CBSE SAMPLE PAPER - 05

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. $\int \frac{(1+\sin x)}{(1-\sin x)} dx = ?$ **[1]**

a) None of these

b) 2 tan x – x + 2 sec x + C

c) 2 tan x + x – 2sec x + C

d) 2 tan x – x – 2sec x + C

2. The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$ are **[1]**

a) parallel

b) intersecting

c) skew

d) coincident

3. If a vector makes angles α, β and γ with the x axis, y axis and z axis respectviely then the value of $\left(\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma\right)$ is **[1]**

a) 2

b) 0

c) 3

d) 1

4. If E_1 and E_2 are two independent events, then P $\left(E_1 \cap E_2\right)$ is equal to **[1]**

a) $P\left(E_1\right) + \ P\left(E_2\right)$

b) $P\left(E_1\right) + P(E_2) + P(E_1 + \cup E_2)$

c) $P\left(E_1\right)P(E_2)$

d) none of these

5. $\int \sqrt{ax+b} dx = ?$ **[1]**

a) $\frac{2(ax+b)^{3/2}}{3a} + C$

b) $\frac{1}{2\sqrt{ax+b}} + c$

c) None of these

d) $\frac{3(ax+b)^{3/2}}{2a} + C$

6. A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots, and C can hit 2 times in 3 shots. The probability that B and C hit and A does not hit is **[1]**

a) None of these

b) $\frac{7}{12}$

c) $\frac{1}{10}$

d) $\frac{2}{5}$

7. The area bounded by $y = 2 - x^2$ and $x + y = 0$ is **[1]**

a) $\frac{9}{2}$ sq. units

b) $\frac{7}{2}$ sq. units

c) None of these

d) 9 sq. units

8. Consider two lines in space as $L_1 : \overrightarrow{\mathrm{r}}_1 = \hat{\mathrm{j}} + 2\hat{\mathrm{k}} + \lambda(3\hat{\mathrm{i}} - \hat{\mathrm{j}} - \hat{\mathrm{k}})$ and $L_2 : \overrightarrow{\mathrm{r}_2} = 4\hat{\mathrm{i}} + 3\hat{\mathrm{j}} + 6\hat{\mathrm{k}} +$ $\mu(\hat{\mathrm{i}} + 2\hat{\mathrm{k}})$. If the shortest distance between these lines is $\sqrt{d}$ then d equals: **[1]**

a) 5

b) 6

c) 8

d) 7

9. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -i + 2j - k$, then a unit vector normal to the vectors $\vec{a} + \vec{b}$ and $\vec{b} - \vec{c}$ is **[1]**

a) $\hat{k}$

b) $\hat{i}$

c) None of these

d) $\hat{j}$

10. The solution of the differential equation cos x sin y dx + sin x cos y dy = 0 is: **[1]**

a) cos x cos y = c

b) sin x + sin y = c

c) sin x sin y = c

d) $\frac{\sin x}{\sin y} = c$

11. The area between x-axis and curve, y = cos x when $0 \leq x \leq 2\pi$ is **[1]**

a) 3

b) 4

c) 0

d) 2

12. $\int \frac{e^{\tan^{-1} x}}{(1+x^2)} dx = ?$ **[1]**

a) $\frac{e^{\tan^{-1} x}}{x} + C$

b) None of these

c) $e^x \tan^{-1} x + C$

d) $e^{\tan^{-1} x} + C$

13. The point on the curve $y^2 = 4x$ which is nearest to the point (2,1) is **[1]**

a) $(1, 2\sqrt{2})$

b) (-2, 1)

c) (1, -2)

d) (1, 2)

14. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^5 =$ **[1]**

a) 16A

b) 10A

c) 5A

d) 32A

15. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij}, then the value of Δ is given by **[1]**

a) $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33}$

b) $a_{21} A_{11} + a_{22} A_{12} + a_{23} A_{13}$

c) $a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31}$

d) $a_{11} A_{11} + a_{12} A_{21} + a_{13} A_{31}$

16. If x, y, z are non-zero real numbers, then the inverse of matrix $A=\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}$ is [1]

a) $\frac{1}{xyz}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

b) $\begin{bmatrix}x^{-1}&0&0\\0&y^{-1}&0\\0&0&z^{-1}\end{bmatrix}$

c) $\frac{1}{xyz}\begin{bmatrix}x&0&0\\0&y&0\\0&0&z\end{bmatrix}$

d) $\frac{5yz}{xyz}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

17. $\cos^{-1}(\cos x)=x$ is satisfied by, [1]

a) $x\in[-1,1]$

b) $x\in[0,\pi]$

c) None of these

d) $x\in[0,1]$

18. The order of the differential equation of all circles of given radius a is: [1]

a) 4

b) 1

c) 2

d) 3

19. **Assertion (A):** The rate of change of area of a circle with respect to its radius r when r = 6 cm is 12π cm^2/cm. [1]

Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The equation of the line joining A(1, 3) and B(0, 0) is given by y = 3x. [1]

Reason (R): The area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) in the form of determinant is

$\triangle=\begin{vmatrix}x_1&y_1&1\\x_2&y_2&1\\x_3&y_3&1\end{vmatrix}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the value of $\sin^{-1}\left(\frac{1}{3}\right)-\cos^{-1}\left(-\frac{1}{3}\right)$ [2]

22. Solve the differential equation: $\frac{dy}{dx}+2x=e^{3x}$ [2]

23. Evaluate $\Delta=\begin{bmatrix}0&\sin\alpha&-\cos\alpha\\-\sin\alpha&0&\sin\beta\\\cos\alpha&-\sin\beta&0\end{bmatrix}$ [2]

OR

Find the matrix X for which: $\begin{bmatrix}3&2\\7&5\end{bmatrix}X\begin{bmatrix}-1&1\\-2&1\end{bmatrix}=\begin{bmatrix}2&-1\\0&4\end{bmatrix}$

24. Find the direction ratios and the direction cosines of the vector $\overrightarrow{r}=2\hat{i}-7\hat{j}-3\hat{k}$ [2]

25. A bag contains 4 white and 5 black balls. Another bag contains 9 white and 7 black balls. A ball is transferred from the first bag to the second and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white. [2]

Section C

26. Evaluate: $\int \frac{x}{x^2+x+1} dx$ **[3]**

27. Find the general solution for the differential equation: $(x^2y - x^2)dx + (xy^2 - y^2)dy = 0$ **[3]**

OR

Solve the initial value problem: $e^{dy/dx} = x + 1$; $y(0) = 3$

28. If the sum of two unit vectors $\hat{a}$ and $\hat{b}$ is a unit vector, show that the magnitude of their difference is $\sqrt{3}$. **[3]**

OR

Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{a}\times\vec{b}|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

29. Prove that: $\int\limits_{a/4}^{3a/4} \frac{\sqrt{x}}{(\sqrt{a-x}+\sqrt{x})} dx = \frac{a}{4}$. **[3]**

OR

Evaluate: $\int \frac{1}{x+\sqrt{x^2-x+1}} dx$

30. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1} x$ when $x \neq 0$ **[3]**

31. Find the area bounded by the curve $y^2 = 4ax$ and the lines $y = 2a$ and y-axis. **[3]**

Section D

32. Solve the Linear Programming Problem graphically: **[5]**

Maximize $Z = 9x + 3y$ Subject to

$2x + 3y \leq 13$

$3x + y \leq 5$

$x, y \geq 0$

33. Let A = R - {3}, B = R - {1}. If $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$ $\forall x \in A$. Then, show that f is bijective. **[5]**

OR

Let A = {1, 2, 3} and R = {(a, b):a, b $\in$ A and $\left|a^2 - b^2\right| \leq 5$. Write R as set of ordered pairs. Mention whether R is

i. reflexive
ii. symmetric
iii. transitive

Give reason in each case.

34. Find the shortest between the l_1 and l_2 whose vector equations are **[5]**

$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$

and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

OR

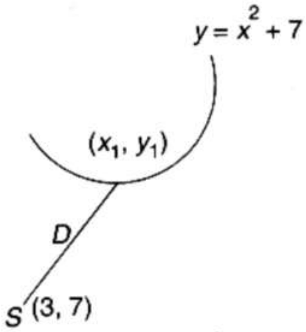
Find the vector equation of the line passing through (1, 2, 3) and parallel to each of the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$.

35. If $y\sqrt{x^2+1} - \log\left(\sqrt{x^2+1} - x\right) = 0$ prove that $\left(x^2+1\right)\frac{dy}{dx} + xy + 1 = 0$ **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

An Apache helicopter of the enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.



(i) If P (x_1, y_1) be the position of a helicopter on curve $y = x^2 + 7$, then find distance D from P to soldier place at (3, 7).

(ii) Find the critical point such that distance is minimum.

(iii) Verify by second derivative test that distance is minimum at (1, 8).

OR

Find the minimum distance between soldier and helicopter?

37. **Read the text carefully and answer the questions:** **[4]**

Three friends Ravi, Raju and Rohit were doing buying and selling of stationery items in a market. The price of per dozen of pen, notebooks and toys are Rupees x, y and z respectively.

Ravi purchases 4 dozen of notebooks and sells 2 dozen of pens and 5 dozen of toys. Raju purchases 2 dozen of toy and sells 3 dozen of pens and 1 dozen of notebooks. Rohit purchases one dozen of pens and sells 3 dozen of notebooks and one dozen of toys.

In the process, Ravi, Raju and Rohit earn ₹1500, ₹100 and ₹400 respectively.

(i) Write the above information in terms of matrix Algebra.

(ii) What is the total price of one dozen of pens and one dozen of notebooks?

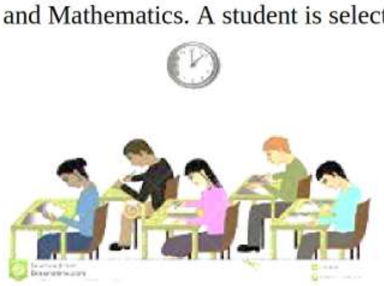
(iii) What is the sale amount of Ravi?

OR

What is the amount of purchases and sales made by all three friends?

38. **Read the text carefully and answer the questions:** **[4]**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.



(i) Find the probability that the selected student has failed in Economics, if it is known that he has failed in Mathematics?

(ii) Find the probability that the selected student has failed in Mathematics, if it is known that he has failed in Economics?

Solution

CBSE SAMPLE PAPER - 05

Class 12 - Mathematics

Section A

1. **(b)** 2 tan x – x + 2 sec x + C

   Explanation: Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \sec^2 x dx = \tan x$

   Therefore ,

   $= \int \frac{1+\sin x(1+\sin x)}{1-\sin x(1+\sin x)} dx$

   $\Rightarrow \int \frac{(1+\sin x)^2}{1-\sin^2 x} dx = \int \frac{1+\sin^2 x+2\sin x}{\cos^2 x} dx$

   $\Rightarrow \int \frac{1}{\cos^2 x} dx + 2\int \frac{\sin x}{\cos^2 x} dx + \int \frac{\sin^2 x}{\cos^2 x} dx$

   $\Rightarrow \int \sec^2 x dx + 2\int \frac{\sin x}{\cos^2 x} dx + \int \tan^2 x dx$

   $\Rightarrow \int \sec^2 x dx + 2\int \frac{\sin x}{\cos^2 x} dx + \int \left(-1+\sec^2 x\right) dx$

   $\Rightarrow 2\int \sec^2 x dx + 2\int \frac{\sin x}{\cos^2 x} dx - \int 1\, dx$

   Put cos x = t

   Therefore $\Rightarrow$ sin x dx = - dt

   $\Rightarrow 2\tan x - 2\int \frac{dt}{t^2} - x + c$

   $\Rightarrow 2\tan x + 2\frac{1}{t} - x + c$

   $\Rightarrow$ 2 tan x + 2 sec x - x + c

2. **(d)** coincident

   Explanation: The equation of the given lines are

   $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ ....(i)

   $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$

   = $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ ....(ii)

   Thus, the two lines are parallel to the vector $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ and pass through the points (0, 0, 0) and (1, 2, 3).

   Now,

   $(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}$ = $(\hat{i} + 2\hat{j} + 3\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

   = $\vec{0}$ [$\because \vec{a} \times \vec{a} = \vec{0}$]

   So, here the distance between the given two parallel lines is 0, the given lines are coincident.

3. **(a)** 2

   Explanation: From the identity, we know that,

   If $\cos\alpha, \cos\beta$ and $\cos\gamma$ be the direction cosines of a vector, then,

   $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

   Using, $\sin^2\theta + \cos^2\theta = 1$

   $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \cos^2\alpha + \cos^2\beta + \cos^2\gamma$ = 1 + 1 + 1

   $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 1 = 3$

   On simplifying, we get,

   $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

4. **(c)** $P(E_1)P(E_2)$

   Explanation: We have, $P(E_1 \cap E_2) = P(E_1).\, P(\frac{E_2}{E_1})$

   Since E_1 and E_2 are independents, therefore

   $P = \left(\frac{E_2}{P(E_1)=P(E_2)}\right)$

   $\therefore P(E_1 \cap E_2) = P(E_1).\, P(E_2)$

5. **(a)** $\frac{2(ax+b)^{3/2}}{3a} + C$

   Explanation: Given integral is $\int \sqrt{ax+b}$

   Let, ax + b = z^2

   $\Rightarrow$ adx = 2zdz

So,
$\int \sqrt{ax+b}dx$
$= \int z\frac{2zdz}{a}$
$= \frac{2}{a}\int z^2 dz$ where, c is the integrating constant.
$= \frac{2}{a}\frac{z^3}{3} + c$
$= \frac{2}{3a}z^3 + c$
$= \frac{2(ax+b)^{3/2}}{3a} + c$
Hence, $\int \sqrt{ax+b} = = \frac{2(ax+b)^{3/2}}{3a} + c$.
Which is the required solution.

6. **(c)** $\frac{1}{10}$
Explanation: $P(A) = \frac{4}{5}, P(B) = \frac{3}{4}, P(C) = \frac{2}{3}$
$P(B \cap C \cap A') = P(B \cap C) - P(B \cap C \cap A)$
As the events are independent,
So, $P(B \cap C) = P(B) \cdot P(C) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$
And $P(B \cap C \cap A) = P(B). P(C). P(A) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$
$P(B \cap C \cap A') = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

7. **(a)** $\frac{9}{2}$ sq. units
Explanation: The area bounded by y = 2 - x^2 and x + y = 0 ⇒ y = -x
2 - x^2 = -x
x^2 - x - 2 = 0
⇒ (x - 2) (x + 1) = 0
⇒ x = 2 or x = -1
$\int_{-1}^{2}\left(2 - x^2 - x\right)dx$
$\left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_1^2$
$2(2+1) - \left(\frac{8}{3} + \frac{1}{3}\right) + \left(2 - \frac{1}{2}\right)$
$6 - 3 + \frac{3}{2}$
$\frac{9}{2}$ sq. units

8. **(b)** 6
Explanation: 6

9. **(b)** $\hat{i}$
Explanation: $\vec{a} = \hat{i} + \hat{j} - \hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$
$\vec{a} + \vec{b} = 3\hat{j} + \vec{k}, \vec{b} - \vec{c} = 3\hat{k}$
$(\vec{a} + \vec{b}) \times (\vec{b} - \vec{c}) = \begin{vmatrix} i & \tilde{j} & \tilde{k} \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{vmatrix}$
$(\vec{a} + \vec{b}) \times (\vec{b} - \vec{c}) = 9\hat{i}$
$|(\vec{a} + \vec{b}) \times (\vec{b} - \vec{c})| = 9$
unit vector perpendicular to both $(\vec{a} + \vec{b})$ and $(\vec{b} - \vec{c})$ is $\frac{(\vec{a}+\vec{b}) \times (\vec{b}-\vec{c})}{|(\vec{a}+\vec{b}) \times (\vec{b}-\vec{c})|}$
$= \frac{9\hat{i}}{9} = \hat{i}$

10. **(c)** sin x sin y = c
Explanation: Given differential equation is
cos x sin y dx + sin x cos y dy = 0
⇒ cos x sin y dx = -sin x cos y dy
⇒ $\frac{\cos x}{\sin x}dx = -\frac{\cos y}{\sin y}dy$
⇒ cot x dx = -cot y dy
On integrating both sides, we get
$\int \cot x dx = \int -\cot y dy$

log Isin xI = - log I sin y I + log C

$\Rightarrow$ log I sin x sin y I = log C

$\Rightarrow$ sin x. sin y = C

11. **(b)** 4

Explanation: Given that y = cos x, $0 \leq x \leq 2\pi$

$\Rightarrow \int_0^{2\pi} y dx = \int_0^{\frac{\pi}{2}} y dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} y dx + \int_{\frac{3\pi}{2}}^{2\pi} y dx$

$\Rightarrow \int_0^{2\pi} y dx = \int_0^{\frac{\pi}{2}} \cos x dx - \int_{\frac{1}{2}}^{\frac{3\pi}{2}} \cos x dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x dx$

$\Rightarrow \int_0^{2\pi} y dx = [\sin x]_0^{\frac{\pi}{2}} - [\sin x]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + [\sin x]_{\frac{3\pi}{2}}^{2\pi}$

$\Rightarrow \int_0^{2\pi} y dx = 1 - 0 - (-1 - 1) + (0 + 1)$

$\Rightarrow \int_0^{2\pi} y dx = 4$ sq. units

12. **(d)** $e^{\tan^{-1} x} + C$

Explanation: Given integral is $\int \frac{e^{\tan^{-1} x}}{(1+x^2)}$

Let, $\tan^{-1}x = z$

$\Rightarrow \frac{1}{1+x^2} dx = dz$

So,

$\int \frac{e^{\tan^{-1} x}}{(1+x^2)} dx$

$=\int e^z dz$

$=e^z+c$

$= e^{\tan^{-1} x} + c$

where c is the integrating constant.

13. **(d)** (1, 2)

Explanation: $y^2 = 4x \Rightarrow x = \frac{y^2}{4}$

$\Rightarrow d = \sqrt{(x-2)^2 + (y-1)^2}$

$\Rightarrow d^2 = (x - 2)^2 + (y - 1)^2$

$\Rightarrow d^2 = \left(\frac{y^2}{4} - 2\right)^2 + (y - 1)^2$

Let $u = \left(\frac{y^2}{4} - 2\right)^2 + (y - 1)^2$

$\Rightarrow \frac{du}{dy} = 2\left(\frac{y^2}{4} - 2\right) \frac{y}{2} + 2(y - 1)$

To find minima

$\frac{du}{dy} = 0$

$2\left(\frac{y^2}{4} - 2\right) \frac{y}{2} + 2(y - 1) = 0$

$\Rightarrow y = 2 \Rightarrow x = 1 \left(x = \frac{y^2}{4}\right)$

$\frac{d^2u}{dy^2} = \frac{3y^2}{4}$

$\Rightarrow \left(\frac{d^2u}{dy^2}\right)_{(1,2)} = 3 > 0$

Hence, nearest point is (1, 2).

14. **(a)** 16A

Explanation: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$$\Rightarrow A = 2\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

We can write as

A = 2I

Hence, $A^5 = (2I)^5$

$A^5 = 32I$

$A^5 = 16 \times 2I = 16A$

15. **(c)** $a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31}$

Explanation: $\Delta = \begin{vmatrix}a_{11} & a_{12} & a_{13}\\a_{21} & a_{22} & a_{23}\\a_{31} & a_{32} & a_{33}\end{vmatrix}$

Expanding along Column 1

$$\Delta = (-1)^{1+1} \times a_{11} \times \begin{vmatrix}a_{22} & a_{23}\\a_{32} & a_{33}\end{vmatrix} + (-1)^{2+1} \times a_{21} \times \begin{vmatrix}a_{12} & a_{13}\\a_{32} & a_{33}\end{vmatrix} + (-1)^{3+1} \times a_{31} \times \begin{vmatrix}a_{12} & a_{13}\\a_{22} & a_{23}\end{vmatrix}$$

$\Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

16. **(b)** $\begin{bmatrix}x^{-1} & 0 & 0\\0 & y^{-1} & 0\\0 & 0 & z^{-1}\end{bmatrix}$

Explanation: Here, $A = \begin{bmatrix}x & 0 & 0\\0 & y & 0\\0 & 0 & z\end{bmatrix}$

Clearly, we can see that

$$adjA = \begin{bmatrix}yz & 0 & 0\\0 & xz & 0\\0 & 0 & xy\end{bmatrix} \quad and \quad |A| = xyz$$

$$\therefore A^{-1} = \frac{adjA}{|A|} = \frac{1}{xyz}\begin{bmatrix}yz & 0 & 0\\0 & xz & 0\\0 & 0 & xy\end{bmatrix}$$

$$= \begin{bmatrix}x^{-1} & 0 & 0\\0 & y^{-1} & 0\\0 & 0 & z^{-1}\end{bmatrix}$$

17. **(b)** $x \in [0, \pi]$

Explanation: $\cos^{-1}(\cos x) = x$ if $0 \leqslant x \leqslant \pi$ i.e. if , $x \in [0, \pi]$

18. **(c)** 2

Explanation: Let the equation of given family be $(x - h)^2 + (y - k)^2 = a^2$. It has two arbitrary constants h and k. Therefore, the order of the given differential equation will be 2.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20. **(c)** A is true but R is false.

Explanation: Assertion: Let P(x, y) be any point on AB.

Then, area of $\triangle$ABP is zero. [since, the three points are collinear]

$$\therefore \frac{1}{2}\begin{vmatrix}1 & 3 & 1\\0 & 0 & 1\\x & y & 1\end{vmatrix} = 0$$

This gives $\frac{1}{2}(3x - y) = 0$

or y = 3x

Which is the equation of required line AB.

Reason: The area of triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is giyen by $\Delta=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$

Section B

21. Given $\sin^{-1}\left(\frac{1}{3}\right)-\cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta)=\pi-\cos^{-1}\theta$

$=\sin^{-1}\left(\frac{1}{3}\right)-\left[\pi-\cos^{-1}\left(\frac{1}{3}\right)\right]$

$=\sin^{-1}\left(\frac{1}{3}\right)-\pi+\cos^{-1}\left(\frac{1}{3}\right)$

$=\sin^{-1}\left(\frac{1}{3}\right)+\cos^{-1}\left(\frac{1}{3}\right)-\pi$

$=\frac{\pi}{2}-\pi$

$=-\frac{\pi}{2}$

Therefore we have,

$\sin^{-1}\left(\frac{1}{3}\right)-\cos^{-1}\left(-\frac{1}{3}\right)=-\frac{\pi}{2}$

22. We have, $\frac{dy}{dx}+2x=e^{3x}$

$\Rightarrow\frac{dy}{dx}=e^{3x}-2x$

$\Rightarrow dy=(e^{3x}-2x)\,dx$

Integrating both sides with respect x, we get

$\Rightarrow\int dy=\int\left(e^{3x}-2x\right)dx$

$\Rightarrow y=\frac{e^{3x}}{3}-2\frac{x^2}{2}+C$

$\Rightarrow y=\frac{e^{3x}}{3}-x^2+C$

$\Rightarrow y+x^2=\frac{e^{3x}}{3}+C$

So, it is defined for all $x\in R$.

Hence, $\Rightarrow y+x^2=\frac{e^{3x}}{3}+C$, where $x\in R$, is the solution to the given differential equation.

23. Expanding along R_1,we get,

$\Delta=0\begin{bmatrix}0 & \sin\beta\\ -\sin\beta & 0\end{bmatrix}-\sin\alpha\begin{vmatrix}-\sin\alpha & \sin\beta\\ \cos\alpha & 0\end{vmatrix}-\cos\alpha\begin{vmatrix}-\sin\alpha & 0\\ \cos\alpha & -\sin\beta\end{vmatrix}$

$=0-sin\alpha(0-cos\alpha sin\beta)-cos\alpha(sin\alpha sin\beta-0)=sin\alpha cos\alpha sin\beta-cos\alpha sin\alpha sin\beta=0$

OR

Let $A=\begin{bmatrix}3 & 2\\ 7 & 5\end{bmatrix}B=\begin{bmatrix}-1 & 1\\ -2 & 1\end{bmatrix}C=\begin{bmatrix}2 & -1\\ 0 & 4\end{bmatrix}$

Then The given equation becomes as

AXB = C

$\Rightarrow X=A^{-1}CB^{-1}$

now |A| = 35 – 14 = 21

and |B| = – 1 + 2 = 1

$\therefore A^{-1}=\frac{adj(A)}{|A|}=\frac{1}{21}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}$

and $B^{-1}=\frac{ad(B)}{|B|}=\frac{1}{1}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$

$\Rightarrow X=A^{-1}CB^{-1}=\frac{1}{21}\begin{bmatrix}5 & -2\\ -7 & 3\end{bmatrix}\begin{bmatrix}2 & -1\\ 0 & 4\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$

$=\frac{1}{21}\begin{bmatrix}10+0 & -5-8\\ -14+0 & 7+12\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$

$=\frac{1}{21}\begin{bmatrix}10 & -13\\ -14 & 19\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$

$=\frac{1}{21}\begin{bmatrix}10-26 & -10+13\\ -14+38 & 14-19\end{bmatrix}$

Hence, $x=\frac{1}{21}\begin{bmatrix}-16 & 3\\ 24 & -5\end{bmatrix}$

24. D.R of $\overrightarrow{r}$ are $2,-7,-3$

$\left|\overrightarrow{r}\right| = \sqrt{4+49+9} = \sqrt{62}$

D.C of $\overrightarrow{r}$ are $\frac{2}{\sqrt{62}}, \frac{-7}{\sqrt{62}}, \frac{-3}{\sqrt{62}}$

25. Here, W_1 = {4 white balls} and B_1= {5 black balls}

And W_2 = {9 white balls} and B_2= {7 black balls}

Let E_1 is the event that ball transferred from the first bag is white and $E_2$is the event that the ball transferred from the first bag is black.

Also, E is the event that the ball drawn from the second bag is white.

$\therefore P\left(\frac{E}{E_1}\right) = \frac{10}{17}, P\left(\frac{E}{E_2}\right) = \frac{9}{17}$

And $P(E_1) = \frac{4}{9}$ and $P(E_2) = \frac{5}{9}$

$\therefore P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$

$= \frac{4}{9} \cdot \frac{10}{17} + \frac{5}{9} \cdot \frac{9}{17}$

$= \frac{40+45}{153} = \frac{85}{153} = \frac{5}{9}$

Section C

26. Let x = $\lambda \frac{d}{dx}$ (x^2 + x +1) + μ. Then, x = λ(2x + 1) + μ

Comparing the coefficients of like powers of x, we get

1 = 2λ and λ + μ = 0 $\Rightarrow \lambda = \frac{1}{2}$ and μ = -λ = $-\frac{1}{2}$

$\therefore \quad I = \int \frac{x}{x^2+x+1} dx$

$\Rightarrow \quad I = \int \frac{1/2(2x+1)-1/2}{x^2+x+1} dx$

by using the values of λ , and μ ,

$\Rightarrow \quad I = \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx - \frac{1}{2} \int \frac{1}{x^2+x+1} dx$

$\Rightarrow \quad I = \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx - \frac{1}{2} \int \frac{1}{\left(x^2+x+\frac{1}{4}\right)+\frac{3}{4}} dx$

$\Rightarrow \quad I = \frac{1}{2} \int \frac{2x+1}{x^2+x+1} dx - \frac{1}{2} \int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} dx$

$\Rightarrow \quad I = \frac{1}{2} \log |x^2 + x + 1| - \frac{1}{2} \times \frac{1}{(\sqrt{3/2})} \tan^{-1}\left(\frac{x+1/2}{\sqrt{3/2}}\right) + C$

$\Rightarrow \quad I = \frac{1}{2} \log |x^2 + x + 1| - \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

27. The given differential equation is,

x^2 (y - 1) dx + y^2(x - 1) dy = 0

$\frac{x^2}{x-1} dx + \frac{y^2}{y-1} dy = 0$

Add and subtract 1 in numerators ,we have,

$\frac{x^2-1+1}{(x-1)} dx + \frac{y^2-1+1}{(y-1)} dy$=0

By the identity (a^2 -b^2) = (a + b).(a - b)

$\frac{(x+1)(x-1)+1}{(x-1)} dx + \frac{(y+1)(y-1)+1}{(y-1)} dy$=0

Splitting the terms,

$(x+1)dx + \frac{1}{(x-1)} dx$ + (y + 1)dy + $\frac{1}{(y-1)} dy$=0

Integrating,we get,

$\int (x+1)dx + \int \frac{1}{(x-1)} dx + \int (y+1)dy + \int \frac{1}{(y-1)} dy = C$

$\frac{x^2}{2} + x + \log|x-1| + \frac{y^2}{2} + y + \log|y-1|$=C

$\frac{1}{2}$.(x^2 + y^2) + (x + y) +log |(x - 1)(y - 1)|=C

This is the required solution.

OR

The given differential equation is,

$e^{dy/dx}$ = x + 1

Taking log on both sides, we get,

$\frac{dy}{dx}$ log e = log (x + 1)

$\Rightarrow \frac{dy}{dx}$ = log (x + 1)

$\Rightarrow$ dy = {log (x + 1)} dx

Integrating both sides, we get

$\int dy$ = $\int$ {log (x + 1) dx

$\Rightarrow y = \int \underset{II}{1} \times \log(\underset{I}{x} + 1)dx$

$\Rightarrow y = \log(x+1)\int 1dx - \int \left[\frac{d}{dx}(\log x + 1)\int 1dx\right] dx$

$\Rightarrow$ y = x log (x + 1) - $\int \frac{x}{x+1}dx$

$\Rightarrow$ y = x log (x + 1) - $\int \left(1 - \frac{1}{x+1}\right) dx$

$\Rightarrow$ y = x log (x + 1) - x + log (x + 1) + C ...(i)

It is given that y(0) = 3

$\therefore$ 3 = 0 $\times$ log (0 + 1) - 0 + log (0 + 1) + C

$\Rightarrow$ C = 3

Substituting the value of C in (i), we get

y = x log (x + 1) + log (x + 1) - x + 3

$\Rightarrow$ y = (x + 1) log (x + 1) - x + 3

Hence, y = (x + 1) log (x + 1) - x + 3 is the solution to the given differential equation.

28. Let $\vec{c} = \hat{a} + \hat{b}$.

Then, according to given condition $\vec{c}$ is a unit vector, i.e. $|\vec{c}| = 1$.

To show $|\hat{a} - \hat{b}| = \sqrt{3}$

Consider, $\vec{c} = \hat{a} + \hat{b}$

$\Rightarrow \quad |\vec{c}| = |\hat{a} + \hat{b}|$

$\Rightarrow \quad 1 = |\hat{a} + \hat{b}|$

$\Rightarrow \quad |\hat{a} + \hat{b}|^2 = 1$

$\Rightarrow \quad (\hat{a} + \hat{b}).(\hat{a} + \hat{b}) = 1$

$\Rightarrow \quad |\hat{a}|^2 + 2\hat{a} \cdot \hat{b} + |\hat{b}|^2 = 1$

$\Rightarrow \quad 1 + 2\hat{a} \cdot \hat{b} + 1 = 1$

$\Rightarrow \quad 2\hat{a}.\hat{b} = -1$ ...(i)

Now consider, $|\hat{a} - \hat{b}|^2 = (\hat{a} - \hat{b}).(\hat{a} - \hat{b})$

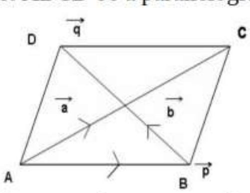
$= \quad |\hat{a}|^2 - 2\hat{a} \cdot \hat{b} + |\hat{b}|^2$

$= \quad 1 - (-1) + 1 = 3$

$\Rightarrow \quad |\hat{a} - \hat{b}| = \sqrt{3}$

OR

Let ABCD be a parallelogram such the



$\overrightarrow{AB} = \vec{p}, \overrightarrow{AD} = \vec{q} \Rightarrow \overrightarrow{BC} = \vec{q}$

$\therefore \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{p} + \vec{q} = \vec{a}$ .....(i) [By triangle law of vectors]

And $\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = -\vec{p} + \vec{q} = \vec{b}$ .....(ii)

Adding (i) and (ii), we get

$\bar{a} + \bar{b} = 2\bar{q}$

Subtracting (ii) from Eq. (i), we get

$\vec{a} - \vec{b} = 2\vec{p}$

$\Rightarrow \vec{p} = \frac{1}{2}(\vec{a} - \vec{b})$

Now, $\vec{p} \times \vec{q} = \frac{1}{4}(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$= \frac{1}{4}(\bar{a} \times \bar{a} + \bar{a} \times \bar{b} - \vec{b} \times \bar{a} - \bar{b} \times \bar{b})$

So, the area of a parallelogram ABCD = $|\vec{p} \times \vec{q}| = \frac{1}{2}|\vec{a} \times \vec{b}|$

Now, area of a parallelogram, whose diagonals are $2i - j + k$ and $i + 3j - \dot{k}$

$= \frac{1}{2}|(2i - j + k) \times (i + 3j - k)|$

$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$

$= \frac{1}{2}|i(1-3) - j(-2-1) + k(6+1)|$

$= \frac{1}{2}|-2\hat{i} + 3\hat{j} + 7\hat{k}| = \frac{1}{2}\sqrt{4+9+49} = \frac{1}{2}\sqrt{62}$ sq. units .

29. Let the given integral be, $y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$ ..... (i)

Use King theorem of definite integral

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{\frac{3a}{4}+\frac{a}{4}-x}}{\sqrt{\frac{3a}{4}+\frac{a}{4}-x}+\sqrt{x}} dx$

$y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$ .... (ii)

Adding eq.(i) and eq.(ii)

$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx + \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$

$2y = \int_{\frac{a}{4}}^{\frac{3a}{4}} \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}} dx$

$y = \frac{1}{2}\int_{\frac{a}{4}}^{\frac{3a}{4}} 1dx$

$y = \frac{1}{2}(x)_{\frac{a}{4}}^{\frac{3a}{4}}$

$y = \frac{a}{4}$

Hence proved..

OR

Let $I = \int \frac{1}{x+\sqrt{x^2-x+1}} dx$

Let $x + \sqrt{x^2 - x + 1}$ = t. Then,

$\sqrt{x^2 - x + 1} = t - x \Rightarrow x^2 - x + 1 = (t-x)^2 \Rightarrow -x + 1 = t^2 - 2tx \Rightarrow x = \frac{t^2-1}{2t-1}$

$\therefore \quad dx = \frac{(2t-1)2t - 2(t^2-1)}{(2t-1)^2} dt = \frac{2t^2-2t+2}{(2t-1)^2} dt$

Substituting these values, we get

$I = \int \frac{1}{t} \times \frac{2t^2-2t+2}{(2t-1)^2} dt = 2\int \frac{t^2-t+1}{t(2t-1)^2} dt$

Let $\frac{t^2-t+1}{t(2t-1)^2} = \frac{A}{t} + \frac{B}{2t-1} + \frac{C}{(2t-1)^2}$ ....(i)

$t^2 - t + 1 = A(2t-1)^2 + B(2t-1)t + Ct$

On equating the coefficients on both sides and on solving, we get

A = 1, B = $-\frac{3}{2}$ and C = $\frac{3}{2}$

Substituting the values of A, B, C in (i), we get

$\frac{t^2-t+1}{t(2t-1)^2} = \frac{1}{t} - \frac{3}{2(2t-1)} + \frac{3}{2}\frac{1}{(2t-1)^2}$

$\therefore \quad I = 2\int \frac{1}{t} dt - 3\int \frac{1}{2t-1} dt + 3\int \frac{1}{(2t-1)^2} dt$

$\Rightarrow \quad I = 2\log t - \frac{3}{2}\log(2t-1) - \frac{3}{4}\frac{1}{(2t-1)} + C$

$I = 2\log\left(x + \sqrt{x^2-x+1}\right) - \frac{3}{2}\log\left\{(2x-1) + 2\sqrt{x^2-x+1}\right\} - \frac{3}{4\left\{(2x-1)+2\sqrt{x^2-x+1}\right\}} + C$

30. Let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $v = \tan^{-1}x$

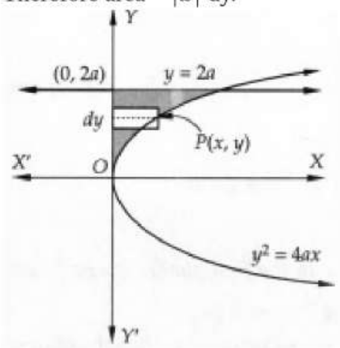
$put\ x = \tan\theta$

$\Rightarrow u = \tan^{-1}\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$

$= \tan^{-1}\frac{(\sec\theta-1)\cos\theta}{\sin\theta}$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$
$= \tan^{-1}\left[\frac{1-1+2\sin^2\theta/2}{2\sin\theta/2.\cos\theta+2}\right]$ $[\because \cos\theta = 1 - 2\sin^2\theta]$
$= \tan^{-1}\left[\tan\frac{\theta}{2}\right]$
$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$
$\therefore \frac{du}{dx} = \frac{1}{2}\frac{d}{dx}\tan^{-1}x = \frac{1}{2}.\frac{1}{1+x^2}$ ...(i)
and $\frac{dv}{dx} = \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$ ...(ii)
$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx}$
$= \frac{1/2(1+x^2)}{1/(1+x^2)} = \frac{(1+x^2)}{2(1+x^2)} = \frac{1}{2}$

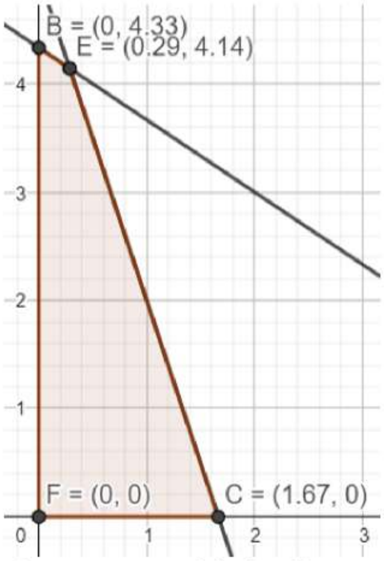
31. Clearly, the equation $y^2 = 4ax$ represents a parabola with vertex (0, 0) and axis as x-axis.
The equation y = 2a represents a straight line parallel to x-axis at a distance of 2a, from it as shown in Figure.
The required region is the shaded portion in the Figure.
To find the area of the shaded region shown in Figure, we slice it into horizontal strips.
We observe that each horizontal strip has its left end on y-axis and the right end on the given parabola $y^2 = 4ax$.
So, the approximating rectangle shown in Fig. has its length = $|x|$ and width = dy
Therefore area = $|x|$ dy.



Since the approximating rectangle can move vertically from y = 0 to y = 2a. So, required area denoted by A, is given by
$A = \int_0^{2a}|x|dy = \int_0^{2a}xdy$ $[\because x \geq 0 \therefore |x| = x]$
$\Rightarrow A = \int_0^{2a}\frac{y^2}{4a}dy$ [$\because$ P(x, y) lies on $y^2 = 4ax \therefore x = \frac{y^2}{4a}$]
$\Rightarrow A = \frac{1}{4a}\left[\frac{y^3}{3}\right]_0^{2a} = \frac{1}{4a}\left(\frac{8a^3}{3} - 0\right) = \frac{2a^2}{3}$ sq. units

Section D

32. First, we will convert the given inequations into equations, we obtain the following equations:
2x + 3y = 13, 3x + y = 5, x = 0 and y = 0
Region represented by 2 x + 3 y $\leq$ 13 :
The line 2 x + 3 y=13 meets the coordinate axes at
$A\left(\frac{13}{2}, 0\right)$ and $B\left(0, \frac{13}{3}\right)$ respectively. By joining these points we obtain the line 2x + 3y = 13
Clearly (0,0) satisfies the inequation 2 x + 3 y $\leq$ 13 . So, the region containing the origin represents the solution set of the inequation 2 x + 3 y $\leq$13
Region represented by 3 x + y $\leq$ 5 :-
The line 3x +y = 5 meets the coordinate axes at
$C\left(\frac{5}{3}, 0\right)$ and D(0, 5) respectviely. By joining these points we obtain the line 3x + y = 5
Clearly (0,0) satisfies the inequation 3x + y $\leq$ 5 . So, the region containing the origin represents the solution set of the inequation 3x + y ≤ 5
Region represented by x $\geq$ 0 and y $\geq$ 0 is the 1st quadrant.
The feasible region determined by the system of constraints, 2 x + 3 y $\leq$ 13,3 x + y $\leq$ 5, x $\leq$ 0, and y $\leq$ 0, are shown in the shaded portion of the graph.



The corner points of the feasible region are O(0,0)

$C\left(\frac{5}{3},0\right)$ $E\left(\frac{2}{7},\frac{29}{7}\right)$ and $B\left(0,\frac{13}{3}\right)$

The values of Z at these corner points are as follows.

We see that the maximum value of the objective function Z is 15 which is at C

Corner point	Z = 9x + 3y
O(0, 0)	9 × 0 + 3 × 0 = 0
$C\left(\frac{5}{3},0\right)$	$9\times\frac{5}{3}+3\times0=15$
$E\left(\frac{2}{7},\frac{29}{7}\right)$	$9\times\frac{2}{7}+3\times\frac{29}{7}=15$
$B\left(0,\frac{13}{3}\right)$	$9\times0+3\times\frac{13}{3}=113$

$\left(\frac{5}{3},0\right)$ and $E\left(\frac{2}{7},\frac{29}{7}\right)$ Thus, the optional value of z is 15.

33. Given that, A = R - {3}, B = R - {1}.

$f:A\to B$ is defined by $f(x)=\frac{x-2}{x-3}\ \forall x\in A$

For injectivity

Let $f(x_1)=f(x_2)\Rightarrow\frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

So, f(x) is an injective function

For surjectivity

Let $y=\frac{x-2}{x-3}\Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = 2 - 3y $\Rightarrow x=\frac{2-3y}{1-y}$

$\Rightarrow x=\frac{3y-2}{y-1}\in A,\ \forall y\in B$ [codomain]

So, f(x) is surjective function.

Hence, f(x) is a bijective function.

OR

Given that

Let A = {1, 2, 3} and R = {(a, b) : a, b ∈ A and $\left|a^2-b^2\right|\leq5$

Put a = 1, b = 1 $\left|1^2-1^2\right|\leq5,(1,1)$ is an ordered pair.

Put a = 1, b = 2 $\left|1^2-2^2\right|\leq5,(1,2)$ is an ordered pair.

Put a = 1, b = 3 $\left|1^2-3^2\right|>5,(1,3)$ is not an ordered pair.

Put a = 2, b = 1 $\left|2^2-1^2\right|\leq5,(2,1)$ is an ordered pair.

Put a = 2, b = 2 $\left|2^2-2^2\right| \leq 5,(2,2)$ is an ordered pair.
Put a = 2, b = 3 $\left|2^2-3^2\right| \leq 5,(2,3)$ is an ordered pair.
Put a = 3, b = 1 $\left|3^2-1^2\right|>5,(3,1)$ is not an ordered pair.
Put a = 3, b = 2 $\left|3^2-2^2\right| \leq 5,(3,2)$ is an ordered pair.
Put a = 3, b = 3 $\left|3^2-3^2\right| \leq 5,(3,3)$ is an ordered pair.
R = {(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3)}

i. For (a, a) ∈ R
$\left|a^2-a^2\right|=0 \leq 5$. Thus, it is reflexive.

ii. Let (a, b) ∈ R
$(a, b) \in R,\left|a^2-b^2\right| \leq 5$
$\left|b^2-a^2\right| \leq 5$
(b, a) ∈ R
Hence, it is symmetric

iii. Put a = 1, b = 2, c = 3
$\left|1^2-2^2\right| \leq 5$
$\left|2^2-3^2\right| \leq 5$
But $\left|1^2-3^2\right|>5$
Thus, it is not transitive

34. $\vec{a}_1=\hat{i}+\hat{j}, \vec{b}_1=2 \hat{i}-\hat{j}+\hat{k}$
$\vec{a}_2=2 \hat{i}+\hat{j}-\hat{k}, \vec{b}_2=3 \hat{i}-5 \hat{j}+2 \hat{k}$
$\vec{a}_2-\vec{a}_1=\hat{i}-\hat{k}$
$\vec{b}_1 \times \vec{b}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$
$=\hat{i}(-2+5)-\hat{j}(4-3)+\hat{k}(-10+3)$
$=3 \hat{i}-\hat{j}-7 \hat{k}$
$\left|\vec{b}_1 \times \vec{b}_2\right|=\sqrt{9+1+49}=\sqrt{59}$
Also, $(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2-\vec{a}_1)=(3 \hat{i}-\hat{j}-7 \hat{k})(\hat{i}-\hat{k})=3+7+0=10$
$d=\left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|=\frac{10}{\sqrt{59}}$

OR

Here the equation of two planes are: $\vec{\mathrm{r}} \cdot(\hat{i}-\hat{j}+2 \hat{k})=5$ and $\vec{r} \cdot(3 \hat{i}+\hat{j}+\hat{k})=6$.
Since the line is parallel to the two planes.
∴ Direction of line $\overrightarrow{b}$ = $(\hat{i}-\hat{j}+2 \hat{k}) \times(3 \hat{i}+\hat{j}+\hat{k})$
$=-3 \hat{i}+5 \hat{j}+4 \hat{k}$
∴ Equation of required line is
$\vec{r}=(\hat{i}+2 \hat{j}+3 \hat{k})+\lambda(-3 \hat{i}+5 \hat{j}+4 \hat{k})$ .... (i)
Any point on line (i) is $(1-3 \lambda, 2+5 \lambda, 3+4 \lambda)$
For this line to intersect the plane $\overrightarrow{r} \cdot(2 \hat{i}+\hat{j}+\hat{k})$ we have
$(1-3 \lambda)2$ + $(2+5 \lambda)1$ + $(3+4 \lambda)1$ = 4
⇒ λ= 1
∴ Point of intersection is (4, -3, -1)

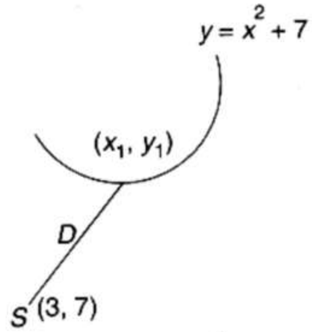
35. $y \sqrt{x^2+1}-\log \left(\sqrt{x^2+1}-x\right)=0$
differentiating both sides w.r.t x
$y. \frac{1}{2 \sqrt{x^2+1}}(2 x)+\sqrt{x^2+1}. \frac{d y}{d x}-\frac{1}{\sqrt{x^2+1}-x}\left[\frac{1(2 x)}{2 \sqrt{x^2+1}}-1\right]=0$
$\frac{x y}{\sqrt{x^2+1}}+\sqrt{x^2+1}. \frac{d y}{d x}-\frac{1}{\sqrt{x^2+1}-x}\left[\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right]=0$
$\frac{x y+\left(x^2+1\right)}{\sqrt{x^2+1}} \frac{d y}{d x}=\frac{-\left(\sqrt{x^2+1}-x\right)}{\left(\sqrt{x^2+1}-x\right) \sqrt{x^2+1}}$
$x y+\left(x^2+1\right) \frac{d y}{d x}=-1$

$(x^2+1)\frac{dy}{dx}+xy+1=0$

Section E

36. **Read the text carefully and answer the questions:**

An Apache helicopter of the enemy is flying along the curve given by y = x^2 + 7. A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.



(i) P(x_1, y_1) is on the curve y = x^2 + 7 ⇒ $y_1 = x_1^2+7$

Distance from p(x_1, x_1^2+7) and (3, 7)

D = $\sqrt{(x_1-3)^2+\left(x_1^2+7-7\right)^2}$

$\Rightarrow \sqrt{(x_1-3)^2+\left(x_1^2\right)^2}$

$\Rightarrow D=\sqrt{x_1^4+x_1^2-6x_1+9}$

(ii) D = $\sqrt{x_1^4+x_1^2-6x_1+9}$

D' = $x_1^4+x_1^2-6x_1+9$

$\frac{dD'}{dx}=4x_1^3+2x_1-6=0$

$\frac{dD'}{dx}=2x_1^3+x_1-3=0$

⇒ (x_1 - 1)$\left(2{x_1}^2+2x_1+3\right)$= 0

x_1 = 1 and $2x_1^2$ + 2x_1 + 3 = 0 gives no real roots

The critical point is (1, 8).

(iii) $\frac{dD'}{dx}=4x_1^3+2x_1-6$

$\frac{d^2D'}{dx^2}=12x_1^2+2$

$\left.\frac{d^2D'}{dx^2}\right]_{x_1=1}$ = 12 + 2 = 14 > 0

Hence distance is minimum at (1, 8).

OR

$D=\sqrt{x_1^4+x_1^2-6x_1+9}$

$D=\sqrt{1+1-6+9}=\sqrt{5}$ units

37. **Read the text carefully and answer the questions:**

Three friends Ravi, Raju and Rohit were doing buying and selling of stationery items in a market. The price of per dozen of pen, notebooks and toys are Rupees x, y and z respectively.

Ravi purchases 4 dozen of notebooks and sells 2 dozen of pens and 5 dozen of toys. Raju purchases 2 dozen of toy and sells 3 dozen of pens and 1 dozen of notebooks. Rohit purchases one dozen of pens and sells 3 dozen of notebooks and one dozen of toys.

In the process, Ravi, Raju and Rohit earn ₹1500, ₹100 and ₹400 respectively.

(i) 2x - 4y + 5z = 1500

3x + y - 2z = 100

-x + 3y + z = 400

$$\begin{bmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1500 \\ 100 \\ 400 \end{bmatrix}$$

(ii) A = $\begin{bmatrix} 2 & -4 & 5 \\ 3 & 1 & -2 \\ -1 & 3 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 1500 \\ 100 \\ 400 \end{bmatrix}$

X = A^{-1}B ...(i)

|A| = 2(1 + 6) + 4(3 - 2) + 5(9 + 1) = 68 $\neq 0$

co-factor matrix A = $\begin{bmatrix} 7 & -1 & 10 \\ 19 & 7 & -2 \\ 3 & 19 & 14 \end{bmatrix}$, adj A = $\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$

A-1 = $\frac{1}{|A|} \cdot adjA$

A-1 = $\frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}$

From (i)

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{68}\begin{bmatrix} 7 & 19 & 3 \\ -1 & 7 & 19 \\ 10 & -2 & 14 \end{bmatrix}\begin{bmatrix} 1500 \\ 100 \\ 400 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{68}\begin{bmatrix} 10500+1900+1200 \\ -1500+700+7600 \\ 15000-200+5600 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{68}\begin{bmatrix} 13600 \\ 6800 \\ 20400 \end{bmatrix}$$

x = 200, y = 100 and z = 300

Total price of one dozen of pens and one dozen of notebooks = 200 + 100 = ₹300

(iii)The sale amount of Ravi = 2x + 5z = 2 $\times$ 200 + 5 $\times$ 300 = 400 + 1500 = ₹1900

OR

The amount of purchases made by all three friends

4y + 2z + x = 4 $\times$ 100 + 2 $\times$ 300 + 200 = ₹1200

The amount of sales made by all three friends

2x + 5z + 3x + y + 3y + z = 5x + 4y + 6z = 5 $\times$ 200 + 4 $\times$ 100 + 6 $\times$ 300 = ₹3200

38. **Read the text carefully and answer the questions:**

In pre-board examination of class XII, commerce stream with Economics and Mathematics of a particular school, 50% of the students failed in Economics, 35% failed in Mathematics and 25% failed in both Economics and Mathematics. A student is selected at random from the class.

(i) Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E $\cap$ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.

Required probability = $P(\frac{E}{M})$

$= \frac{P(E \cap M)}{P(M)} = \frac{\frac{1}{4}}{\frac{7}{20}} = \frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$

(ii) Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E $\cap$ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.

Required probability = P(M/E)

$= \frac{P(M \cap E)}{P(E)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$