

# PROBABILITY

## 16.1 Overview

Probability is defined as a quantitative measure of uncertainty – a numerical value that conveys the strength of our belief in the occurrence of an event. The probability of an event is always a number between 0 and 1 both 0 and 1 inclusive. If an event's probability is nearer to 1, the higher is the likelihood that the event will occur; the closer the event's probability to 0, the smaller is the likelihood that the event will occur. If the event cannot occur, its probability is 0. If it must occur (i.e., its occurrence is certain), its probability is 1.

**16.1.1 Random experiment** An experiment is random means that the experiment has more than one possible outcome and it is not possible to predict with certainty which outcome that will be. For instance, in an experiment of tossing an ordinary coin, it can be predicted with certainty that the coin will land either heads up or tails up, but it is not known for sure whether heads or tails will occur. If a die is thrown once, any of the six numbers, i.e., 1, 2, 3, 4, 5, 6 may turn up, not sure which number will come up.

(i) **Outcome** A possible result of a random experiment is called its **outcome** for example if the experiment consists of tossing a coin twice, some of the outcomes are HH, HT etc.

(ii) **Sample Space** A sample space is the set of all possible outcomes of an experiment. In fact, it is the universal set  $S$  pertinent to a given experiment.

The sample space for the experiment of tossing a coin twice is given by

$$S = \{HH, HT, TH, TT\}$$

The sample space for the experiment of drawing a card out of a deck is the set of all cards in the deck.

**16.1.2 Event** An event is a subset of a sample space  $S$ . For example, the event of drawing an ace from a deck is

$$A = \{\text{Ace of Heart, Ace of Club, Ace of Diamond, Ace of Spade}\}$$

## 16.1.3 Types of events

(i) **Impossible and Sure Events** The empty set  $\phi$  and the sample space  $S$  describe events. In fact  $\phi$  is called an impossible event and  $S$ , i.e., the whole sample space is called a sure event.

- (ii) **Simple or Elementary Event** If an event  $E$  has only one sample point of a sample space, i.e., a single outcome of an experiment, it is called a **simple** or **elementary** event. The sample space of the experiment of tossing two coins is given by

$$S = \{HH, HT, TH, TT\}$$

The event  $E_1 = \{HH\}$  containing a single outcome  $HH$  of the sample space  $S$  is a simple or elementary event. If one card is drawn from a well shuffled deck, any particular card drawn like 'queen of Hearts' is an elementary event.

- (iii) **Compound Event** If an event has more than one sample point it is called a **compound event**, for example,  $S = \{HH, HT\}$  is a compound event.
- (iv) **Complementary event** Given an event  $A$ , the complement of  $A$  is the event consisting of all sample space outcomes that do not correspond to the occurrence of  $A$ .

The complement of  $A$  is denoted by  $A'$  or  $\bar{A}$ . It is also called the event 'not  $A$ '. Further  $P(\bar{A})$  denotes the probability that  $A$  will not occur.

$$A' = \bar{A} = S - A = \{w : w \in S \text{ and } w \notin A\}$$

**16.1.4 Event 'A or B'** If  $A$  and  $B$  are two events associated with same sample space, then the event 'A or B' is same as the event  $A \cup B$  and contains all those elements which are either in  $A$  or in  $B$  or in both. Further more,  $P(A \cup B)$  denotes the probability that  $A$  or  $B$  (or both) will occur.

**16.1.5 Event 'A and B'** If  $A$  and  $B$  are two events associated with a sample space, then the event 'A and B' is same as the event  $A \cap B$  and contains all those elements which are common to both  $A$  and  $B$ . Further more,  $P(A \cap B)$  denotes the probability that both  $A$  and  $B$  will simultaneously occur.

**16.1.6 The Event 'A but not B' (Difference  $A - B$ )** An event  $A - B$  is the set of all those elements of the same space  $S$  which are in  $A$  but not in  $B$ , i.e.,  $A - B = A \cap B'$ .

**16.1.7 Mutually exclusive** Two events  $A$  and  $B$  of a sample space  $S$  are **mutually exclusive** if the occurrence of any one of them excludes the occurrence of the other event. Hence, the two events  $A$  and  $B$  cannot occur simultaneously, and thus  $P(A \cap B) = 0$ .

**Remark** Simple or elementary events of a sample space are always mutually exclusive. For example, the elementary events  $\{1\}$ ,  $\{2\}$ ,  $\{3\}$ ,  $\{4\}$ ,  $\{5\}$  or  $\{6\}$  of the experiment of throwing a dice are mutually exclusive.

Consider the experiment of throwing a die once.

The events  $E$  = getting a even number and  $F$  = getting an odd number are mutually exclusive events because  $E \cap F = \phi$ .

**Note** For a given sample space, there may be two or more mutually exclusive events.

**16.1.8 Exhaustive events** If  $E_1, E_2, \dots, E_n$  are  $n$  events of a sample space  $S$  and if

$$E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n = \bigcup_{i=1}^n E_i = S$$

then  $E_1, E_2, \dots, E_n$  are called exhaustive events.

In other words, events  $E_1, E_2, \dots, E_n$  of a sample space  $S$  are said to be exhaustive if atleast one of them necessarily occur whenever the experiment is performed.

Consider the example of rolling a die. We have  $S = \{1, 2, 3, 4, 5, 6\}$ . Define the two events

A : ‘a number less than or equal to 4 appears.’

B : ‘a number greater than or equal to 4 appears.’

Now  $A : \{1, 2, 3, 4\}, B = \{4, 5, 6\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\} = S$$

Such events  $A$  and  $B$  are called exhaustive events.

**16.1.9 Mutually exclusive and exhaustive events** If  $E_1, E_2, \dots, E_n$  are  $n$  events of a sample space  $S$  and if  $E_i \cap E_j = \phi$  for every  $i \neq j$ , i.e.,  $E_i$  and  $E_j$  are pairwise disjoint

and  $\bigcup_{i=1}^n E_i = S$ , then the events  $E_1, E_2, \dots, E_n$  are called mutually exclusive and exhaustive events.

Consider the example of rolling a die.

We have  $S = \{1, 2, 3, 4, 5, 6\}$

Let us define the three events as

A = a number which is a perfect square

B = a prime number

C = a number which is greater than or equal to 6

Now  $A = \{1, 4\}, B = \{2, 3, 5\}, C = \{6\}$

Note that  $A \cup B \cup C = \{1, 2, 3, 4, 5, 6\} = S$ . Therefore,  $A, B$  and  $C$  are exhaustive events.

Also  $A \cap B = B \cap C = C \cap A = \phi$

Hence, the events are pairwise disjoint and thus mutually exclusive.

Classical approach is useful, when all the outcomes of the experiment are equally likely. We can use logic to assign probabilities. To understand the classical method consider the experiment of tossing a fair coin. Here, there are two equally likely

outcomes - head (H) and tail (T). When the elementary outcomes are taken as equally likely, we have a uniform probability model. If there are  $k$  elementary outcomes in  $S$ , each is assigned the probability of  $\frac{1}{k}$ . Therefore, logic suggests that the probability of

observing a head, denoted by  $P(H)$ , is  $\frac{1}{2} = 0.5$ , and that the probability of observing a

tail, denoted  $P(T)$ , is also  $\frac{1}{2} = 0.5$ . Notice that each probability is between 0 and 1.

Further  $H$  and  $T$  are all the outcomes of the experiment and  $P(H) + P(T) = 1$ .

**16.1.10 Classical definition** If all of the outcomes of a sample space are equally likely, then the probability that an event will occur is equal to the ratio :

$$\frac{\text{The number of outcomes favourable to the event}}{\text{The total number of outcomes of the sample space}}$$

Suppose that an event  $E$  can happen in  $h$  ways out of a total of  $n$  possible equally likely ways.

Then the classical probability of occurrence of the event is denoted by

$$P(E) = \frac{h}{n}$$

The probability of non occurrence of the event  $E$  is denoted by

$$P(\text{not } E) = \frac{n-h}{n} = 1 - \frac{h}{n} = 1 - P(E)$$

Thus  $P(E) + P(\text{not } E) = 1$

The event 'not  $E$ ' is denoted by  $\bar{E}$  or  $E'$  (complement of  $E$ )

Therefore  $P(\bar{E}) = 1 - P(E)$

**16.1.11 Axiomatic approach to probability** Let  $S$  be the sample space of a random experiment. The probability  $P$  is a real valued function whose domain is the power set of  $S$ , i.e.,  $P(S)$  and range is the interval  $[0, 1]$  i.e.  $P : P(S) \rightarrow [0, 1]$  satisfying the following axioms.

- (i) For any event  $E$ ,  $P(E) \geq 0$ .
- (ii)  $P(S) = 1$
- (iii) If  $E$  and  $F$  are mutually exclusive events, then  $P(E \cup F) = P(E) + P(F)$ .

It follows from (iii) that  $P(\phi) = 0$ .

Let  $S$  be a sample space containing elementary outcomes  $w_1, w_2, \dots, w_n$ ,  
i.e.,  $S = \{w_1, w_2, \dots, w_n\}$

It follows from the axiomatic definition of probability that

$$(i) \quad 0 \leq P(w_i) \leq 1 \text{ for each } w_i \in S$$

$$(ii) \quad P(w_1) + P(w_2) + \dots + P(w_n) = 1$$

$$(iii) \quad P(A) = \sum P(w_i) \text{ for any event } A \text{ containing elementary events } w_i.$$

For example, if a fair coin is tossed once

$$P(H) = P(T) = \frac{1}{2} \text{ satisfies the three axioms of probability.}$$

Now suppose the coin is not fair and has double the chances of falling heads up as

compared to the tails, then  $P(H) = \frac{2}{3}$  and  $P(T) = \frac{1}{3}$ .

This assignment of probabilities are also valid for  $H$  and  $T$  as these satisfy the axiomatic definitions.

**16.1.12 Probabilities of equally likely outcomes** Let a sample space of an experiment be  $S = \{w_1, w_2, \dots, w_n\}$  and suppose that all the outcomes are equally likely to occur i.e., the chance of occurrence of each simple event must be the same

i.e.,  $P(w_i) = p$  for all  $w_i \in S$ , where  $0 \leq p \leq 1$

$$\text{Since } \sum_{i=1}^n P(w_i) = 1$$

$$\text{i.e., } p + p + p + \dots + p \text{ (n times)} = 1$$

$$\Rightarrow np = 1, \quad \text{i.e. } p = \frac{1}{n}$$

Let  $S$  be the sample space and  $E$  be an event, such that  $n(S) = n$  and  $n(E) = m$ . If each outcome is equally likely, then it follows that

$$P(E) = \frac{m}{n} = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of possible outcomes}}$$

**16.1.13 Addition rule of probability** If  $A$  and  $B$  are any two events in a sample

space  $S$ , then the probability that atleast one of the events  $A$  or  $B$  will occur is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Similarly, for three events  $A$ ,  $B$  and  $C$ , we have

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

**16.1.14 Addition rule for mutually exclusive events** If  $A$  and  $B$  are disjoint sets, then

$$P(A \cup B) = P(A) + P(B) \text{ [since } P(A \cap B) = P(\phi) = 0, \text{ where } A \text{ and } B \text{ are disjoint].}$$

The addition rule for mutually exclusive events can be extended to more than two events.

## 16.2 Solved Examples

### Short Answer Type (S.A.)

**Example 1** An ordinary deck of cards contains 52 cards divided into four suits. The red suits are diamonds and hearts and black suits are clubs and spades. The cards J, Q, and K are called face cards. Suppose we pick one card from the deck at random.

- What is the sample space of the experiment?
- What is the event that the chosen card is a black face card?

### Solution

- The outcomes in the sample space  $S$  are 52 cards in the deck.
- Let  $E$  be the event that a black face card is chosen. The outcomes in  $E$  are Jack, Queen, King or spades or clubs. Symbolically

$$E = \{J, Q, K, \text{ of spades and clubs}\} \text{ or } E = \{J\clubsuit, Q\clubsuit, K\clubsuit, J\spadesuit, Q\spadesuit, K\spadesuit\}$$

**Example 2** Suppose that each child born is equally likely to be a boy or a girl. Consider a family with exactly three children.

- List the eight elements in the sample space whose outcomes are all possible genders of the three children.
- Write each of the following events as a set and find its probability :
  - The event that exactly one child is a girl.
  - The event that at least two children are girls
  - The event that no child is a girl

### Solution

- All possible genders are expressed as :  
 $S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$

- (b) (i) Let A denote the event : ‘exactly one child is a girl’

$$A = \{BBG, BGB, GBB\}$$

$$P(A) = \frac{3}{8}$$

- (ii) Let B denote the event that at least two children are girls.

$$B = \{GGB, GBG, BGG, GGG\}, P(B) = \frac{4}{8}.$$

- (iii) Let C denote the event : ‘no child is a girl’.

$$C = \{BBB\}$$

$$\therefore P(C) = \frac{1}{8}$$

### Example 3

- (a) How many two-digit positive integers are multiples of 3?  
 (b) What is the probability that a randomly chosen two-digit positive integer is a multiple of 3?

### Solution

- (a) 2 digit positive integers which are multiples of 3 are 12, 15, 18, ... , 99. Thus, there are 30 such integers.  
 (b) 2-digit positive integers are 10, 11, 12, ... , 99. Thus, there are 90 such numbers. Since out of these, 30 numbers are multiple of 3, therefore, the probability that a

randomly chosen positive 2-digit integer is a multiple of 3, is  $\frac{30}{90} = \frac{1}{3}$ .

**Example 4** A typical PIN (personal identification number) is a sequence of any four symbols chosen from the 26 letters in the alphabet and the ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?

**Solution** A PIN is a sequence of four symbols selected from 36 (26 letters + 10 digits) symbols.

By the fundamental principle of counting, there are  $36 \times 36 \times 36 \times 36 = 36^4 = 1,679,616$  PINs in all. When repetition is not allowed the multiplication rule can be applied to conclude that there are

$$36 \times 35 \times 34 \times 33 = 1,413,720 \text{ different PINs}$$

The number of PINs that contain at least one repeated symbol

$$= 1,679,616 - 1,413,720 = 2,65,896$$

Thus, the probability that a randomly chosen PIN contains a repeated symbol is

$$\frac{265,896}{1,679,616} = .1583$$

**Example 5** An experiment has four possible outcomes A, B, C and D, that are mutually exclusive. Explain why the following assignments of probabilities are not permissible:

$$(a) \quad P(A) = .12, \quad P(B) = .63, \quad P(C) = 0.45, \quad P(D) = -0.20$$

$$(b) \quad P(A) = \frac{9}{120}, \quad P(B) = \frac{45}{120}, \quad P(C) = \frac{27}{120}, \quad P(D) = \frac{46}{120}$$

**Solution**

$$(a) \quad \text{Since } P(D) = -0.20, \text{ this is not possible as } 0 \leq P(A) \leq 1 \text{ for any event A.}$$

$$(b) \quad P(S) = P(A \cup B \cup C \cup D) = \frac{9}{120} + \frac{45}{120} + \frac{27}{120} + \frac{46}{120} = \frac{127}{120} \neq 1.$$

This violates the condition that  $P(S) = 1$ .

**Example 6** Probability that a truck stopped at a roadblock will have faulty brakes or badly worn tires are 0.23 and 0.24, respectively. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes and/or badly working tires. What is the probability that a truck stopped at this roadblock will have faulty breaks as well as badly worn tires?

**Solution** Let B be the event that a truck stopped at the roadblock will have faulty brakes and T be the event that it will have badly worn tires. We have  $P(B) = 0.23$ ,  $P(T) = 0.24$  and  $P(B \cup T) = 0.38$

$$\text{and} \quad P(B \cup T) = P(B) + P(T) - P(B \cap T)$$

$$\text{So} \quad 0.38 = 0.23 + 0.24 - P(B \cap T)$$

$$\Rightarrow \quad P(B \cap T) = 0.23 + 0.24 - 0.38 = 0.09$$

**Example 7** If a person visits his dentist, suppose the probability that he will have his teeth cleaned is 0.48, the probability that he will have a cavity filled is 0.25, the probability that he will have a tooth extracted is 0.20, the probability that he will have a teeth cleaned and a cavity filled is 0.09, the probability that he will have his teeth cleaned and a tooth extracted is 0.12, the probability that he will have a cavity filled and a tooth extracted is 0.07, and the probability that he will have his teeth cleaned, a cavity filled, and a tooth extracted is 0.03. What is the probability that a person visiting his dentist



will have atleast one of these things done to him?

**Solution** Let C be the event that the person will have his teeth cleaned and F and E be the event of getting cavity filled or tooth extracted, respectively. We are given

$$P(C) = 0.48, P(F) = 0.25, P(E) = .20, P(C \cap F) = .09,$$

$$P(C \cap E) = 0.12, P(E \cap F) = 0.07 \text{ and } P(C \cap F \cap E) = 0.03$$

$$\text{Now, } P(C \cup F \cup E) = P(C) + P(F) + P(E)$$

$$- P(C \cap F) - P(C \cap E) - P(F \cap E)$$

$$+ P(C \cap F \cap E)$$

$$= 0.48 + 0.25 + 0.20 - 0.09 - 0.12 - 0.07 + 0.03$$

$$= 0.68$$

### Long Answer Type

**Example 8** An urn contains twenty white slips of paper numbered from 1 through 20, ten red slips of paper numbered from 1 through 10, forty yellow slips of paper numbered from 1 through 40, and ten blue slips of paper numbered from 1 through 10. If these 80 slips of paper are thoroughly shuffled so that each slip has the same probability of being drawn. Find the probabilities of drawing a slip of paper that is

- blue or white
- numbered 1, 2, 3, 4 or 5
- red or yellow and numbered 1, 2, 3 or 4
- numbered 5, 15, 25, or 35;
- white and numbered higher than 12 or yellow and numbered higher than 26.

### Solution

$$(a) P(\text{Blue or White}) = P(\text{Blue}) + P(\text{White}) \quad (\text{Why?})$$

$$= \frac{10}{80} + \frac{20}{80} = \frac{30}{80} = \frac{3}{8}$$

$$(b) P(\text{numbered 1, 2, 3, 4 or 5})$$

$$= P(1 \text{ of any colour}) + P(2 \text{ of any colour})$$

$$+ P(3 \text{ of any colour}) + P(4 \text{ of any colour}) + P(5 \text{ of any colour})$$

$$= \frac{4}{80} + \frac{4}{80} + \frac{4}{80} + \frac{4}{80} + \frac{4}{80} = \frac{20}{80} = \frac{2}{8} = \frac{1}{4}$$

$$(c) P(\text{Red or yellow and numbered 1, 2, 3 or 4})$$

$$= P(\text{Red numbered 1, 2, 3 or 4}) + P(\text{yellow numbered 1, 2, 3 or 4})$$

$$= \frac{4}{80} + \frac{4}{80} = \frac{8}{80} = \frac{1}{10}$$

(d) P (numbered 5, 15, 25 or 35)

$$= P(5) + P(15) + P(25) + P(35)$$

$$= P(5 \text{ of White, Red, Yellow, Blue}) + P(15 \text{ of White, Yellow}) + P(25 \text{ of Yellow}) \\ + P(35 \text{ of Yellow})$$

$$= \frac{4}{80} + \frac{2}{80} + \frac{1}{80} + \frac{1}{80} = \frac{8}{80} = \frac{1}{10}$$

(e) P (White and numbered higher than 12 or Yellow and numbered higher than 26)

$$= P(\text{White and numbered higher than 12})$$

$$+ P(\text{Yellow and numbered higher than 26})$$

$$= \frac{8}{80} + \frac{14}{80} = \frac{22}{80} = \frac{11}{40}$$

### Objective Type Questions

Choose the correct answer from given four options in each of the Examples 9 to 15 (M.C.Q.).

**Example 9** In a leap year the probability of having 53 Sundays or 53 Mondays is

(A)  $\frac{2}{7}$

(B)  $\frac{3}{7}$

(C)  $\frac{4}{7}$

(D)  $\frac{5}{7}$

**Solution** (B) is the correct answer. Since a leap year has 366 days and hence 52 weeks and 2 days. The 2 days can be SM, MT, TW, WTh, ThF, FSt, StS.

Therefore,  $P(53 \text{ Sundays or } 53 \text{ Mondays}) = \frac{3}{7}$ .

**Example 10** Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits?

(A)  $\frac{1}{16}$

(B)  $\frac{16}{25}$

(C)  $\frac{1}{645}$

(D)  $\frac{1}{25}$

**Solution** (D) is the correct answer. Since a 3-digit number cannot start with digit 0, the hundredth place can have any of the 4 digits. Now, the tens and units place can have all the 5 digits. Therefore, the total possible 3-digit numbers are  $4 \times 5 \times 5$ , i.e., 100.

The total possible 3 digit numbers having all digits same = 4

$$\text{Hence, } P(\text{3-digit number with same digits}) = \frac{4}{100} = \frac{1}{25}.$$

**Example 11** Three squares of chess board are selected at random. The probability of getting 2 squares of one colour and other of a different colour is

- (A)  $\frac{16}{21}$                       (B)  $\frac{8}{21}$                       (C)  $\frac{3}{32}$                       (D)  $\frac{3}{8}$

**Solution** (A) is the correct answer. In a chess board, there are 64 squares of which 32 are white and 32 are black. Since 2 of one colour and 1 of other can be 2W, 1B, or 1W, 2B, the number of ways is  $({}^{32}C_2 \times {}^{32}C_1) \times 2$  and also, the number of ways of choosing any 3 boxes is  ${}^{64}C_3$ .

$$\text{Hence, the required probability} = \frac{{}^{32}C_2 \times {}^{32}C_1 \times 2}{{}^{64}C_3} = \frac{16}{21}.$$

**Example 12** If A and B are any two events having  $P(A \cup B) = \frac{1}{2}$  and  $P(\bar{A}) = \frac{2}{3}$ , then the probability of  $\bar{A} \cap B$  is

- (A)  $\frac{1}{2}$                       (B)  $\frac{2}{3}$                       (C)  $\frac{1}{6}$                       (D)  $\frac{1}{3}$

**Solution** (C) is the correct answer. We have  $P(A \cup B) = \frac{1}{2}$

$$\Rightarrow P(A \cup (B - A)) = \frac{1}{2}$$

$$\Rightarrow P(A) + P(B - A) = \frac{1}{2} \text{ (since A and B - A are mutually exclusive)}$$

$$\Rightarrow 1 - P(\bar{A}) + P(B - A) = \frac{1}{2}$$

$$\Rightarrow 1 - \frac{2}{3} + P(B - A) = \frac{1}{2}$$

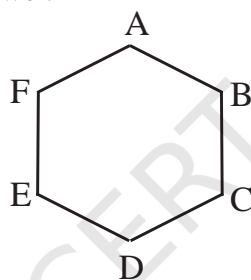
$$\Rightarrow P(B - A) = \frac{1}{6}$$

$$\Rightarrow P(\bar{A} \cap B) = \frac{1}{6} \quad (\text{since } \bar{A} \cap B \equiv B - A)$$

**Example 13** Three of the six vertices of a regular hexagon are chosen at random. What is the probability that the triangle with these vertices is equilateral?

- (A)  $\frac{3}{10}$                       (B)  $\frac{3}{20}$                       (C)  $\frac{1}{20}$                       (D)  $\frac{1}{10}$

**Solution** (D) is the correct answer.



**Fig. 16.1**

ABCDEF is a regular hexagon. Total number of triangles  ${}^6C_3 = 20$ . (Since no three points are collinear). Of these only  $\triangle ACE$ ;  $\triangle BDF$  are equilateral triangles.

Therefore, required probability =  $\frac{2}{20} = \frac{1}{10}$ .

**Example 14** If A, B, C are three mutually exclusive and exhaustive events of an experiment such that

$3P(A) = 2P(B) = P(C)$ , then  $P(A)$  is equal to

- (A)  $\frac{1}{11}$                       (B)  $\frac{2}{11}$                       (C)  $\frac{5}{11}$                       (D)  $\frac{6}{11}$

**Solution** (B) is the correct answer. Let  $3P(A) = 2P(B) = P(C) = p$  which gives  $p(A)$

$$= \frac{p}{3}, P(B) = \frac{p}{2} \text{ and } P(C) = p$$

Now since A, B, C are mutually exclusive and exhaustive events, we have

$$P(A) + P(B) + P(C) = 1$$

$$\Rightarrow \frac{p}{3} + \frac{p}{2} + p = 1 \quad \Rightarrow \quad p = \frac{6}{11}$$

$$\text{Hence, } P(A) = \frac{p}{3} = \frac{2}{11}$$

**Example 15** One mapping (function) is selected at random from all the mappings of the set  $A = \{1, 2, 3, \dots, n\}$  into itself. The probability that the mapping selected is one to one is

- (A)  $\frac{1}{n^n}$       (B)  $\frac{1}{\lfloor n \rfloor}$       (C)  $\frac{\lfloor n-1 \rfloor}{n^{n-1}}$       (D) none of these

**Solution** (C) is the correct answer. Total number of mappings from a set  $A$  having  $n$  elements onto itself is  $n^n$

Now, for one to one mapping the first element in  $A$  can have any of the  $n$  images in  $A$ ; the 2<sup>nd</sup> element in  $A$  can have any of the remaining  $(n-1)$  images, counting like this, the  $n^{\text{th}}$  element in  $A$  can have only 1 image.

Therefore, the total number of one to one mappings is  $\lfloor n \rfloor$ .

$$\text{Hence the required probability is } \frac{\lfloor n \rfloor}{n^n} = \frac{n \lfloor n-1 \rfloor}{n n^{n-1}} = \frac{\lfloor n-1 \rfloor}{n^{n-1}}.$$

### 16.3 EXERCISE

#### Short Answer Type

- If the letters of the word **ALGORITHM** are arranged at random in a row what is the probability the letters **GOR** must remain together as a unit?
- Six new employees, two of whom are married to each other, are to be assigned six desks that are lined up in a row. If the assignment of employees to desks is made randomly, what is the probability that the married couple will have nonadjacent desks?  
[**Hint:** First find the probability that the couple has adjacent desks, and then subtract it from 1.]
- Suppose an integer from 1 through 1000 is chosen at random, find the probability that the integer is a multiple of 2 or a multiple of 9.
- An experiment consists of rolling a die until a 2 appears.
  - How many elements of the sample space correspond to the event that the 2 appears on the  $k^{\text{th}}$  roll of the die?

- (ii) How many elements of the sample space correspond to the event that the 2 appears not later than the  $k^{\text{th}}$  roll of the die?

[**Hint:**(a) First  $(k - 1)$  rolls have 5 outcomes each and  $k^{\text{th}}$  rolls should result in 1 outcomes. (b)  $1 + 5 + 5^2 + \dots + 5^{k-1}$ .]

5. A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find  $P(G)$ , where  $G$  is the event that a number greater than 3 occurs on a single roll of the die.
6. In a large metropolitan area, the probabilities are .87, .36, .30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set, or both kinds of sets. What is the probability that a family owns either anyone or both kinds of sets?
7. If  $A$  and  $B$  are mutually exclusive events,  $P(A) = 0.35$  and  $P(B) = 0.45$ , find
  - (a)  $P(A')$       (b)  $P(B')$       (c)  $P(A \cup B)$       (d)  $P(A \cap B)$
  - (e)  $P(A \cap B')$       (f)  $P(A' \cap B')$
8. A team of medical students doing their internship have to assist during surgeries at a city hospital. The probabilities of surgeries rated as very complex, complex, routine, simple or very simple are respectively, 0.15, 0.20, 0.31, 0.26, .08. Find the probabilities that a particular surgery will be rated
  - (a) complex or very complex;
  - (b) neither very complex nor very simple;
  - (c) routine or complex
  - (d) routine or simple
9. Four candidates  $A, B, C, D$  have applied for the assignment to coach a school cricket team. If  $A$  is twice as likely to be selected as  $B$ , and  $B$  and  $C$  are given about the same chance of being selected, while  $C$  is twice as likely to be selected as  $D$ , what are the probabilities that
  - (a)  $C$  will be selected?
  - (b)  $A$  will not be selected?
10. One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently the sample space consists of four elementary outcomes  $S = \{\text{John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}\}$   
 You are told that the chances of John's promotion is same as that of Gurpreet, Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.
  - (a) Determine  $P(\text{John promoted})$

$P$  (Rita promoted)

$P$  (Aslam promoted)

$P$  (Gurpreet promoted)

(b) If  $A = \{\text{John promoted or Gurpreet promoted}\}$ , find  $P(A)$ .

11. The accompanying Venn diagram shows three events,  $A$ ,  $B$ , and  $C$ , and also the probabilities of the various intersections (for instance,  $P(A \cap B) = .07$ ). Determine

- $P(A)$
- $P(B \cap \bar{C})$
- $P(A \cup B)$
- $P(A \cap \bar{B})$
- $P(B \cap C)$
- Probability of exactly one of the three occurs.

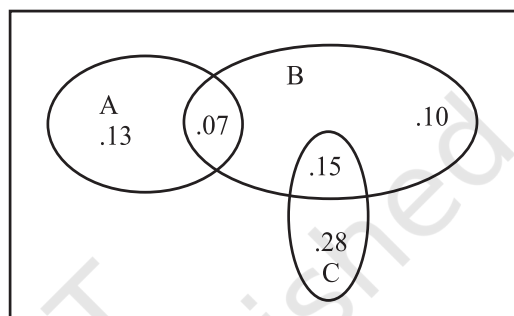


Fig. 16.2

### Long Answer Type

12. One urn contains two black balls (labelled  $B_1$  and  $B_2$ ) and one white ball. A second urn contains one black ball and two white balls (labelled  $W_1$  and  $W_2$ ). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.
- Write the sample space showing all possible outcomes
  - What is the probability that two black balls are chosen?
  - What is the probability that two balls of opposite colour are chosen?
13. A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the Probability that
- All the three balls are white
  - All the three balls are red
  - One ball is red and two balls are white
14. If the letters of the word ASSASSINATION are arranged at random. Find the Probability that
- Four S's come consecutively in the word
  - Two I's and two N's come together
  - All A's are not coming together
  - No two A's are coming together.

15. A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card.
16. A sample space consists of 9 elementary outcomes  $e_1, e_2, \dots, e_9$  whose probabilities are  
 $P(e_1) = P(e_2) = .08$ ,  $P(e_3) = P(e_4) = P(e_5) = .1$   
 $P(e_6) = P(e_7) = .2$ ,  $P(e_8) = P(e_9) = .07$   
 Suppose  $A = \{e_1, e_5, e_8\}$ ,  $B = \{e_2, e_5, e_8, e_9\}$
- Calculate  $P(A)$ ,  $P(B)$ , and  $P(A \cap B)$
  - Using the addition law of probability, calculate  $P(A \cup B)$
  - List the composition of the event  $A \cup B$ , and calculate  $P(A \cup B)$  by adding the probabilities of the elementary outcomes.
  - Calculate  $P(\bar{B})$  from  $P(B)$ , also calculate  $P(\bar{B})$  directly from the elementary outcomes of  $\bar{B}$
17. Determine the probability  $p$ , for each of the following events.
- An odd number appears in a single toss of a fair die.
  - At least one head appears in two tosses of a fair coin.
  - A king, 9 of hearts, or 3 of spades appears in drawing a single card from a well shuffled ordinary deck of 52 cards.
  - The sum of 6 appears in a single toss of a pair of fair dice.

### Objective Type Questions

Choose the correct answer out of four given options in each of the Exercises 18 to 29 (M.C.Q.).

18. In a non-leap year, the probability of having 53 tuesdays or 53 wednesdays is  
 (A)  $\frac{1}{7}$  (B)  $\frac{2}{7}$  (C)  $\frac{3}{7}$  (D) none of these
19. Three numbers are chosen from 1 to 20. Find the probability that they are not consecutive  
 (A)  $\frac{186}{190}$  (B)  $\frac{187}{190}$  (C)  $\frac{188}{190}$  (D)  $\frac{18}{{}^{20}C_3}$
20. While shuffling a pack of 52 playing cards, 2 are accidentally dropped. Find the probability that the missing cards to be of different colours



- (A)  $\frac{29}{52}$  (B)  $\frac{1}{2}$  (C)  $\frac{26}{51}$  (D)  $\frac{27}{51}$

21. Seven persons are to be seated in a row. The probability that two particular persons sit next to each other is

- (A)  $\frac{1}{3}$  (B)  $\frac{1}{6}$  (C)  $\frac{2}{7}$  (D)  $\frac{1}{2}$

22. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is

- (A)  $\frac{1}{5}$  (B)  $\frac{4}{5}$  (C)  $\frac{1}{30}$  (D)  $\frac{5}{9}$

23. If A and B are mutually exclusive events, then

- (A)  $P(A) \leq P(\bar{B})$  (B)  $P(A) \geq P(\bar{B})$   
 (C)  $P(A) < P(\bar{B})$  (D) none of these

24. If  $P(A \cup B) = P(A \cap B)$  for any two events A and B, then

- (A)  $P(A) = P(B)$  (B)  $P(A) > P(B)$   
 (C)  $P(A) < P(B)$  (D) none of these

25. 6 boys and 6 girls sit in a row at random. The probability that all the girls sit together is

- (A)  $\frac{1}{432}$  (B)  $\frac{12}{431}$  (C)  $\frac{1}{132}$  (D) none of these

26. A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel is

- (A)  $\frac{1}{3}$  (B)  $\frac{4}{11}$  (C)  $\frac{2}{11}$  (D)  $\frac{3}{11}$

27. If the probabilities for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is

- (A)  $> .5$  (B)  $.5$  (C)  $\leq .5$  (D) 0

28. The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then  $P(\bar{A}) + P(\bar{B})$  is

- (A) 0.4 (B) 0.8 (C) 1.2 (D) 1.6

29. If M and N are any two events, the probability that at least one of them occurs is
- (A)  $P(M) + P(N) - 2P(M \cap N)$       (B)  $P(M) + P(N) - P(M \cap N)$   
 (C)  $P(M) + P(N) + P(M \cap N)$       (D)  $P(M) + P(N) + 2P(M \cap N)$

State whether the statements are True or False in each of the Exercises 30 to 36.

30. The probability that a person visiting a zoo will see the giraffee is 0.72, the probability that he will see the bears is 0.84 and the probability that he will see both is 0.52.
31. The probability that a student will pass his examination is 0.73, the probability of the student getting a compartment is 0.13, and the probability that the student will either pass or get compartment is 0.96.
32. The probabilities that a typist will make 0, 1, 2, 3, 4, 5 or more mistakes in typing a report are, respectively, 0.12, 0.25, 0.36, 0.14, 0.08, 0.11.
33. If A and B are two candidates seeking admission in an engineering College. The probability that A is selected is .5 and the probability that both A and B are selected is at most .3. Is it possible that the probability of B getting selected is 0.7?
34. The probability of intersection of two events A and B is always less than or equal to those favourable to the event A.
35. The probability of an occurrence of event A is .7 and that of the occurrence of event B is .3 and the probability of occurrence of both is .4.
36. The sum of probabilities of two students getting distinction in their final examinations is 1.2.

Fill in the blanks in the Exercises 37 to 41.

37. The probability that the home team will win an upcoming football game is 0.77, the probability that it will tie the game is 0.08, and the probability that it will lose the game is \_\_\_\_\_.
38. If  $e_1, e_2, e_3, e_4$  are the four elementary outcomes in a sample space and  $P(e_1) = .1, P(e_2) = .5, P(e_3) = .1$ , then the probability of  $e_4$  is \_\_\_\_\_.
39. Let  $S = \{1, 2, 3, 4, 5, 6\}$  and  $E = \{1, 3, 5\}$ , then  $\bar{E}$  is \_\_\_\_\_.
40. If A and B are two events associated with a random experiment such that  $P(A) = 0.3, P(B) = 0.2$  and  $P(A \cap B) = 0.1$ , then the value of  $P(A \cap \bar{B})$  is \_\_\_\_\_.
41. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B is \_\_\_\_\_.

42. Match the proposed probability under Column  $C_1$  with the appropriate written description under column  $C_2$ :

$C_1$ Probability	$C_2$ Written Description
(a) 0.95	(i) An incorrect assignment
(b) 0.02	(ii) No chance of happening
(c) $-0.3$	(iii) As much chance of happening as not.
(d) 0.5	(iv) Very likely to happen
(e) 0	(v) Very little chance of happening

43. Match the following

- |                                                                         |                                               |
|-------------------------------------------------------------------------|-----------------------------------------------|
| (a) If $E_1$ and $E_2$ are the two mutually exclusive events            | (i) $E_1 \cap E_2 = E_1$                      |
| (b) If $E_1$ and $E_2$ are the mutually exclusive and exhaustive events | (ii) $(E_1 - E_2) \cup (E_1 \cap E_2) = E_1$  |
| (c) If $E_1$ and $E_2$ have common outcomes, then                       | (iii) $E_1 \cap E_2 = \phi, E_1 \cup E_2 = S$ |
| (d) If $E_1$ and $E_2$ are two events such that $E_1 \subset E_2$       | (iv) $E_1 \cap E_2 = \phi$                    |

