of earth its apparent weight can be given by $\overrightarrow{mg'} = \overrightarrow{mg} + \overrightarrow{F_c}$

or $$mg' = \sqrt{(mg)^2 + (F_c)^2 + 2mg\, F_c \cos(180 - \lambda)}$$

$\Rightarrow$ $$mg' = \sqrt{(mg)^2 + (m\omega^2 R\cos\lambda)^2 + 2mg\; m\omega^2 R\cos\lambda\,(-\cos\lambda)}$$ [As $F_c = m\omega^2 r = m\omega^2 R\cos\lambda$]

By solving we get $g' = g - \omega^2 R\cos^2\lambda$

Note : ❑ The latitude at a point on the surface of the earth is defined as the angle, which the line joining that point to the centre of earth makes with equatorial plane. It is denoted by λ.

❑ For the poles $\lambda = 90^o$ and for equator $\lambda = 0^o$

Important points

(i) Substituting $\lambda = 90^o$ in the above expression we get $g_{pole} = g - \omega^2 R\cos^2 90^o$

$\therefore \quad g_{pole} = g$ .....(i)

i.e., there is no effect of rotational motion of the earth on the value of g at the poles.

(ii) Substituting $\lambda = 0^o$ in the above expression we get $g_{eqator} = g - \omega^2 R\cos^2 0^o$

$\therefore \quad g_{equator} = g - \omega^2 R$ .....(ii)

i.e., the effect of rotation of earth on the value of g at the equator is maximum.

From equation (i) and (ii) $g_{pole} - g_{equator} = R\omega^2 = 0.034\, m/s^2$

(iii) When a body of mass m is moved from the equator to the poles, its weight increases by an amount

$$m(g_p - g_e) = m\omega^2 R$$

(iv) Weightlessness due to rotation of earth : As we know that apparent weight of the body decreases due to rotation of earth. If ω is the angular velocity of rotation of earth for which a body at the equator will become weightless

$$g' = g - \omega^2 R\cos^2\lambda$$

$\Rightarrow \quad 0 = g - \omega^2 R\cos^2 0^o$ [As $\lambda = 0^o$ for equator]

$\Rightarrow \quad g - \omega^2 R$

$\therefore \quad \omega = \sqrt{\dfrac{g}{R}}$

or time period of rotation of earth $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R}{g}}$

Substituting the value of $R = 6400 \times 10^3 m$ and $g = 10 m/s^2$ we get

$$\omega = \frac{1}{800} = 1.25 \times 10^{-3}\frac{rad}{sec} \quad \text{and} \quad T = 5026.5\ sec = 1.40\ hr.$$

Note : ❑ This time is about $\frac{1}{17}$ times the present time period of earth. Therefore if earth starts rotating 17 times faster then all objects on equator will become weightless.

❑ If earth stops rotation about its own axis then at the equator the value of g increases by $\omega^2 R$ and consequently the weight of body lying there increases by $m\omega^2 R$.

❑ After considering the effect of rotation and elliptical shape of the earth, acceleration due to gravity at the poles and equator are related as

$$g_p = g_e + 0.034 + 0.018\, m/s^2 \quad \therefore g_p = g_e + 0.052\, m/s^2$$

Sample problems based on variation in g due to rotation of the earth

Problem 23. The angular velocity of the earth with which it has to rotate so that acceleration due to gravity on 60° latitude becomes zero is (Radius of earth = 6400 *km*. At the poles g = 10 ms^{-2}) **[EAMCET 2000]**

(a) $2.5\times10^{-3} rad/sec$ (b) $5.0\times10^{-1} rad/sec$ (c) $10\times10^{1} rad/sec$ (d) $7.8\times10^{-2} rad/sec$

Solution : (a) Effective acceleration due to gravity due to rotation of earth $g' = g - \omega^2 R\cos^2\lambda$

$$\Rightarrow 0 = g - \omega^2 R\cos^2 60^o \Rightarrow \frac{\omega^2 R}{4} = g \Rightarrow \omega = \sqrt{\frac{4g}{R}} = 2\sqrt{\frac{g}{R}} = \frac{2}{800}\frac{rad}{sec} \quad \text{[As } g' = 0 \text{ and } \lambda = 60^o\text{]}$$

$$\Rightarrow \omega = \frac{1}{400} = 2.5\times10^{-3}\frac{rad}{\sec}.$$

Problem 24. If earth stands still what will be its effect on man's weight

(a) Increases (b) Decreases (c) Remains same (d) None of these

Solution : (a) When earth stops suddenly, centrifugal force on the man becomes zero so its effective weight increases.

Problem 25. If the angular speed of earth is increased so much that the objects start flying from the equator, then the length of the day will be nearly

(a) 1.5 hours (b) 8 hours (c) 18 hours (d) 24 hours

Solution : (a) Time period for the given condition $T = 2\pi\sqrt{\frac{R}{g}} = 1.40\, hr \approx 1.5\, hr$ nearly.

8.9 Mass and Density of Earth

Newton's law of gravitation can be used to estimate the mass and density of the earth.

As we know $g = \frac{GM}{R^2}$, so we have $M = \frac{gR^2}{G}$

$$\therefore \quad M = \frac{9.8\times(6.4\times10^6)^2}{6.67\times10^{-11}} = 5.98\times10^{24} kg \approx 10^{25} kg$$

and as we know $g = \frac{4}{3}\pi\rho GR$, so we have $\rho = \frac{3g}{4\pi GR}$

$$\therefore \qquad \rho = \frac{3 \times 9.8}{4 \times 3.14 \times 6.67 \times 10^{-11} \times 6.4 \times 10^{6}} = 5478.4\ kg/m^3$$

8.10 Inertial and Gravitational Masses

(1) **Inertial mass** : It is the mass of the material body, which measures its inertia.

If an external force F acts on a body of mass m_i, then according to Newton's second law of motion

$$F = m_i a \text{ or } m_i = \frac{F}{a}$$

Hence inertial mass of a body may be measured as the ratio of the magnitude of the external force applied on it to the magnitude of acceleration produced in its motion.

Important points

(i) It is the measure of ability of the body to oppose the production of acceleration in its motion by an external force.

(ii) Gravity has no effect on inertial mass of the body.

(iii) It is proportional to the quantity of matter contained in the body.

(iv) It is independent of size, shape and state of body.

(v) It does not depend on the temperature of body.

(vi) It is conserved when two bodies combine physically or chemically.

(vii) When a body moves with velocity v, its inertial mass is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$, where m_0 = rest mass of body, c = velocity of light in vacuum,

(2) **Gravitational Mass :** It is the mass of the material body, which determines the gravitational pull acting upon it.

If M is the mass of the earth and R is the radius, then gravitational pull on a body of mass m_g is given by

$$F = \frac{GMm_g}{R^2} \text{ or } m_g = \frac{F}{(GM/R^2)} = \frac{F}{E}$$

Here m_g is the gravitational mass of the body, if $E = 1$ then $m_g = F$

Thus the gravitational mass of a body is defined as the gravitational pull experienced by the body in a gravitational field of unit intensity,

(3) **Comparison between inertial and gravitational mass**

(i) Both are measured in the same units.

(ii) Both are scalars

(iii) Both do not depends on the shape and state of the body

(iv) Inertial mass is measured by applying Newton's second law of motion where as gravitational mass is measured by applying Newton's law of gravitation.

(v) Spring balance measure gravitational mass and inertial balance measure inertial mass.

(4) **Comparison between mass and weight of the body**

Mass *(m)*	**Weight *(W)***
It is a quantity of matter contained in a body.	It is the attractive force exerted by earth on any body.
Its value does not change with g	Its value changes with g.
Its value can never be zero for any material particle.	At infinity and at the centre of earth its value is zero.
Its unit is kilogram and its dimension is $[M]$.	Its unit is Newton or $kg\text{-}wt$ and dimension are $[MLT^{-2}]$
It is determined by a physical balance.	It is determined by a spring balance.
It is a scalar quantity.	It is a vector quantity.

Sample problems based on inertial and gravitational mass

***Problem* 26.** Gravitational mass is proportional to gravitational

(a) Field (b) Force (c) Intensity (d) All of these

Solution : (d)

***Problem* 27.** The ratio of the inertial mass to gravitational mass is equal to **[CPMT 1978]**

(a) 1/2 (b) 1 (c) 2 (d) No fixed number

Solution : (b)

8.11 Gravitational Field

The space surrounding a material body in which gravitational force of attraction can be experienced is called its gravitational field.

Gravitational field intensity : The intensity of the gravitational field of a material body at any point in its field is defined as the force experienced by a unit mass (test mass) placed at that point, provided the unit mass (test mass) itself does not produce any change in the field of the body.

So if a test mass m at a point in a gravitational field experiences a force $\vec{F}$ then

$$\vec{I} = \frac{\vec{F}}{m}$$

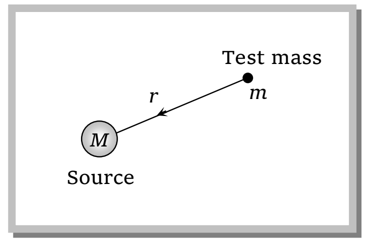
Important points

(i) It is a vector quantity and is always directed towards the centre of gravity of body whose gravitational field is considered.

(ii) Units : *Newton/kg* or m/s^2

(iii) Dimension : $[M^0LT^{-2}]$

(iv) If the field is produced by a point mass M and the test mass m is at a distance r from it then by Newton's law of gravitation $F = \frac{GMm}{r^2}$

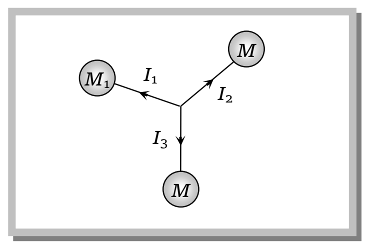


then intensity of gravitational field $I = \frac{F}{m} = \frac{GMm/r^2}{m}$

$$\therefore \qquad I = \frac{GM}{r^2}$$

(v) As the distance (r) of test mass from the point mass (M), increases, intensity of gravitational field decreases

$$I = \frac{GM}{r^2}; \quad \therefore \quad I \propto \frac{1}{r^2}$$



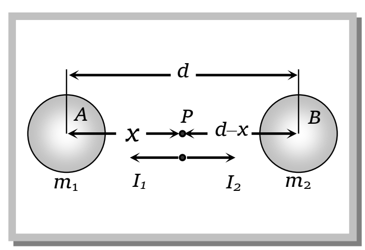
(vi) Intensity of gravitational field $I = 0$, when $r = \infty$.

(vii) Intensity at a given point (P) due to the combined effect of different point masses can be calculated by vector sum of different intensities

$$\overrightarrow{I_{net}} = \overrightarrow{I_1} + \overrightarrow{I_2} + \overrightarrow{I_3} + ...$$

(viii) Point of zero intensity : If two bodies A and B of different masses m_1 and m_2 are d distance apart.

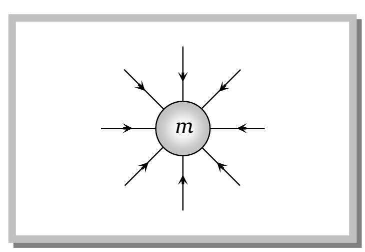
Let P be the point of zero intensity *i.e.*, the intensity at this point is equal and apposite due to two bodies A and B and if any test mass placed at this point it will not experience any force.



For point P $\quad \overrightarrow{I_1} + \overrightarrow{I_2} = 0 \;\Rightarrow\; \frac{-Gm_1}{x^2} + \frac{Gm_2}{(d-x)^2} = 0$

By solving $\quad x = \frac{\sqrt{m_1}\, d}{\sqrt{m_1} + \sqrt{m_2}}$ and $(d - x) = \frac{\sqrt{m_2}\, d}{\sqrt{m_1} + \sqrt{m_2}}$

(ix) Gravitational field line is a line, straight or curved such that a unit mass placed in the field of another mass would always move along this line. Field lines for an isolated mass m are radially inwards.



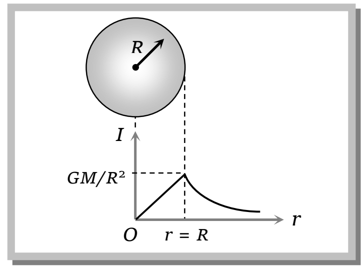
(x) As $I = \frac{GM}{r^2}$ and also $g = \frac{GM}{R^2}$ $\therefore$ $I = g$

Thus the intensity of gravitational field at a point in the field is equal to acceleration of test mass placed at that point.

8.12 Gravitational Field Intensity for Different Bodies

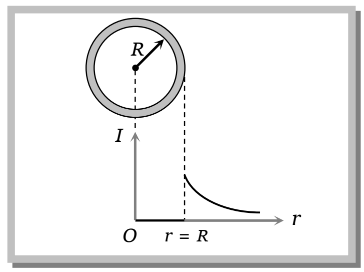
(1) **Intensity due to uniform solid sphere**

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = \frac{GMr}{R^3}$



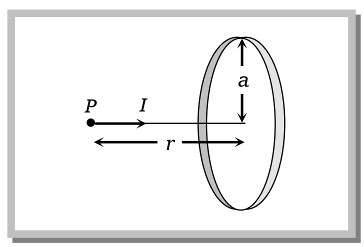
(2) **Intensity due to spherical shell**

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$I = \frac{GM}{r^2}$	$I = \frac{GM}{R^2}$	$I = 0$



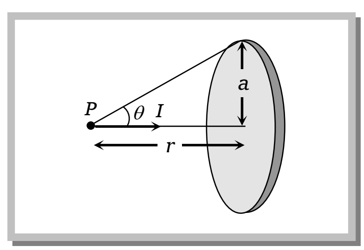
(3) **Intensity due to uniform circular ring**

At a point on its axis	At the centre of the ring
$I = \frac{GMr}{(a^2 + r^2)^{3/2}}$	$I = 0$



(4) **Intensity due to uniform disc**

At a point on its axis	At the centre of the disc
$I = \frac{2GMr}{a^2}\left[\frac{1}{r} - \frac{1}{\sqrt{r^2 + a^2}}\right]$ or $I = \frac{2GM}{a^2}(1 - \cos\theta)$	$I = 0$

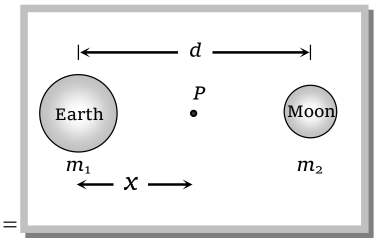


Sample problems based on gravitational field

Problem **28.** Knowing that mass of Moon is $\frac{M}{81}$ where M is the mass of Earth, find the distance of the point where gravitational field due to Earth and Moon cancel each other, from the Moon. Given that distance between Earth and Moon is 60 R. Where R is the radius of Earth

(a) 2 R (b) 4 R (c) 6 R (d) 8 R

Solution : (c) Point of zero intensity $x = \frac{\sqrt{m_1}\, d}{\sqrt{m_1} + \sqrt{m_2}}$



mass of the earth $m_1 = M$, Mass of the moon $m_2 = \frac{M}{81}$

and distance between earth & moon $d = 60\,R$

Point of zero intensity from the Earth $x = \frac{\sqrt{M} \times 60\,R}{\sqrt{M} + \sqrt{\frac{M}{81}}} = \frac{9}{10} \times 60\,R =$

So distance from the moon $= 60\,R - 54\,R = 6R$.

Problem **29.** The gravitational potential in a region is given by $V = (3x + 4y + 12z)\,J/kg$. The modulus of the gravitational field at $(x = 1, y = 0, z = 3)$ is

(a) $20\ N\,kg^{-1}$ (b) $13\ N\,kg^{-1}$ (c) $12\ N\,kg^{-1}$ (d) $5\ N\,kg^{-1}$

Solution : (b) $I = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right) = -\left(3\hat{i} + 4\hat{j} + 12\hat{k}\right)$ [As $V = (3x + 4y + 12z)$ (given)]

It is uniform field Hence its value is same every where $|I| = \sqrt{3^2 + 4^2 + 12^2} = 13\ Nkg^{-1}$.

Problem **30.** The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(b) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

Solution : (a, b) We know that gravitational force $\propto$ Intensity $\propto \frac{1}{r^2}$ when $r > R$ [As $I = \frac{GM}{r^2}$]

$\therefore \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

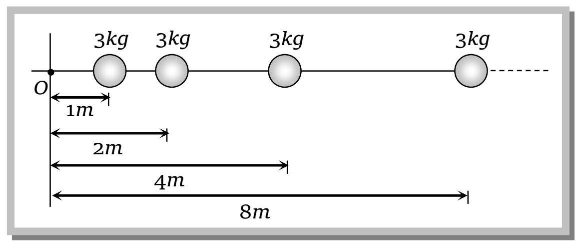
and gravitational force $\propto$ Intensity $\propto r$ when $r < R$ [As $I = \frac{4}{3}\pi\rho G r$]

$\therefore \ \frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$.

***Problem* 31.** Infinite bodies, each of mass 3*kg* are situated at distances 1*m*, 2*m*, 4*m*, 8*m*....... respectively on *x*-axis. The resultant intensity of gravitational field at the origin will be

(a) G (b) $2G$ (c) $3G$ (d) $4G$

Solution : (d) Intensity at the origin $I = I_1 + I_2 + I_3 + I_4 + ...$



$$= \frac{GM}{r_1^2} + \frac{GM}{r_2^2} + \frac{GM}{r_3^2} + \frac{GM}{r_4^2} + ...$$

$$= GM\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + ...\right]$$

$$= GM\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + ...\right]$$

$$= GM\left(\frac{1}{1 - \frac{1}{4}}\right) \qquad \left[\text{As sum of G.P.} = \frac{a}{1-r}\right]$$

$$= GM \times \frac{4}{3} = G \times 3 \times \frac{4}{3} = 4G \qquad [\text{As } M = 3kg \text{ given}]$$

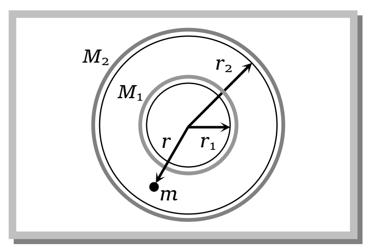
***Problem* 32.** Two concentric shells of mass M_1 and M_2 are having radii r_1 and r_2. Which of the following is the correct expression for the gravitational field on a mass *m*.

(a) $I = \frac{G(M_1 + M_2)}{r^2}$ for $r < r_1$

(b) $I = \frac{G(M_1 + M_2)}{r^2}$ for $r < r_2$

(c) $I = G\frac{M_2}{r^2}$ for $r_1 < r < r_2$

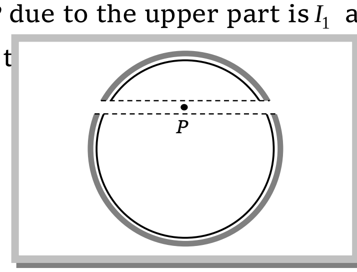
(d) $I = \frac{GM_1}{r^2}$ for $r_1 < r < r_2$



Solution : (d) Gravitational field on a mass *m* due to outer shell (radius r_2) will be zero because the mass is placed inside this shell. But the inner shell (radius r_1) behaves like point mass placed at the centre so $I = \frac{GM_1}{r^2}$ for $r_1 < r < r_2$

***Problem* 33.** A spherical shell is cut into two pieces along a chord as shown in the figure. *P* is a point on the plane of the chord. The gravitational field at *P* due to the upper part is I_1 and that due to the lower part is I_2. What is the relation between t

(a) $I_1 > I_2$



(b) $I_1 < I_2$

(c) $I_1 = I_2$

(d) No definite relation

Solution : (c) Intensity at P due to upper part $= I_1$ and Intensity at P due to lower part $= I_2$

Net Intensity at p due to spherical shell $\vec{I_1} + \vec{I_2} = 0$

$\therefore \vec{I_1} = -\vec{I_2}$

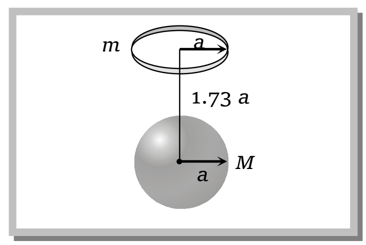
Problem **34.** A uniform ring of mass m is lying at a distance 1.73 a from the centre of a sphere of mass M just over the sphere where a is the small radius of the ring as well as that of the sphere. Then gravitational force exerted is

(a) $\frac{GMm}{8a^2}$

(b) $\frac{GMm}{(1.73\,a)^2}$

(c) $\sqrt{3}\frac{GMm}{a^2}$

(d) $1.73\frac{GMm}{8a^2}$



Solution : (d) Intensity due to uniform circular ring at a point on its axis $I = \frac{Gmr}{(a^2 + r^2)^{3/2}}$

$\therefore$ Force on sphere $F = \frac{GMmr}{(a^2 + r^2)^{3/2}} = \frac{GMm\sqrt{3}a}{(a^2 + (\sqrt{3}a)^2)^{3/2}} = \frac{GMm\sqrt{3}a}{(4a^2)^{3/2}} = \frac{\sqrt{3}GMm}{8a^2}$ [As $r = \sqrt{3}a$]

8.13 Gravitational Potential

At a point in a gravitational field potential V is defined as negative of work done per unit mass in shifting a test mass from some reference point (usually at infinity) to the given point *i.e.,*

$$V = -\frac{W}{m} = -\int \frac{\vec{F}.\vec{dr}}{m} = -\int \vec{I}.\vec{dr} \qquad \left[\text{As } \frac{F}{m} = I\right]$$

$$\therefore \qquad I = -\frac{dV}{dr}$$

i.e., negative gradient of potential gives intensity of field or potential is a scalar function of position whose space derivative gives intensity. Negative sign indicates that the direction of intensity is in the direction where the potential decreases.

Important points

(i) It is a scalar quantity because it is defined as work done per unit mass.

(ii) Unit : *Joule/kg* or m^2/sec^2

(iii) Dimension : $[M^0L^2T^{-2}]$

(iv) If the field is produced by a point mass then $V = -\int I\, dr = -\int \left(-\frac{GM}{r^2}\right) dr$ [As $I = -\frac{GM}{r^2}$]

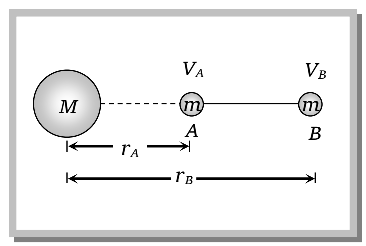
$\therefore$ $V = -\frac{GM}{r} + c$ [Here c = constant of integration]

Assuming reference point at ∞ and potential to be zero there we get

$$0 = -\frac{GM}{\infty} + c \Rightarrow c = 0$$

$\therefore$ Gravitational potential $V = -\frac{GM}{r}$

(v) Gravitational potential difference : It is defined as the work done to move a unit mass from one point to the other in the gravitational field. The gravitational potential difference in bringing unit test mass *m* from point *A* to point *B* under the gravitational influence of source mass *M* is



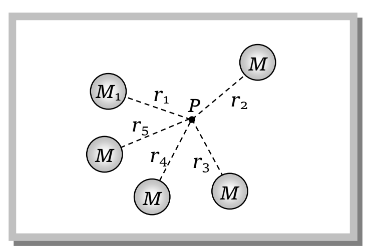
$$\Delta V = V_B - V_A = \frac{W_{A\to B}}{m} = -GM\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

(vi) Potential due to large numbers of particle is given by scalar addition of all the potentials.

$$V = V_1 + V_2 + V_3 + \dots\dots\dots$$

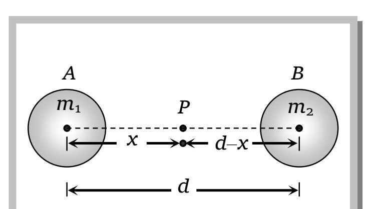
$$= -\frac{GM}{r_1} - \frac{GM}{r_2} - \frac{GM}{r_3} \dots\dots\dots$$

$$= -G\sum_{i=1}^{i=n} \frac{M_i}{r_i}$$



(vii) Point of zero potential : It is that point in the gravitational field, if the unit mass is shifted from infinity to that point then net work done will be equal to zero.

Let m_1 and m_2 are two masses placed at *d* distance apart and *P* is the point of zero potential in between the two masses.



Net potential for point $P = V_A + V_B = 0$

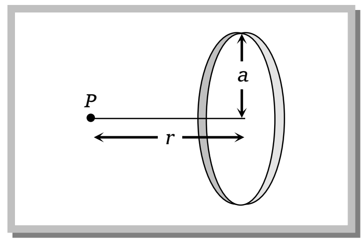
$$\Rightarrow \quad -\frac{Gm_1}{x} - \frac{Gm_2}{d-x} = 0$$

By solving $x = \dfrac{m_1 d}{m_1 - m_2}$

8.14 Gravitational Potential for Different Bodies

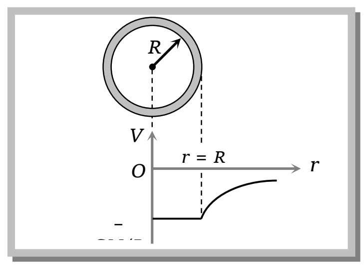
(1) **Potential due to uniform ring**

At a point on its axis	At the centre
$V = -\dfrac{GM}{\sqrt{a^2 + r^2}}$	$V = -\dfrac{GM}{a}$



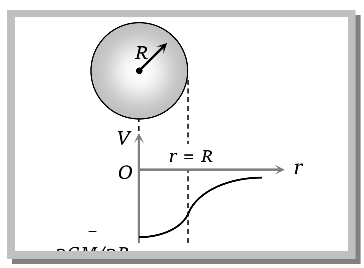
(2) **Potential due to spherical shell**

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$V = \dfrac{-GM}{r}$	$V = \dfrac{-GM}{R}$	$V = \dfrac{-GM}{R}$



(3) **Potential due to uniform solid sphere**

Outside the surface $r > R$	On the surface $r = R$	Inside the surface $r < R$
$V = \dfrac{-GM}{r}$	$V_{surface} = \dfrac{-GM}{R}$	$V = \dfrac{-GM}{2R}\left[3 - \left(\dfrac{r}{R}\right)^2\right]$ at the centre (r = 0) $V_{centre} = \dfrac{-3}{2}\dfrac{GM}{R}$ (*max.*) $V_{centre} = \dfrac{3}{2} V_{surface}$



Sample problems based on gravitational potential

***Problem* 35.** In some region, the gravitational field is zero. The gravitational potential in this region **[BVP 2003]**

(a) Must be variable (b) Must be constant (c) Cannot be zero (d)

Solution : (b) As $I = -\frac{dV}{dx}$, if $I = 0$ then V = constant.

***Problem* 36.** The gravitational field due to a mass distribution is $E = K/x^3$ in the x - direction (K is a constant). Taking the gravitational potential to be zero at infinity, its value at a distance x is **[MP PET 1994]**

(a) K/x (b) $K/2x$ (c) K/x^2 (d) $K/2x^2$

Solution : (d) $V = -\int E\,dx = -\int \frac{K}{x^3}dx = \frac{K}{2x^2}$.

***Problem* 37.** The intensity of gravitational field at a point situated at a distance of 8000 *km* from the centre of the earth is $6N/kg$. The gravitational potential at that point is – (in *Joule / kg*)

(a) 8×10^6 (b) 2.4×10^3 (c) 4.8×10^7 (d) 6.4×10^{14}

Solution : (c) Gravitational intensity at point *P*, $I = \frac{GM}{r^2}$ and gravitational potential $V = -\frac{GM}{r}$

$\therefore\ V = I\times r = 6\ N/kg\times 8000\ km = 4.8\times10^7\ \frac{Joule}{kg}$.

***Problem* 38.** The gravitational potential due to the earth at infinite distance from it is zero. Let the gravitational potential at a point *P* be $-5J/kg$. Suppose, we arbitrarily assume the gravitational potential at infinity to be + 10 J/kg, then the gravitational potential at *P* will be

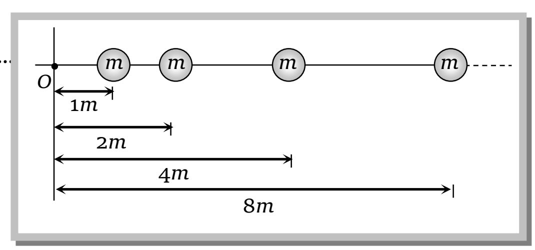
(a) $-5\ J/kg$ (b) $+5\ J/kg$ (c) $-15\ J/kg$ (d) $+15\ J/kg$

Solution : (b) Potential increases by $+10J/kg$ every where so it will be $+10-5=+5J/kg$ at P

***Problem* 39.** An infinite number of point masses each equal to *m* are placed at *x* =1. *x* = 2, *x* = 4, *x* = 8 .... What is the total gravitational potential at *x* = 0

(a) $-Gm$ (b) $-2Gm$ (c) $-4Gm$ (d) $-8Gm$

Solution : (b) Net potential at origin $V = -\left[\frac{Gm}{r_1}+\frac{Gm}{r_2}+\frac{Gm}{r_3}+........\right]$



$= -Gm\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right] = -Gm\left(\frac{1}{1-\frac{1}{2}}\right) = -2Gm$

***Problem* 40.** Two bodies of masses *m* and *M* are placed a distance *d* apart. The gravitational potential at the position where the gravitational field due to them is zero is *V*, then

(a) $V = -\frac{G}{d}(m+M)$ (b) $V = -\frac{Gm}{d}$ (c) $V = -\frac{GM}{d}$ (d) $V = -\frac{G}{d}(\sqrt{m}+\sqrt{M})^2$

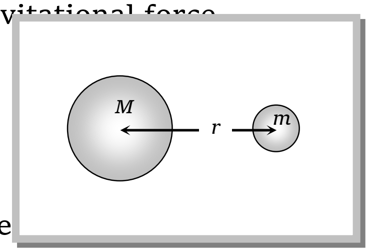
Solution : (d) If P is the point of zero intensity, then $x = \frac{\sqrt{M}}{\sqrt{M}+\sqrt{m}}.d$ and $d-x = \frac{\sqrt{m}}{\sqrt{M}+\sqrt{m}}d$

Now potential at point *P*, $V = V_1 + V_2 = -\frac{GM}{x} - \frac{GM}{d-x}$

Substituting the value of x and $d - x$ we get $V = -\frac{G}{d}\left(\sqrt{m} + \sqrt{M}\right)^2$.

8.15 Gravitational Potential Energy

The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the gravitational force.



$$W = \int_{\infty}^{r} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$W = -\frac{GMm}{r}$$

This work done is stored inside the body as its gravitational pote

$$\therefore U = -\frac{GMm}{r}$$

Important points

(i) Potential energy is a scalar quantity.

(ii) *Unit* : Joule

(iii) *Dimension* : $[ML^2T^{-2}]$

(iv) Gravitational potential energy is always negative in the gravitational field because the force is always attractive in nature.

(v) As the distance r increases, the gravitational potential energy becomes less negative i.e., it increases.

(vi) If $r = \infty$ then it becomes zero (maximum)

(vii) In case of discrete distribution of masses

Gravitational potential energy $U = \sum u_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + ...\right]$

(viii) If the body of mass m is moved from a point at a distance r_1 to a point at distance $r_2 (r_1 > r_2)$ then change in potential energy $\Delta U = \int_{r_1}^{r_2} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$ or $\Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

As r_1 is greater than r_2, the change in potential energy of the body will be negative. It means that if a body is brought closer to earth it's potential energy decreases.

(ix) Relation between gravitational potential energy and potential $U = -\frac{GMm}{r} = m\left[\frac{-GM}{r}\right]$

$\therefore$ $U = mV$

(x) Gravitational potential energy at the centre of earth relative to infinity.

$$U_{centre} = m\,V_{centre} = m\left(-\frac{3}{2}\frac{GM}{R}\right) = -\frac{3}{2}\frac{GMm}{R}$$

(xi) Gravitational potential energy of a body at height h from the earth surface is given by

$$U_h = -\frac{GMm}{R+h} = -\frac{gR^2m}{R+h} \equiv -\frac{mgR}{1+\frac{h}{R}}$$

8.16 Work Done Against Gravity

If the body of mess m is moved from the surface of earth to a point at distance h above the surface of earth, then change in potential energy or work done against gravity will be

$$W = \Delta U = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$\Rightarrow$ $$W = GMm\left[\frac{1}{R} - \frac{1}{R+h}\right]$$ [As $r_1 = R$ and $r_2 = R+h$]

$\Rightarrow$ $$W = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)} = \frac{mgh}{1+\frac{h}{R}}$$ [As $\frac{GM}{R^2} = g$]

Important points

(i) When the distance h is not negligible and is comparable to radius of the earth, then we will use above formula.

(ii) If $h = nR$ then $W = mgR\left(\frac{n}{n+1}\right)$

(iii) If $h = R$ then $W = \frac{1}{2}mgR$

(iv) If h is very small as compared to radius of the earth then term h/R can be neglected

From $$W = \frac{mgh}{1+h/R} = mgh$$ $\left[\text{As } \frac{h}{R} \to 0\right]$

Sample problems based on potential Energy

Problem 41. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

(a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{3R^2}$ (c) $\frac{GMm}{8R}$ (d) $\frac{GMm}{6R}$

Solution : (d) Work done = Change in potential energy $= U_2 - U_1 = \left[-\frac{GMm}{r_2}\right] - \left[-\frac{GMm}{r_1}\right] = -\frac{GMm}{3R} + \frac{GMm}{2R}$

$= \frac{GMm}{6R}$.

Problem 42. A body of mass m kg. starts falling from a point $2R$ above the earth's surface. Its kinetic energy when it has fallen to a point 'R' above the earth's surface [R-Radius of earth, M-Mass of earth, G-Gravitational constant]

[MP PMT 2002]

(a) $\frac{1}{2}\frac{GMm}{R}$ (b) $\frac{1}{6}\frac{GMm}{R}$ (c) $\frac{2}{3}\frac{GMm}{R}$ (d) $\frac{1}{3}\frac{GMm}{R}$

Solution : (b) When body starts falling toward earth's surface its potential energy decreases so kinetic energy increases.

Increase in kinetic energy = Decrease in potential energy

Final kinetic energy – Initial kinetic energy = Initial potential energy – Final potential energy

Final kinetic energy – 0 $= \left(-\frac{GMm}{r_1}\right) - \left(-\frac{GMm}{r_2}\right)$

$\therefore$ Final kinetic energy $= \left(\frac{-GMm}{R+h_1}\right) - \left(-\frac{GMm}{R+h_2}\right)$

$= \left(-\frac{GMm}{R+2R}\right) - \left(-\frac{GMm}{R+R}\right) = -\frac{GMm}{3R} + \frac{GMm}{2R} = \frac{1}{6}\frac{GMm}{R}$.

Problem **43.** A body of mass m is taken from earth surface to the height h equal to radius of earth, the increase in potential energy will be

Haryana CEE 1996; CEET Bihar 1995; MNR 1998; RPET 2000]

(a) mgR (b) $\frac{1}{2}mgR$ (c) $2mgR$ (d) $\frac{1}{4}mgR$

Solution : (b) Work done $= \frac{mgh}{1+h/R}$, If $h = R$ then work done $= \frac{mgR}{1+R/R} = \frac{1}{2}mgR$.

Problem **44.** If mass of earth is M, radius is R and gravitational constant is G, then work done to take 1 kg mass from earth surface to infinity will be

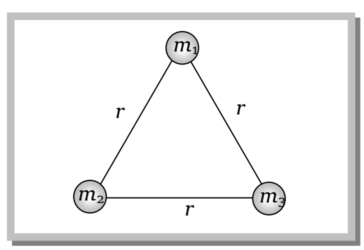
(a) $\sqrt{\frac{GM}{2R}}$ (b) $\frac{GM}{R}$ (c) $\sqrt{\frac{2GM}{R}}$ (d) $\frac{GM}{2R}$

Solution : (b) Work done $= U_{final} - U_{initial} = U_{\infty} - U_R = 0 - \left(-\frac{GMm}{R}\right) = \frac{GM}{R}$ [As $m = 1kg$]

Problem **45.** Three particles each of mass 100 gm are brought from a very large distance to the vertices of an equilateral triangle whose side is 20 cm in length. The work done will be

(a) 0.33×10^{-11} *Joule* (b) -0.33×10^{-11} *Joule* (c) 1.00×10^{-11} *Joule* (d) -1.00×10^{-11} *Joule*

Solution : (d) Potential energy of three particles system



$$U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_2m_3}{r_{23}} - \frac{Gm_1m_3}{r_{13}}$$

Given $m_1 = m_2 = m_3 = 100\ gm$ and $r_{12} = r_{23} = r_{13} = 2cm$

$$\therefore\ U = 3\left[\frac{-6.67\times10^{-11}\times(10^{-1})\times(10^{-1})}{20\times10^{-2}}\right] = -1.00\times10^{-11}\ Joule.$$

Problem **46.** A boy can jump to a height h on ground level. What should be the radius of a sphere of density d such that on jumping on it, he escapes out of the gravitational field of the sphere

(a) $\left[\frac{4\pi}{3}\frac{Gd}{gh}\right]^{1/2}$ (b) $\left[\frac{4\pi}{3}\frac{gh}{Gd}\right]^{1/2}$ (c) $\left[\frac{3}{4\pi}\frac{gh}{Gd}\right]^{1/2}$ (d) $\left[\frac{3}{4\pi}\frac{Gd}{gh}\right]^{1/2}$

Solution : (c) When a boy jumps from a ground level up to height h then its velocity of jumping $v = \sqrt{2gh}$ .....(i) and for the given condition this will become equal to escape velocity $v_{\text{escape}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\left(\frac{4}{3}\pi R^3.d\right)}$ .....(ii)

Equating (i) and (ii) $\sqrt{2gh} = R\sqrt{\frac{8}{3}G\pi d} \Rightarrow R = \left[\frac{3}{4\pi}\frac{gh}{Gd}\right]^{1/2}$.

8.17 Escape Velocity

The minimum velocity with which a body must be projected up so as to enable it to just overcome the gravitational pull, is known as escape velocity.

The work done to displace a body from the surface of earth ($r = R$) to infinity ($r = \infty$) is

$$W = \int_R^{\infty} \frac{GMm}{x^2} dx = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

$$\Rightarrow \qquad W = \frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth.

If v_e is the required escape velocity, then kinetic energy which should be given to the body is $\frac{1}{2}mv_e^2$

$$\therefore \qquad \frac{1}{2}mv_e^2 = \frac{GMm}{R} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow v_e = \sqrt{2gR} \qquad [\text{As } GM = gR^2]$$

$$\text{or} \qquad v_e = \sqrt{2 \times \frac{4}{3}\pi\rho GR \times R} \Rightarrow v_e = R\sqrt{\frac{8}{3}\pi G\rho} \qquad [\text{As } g = \frac{4}{3}\pi\rho GR]$$

Important points

(i) Escape velocity is independent of the mass and direction of projection of the body.

(ii) Escape velocity depends on the reference body. Greater the value of (M/R) or (gR) for a planet, greater will be escape velocity.

(iii) For the earth as $g = 9.8 m/s^2$ and $R = 6400\ km$

$$\therefore v_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = 11.2 km/\text{sec}$$

(iv) A planet will have atmosphere if the velocity of molecule in its atmosphere $\left[v_{rms} = \sqrt{\frac{3RT}{M}}\right]$ is lesser than escape velocity. This is why earth has atmosphere (as at earth $v_{rms} < v_e$) while moon has no atmosphere (as at moon $v_{rms} < v_e$)

(v) If body projected with velocity lesser than escape velocity ($v < v_e$) it will reach a certain maximum height and then may either move in an orbit around the planet or may fall down back to the planet.

(vi) Maximum height attained by body : Let a projection velocity of body (mass m) is v , so that is attains a maximum height h. At maximum height, the velocity of particle is zero, so kinetic energy is zero.

By the law of conservation of energy

Total energy at surface = Total energy at height h.

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{R+h}+0$$

$$\Rightarrow \quad \frac{v^2}{2} = GM\left[\frac{1}{R}-\frac{1}{R+h}\right] = \frac{GMh}{R(R+h)}$$

$$\Rightarrow \quad \frac{2GM}{v^2R} = \frac{R+h}{h} = 1+\frac{R}{h}$$

$$\Rightarrow \quad h = \frac{R}{\left(\frac{2GM}{v^2R}-1\right)} = \frac{R}{\frac{v_e^2}{v^2}-1} = R\left[\frac{v^2}{v_e^2-v^2}\right] \qquad \left[\text{As } v_e = \sqrt{\frac{2GM}{R}} \;\therefore\; \frac{2GM}{R} = v_e^2\right]$$

(vii) If a body is project with velocity greater than escape velocity $(v > v_e)$ then by conservation of energy.

Total energy at surface = Total energy at infinite

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}m(v')^2 + 0$$

i.e., $$(v')^2 = v^2 - \frac{2GM}{R} \Rightarrow v'^2 = v^2 - v_e^2 \qquad \left[\text{As } \frac{2GM}{R} = v_e^2\right]$$

$$\therefore \quad v' = \sqrt{v^2 - v_e^2}$$

i.e, the body will move in interplanetary or inter stellar space with velocity $\sqrt{v^2 - v_e^2}$.

(viii) Energy to be given to a stationary object on the surface of earth so that its total energy becomes zero, is called escape energy.

Total energy at the surface of the earth $= KE + PE = 0 - \frac{GMm}{R}$

$$\therefore \quad \text{Escape energy } = \frac{GMm}{R}$$

(ix) If the escape velocity of a body is equal to the velocity of light then from such bodies nothing can escape, not even light. Such bodies are called black holes.

The radius of a black hole is given as

$$R = \frac{2GM}{C^2} \qquad [\text{As } C = \sqrt{\frac{2GM}{R}}, \text{ where } C \text{ is the velocity of light}]$$

Sample problems based on escape velocity

***Problem* 47.** For a satellite escape velocity is $11\,km/s$. If the satellite is launched at an angle of 60^o with the vertical, then escape velocity will be

(a) $11\,km/s$ (b) $11\sqrt{3}\,km/s$ (c) $\frac{11}{\sqrt{3}}km/s$ (d) $33\,km/s$

Solution : (a) Escape velocity does not depend upon the angle of projection.

***Problem* 48.** The escape velocity from the earth is about $11\,km/s$. The escape velocity from a planet having twice the radius and the same mean density as the earth, is **[MP PMT 1987; UPSEAT 1999; AIIMS 2001; MP**

(a) 22 *km/s* (b) 11 *km/s* (c) 5.5 *km/s* (d) 15.5 *km/s*

Solution : (a) $v_e = \sqrt{\frac{2Gm}{R}} = \sqrt{\frac{8}{3}\pi\rho GR^2}$ $\quad \therefore v_e \propto R$ if $\rho =$ constant. Since the planet having double radius in comparison to earth therefore the escape velocity becomes twice *i.e.* $22\,km/s$.

***Problem* 49.** A projectile is projected with velocity kv_e in vertically upward direction from the ground into the space. (v_e is escape velocity and $k < 1$). If air resistance is considered to be negligible then the maximum height from the centre of earth to which it can go, will be (R = radius of earth) **[Roorkee 1999; RPET 1999]**

(a) $\frac{R}{k^2+1}$ (b) $\frac{R}{k^2-1}$ (c) $\frac{R}{1-k^2}$ (d) $\frac{R}{k+1}$

Solution : (c) From the law of conservation of energy

Difference in potential energy between ground and maximum height = Kinetic energy at the point of projection

$$\frac{mgh}{1+h/R} = \frac{1}{2}m(kv_e)^2 = \frac{1}{2}mk^2v_e^2 = \frac{1}{2}mk^2(\sqrt{2gR})^2 \qquad [\text{As } v_e = \sqrt{2gR}\,]$$

By solving height from the surface of earth $h = \frac{Rk^2}{1-k^2}$

So height from the centre of earth $r = R + h = R + \frac{Rk^2}{1-k^2} = \frac{R}{1-k^2}$.

***Problem* 50.** If the radius of earth reduces by 4% and density remains same then escape velocity will **[MP PET 1991; MP PMT 1995]**

(a) Reduce by 2% (b) Increase by 2% (c) Reduce by 4% (d) Increase by 4%

Solution : (c) Escape velocity $v_e \propto R\sqrt{\rho}$ and if density remains constant $v_e \propto R$

So if the radius reduces by 4% then escape velocity also reduces by 4%.

***Problem* 51.** A rocket of mass *M* is launched vertically from the surface of the earth with an initial speed *V*. Assuming the radius of the earth to be *R* and negligible air resistance, the maximum height attained by the rocket above the surface of the earth is

(a) $\frac{R}{\left(\frac{gR}{2V^2}-1\right)}$ (b) $R\left(\frac{gR}{2V^2}-1\right)$ (c) $\frac{R}{\left(\frac{2gR}{V^2}-1\right)}$ (d) $R\left(\frac{2gR}{V^2}-1\right)$

Solution : (c) Kinetic energy given to rocket at the surface of earth = Change in potential energy of the rocket in reaching from ground to highest point

$$\Rightarrow \frac{1}{2}mv^2 = \frac{mgh}{1+h/R} \Rightarrow \frac{v^2}{2} = \frac{g}{\frac{1}{h}+\frac{1}{R}} \Rightarrow \frac{1}{h}+\frac{1}{R} = \frac{2g}{v^2} \Rightarrow \frac{1}{h} = \frac{2g}{v^2} - \frac{1}{R} \Rightarrow \frac{1}{h} = \frac{2gR-v^2}{v^2R} \Rightarrow h = \frac{v^2R}{2gR-v^2}$$

$$\Rightarrow h = \frac{R}{\left(\frac{2gR}{v^2}-1\right)}$$

***Problem* 52.** A body of mass m is situated at a distance $4R_e$ above the earth's surface, where R_e is the radius of earth. How much minimum energy be given to the body so that it may escape

(a) mgR_e (b) $2mgR_e$ (c) $\frac{mgR_e}{5}$ (d) $\frac{mgR_e}{16}$

Solution : (c) Potential energy of the body at a distance $4R_e$ from the surface of earth

$$U = -\frac{mgR_e}{1+h/R_e} = -\frac{mgR_e}{1+4} = -\frac{mgR_e}{5} \qquad \text{[As } h = 4R_e \text{ (given)]}$$

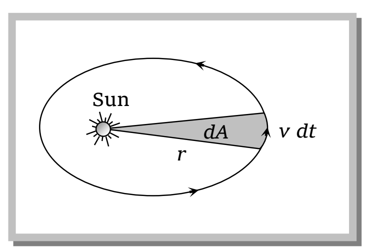
So minimum energy required to escape the body will be $\frac{mgR_e}{5}$.

8.18 Kepler's Laws of Planetary Motion

Planets are large natural bodies rotating around a star in definite orbits. The planetary system of the star sun called solar system consists of nine planets, viz., Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto. Out of these planets Mercury is the smallest, closest to the sun and so hottest. Jupiter is largest and has maximum moons (12). Venus is closest to Earth and brightest. Kepler after a life time study work out three empirical laws which govern the motion of these planets and are known as *Kepler's laws of planetary motion*. These are,

(1) **The law of Orbits** : Every planet moves around the sun in an elliptical orbit with sun at one of the foci.

(2) **The law of Area** : The line joining the sun to the planet sweeps out equal areas in equal interval of time. *i.e.* areal velocity is constant. According to this law planet will move slowly when it is farthest from sun and more rapidly when it is nearest to sun. It is similar to law of conservation of angular momentum.



Areal velocity $= \frac{dA}{dt} = \frac{1}{2}\frac{r(vdt)}{dt} = \frac{1}{2}rv$

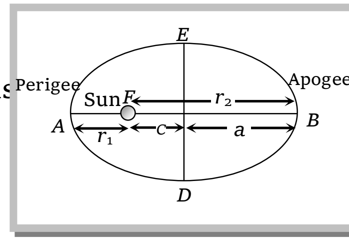
$\therefore \qquad \frac{dA}{dt} = \frac{L}{2m} \qquad$ [As $L = mvr$; $rv = \frac{L}{m}$]

(3) **The law of periods** : The square of period of revolution (T) of any planet around sun is directly proportional to the cube of the semi-major axis of the orbit.

$$T^2 \propto a^3 \text{ or } T^2 \propto \left(\frac{r_1 + r_2}{2}\right)^3$$

Proof : From the figure $AB = AF + FB$

$$2a = r_1 + r_2 \quad \therefore a = \frac{r_1 + r_2}{2}$$ where a = semi-major axis



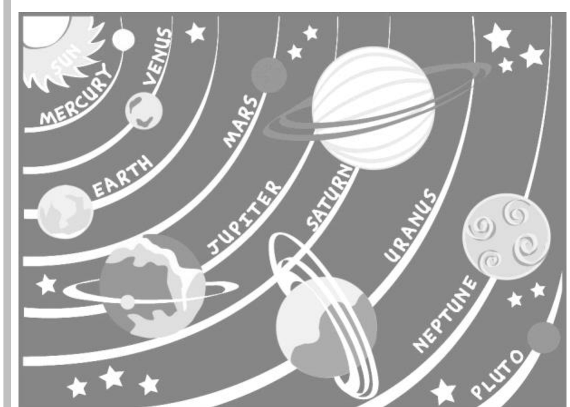
r_1 = Shortest distance of planet from sun (perigee).

r_2 = Largest distance of planet from sun (apogee).

Important data

Planet	Semi-major axis a (10^{10} *meter*)	Period *T*(*year*)	T^2/a^3 (10^{-34} *year*^{2}/*meter*^{3})
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Solar System



Note: ❑ Kepler's laws are valid for satellites also.

8.19 Velocity of a Planet in Terms of Eccentricity

Applying the law of conservation of angular momentum at perigee and apogee

$$mv_p r_p = mv_a r_a$$

$$\Rightarrow \quad \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{a+c}{a-c} = \frac{1+e}{1-e} \qquad \left[\text{As } r_p = a-c, \quad r_a = a+c \text{ and eccentricity } e = \frac{c}{a}\right]$$

Applying the conservation of mechanical energy at perigee and apogee

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a} \Rightarrow v_p^2 - v_a^2 = 2GM\left[\frac{1}{r_p} - \frac{1}{r_a}\right]$$

$$\Rightarrow \quad v_a^2\left[\frac{r_a^2 - r_p^2}{r_p^2}\right] = 2GM\left[\frac{r_a - r_p}{r_a r_p}\right] \qquad \left[\text{As } v_p = \frac{v_a r_a}{r_p}\right]$$

$$\Rightarrow \quad v_a^2 = \frac{2GM}{r_a + r_p}\left[\frac{r_p}{r_a}\right] \Rightarrow v_a^2 = \frac{2GM}{a}\left(\frac{a-c}{a+c}\right) = \frac{2GM}{a}\left(\frac{1-e}{1+e}\right)$$

Thus the speeds of planet at apogee and perigee are

$$v_a = \sqrt{\frac{2GM}{a}\left(\frac{1-e}{1+e}\right)}, \qquad v_p = \sqrt{\frac{2GM}{a}\left(\frac{1+e}{1-e}\right)}$$

Note: ❑ The gravitational force is a central force so torque on planet relative to sun is always zero, hence angular momentum of a planet or satellite is always constant irrespective of shape of orbit.

8.20 Some Properties of the Planet

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mean distacne from sun, 10^6 *km*	57.9	108	150	228	778	1430	2870	4500	5900
Period of revolution, *year*	0.241	0.615	1.00	1.88	11.9	29.5	84.0	165	248
Orbital speed, *km/s*	47.9	35.0	29.8	24.1	13.1	9.64	6.81	5.43	4.74
Equatiorial diameter, *km*	4880	12100	12800	6790	143000	120000	51800	49500	2300
Mass (Earth =1)	0.0558	0.815	1.000	0.107	318	95.1	14.5	17.2	0.002
Density (Water =1)	5.60	5.20	5.52	3.95	1.31	0.704	1.21	1.67	2.03
Surface value of *g*, m/s^2	3.78	8.60	9.78	3.72	22.9	9.05	7.77	11.0	0.5
Escape velocity, *km/s*	4.3	10.3	11.2	5.0	59.5	35.6	21.2	23.6	1.1
Known satellites	0	0	1	2	16+ring	18+rings	17+rings	8+rings	1

Sample problems based on Kepler's law

Problem 53. The distance of a planet from the sun is 5 times the distance between the earth and the sun. The Time period of the planet is

(a) $5^{3/2}$ *years* (b) $5^{2/3}$ *years* (c) $5^{1/3}$ *years* (d) $5^{1/2}$ *years*

Solution : (a) According to Kepler's law $T \propto R^{3/2}$ $\therefore$ $T_{planet} = (5)^{3/2}\, T_{earth} = 5^{(3/2)} \times 1\, year = 5^{3/2}$ *years* .

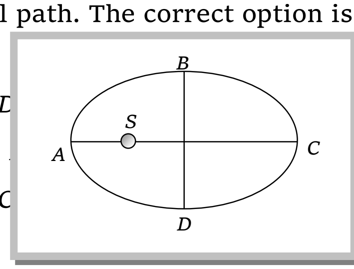
Problem 54. In planetary motion the areal velocity of position vector of a planet depends on angular velocity (ω) and the distance of the planet from sun (r). If so the correct relation for areal velocity is **[EAMCET 2003]**

(a) $\frac{dA}{dt} \propto \omega r$ (b) $\frac{dA}{dt} \propto \omega^2 r$ (c) $\frac{dA}{dt} \propto \omega r^2$ (d) $\frac{dA}{dt} \propto \sqrt{\omega r}$

Solution : (c) $\frac{dA}{dt} = \frac{L}{2m} = \frac{mvr}{2m} = \frac{1}{2}\omega r^2$ [As Angular momentum $L = mvr$ and $v = r\omega$]

$\therefore \frac{dA}{dt} \propto \omega r^2$.

Problem 55. The planet is revolving around the sun as shown in elliptical path. The correct option is **[UPSEAT 2(**



(a) The time taken in travelling *DAB* is less than that for *BCD*

(b) The time taken in travelling *DAB* is greater than that for *BCD*

(c) The time taken in travelling *CDA* is less than that for *ABC*

(d) The time taken in travelling *CDA* is greater than that for *ABC*

Solution : (a) When the planet passes nearer to sun then it moves fast and vice-versa. Hence the time taken in travelling *DAB* is less than that for *BCD*.

***Problem* 56.** The distance of Neptune and Saturn from sun are nearly 10^{13} and 10^{12} meters respectively. Assuming that they move in circular orbits, their periodic times will be in the ratio **[NCERT 1975; C**

(a) $\sqrt{10}$ (b) 100 (c) $10\sqrt{10}$ (d) $1/\sqrt{10}$

Solution : (c) Kepler's third law $T^2 \propto R^3$ $\therefore \frac{T_{Neptune}}{T_{Saturn}} = \left(\frac{R_{Neptune}}{R_{Saturn}}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10\sqrt{10}$.

***Problem* 57.** The maximum and minimum distance of a comet from the sun are $8 \times 10^{12}\, m$ and $1.6 \times 10^{12}\, m$. If its velocity when nearest to the sun is 60 *m/s,* what will be its velocity in *m/s* when it is farthest **[Orissa JEE 2001]**

(a) 12 (b) 60 (c) 112 (d) 6

Solution : (a) According to conservation of angular momentum $mv_{\min} r_{\max} = mv_{\max} r_{\min}$ = constant

$$\therefore\ v_{\min} = v_{\max} \times \frac{r_{\min}}{r_{\max}} = 60 \times \left(\frac{1.6 \times 10^{12}}{8 \times 10^{12}}\right) = 12\, m/s$$

***Problem* 58.** A satellite *A* of mass *m* is at a distance of *r* from the centre of the earth. Another satellite *B* of mass 2*m* is at distance of 2*r* from the earth's centre. Their time periods are in the ratio of **[CBSE PMT 1993]**

(a) 1 : 2 (b) 1 : 16 (c) 1 : 32 (d) $1 : 2\sqrt{2}$

Solution : (d) Time period does not depend upon the mass of satellite, it only depends upon the orbital radius.

According to Kepler's law $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \frac{1}{2\sqrt{2}}$.

***Problem* 59.** A planet moves around the sun. At a given point *P,* it is closed from the sun at a distance d_1 and has a speed v_1. At another point *Q*. when it is farthest from the sun at a distance d_2, its speed will be **[MP PMT 1987]**

(a) $\frac{d_1^2 v_1}{d_2^2}$ (b) $\frac{d_2 v_1}{d_1}$ (c) $\frac{d_1 v_1}{d_2}$ (d) $\frac{d_2^2 v_1}{d_1^2}$

Solution : (c) According to law of conservation of angular momentum $mv_1 d_1 = mv_2 d_2$ $\therefore\ v_2 = \frac{d_1 v_1}{d_2}$.

8.21 Orbital Velocity of Satellite

Satellites are natural or artificial bodies describing orbit around a planet under its gravitational attraction. Moon is a natural satellite while INSAT-1B is an artificial satellite of earth. Condition for establishment of artificial satellite is that the centre of orbit of satellite must coincide with centre of earth or satellite must move around great circle of earth.

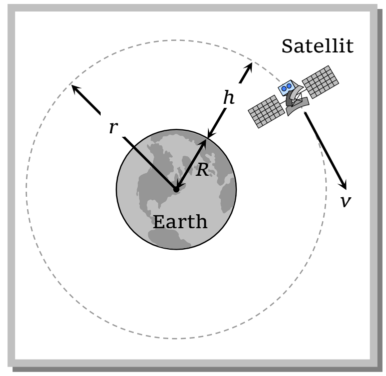
Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth.

For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$\Rightarrow$ $$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$$ [As $GM = gR^2$ and $r = R+h$]



Important points

(i) Orbital velocity is independent of the mass of the orbiting body and is always along the tangent of the orbit *i.e.,* satellites of deferent masses have same orbital velocity, if they are in the same orbit.

(ii) Orbital velocity depends on the mass of central body and radius of orbit.

(iii) For a given planet, greater the radius of orbit, lesser will be the orbital velocity of the satellite $(v \propto 1/\sqrt{r})$.

(iv) Orbital velocity of the satellite when it revolves very close to the surface of the planet

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} \quad \therefore\ v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$ [As $h = 0$ and $GM = gR^2$]

For the earth $v = \sqrt{9.8 \times 6.4 \times 10^6} = 7.9\, km/s \approx 8\, km/sec$

(v) Close to the surface of planet $v = \sqrt{\frac{GM}{R}}$ [As $v_e = \sqrt{\frac{2GM}{R}}$]

$\therefore$ $v = \frac{v_e}{\sqrt{2}}$ *i.e.,* $v_{escape} = \sqrt{2}\ v_{orbital}$

It means that if the speed of a satellite orbiting close to the earth is made $\sqrt{2}$ times (or increased by 41%) then it will escape from the gravitational field.

(vi) If the gravitational force of attraction of the sun on the planet varies as $F \propto \frac{1}{r^n}$ then the orbital velocity varies as $v \propto \frac{1}{\sqrt{r^n - 1}}$.

Sample problems based on orbital velocity

***Problem* 60.** Two satellites *A* and *B* go round a planet *P* in circular orbits having radii 4*R* and *R* respectively. If the speed of the satellite *A* is 3*V*, the speed of the satellite *B* will be

(a) 12 *V* (b) 6 *V* (c) 3/2 *V* (d) 3/2 *V*

Solution : (b) Orbital velocity of satellite $v = \sqrt{\frac{GM}{r}}$ $\therefore\ v \propto \frac{1}{\sqrt{r}} \Rightarrow \frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} \Rightarrow \frac{v_B}{3V} = \sqrt{\frac{4R}{R}} \Rightarrow v_B = 6V$.

***Problem* 61.** A satellite is moving around the earth with speed *v* in a circular orbit of radius *r*. If the orbit radius is decreased by 1%, its speed will

(a) Increase by 1% (b) Increase by 0.5% (c) Decrease by 1% (d)

Solution : (b) Orbital velocity $v = \sqrt{\frac{Gm}{r}}$ $\therefore$ $v \propto \frac{1}{\sqrt{r}}$ [If *r* decreases then *v* increases]

Percentage change in $v = \frac{1}{2}$ (Percentage change in *r*) $= \frac{1}{2}$ (1%) $= 0.5\%$ $\therefore$ orbital velocity increases by 0.5%.

***Problem* 62.** If the gravitational force between two objects were proportional to 1/*R*; where *R* is separation between them, then a particle in circular orbit under such a force would have its orbital speed *v* proportional to

[CBSE PMT 1994; JIPMER 2001, 02]

(a) $1/R^2$ (b) R^0 (c) R^1 (d) $1/R$

Solution : (b) If $F \propto \frac{1}{R^n}$ then $v \propto \frac{1}{\sqrt{R^{n-1}}}$; here $n = 1$ $\therefore v \propto \frac{1}{\sqrt{R^{1-1}}} \propto R^0$.

***Problem* 63.** The distance between centre of the earth and moon is 384000 *km*. If the mass of the earth is 6×10^{24} *kg* and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2. The speed of the moon is nearly

(a) 1 *km / sec* (b) 4 *km / sec* (c) 8 *km / sec* (d) 11.2 *km / sec*

Solution : (a) Orbital velocity $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{38400 \times 10^3}}$ $v = 1.02$ $km/sec = 1 km/sec$ (Approx.)

8.22 Time Period of Satellite

It is the time taken by satellite to go once around the earth.

$\therefore \quad T = \dfrac{\text{Circumference of the orbit}}{\text{orbital velocity}}$

$\Rightarrow \quad T = \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM}}$ $\quad$ [As $v = \sqrt{\dfrac{GM}{r}}$]

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{r^3}{GM}} = 2\pi\sqrt{\dfrac{r^3}{gR^2}}$ $\quad$ [As $GM = gR^2$]

$\Rightarrow \quad T = 2\pi\sqrt{\dfrac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\dfrac{R}{g}}\left(1+\dfrac{h}{R}\right)^{3/2}$ $\quad$ [As $r = R + h$]

Important points

(i) From $T = 2\pi\sqrt{\dfrac{r^3}{GM}}$ it is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit

(ii) $\quad T = 2\pi\sqrt{\dfrac{r^3}{GM}}$

$\Rightarrow \quad T^2 = \dfrac{4\pi^2}{GM}r^3$ *i.e.*, $T^2 \propto r^3$

This is in accordance with Kepler's third law of planetary motion r becomes a (semi major axis) if the orbit is elliptic.

(iii) Time period of nearby satellite,

From $\quad T = 2\pi\sqrt{\dfrac{r^3}{GM}} = 2\pi\sqrt{\dfrac{R^3}{gR^2}} = 2\pi\sqrt{\dfrac{R}{g}}$ $\quad$ [As $h = 0$ and $GM = gR^2$]

For earth $R = 6400\,km$ and $g = 9.8m/s^2$

$T = 84.6 \text{ minute} \approx 1.4\ hr$

(iv) Time period of nearby satellite in terms of density of planet can be given as

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{R^3}{GM}} = \frac{2\pi(R^3)^{1/2}}{\left[G.\frac{4}{3}\pi R^3\rho\right]^{1/2}} = \sqrt{\frac{3\pi}{G\rho}}$$

(v) If the gravitational force of attraction of the sun on the planet varies as $F \propto \dfrac{1}{r^n}$ then the time period varies as $T \propto r^{\frac{n+1}{2}}$

(vi) If there is a satellite in the equatorial plane rotating in the direction of earth's rotation from west to east, then for an observer, on the earth, angular velocity of satellite will be $(\omega_S - \omega_E)$. The time interval between the two consecutive appearances overhead will be

$$T = \frac{2\pi}{\omega_s - \omega_E} = \frac{T_S T_E}{T_E - T_S} \qquad \left[\text{As } T = \frac{2\pi}{\omega}\right]$$

If $\omega_S = \omega_E$, $T = \infty$ *i.e.* satellite will appear stationary relative to earth. Such satellites are called geostationary satellites.

Sample problems based on time period

***Problem* 64.** A satellite is launched into a circular orbit of radius 'R' around earth while a second satellite is launched into an orbit of radius 1.02 R. The percentage difference in the time periods of the two satellites is **[EAMCET 2003]**

(a) 0.7 (b) 1.0 (c) 1.5 (d) 3

Solution : (d) Orbital radius of second satellite is 2% more than first satellite

So from $T \propto (r)^{3/2}$, Percentage increase in time period $= \frac{3}{2}$ (Percentage increase in orbital radius)

$$= \frac{3}{2} \text{ (2\%) = 3\%.}$$

***Problem* 65.** Periodic time of a satellite revolving above Earth's surface at a height equal to R, where R the radius of Earth, is **[**g is acceleration due to gravity at Earth's surface**]**

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $4\sqrt{2}\pi\sqrt{\frac{R}{g}}$ (c) $2\pi\sqrt{\frac{R}{g}}$ (d) $8\pi\sqrt{\frac{R}{g}}$

Solution : (b) $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+R)^3}{gR^2}} = 2\pi\sqrt{\frac{8R}{g}} = 4\sqrt{2}\pi\sqrt{\frac{R}{g}}$ [As $h = R$ (given)].

***Problem* 66.** An earth satellite S has an orbit radius which is 4 times that of a communication satellite C. The period of revolution of S is

(a) 4 days (b) 8 days (c) 16 days (d) 32 days

Solution : (b) Orbital radius of satellite $r_s = 4r_c$ (given)

From Kepler's law $T \propto r^{3/2}$ $\therefore$ $\frac{T_s}{T_c} = \left(\frac{r_s}{s_c}\right)^{3/2} = (4)^{3/2} \Rightarrow T_s = 8T_c = 8 \times 1$ day = 8 days.

***Problem* 67.** One project after deviation from its path, starts moving round the earth in a circular path at radius equal to nine times the radius at earth R, its time period will be

(a) $2\pi\sqrt{\frac{R}{g}}$ (b) $27 \times 2\pi\sqrt{\frac{R}{g}}$ (c) $\pi\sqrt{\frac{R}{g}}$ (d) $8 \times 2\pi\sqrt{\frac{R}{g}}$

Solution : (b) $T = 2\pi\sqrt{\frac{r^3}{gR^2}} = 2\pi\sqrt{\frac{(9R)^3}{gR^2}} = 2\pi(9)^{3/2}\sqrt{\frac{R}{g}} = 27 \times 2\pi\sqrt{\frac{R}{g}}$ [As $r = 9R$ (given)].

***Problem* 68.** A satellite A of mass m is revolving round the earth at a height 'r' from the centre. Another satellite B of mass $2m$ is revolving at a height $2r$. The ratio of their time periods will be

(a) 1 : 2 (b) 1 : 16 (c) 1 : 32 (d) $1:2\sqrt{2}$

Solution : (d) Time period depends only upon the orbital radius $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{r}{2r}\right)^{3/2} = \frac{1}{2\sqrt{2}}$.

8.23 Height of Satellite

As we know, time period of satellite $T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

By squaring and rearranging both sides $\frac{gR^2T^2}{4\pi^2} = (R+h)^3$

$$\Rightarrow \quad h = \left(\frac{T^2 g R^2}{4\pi^2}\right)^{1/3} - R$$

By knowing the value of time period we can calculate the height of satellite the surface of the earth.

Sample problems based on height

***Problem* 69.** Given radius of earth 'R' and length of a day 'T' the height of a geostationary satellite is

[G – Gravitational constant, M – Mass of earth]

(a) $\left(\frac{4\pi^2 GM}{T^2}\right)^{1/3}$ (b) $\left(\frac{4\pi GM}{R^2}\right)^{1/3} - R$ (c) $\left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$ (d) $\left(\frac{GMT^2}{4\pi^2}\right)^{1/3} + R$

Solution : (c) From the expression $h = \left(\frac{T^2gR^2}{4\pi^2}\right)^{1/3} - R \quad \therefore \; h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$ [As $gR^2 = GM$]

***Problem* 70.** A satellite is revolving round the earth in circular orbit at some height above surface of earth. It takes 5.26×10^3 seconds to complete a revolution while its centripetal acceleration is $9.92\, m/s^2$. Height of satellite above surface of earth is (Radius of earth 6.37×10^6 m)

(a) 70 *km* (b) 120 *km* (c) 170 *km* (d) 220 *km*

Solution : (c) Centripetal acceleration $(a_c) = \frac{v^2}{r}$ and $T = \frac{2\pi r}{v}$

From equation (i) and (ii) $r = \frac{a_c T^2}{4\pi^2} \Rightarrow R + h = \frac{9.32 \times (5.26 \times 10^3)^2}{4 \times \pi^2}$

$$h = 6.53\times10^{6} - R\ = 6.53\times10^{6} - 6.37\times10^{6} = 160\times10^{3}\,m\ = 160\ km\ \approx 170\ km.$$

8.24 Geostationary Satellite

The satellite which appears stationary relative to earth is called geostationary or geosynchronous satellite, communication satellite.

A geostationary satellite always stays over the same place above the earth such a satellite is never at rest. Such a satellite appears stationary due to its zero relative velocity *w.r.t.* that place on earth.

The orbit of a geostationary satellite is known as the parking orbit.

Important points

(i) It should revolve in an orbit concentric and coplanar with the equatorial plane.

(ii) It sense of rotation should be same as that of earth about its own axis *i.e.,* in anti-clockwise direction (from west to east).

(iii) Its period of revolution around the earth should be same as that of earth about its own axis.

$$\therefore\ T = 24\ hr = 86400\ \sec$$

(iv) Height of geostationary satellite

$$\text{As}\qquad T = 2\pi\sqrt{\frac{r^3}{GM}}\ \Rightarrow\ 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 24\,hr$$

Substituting the value of G and M we get $R + h = r = 42000\ km = 7R$

$\therefore$ height of geostationary satellite from the surface of earh $h = 6R = 36000\ km$

(v) Orbital velocity of geo stationary satellite can be calculated by $v = \sqrt{\frac{GM}{r}}$

Substituting the value of G and M we get $v = 3.08\ km/\sec$

8.25 Angular Momentum of Satellite

Angular momentum of satellite $L = mvr$

$$\Rightarrow\qquad L = m\sqrt{\frac{GM}{r}}\,r\qquad \left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$$

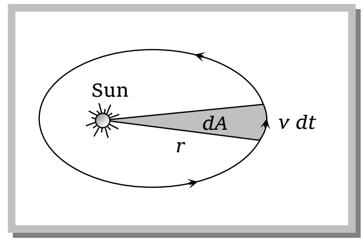
$$\therefore\qquad L = \sqrt{m^2GMr}$$

i.e., Angular momentum of satellite depend on both the mass of orbiting and central body as well as the radius of orbit.

Important points

(i) In case of satellite motion, force is central so torque = 0 and hence angular momentum of satellite is conserved *i.e.,* $L =$ constant

(ii) In case of satellite motion as areal velocity



$$\frac{dA}{dt} = \frac{1}{2}\frac{(r)(vdt)}{dt} = \frac{1}{2}rv$$

$$\Rightarrow \quad \frac{dA}{dt} = \frac{L}{2m} \qquad [\text{As } L = mvr\ ]$$

But as $L =$ constant, $\therefore$ areal velocity (dA/dt) = constant which is Kepler's II law

i.e., Kepler's II law or constancy of areal velocity is a consequence of conservation of angular momentum.

Sample problems based on angular momentum

***Problem* 71.** The orbital angular momentum of a satellite revolving at a distance *r* from the centre is *L*. If the distance is increased to 16*r*, then new angular momentum will be

(a) 16 L (b) 64 L (c) $\frac{L}{4}$ (d) 4 L

Solution : (d) Angular momentum $L = \sqrt{m^2GMr}$ $\therefore$ $L \propto \sqrt{r}$

$$\frac{L_2}{L_1} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{16\,r}{r}} = 4$$

$$L_2 = 4L_1 = 4L$$

***Problem* 72.** Angular momentum of a planet of mass *m* orbiting around sun is *J*, areal velocity of its radius vector will be

(a) $\frac{1}{2}mJ$ (b) $\frac{J}{2m}$ (c) $\frac{m}{2J}$ (d) $\frac{1}{2mJ}$

Solution : (b)

8.26 Energy of Satellite

When a satellite revolves around a planet in its orbit, it possesses both potential energy (due to its position against gravitational pull of earth) and kinetic energy (due to orbital motion).

(1) Potential energy : $U = mV = \frac{-GMm}{r} = \frac{-L^2}{mr^2}$ $\left[\text{As } V = \frac{-GM}{r}, L^2 = m^2GMr\right]$

(2) Kinetic energy : $K = \frac{1}{2}mv^2 = \frac{GMm}{2r} = \frac{L^2}{2mr^2}$ $\left[\text{As } v = \sqrt{\frac{GM}{r}}\right]$

(3) Total energy : $E = U + K = \frac{-GMm}{r} + \frac{GMm}{2r} = \frac{-GMm}{2r} = \frac{-L^2}{2mr^2}$

Important points

(i) Kinetic energy, potential energy or total energy of a satellite depends on the mass of the satellite and the central body and also on the radius of the orbit.

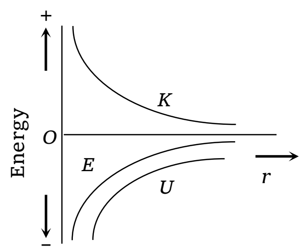
(ii) From the above expressions we can say that

Kinetic energy (K) = – (Total energy)

Potential energy (U) = 2 (Total energy)

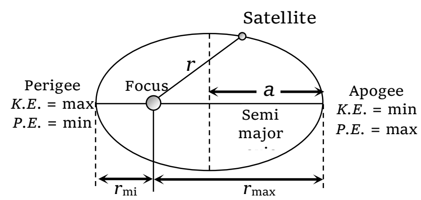
Potential energy (K) = – 2 (Kinetic energy)

(iii) Energy graph for a satellite



(A)

(iv) Energy distribution in elliptical orbit



(B)

(v) If the orbit of a satellite is elliptic then

(a) Total energy $(E) = \frac{-GMm}{2a}$ = constant ; where a is semi-major axis .

(b) Kinetic energy (K) will be maximum when the satellite is closest to the central body (at perigee) and maximum when it is farthest from the central body (at apogee)

(c) Potential energy (U) will be minimum when kinetic energy = maximum i.e., the satellite is closest to the central body (at perigee) and maximum when kinetic energy = minimum i.e., the satellite is farthest from the central body (at apogee).

(vi) Binding Energy : Total energy of a satellite in its orbit is negative. Negative energy means that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity. The energy required to remove the satellite from its orbit to infinity is called Binding Energy of the system, *i.e.*,

Binding Energy (B.E.) $= -E = \frac{GMm}{2r}$

8.27 Chanae in the Orbit of Satellite

When the satellite is transferred to a higher orbit $(r_2 > r_1)$ then variation in different quantities can be shown by the following table

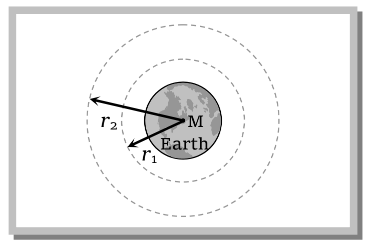
Quantities	Variation	Relation with *r*
Orbital velocity	Decreases	$v \propto \frac{1}{\sqrt{r}}$
Time period	Increases	$T \propto r^{3/2}$

Linear momentum	Decreases	$P \propto \frac{1}{\sqrt{r}}$
Angular momentum	Increases	$L \propto \sqrt{r}$
Kinetic energy	Decreases	$K \propto \frac{1}{r}$
Potential energy	Increases	$U \propto -\frac{1}{r}$
Total energy	Increases	$E \propto -\frac{1}{r}$
Binding energy	Decreases	$BE \propto \frac{1}{r}$

Note: ❑ Work done in changing the orbit $W = E_2 - E_1$

$$W = \left(-\frac{GMm}{2r_2}\right) - \left(-\frac{GMm}{2r_1}\right)$$

$$W = \frac{GMm}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$



Sample problems based on Energy

***Problem* 73.** Potential energy of a satellite having mass 'm' and rotating at a height of $6.4 \times 10^6\ m$ from the earth centre is

[AIIMS 2000; CBSE PMT 2001; BHU 2001]

(a) $-0.5\,mgR_e$ (b) $-mgR_e$ (c) $-2\,mgR_e$ (d) $4\,mgR_e$

Solution : (a) Potential energy $= -\frac{GMm}{r} = -\frac{GMm}{R_e + h} = -\frac{GMm}{2R_e}$ [As $h = R_e$ (given)]

$\therefore$ Potential energy $= -\frac{gR_e^2 m}{2R_e} = -0.5 mgR_e$ [As $GM = gR^2$]

***Problem* 74.** In a satellite if the time of revolution is T, then kinetic energy is proportional to

(a) $\frac{1}{T}$ (b) $\frac{1}{T^2}$ (c) $\frac{1}{T^3}$ (d) $T^{-2/3}$

Solution : (d) Time period $T \propto r^{3/2} \Rightarrow r \propto T^{2/3}$ and Kinetic energy $\propto \frac{1}{r} \propto \frac{1}{T^{2/3}} \propto T^{-2/3}$.

***Problem* 75.** Two satellites are moving around the earth in circular orbits at height R and $3R$ respectively, R being the radius of the earth, the ratio of their kinetic energies is

(a) 2 (b) 4 (c) 8 (d) 16

Solution : (a) $r_1 = R + h_1 = R + R = 2R$ and $r_2 = R + h_2 = R + 3R = 4R$

Kinetic energy $\propto \frac{1}{r}$ $\quad \therefore \frac{(KE)_1}{(KE)_2} = \frac{r_2}{r_1} = \frac{4R}{2R} = \frac{2}{1}$.

8.28 Weightlessness

The weight of a body is the force with which it is attracted towards the centre of earth. When a body is stationary with respect to the earth, its weight equals the gravity. This weight of the body is known as its static or true weight.

We become conscious of our weight, only when our weight (which is gravity) is opposed by some other object. Actually, the secret of measuring the weight of a body with a weighing machine lies in the fact that as we place the body on the machine, the weighing machine opposes the weight of the body. The reaction of the weighing machine to the body gives the measure of the weight of the body.

The state of weightlessness can be observed in the following situations.

(1) **When objects fall freely under gravity** : For example, a lift falling freely, or an airship showing a feat in which it falls freely for a few seconds during its flight, are in state of weightlessness.

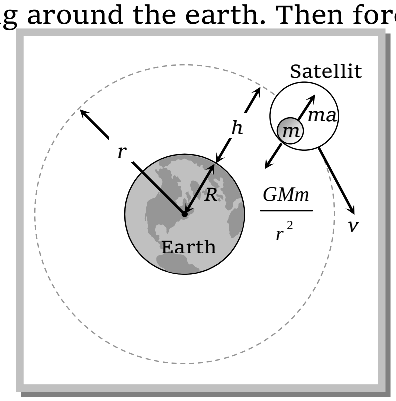
(2) **When a satellite revolves in its orbit around the earth** : Weightlessness poses many serious problems to the astronauts. It becomes quite difficult for them to control their movements. Everything in the satellite has to be kept tied down. Creation of artificial gravity is the answer to this problem.

(3) **When bodies are at null points in outer space** : On a body projected up, the pull of the earth goes on decreasing, but at the same time the gravitational pull of the moon on the body goes on increasing. At one particular position, the two gravitational pulls may be equal and opposite and the net pull on the body becomes zero. This is zero gravity region or the null point and the body in question is said to appear weightless.

8.29 Weightlessness in a Satellite

A satellite, which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. The acceleration of satellite is $\frac{GM}{r^2}$ towards the centre of earth.

If a body of mass m placed on a surface inside a satellite moving around the earth. Then force on the body are



(i) The gravitational pull of earth $= \frac{GMm}{r^2}$

(ii) The reaction by the surface $= R$

By Newton's law $\quad \frac{GmM}{r^2} - R = m\,a$

$$\frac{GmM}{r^2} - R = m\left(\frac{GM}{r^2}\right)$$

$\therefore \quad R = 0$

Thus the surface does not exert any force on the body and hence its apparent weight is zero.

A body needs no support to stay at rest in the satellite and hence all position are equally comfortable. Such a state is called weightlessness.

Important points

(i) One will find it difficult to control his movement, without weight he will tend to float freely. To get from one spot to the other he will have to push himself away from the walls or some other fixed objects.

(ii) As everything is in free fall, so objects are at rest relative to each other, *i.e.,* if a table is withdrawn from below an object, the object will remain where it was without any support.

(iii) If a glass of water is tilted and glass is pulled out, the liquid in the shape of container will float and will not flow because of surface tension.

(iv) If one tries to strike a match, the head will light but the stick will not burn. This is because in this situation convection currents will not be set up which supply oxygen for combustion

(v) If one tries to perform simple pendulum experiment, the pendulum will not oscillate. It is because there will not be any restoring torque and so $T = 2\pi\sqrt{(L/g')} = \infty$. [As $g' = 0$]

(vi) Condition of weightlessness can be experienced only when the mass of satellite is negligible so that it does not produce it own gravity.

e.g. Moon is a satellite of earth but due to its own weight it applies gravitational force of attraction on the body placed on its surface and hence weight of the body will not be equal to zero at the surface of the moon.

Sample problems based on weightlessness in satellite

Problem 76. The time period of a simple pendulum on a freely moving artificial satellite is

(a) Zero (b) 2 *sec* (c) 3 *sec* (d) Infinite

Solution : (d) $T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{0}} = \infty$ [As $g' = 0$ in the satellite]

Problem 77. The weight of an astronaut, in an artificial satellite revolving around the earth, is

(a) Zero (b) Equal to that on the earth

(c) More than that on the earth (d) Less than that on the earth

Solution : (a)

Problem 78. A ball is dropped from a spacecraft revolving around the earth at a height of 120 *km*. What will happen to the ball

(a) It will continue to move with velocity *v* along the original orbit of spacecraft

(b) If will move with the same speed tangentially to the spacecraft

(c) It will fall down to the earth gradually

(d) It will go very far in the space

Solution : (a) Because ball possess same initial tangential speed as that of space craft.. So it also feels the condition of weightlessness.

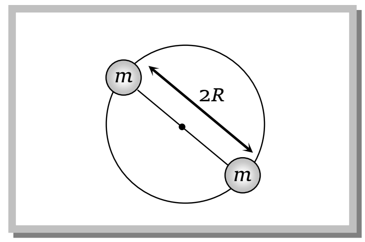
Sample problems (Miscellaneous)

Problem **79.** Two particles of equal mass go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $v = \frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (b) $v = \sqrt{\frac{Gm}{2R}}$ (c) $v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$ (d) $v = \sqrt{\frac{4Gm}{R}}$

Solution : (c) Both the particles moves diametrically opposite position along the circular path of radius R and the gravitational force provides required centripetal force

$$\frac{mv^2}{R} = \frac{Gmm}{(2R)^2} \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$



Problem **80.** Two types of balances, the beam balance and the spring balance are commonly used for measuring weight in shops. If we are on the moon, we can continue to use

(a) Only the beam type balance without any change

(b) Only the spring balance without any change

(c) Both the balances without any change

(d) Neither of the two balances without making any change

Solution : (a) Because in beam type balance effect of less gravitation force works on both the Pans. So it is neutralizes but in spring balance weight of the body decreases so apparent weight varies with actual weight.

Problem **81.** During a journey from earth to the moon and back, the greatest energy required from the space-ship rockets is to overcome

(a) The earth's gravity at take off

(b) The moon's gravity at lunar landing

(c) The moon's gravity at lunar take off

(d) The point where the pull of the earth and moon are equal but opposite

Solution : (a)

Problem **82.** If the radius of earth contracts $\frac{1}{n}$ of its present value, the length of the day will be approximately

(a) $\frac{24}{n}h$ (b) $\frac{24}{n^2}h$ (c) $24\,n\,h$ (d) $24\,n^2\,h$

Solution : (b) Conservation of angular momentum $L = I\omega = = \frac{2}{5}MR^2 \times \frac{2\pi}{T} =$ constant $\therefore\ T \propto R^2$ [If M remains same]

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^2 = \left(\frac{R/n}{R}\right)^2 = \frac{1}{n^2} \Rightarrow T_2 = \frac{24}{n^2}hr \qquad [\text{As } T_1 = 24\,hr].$$

***Problem* 83.** A body released from a height h takes time t to reach earth's surface. The time taken by the same body released from the same height to reach the moon's surface is

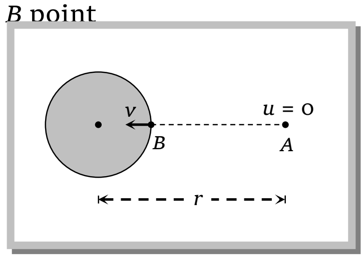
(a) t (b) $6t$ (c) $\sqrt{6}t$ (d) $\frac{t}{6}$

Solution : (c) If body falls from height h then time of descent $t = \sqrt{\frac{2h}{g}}$ $\Rightarrow$ $\frac{t_{moon}}{t_{earth}} = \sqrt{\frac{g_{earth}}{g_{moon}}} = \sqrt{6}$ $\Rightarrow$ $t_{moon} = \sqrt{6}\ t$.

***Problem* 84.** A satellite is revolving round the earth with orbital speed v_0. If it stops suddenly, the speed with which it will strike the surface of earth would be (v_e = escape velocity of a particle on earth's surface)

(a) $\frac{v_e^2}{v_0}$ (b) v_0 (c) $\sqrt{v_e^2 - v_0^2}$ (d) $\sqrt{v_e^2 - 2v_0^2}$

Solution : (d) Applying conservation of mechanical energy between A and B point



$$-\frac{GMm}{r} = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right); \quad \frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{r}$$

$$v^2 = \frac{2Gm}{R} - \frac{2Gm}{r} = v_e^2 - 2v_0^2 \Rightarrow v = \sqrt{v_e^2 - 2v_0^2}$$

[As escape velocity $v_e = \sqrt{\frac{2Gm}{R}}$, orbital velocity $v_0 = \sqrt{\frac{Gm}{r}}$]

***Problem* 85.** The escape velocity for a planet is v_e. A tunnel is dug along a diameter of the planet and a small body is dropped into it at the surface. When the body reaches the centre of the planet, its speed will be

(a) v_e (b) $\frac{v_e}{\sqrt{2}}$ (c) $\frac{v_e}{2}$ (d) Zero

Solution : (b) Gravitational potential at the surface of the earth $V_s = -\frac{GM}{R}$

Gravitational potential at the centre of earth $V_c = -\frac{3GM}{2R}$

By the conservation of energy $\frac{1}{2}mv^2 = m(V_s - V_c)$

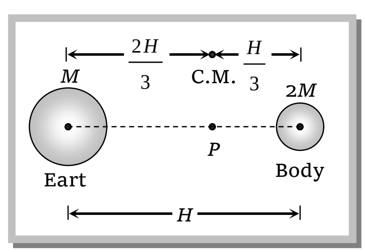
$$v^2 = 2\frac{GM}{R}\left(\frac{3}{2} - 1\right) = \frac{GM}{R} = gR = \frac{v_e^2}{2} \quad \text{[As } v_e = \sqrt{2gR}\text{]}$$

$$\therefore\ v = \frac{v_e}{\sqrt{2}}$$

***Problem* 86.** A small body of superdense material, whose mass is twice the mass of the earth but whose size is very small compared to the size of the earth, starts from rest at a height $H \ll R$ above the earth's surface, and reaches the earth's surface in time t. Then t is equal to

(a) $\sqrt{2H/g}$ (b) $\sqrt{H/g}$ (c) $\sqrt{2H/3g}$ (d) $\sqrt{4H/3g}$

Solution : (c) As the masses of the body and the earth are comparable, they will move towards their centre of mass, which remains stationary.



Hence the body of mass 2*m* move through distance $\frac{H}{3}$.

and time to reach the earth surface $= \sqrt{\frac{2h}{g}} = \sqrt{\frac{2H/3}{g}} = \sqrt{\frac{2H}{3g}}$