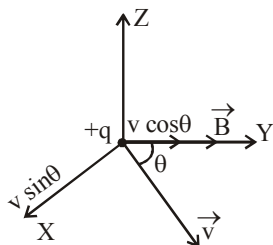# MOVING CHARGES AND MAGNETISM

GENERAL KEY CONCEPT

1. **Force on a moving charge:–** A moving charge is a source of magnetic field.

    Let a positive charge q is moving in a uniform magnetic field $\vec{B}$ with velocity $\vec{v}$.'

    $F \propto q \Rightarrow F \propto v \sin\theta \Rightarrow F \propto B$

    $\therefore \quad F \propto qBv \sin\theta \Rightarrow F = kq \, Bv \sin\theta \; [k = constant]$

    k = 1 in S.I. system.

    $\therefore \quad F = qBv \sin\theta$ and $\quad \vec{F} = q(\vec{v} \times \vec{B})$

2. **Magnetic field strength** $(\vec{B})$:

    In the equation $F = qBv \sin\theta$, if q = 1, v = 1,

    $\sin\theta = 1$ i.e. $\theta = 90°$ then F = B.

    $\therefore \quad$ Magnetic field strength is defined as the force experienced by a unit charge moving with unit velocity perpendicular to the direction of magnetic field.

    Special Cases:

    (1) It $\theta = 0°$ or $180°$, $\sin\theta = 0$

    $\therefore$ F = 0

    A charged particle moving parallel to the magnetic field, will not experience any force.

    (2) If v = 0, F = 0

    A charged particle at rest in a magnetic field will not experience any force.

    (3) If $\theta = 90°$, $\sin\theta = 1$ then the force is maximum

    $F_{max.} = qvB$

    A charged particle moving perpendicular to magnetic field will experience maximum force.

3. **S.I. unit of magnetic field intensity.** It is called tesla (T).

    $B = \dfrac{F}{qv\sin\theta}$

    If q = 1C, v = 1m/s, $\theta = 90°$ i.e. $\sin\theta = 1$ and F = 1N

    Then B = 1T.

The strength of magnetic field at a point is said to be 1T if a charge of 1C while moving at right angle to a magnetic field, with a velocity of 1 m/s experiences a force of 1N at that point.

4. **Biot-Savart's law:–** The strength of magnetic field or magnetic flux density at a point P (dB) due to current element dl depends on,

(i) $dB \propto I$

(ii) $dB \propto dl$

(iii) $dB \propto \sin\theta$

(iv) $dB \propto \dfrac{1}{r^2}$,
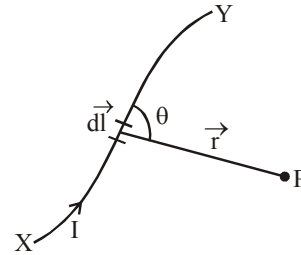
Combining, $dB \propto \dfrac{Idl\sin\theta}{r^2} \Rightarrow dB = k\dfrac{Idl\sin\theta}{r^2}$ [k = Proportionality constant]

In S.I. units, $k = \dfrac{\mu_0}{4\pi}$ where μ_0 is called permeability of free space.

$\mu_0 = 4\pi \times 10^{-7}$ $TA^{-1}m$

$\therefore$ $dB = \dfrac{\mu_0}{4\pi}\dfrac{Idl\sin\theta}{r^2}$ and $d\vec{B} = \dfrac{\mu_0}{4\pi}I\dfrac{(\vec{dl}\times\vec{r})}{r^3}$

$d\vec{B}$ is perpendicular to the plane containing $\vec{dl}$ and $\vec{r}$ and is directed inwards.

5. **Applications of Biot-Savart's law:–**

(a) Magnetic field (B) at the Centre of a Circular Coil Carrying Current.

$B = \dfrac{\mu_0 nI}{2r}$

where n is the number of turns of the coil. I is the current in the coil and r is the radius of the coil.

(b) Magnetic field due to a straight conductor carrying current.

$B = \dfrac{\mu_0 I}{4\pi a}(\sin\phi_2 + \sin\phi_1)$

where a is the perpendicular distance of the conductor from the point where field is to the measured.

ϕ_1 and ϕ_2 are the angles made by the two ends of the conductor with the point.

(c) For an infinitely long conductor, $\phi_1 = \phi_2 = \pi/2$
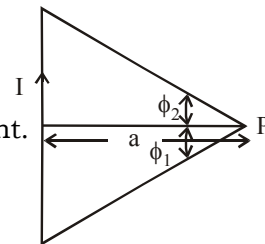
$\therefore$ $B = \dfrac{\mu_0}{4\pi}\cdot\dfrac{2I}{a}$

(d) Magnetic field at a point on the axis of a Circular Coil Carrying Current.

when point P lies far away from the centre of the coil.

B $\dfrac{\mu_0}{4}\cdot\dfrac{2M}{x^3}$

where M = nIA = magnetic dipole moment of the coil.

x is the distance of the point where the field is to the measured, n is the number of turns, I is the current and A is the area of the coil.

6. **Ampere's circuital law:–**

The line integral of magnetic field $\vec{B}$ around any closed path in vacuum is μ_0 times the total current through the closed path. i.e. $\oint \vec{B}.d\vec{l} = \mu_0 I$

7. **Application of Ampere's circuital law:–**

(a) Magnetic field due to a current carrying solenoid, $B = \mu_0 nI$

n is the number of turns per unit length of the solenoid.

At the edge of a short solenoid, $B = \dfrac{\mu_0 nI}{2}$

(b) Magnetic field due to a toroid or endless solenoid

$B = \mu_0 nI$

8. **Motion of a charged particle in uniform electric field:–**

The path of a charged particle in an electric field is a parabola.

Equation of the parabola is $x^2 = \dfrac{2mv^2}{qE} y$

where x is the width of the electric field.

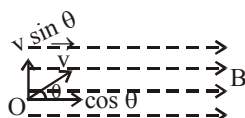y is the displacement of the particle from its straight path.

v is the speed of the charged particle.

q is the charge of the particle

E is the electric field intensity.

m is the mass of the particle.

9. Motion of the charged particle in a magnetic field. The path of a charged particle moving in a uniform magnetic field $(\vec{B})$ with a velocity $\vec{v}$ making an angle θ with $\vec{B}$ is a helix.

The component of velocity $v\cos\theta$ will not provide a force to the charged particle, so under this velocity the particle with move forward with a constant velocity along the direction of $\vec{B}$. The other component $v\sin\theta$ will produce the force $F = q$ $Bv\sin\theta$, which will supply the necessary centripetal force to the charged particle in moving along a circular path of radius r.

$\because$ Centripetal force = $\dfrac{m(v\sin\theta)^2}{r}$ = $B\,qv\,\sin\theta$

$\therefore v\sin\theta = \dfrac{Bqr}{m}$

Angular velocity of rotation = w = $\dfrac{v\sin\theta}{r} = \dfrac{Bq}{m}$

Frequency of rotation = $v = \dfrac{\omega}{2\pi} = \dfrac{Bq}{2\pi m}$

Time period of revolution = T = $\dfrac{1}{v} = \dfrac{2\pi m}{Bq}$

10. Cyclotron: It is a device used to accelerate and hence energies the positively charged particle. This is done by placing the particle in an oscillating electric field and a perpendicular magnetic field. The particle moves in a circular path.

∴ Centripetal force = magnetic Lorentz force

$$\Rightarrow \frac{mv^2}{r} = Bqv \Rightarrow \frac{mv}{Bq} = r \leftarrow \text{radius of the circular path}$$

Time to travel a semicircular path $= \dfrac{\pi r}{v} = \dfrac{\pi m}{Bq} = $ constant.

If v_0 be the maximum velocity of the particle and r_0 be the maximum radius of its path then

$$\frac{mv_0^2}{r_0} = Bqv_0 \Rightarrow v_0 = \frac{Bqr_0}{m}$$

Max. K.E. of the particle $= \dfrac{1}{2}mv_0^2 = \dfrac{1}{2}m\left(\dfrac{Bqr_0}{m}\right)^2 \Rightarrow (\text{K.E.})_{\text{max.}} = \dfrac{B^2q^2r_0^2}{2m}$

Time period of the oscillating electric field $\Rightarrow T = \dfrac{2\pi m}{Bq}$.

Time period is independent of the speed and radius.

Cyclotron frequency $= v = \dfrac{1}{T} = \dfrac{Bq}{2\pi m}$

Cyclotron angular frequency $= \omega_0 = 2\pi v = \dfrac{Bq}{m}$

11. Force on a current carrying conductor placed in a magnetic field:

$$\overrightarrow{F} = I\left|\vec{\ell} \times \vec{B}\right| \text{ or } F = I\ell B \sin\theta$$

where I is the current through the conductor

B is the magnetic field intensity.

l is the length of the conductor.

θ is the angle between the direction of current and magnetic field.

(i) When $\theta = 0°$ or $180°$, $\sin\theta = 0 \Rightarrow F = 0$

∴ When a conductor is placed along the magnetic field, no force will act on the conductor.

(ii) When $\theta = 90°$, $\sin\theta = 1$, F is maximum.

$F_{\text{max}} = I\ell B$

when the conductor is placed perpendicular to the magnetic field, it will experience maximum force.

12. Force between two parallel conductors carrying current:–

(a) When the current is in same direction the two conductors will attract each other with a force

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} \text{ per unit length of the conductor}$$

(b) When the current is in opposite direction the two conductors will repel each other with the same force.

(c) S.I. unit of current is 1 ampere. (A).

1A is the current which on flowing through each of the two parallel uniform linear conductor placed in free space at a distance of 1 m from each other produces a force of 2×10^{-7} N/m along their lengths.

13. Torque on a current carrying coil placed in a magnetic field:–

$\vec{\tau} = \vec{M} \times \vec{B} \Rightarrow \tau = MB \sin\alpha = nIBA \sin\alpha$ where M is the magnetic dipole moment of the coil.

M = nIA

where n is the number of turns of the coil.

I is the current through the coil.

B is the magnetic field intensity.

A is the area of the coil.

α is the angle between the magnetic field $\left(\vec{B}\right)$ and the perpendicular to the plane of the coil.

Special Cases:

(i) If the coil is placed parallel to magnetic field $\theta = 0°$, $\cos\theta = 1$ then torque is maximum.

$\tau_{max.} = nIBA$

(ii) If the coil is placed perpendicular to magnetic field, $\theta = 90°$, $\cos\theta = 0$

$\therefore \tau = 0$

14. Moving coil galvanometer:– This is based on the principle that when a current carrying coil is placed in a magnetic field it experiences a torque. There is a restoring torque due to the phosphor bronze strip which brings back the coil to its normal position.

In equilibrium, Deflecting torque = Restoring torque

nIBA = $k\theta$ [k = restoring torque/unit twist of the phosphor bronze strip]

$I = \dfrac{k}{nBA}\theta = G\theta$ where $G = \dfrac{k}{nBA}$ = Galvanometer constant

$\therefore \quad I \propto \theta$

Current sensitivity of the galvanometer is the deflection produced when unit current is passed through the galvanometer.

$I_s = \dfrac{\theta}{I} = \dfrac{nBA}{k}$

Voltage sensitivity is defined as the deflection produced when unit potential difference is applied across the galvanometer.

$V_s = \dfrac{\theta}{V} = \dfrac{\theta}{IR} = \dfrac{nBA}{kR}$ [R = Resistance of the galvanometer]

15. Condition for the maximum sensitivity of the galvanometer:-

The galvanometer is said to be sensitive if a small current produces a large deflection.

$$\because \quad \theta = \frac{nBA}{k}I$$

$\therefore \quad \theta$ will be large if (i) n is large, (ii) B is large (iii) A is large and (iv) k is small.

16. Conversion of galvanometer into voltmeter and ammeter

(a) A galvanometer is converted to voltmeter by putting a high resistance in series with it.

Total resistance of voltmeter = R_g + R where R_g is the galvonometer resistance. R is the resistance added in series.

Current through the galvanometer = I_g = $\dfrac{V}{Rg+R}$

where V is the potential difference across the voltmeter.
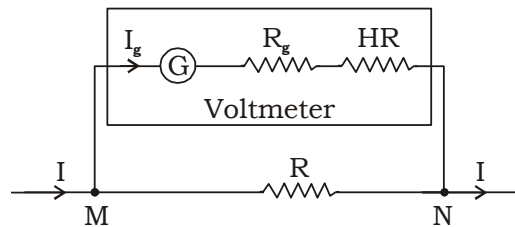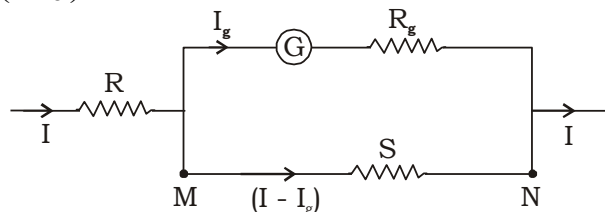

$$\therefore \qquad R = \frac{V}{I_g} - G$$

Range of the voltmeter: 0 – V volt.

(b) A galvanometer is converted into an ammeter by connecting a low resistance in parallel with it (shunt)

Shunt = $S = \left(\dfrac{I_g}{I-I_g}\right)R_g$ where R_g is the galvanometere resistance.

I is the total current through the ammeter.

I_g is the current through the ammeter. Effective resistance of the ammeter

$$R = \frac{R_g}{R_g + S}$$

The range of the ammeter is 0 – I A. An ideal ammeter has zero resistance.