Date: 05/05/2022

Question Paper Code

430/2/1

Time: 2 Hrs.

Class-X

Max. Marks: 40

MATHEMATICS (Basic) Term-II

(CBSE 2022)

GENERAL INSTRUCTIONS

(i) *This question paper contains **14** questions. **All** questions are compulsory.*

(ii) *This question paper is divided into 3 Sections – **Section A, B** and **C**.*

(iii) ***Section-A** comprises of **6** questions (Q. Nos. **1** to **6**) of **2** marks each.*

*Internal choice has been provided in **two** questions.*

(iv) ***Section-B** comprises of **4** questions (Q. Nos. **7** to **10**) of **3** marks each.*

*Internal choice has been provided in **one** question.*

(v) ***Section-C** comprises of **4** questions (Q. Nos. **11** to **14**) of **4** marks each. An internal choice has been provided in **one** question. It also contains **two** case study based questions.*

(vi) *Use of calculator is not permitted.*

SECTION-A

Question Numbers 1 to 6 carry 2 marks each.

1. Find the nature of the roots of the quadratic equation : **[2]**

   $4x^2 - 5x - 1 = 0$

Solution

$4x^2 - 5x - 1 = 0$

$D = b^2 - 4ac$, where $a = 4$, $b = -5$ and $c = -1$ **[½]**

$\Rightarrow$ $D = 25 + 16 = 41$ **[½]**

$\Rightarrow$ $D > 0$ **[½]**

$\therefore$ The given equation has real and distinct roots **[½]**

2. **(a)** Which term of the A.P. 3, 8, 13, 18, … is 78? **[2]**

OR

(b) Find the common difference of an A.P. whose n^{th} term is given by **[2]**

$a_n = 6n - 5$.

Solution

(a) Given A.P. is 3, 8, 13, 18, … .

Here, $a = 3$ and $d = 8 - 3 = 5$ **[½]**

$a_n = a + (n - 1)d$ [n^{th} term] **[½]**

$\Rightarrow$ $78 = 3 + (n - 1)5$

$\Rightarrow$ $\frac{75}{5} = n - 1$ **[½]**

$\Rightarrow$ $n = 16$

$\therefore$ 78 is 16^{th} term of the given A.P. **[½]**

OR

(b) n^{th} term of A.P. is

$a_n = 6n - 5$

if $n = 1$,

$\Rightarrow$ $a_1 = 6 - 5 = 1$ **[½]**

if $n_2 = 1$,

$a_2 = 6 \times 2 - 5 = 7$ **[½]**

$\therefore$ Common difference $(d) = 7 - 1$ **[½]**

$= 6$ **[½]**

3. 3 cubes each of 8 cm edge are joined end to end. Find the total surface area of the cuboid so formed. **[2]**

Solution

Dimensions of cuboid are 24 cm, 8 cm, 8 cm

T.S.A of cuboid $= 2(lb + bh + lh)$ **[½]**

$= 2[24(8) + 8(8) + 24(8)]$ **[½]**

$= 2[192 + 64 + 192]$ **[½]**

$= 2[448] = 896 \text{ cm}^2$ **[½]**

4. (a) In Fig. 1, perimeter of ΔPQR is 20 cm. Find the length of tangent PA. **[2]**

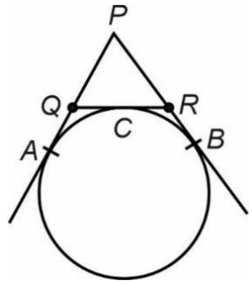


Fig. 1

OR

(b) In Fig. 2, BC is tangent to the circle at point B of circle centred at O. BD is a chord of the circle so that $\angle BAD = 55°$. Find $m\angle DBC$. **[2]**

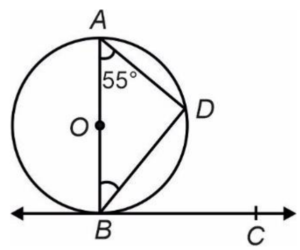
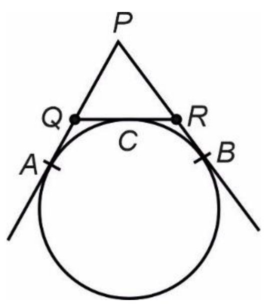


Fig. 2

Solution

(a)



$PA = PQ + QA$

$= PQ + QC$ ...(i) $[\because QA = QC]$ **[½]**

and $PB = PR + BR$ **[½]**

$= PR + CR$ ...(ii) $[\because BR = CR]$

Adding (i) and (ii), we get

$PA + PB = PQ + QC + CR + PR$

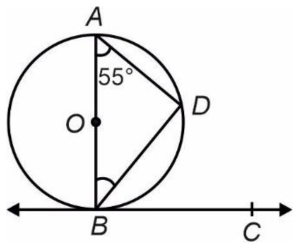
$\Rightarrow 2PA = PQ + QR + PR$ $[\because PA = PB]$ **[½]**

$\Rightarrow PA = \dfrac{\text{Perimeter of } \Delta PQR}{2}$

$= \dfrac{20}{2}$ **[½]**

$= 10$ cm

OR

(b) 

$\angle ADB$ = 90° [Angle in semi-circle] **[½]**

$\angle ABD$ = 90° – $\angle BAD$ [Angle sum property of ΔABD]

= 90° – 55°

= 35° **[½]**

Now, $\angle DBC$ = 90° – $\angle ABD$ [$\because$ $AB \perp BC$] **[½]**

= 90° – 35°

= 55° **[½]**

5. Find the mode of the following frequency distribution: **[2]**

Class :	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency :	25	30	45	42	35

Solution

Mode = $l + \dfrac{(f_m - f_1)}{(2f_m - f_1 - f_2)} \times h$ **[½]**

$\Rightarrow$ f_m = 45

f_1 = 30

f_2 = 42

h = 10 **[½]**

l = 40

$\therefore$ Mode $= 40 + \left(\dfrac{45-30}{90-72}\right) \times 10$ **[½]**

$= 40 + \left(\dfrac{15}{18} \times 10\right) = 40 + \left(\dfrac{150}{18}\right) = 40 + 8.33 = 48.33$ **[½]**

6. Find the sum of the first fifteen multiples of 8. **[2]**

Solution

First fifteen multiples of 8 are

8, 16, 24….. **[½]**

Here, a = 8 and d = 8

$S_{15} = \dfrac{15}{2}[2 \times 8 + (15-1)8]$ **[½]**

$= \dfrac{15}{2}[16 + 112]$ **[½]**

$= \dfrac{15 \times 128}{2}$

= 960

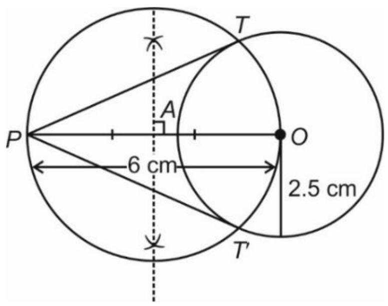
$\therefore$ Sum of first fifteen multiples of 8 is 960. **[½]**

SECTION-B

Question Numbers 7 to 10 carry 3 marks each.

7. Draw a circle of radius 2.5 cm. Construct a pair of tangents from a point P at a distance of 6 cm from the centre of the circle. **[3]**

Solution



$\therefore$ PT and PT' are the required tangents. **[3]**

8. (a) As observed from the top of a light house 100 m above sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 45°. Determine the distance travelled by the ship during this time. **[3]**

$\left(\text{Use } \sqrt{3} = 1.73\right)$

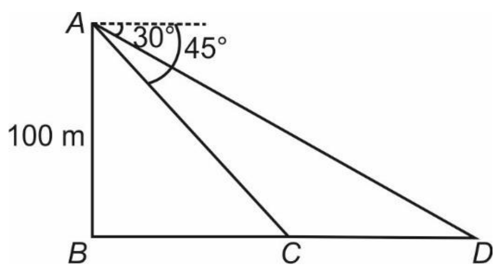
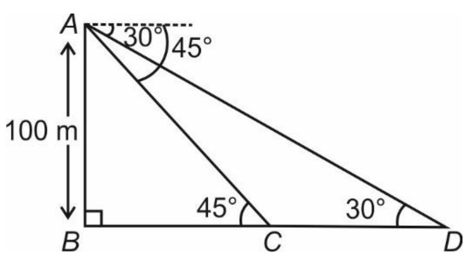


Fig. 3

Solution



In ΔABC,

$$\frac{AB}{BC} = \tan 45° = 1$$ **[½]**

$\Rightarrow$ $AB = BC = 100$ m ...(i) **[½]**

In ΔABD,

$$\frac{AB}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $BD = AB \times \sqrt{3}$

$= 100\sqrt{3}$ m ...(ii) **[½]**

$\therefore$ $CD = BD - BC$

$= \left(100\sqrt{3} - 100\right)$ m [From (i) and (ii)] **[½]**

$= 100\left(\sqrt{3} - 1\right)$ m

= 100 (1.73 –1) m

= 100 × 0.73 m **[½]**

= 73 m

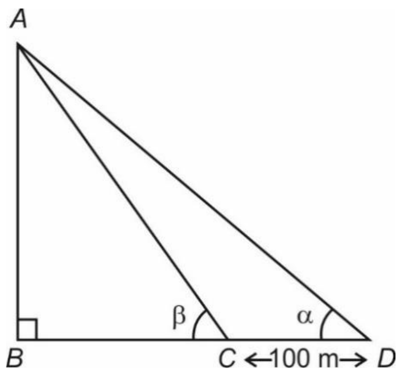
$\therefore$ Ship will travel 73 m during the given time. **[½]**

OR

(b) At a point on level ground, the angle of elevation of a vertical tower is, found to be α such that $\tan\alpha = \frac{1}{3}$.

After walking 100 m towards the tower, the angle of elevation β becomes such that $\tan\beta = \frac{3}{4}$. Find the height of the tower. **[3]**

Solution



Let AB represents the tower. Observer is moving from D to C.

In ΔABC,

$$\tan\beta = \frac{AB}{BC} = \frac{3}{4} \qquad \ldots\text{(i)}$$ **[½]**

and in ΔABD,

$$\tan\alpha = \frac{AB}{BD} = \frac{1}{3} \qquad \ldots\text{(ii)}$$ **[½]**

From (i) and (ii), we get

$$BC = \frac{4AB}{3} \text{ and } BD = 3AB$$ **[½]**

$\Rightarrow$ $CD = BD - BC$ **[½]**

$\Rightarrow$ $100 = 3AB - \frac{4AB}{3}$

$\Rightarrow$ $100 = \frac{9AB - 4AB}{3}$ **[½]**

$\Rightarrow$ $300 = 5AB$

$\Rightarrow$ $AB = 60$ m

$\therefore$ Height of tower is 60 m. **[½]**

9. Find the mean of the following frequency distribution : **[3]**

Class :	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency :	4	10	5	6	5

Solution

Class	Frequency (f_i)	Class Marks (x_i)	Product ($f_i x_i$)
10–15	4	12.5	50.00
15–20	10	17.5	175.00
20–25	5	22.5	112.50
25–30	6	27.5	165.00
30–35	5	32.5	162.50
Total	$N = 30$		$\sum f_i x_i = \mathbf{665.00}$

[1]

Mean $(\bar{x}) = \frac{1}{N}\sum_{i=1}^{k} f_i x_i$ **[1]**

$$= \frac{\sum_{i=1}^{5} f_i x_i}{N} = \frac{665.0}{30}$$ **[1]**

= 22.17 (approx.)

10. The median of following frequency distribution is 25. Find the value of x. **[3]**

Class :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency :	6	9	10	8	x

Solution

Class	Frequency	c.f.
0–10	6	6
10–20	9	15
20–30	10	25
30–40	8	33
40–50	x	$33+x$

[½]

Median = 25

$\Rightarrow$ Median class is 20–30

$\Rightarrow f = 10$, c.f. = 15, $N = 33 + x$, $h = 10$ and $l = 20$ **[½]**

Median $= l + \left(\frac{\frac{N}{2} - \text{cf}}{f}\right) \times h$ **[½]**

$$\Rightarrow \quad 25 = 20 + \left(\frac{\frac{33+x}{2} - 15}{10} \times 10\right)$$ **[½]**

$\Rightarrow \quad 5 = \dfrac{33 + x - 30}{2}$ **[½]**

$\Rightarrow \quad 10 = 3 + x$

$\therefore \quad x = 7$ **[½]**

SECTION-C

Question Numbers 11 to 14 carry 4 marks each.

11. (a) Prove that a parallelogram circumscribing a circle is a rhombus. **[4]**

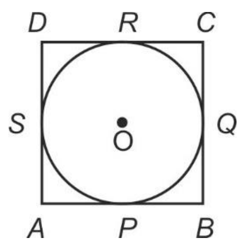
OR

(b) Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle. **[4]**

Solution (a)

Given : A circle with centre *O*.

A parallelogram *ABCD* touching the circle at Points *P*, *Q*, *R* and *S*



To Prove: *ABCD* is a rhombus

Proof: A rhombus is a parallelogram with all sides equal

In parallelogram *ABCD*

$AB = CD$ and $BC = AD$ **[1]**

We know that the lengths of tangents from an external point are equal

$\therefore \quad AP = AS$ …(i)

$BP = BQ$ …(ii)

$CQ = CR$ …(iii)

$DR = DS$ …(iv) **[1]**

Adding (i), (ii), (iii) and (iv), we get

$\Rightarrow AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow AB + (CR + DR) = AS + BQ + CQ + DS$

$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC$

$\Rightarrow CD + CD = BC + BC$

$[\because AB = CD$ and $AD = BC]$ **[1]**

$\Rightarrow CD = BC$

$\therefore AB = CD = BC = AD$

All sides are equal

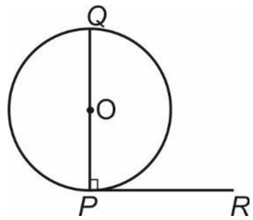
$\Rightarrow$ Hence, *ABCD* is a rhombus **[1]**

OR

Solution (b)

Let, O is the centre of the given circle. A segment PR has been drawn touching the circle at point P. **[½]**

Draw $QP \perp RP$ at point P, such that point Q lies on the circle. **[½]**



$\angle OPR = 90°$ [Radius $\perp$ Tangent] **[½]**

Also, $\angle QPR = 90°$ [given] **[½]**

$\therefore \angle OPR = \angle QPR$ **[½]**

Now, the above case is possible only when centre O lies on the line QP. **[1]**

Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle. **[½]**

12. The sum of the ages of a boy and his sister (in years) is 25 and product of their ages is 150. Find their present ages. **[4]**

Solution

Let age of boy be x years, then age of his sister will be $(25 - x)$ years **[½]**

Product of their ages, $(x)(25 - x) = 150$ **[½]**

$\Rightarrow 25x - x^2 = 150$ **[½]**

$\Rightarrow x^2 - 25x + 150 = 0$ **[½]**

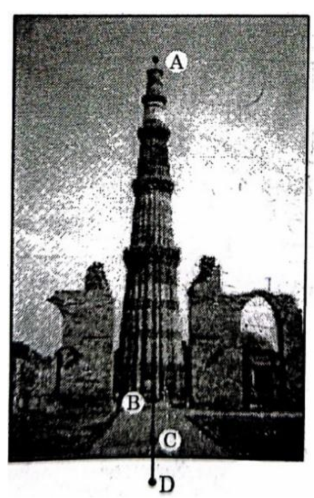
$\Rightarrow (x - 15)(x - 10) = 0$ **[1]**

$\Rightarrow x = 10$ and 15 **[½]**

Hence, their present age's are 10 years and 15 years. **[½]**

Case Study - 1

13. Qutub Minar, located in South Delhi, India, was built in the year 1193. It is 72 m high tower. Working on a school project, Charu and Daljeet visited the monument. They used trigonometry to find their distance from the tower. Observe the picture given below. Points C and D represent their positions on the ground in line with the base of tower, the angles of elevation of top of the tower (Point A) are 60° and 45° from points C and D respectively.

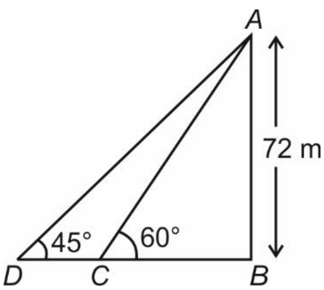


(i) Based on above information, draw a well-labelled diagram. **[1]**

(ii) Find the distances *CD*, *BC* and *BD*. (use $\sqrt{3}$ = 1.73) **[3]**

Solution

(i) Let positions of Charu and Daljeet be *C* and *D* respectively, **[1]**



Charu is nearer to Qutub Minar as its angle of elevation is greater.

(ii) In ΔABC,

$\tan 60^\circ = \dfrac{AB}{BC}$ **[½]**

$\Rightarrow \sqrt{3} = \dfrac{72}{BC}$ **[½]**

$\Rightarrow BC = 41.52$ m **[½]**

In ΔABD,

$\tan 45^\circ = \dfrac{AB}{BD}$ **[½]**

$\Rightarrow 1 = \dfrac{72}{BD}$

$\Rightarrow BD = 72$ m **[½]**

$CD = BD - BC$

$CD = (72 - 41.52)$ m

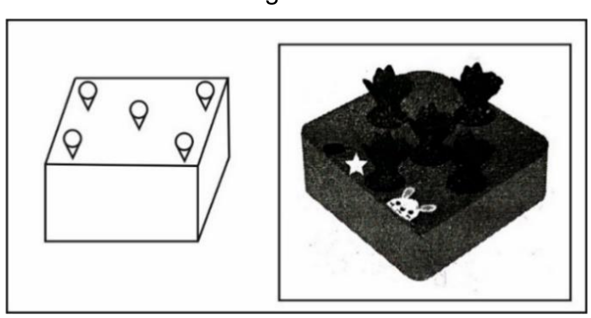
= 30.48 m **[½]**

Case Study - 2

14. A solid cuboidal toy is made of wood. It has five cone shaped cavities to hold toy carrots.

The dimensions of the toy are cuboid – 10 cm × 10 cm × 8 cm.

Each cone carved out – Radius = 2.1 cm and Height = 6 cm.



(i) Find the volume of wood carved out to make five conical cavities. **[2]**

(ii) Find the volume of the wood in the final product. **[2]**

Solution

(i) Dimensions of cuboid = 10 cm × 10 cm × 8 cm

Dimensions of cone,

Radius, R = 2.1 cm

Height, H = 6 cm

Volume of wood carved out = Volume of 5 cones = $\frac{1}{3}(\pi)R^2H \times 5$ **[1]**

$= 5 \times \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 6$ = 138.6 cm^3 **[1]**

(ii) Volume of the wood in the final product = Volume of cuboid – Volume of wood carved out **[1]**

= (10 × 10 × 8 – 138.6) cm^3 **[½]**

= 661.4 cm^3 **[½]**

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