Class- X Session- 2022-23

Subject- Mathematics (Standard)

Sample Question Paper - 7

with Solution

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. If α, β are the zeros of the polynomial $x^2 + 6x + 2$ then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = ?$ **[1]**

a) -3 b) -12

c) 3 d) 12

2. If $\triangle ABC \sim \triangle DEF$ then which of the following is true? **[1]**

a) BC.DE = AB.FD b) AB.EF = AC.DE

c) BC.EF = AC.FD d) BC.DE = AB.EF

3. If a pair of linear equations in two variables is consistent, then the lines represented by two equations are **[1]**

a) parallel b) always coincident

c) intersecting d) intersecting or coincident

4. Half the perimeter of a rectangular garden, whose length is 4m more than its width is 36m. The area of the garden is **[1]**

a) $320\ m^2$ b) $300\ m^2$

c) $400\ m^2$ d) $360\ m^2$

5. Three coins are tossed once. The probability of getting two heads and one tale is **[1]**

a) $\frac{3}{8}$ b) $\frac{1}{8}$

c) $\frac{5}{8}$ d) $\frac{1}{4}$

6. The value of $\frac{\tan 30^\circ}{\cot 60^\circ}$ is [1]

a) 1 b) $\frac{1}{\sqrt{3}}$

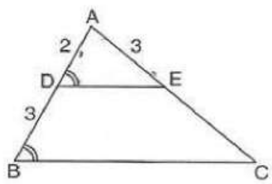
c) $\frac{1}{\sqrt{2}}$ d) $\sqrt{3}$

7. If in $\triangle$ ABC and $\triangle$ DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then $\triangle$ ABC $\sim$ $\triangle$DEF when [1]

a) $\angle$B = $\angle$E b) $\angle$B = $\angle$D

c) $\angle$A = $\angle$F d) $\angle$A = $\angle$D

8. In the given figure if $\angle ADE = \angle ABC$, then CE is equal to [1]



a) 2. b) 9/2

c) 5. d) 3.

9. If x_i is changed to $x_i + a$, then $\overline{x}$ is changed to [1]

a) $\overline{x}$ + a b) a$\overline{x}$

c) $\overline{x}$ - a d) $\frac{\overline{x}}{a}$

10. Let b = a + c. Then the equation $ax^2 + bx + c = 0$ has equal roots if [1]

a) a = -c b) a = c

c) a = -2c d) a = 2c

11. Which point on x-axis is equidistant from the points A(7, 6) and B(-3, 4)? [1]

a) (-4, 0) b) (0, 4)

c) (0, 3) d) (3, 0)

12. 2 - $\sqrt{3}$ is [1]

a) an irrational number b) an integer

c) a rational number d) a whole number

13. If 35 is removed from the data: 30, 34, 35, 36, 37, 38, 39, 40, then the median increases by: [1]

a) 0.5 b) 1.5

c) 2 d) 1

14. Two men are on opposite sides of a tower. They observe the angles of elevation of the top of the tower as 60° and 45° respectively. If the height of the tower is 60m, then the distance between them is **[1]**

a) $20(3-\sqrt{3})m$ b) $20(\sqrt{3}-3)m$

c) None of these d) $20(\sqrt{3}+3)m$

15. $\cos^2 30^{\circ} \cos^2 45^{\circ} + 4\sec^2 60^{\circ} + \frac{1}{2}\cos^2 90^{\circ} - 2\tan^2 60^{\circ}$ = ? **[1]**

a) $\frac{75}{8}$ b) $\frac{73}{8}$

c) $\frac{83}{8}$ d) $\frac{81}{8}$

16. $\triangle$PQR $\sim$ $\triangle$XYZ and the perimeters of $\triangle$PQR and $\triangle$XYZ are 30 cm and 18 cm respectively. If QR = 9 cm, then, YZ is equal to **[1]**

a) 4.5 cm. b) 5.4 cm.

c) 12.5 cm. d) 9.5 cm.

17. To draw a pair of tangents to a circle, which are inclined to each other at an angle of 45°, we have to draw tangents at the endpoints of those two radii, the angle between which is **[1]**

a) 135° b) 145°

c) 105° d) 140°

18. **Assertion (A):** A quadratic polynomial with zeroes 2, 3 and -3 is $x^2 - 5x + 6$. **[1]**
Reason (R): If α and β are zeroes of a quadratic polynomial then polynomial is given by $x^2 - (\alpha + \beta)x + \alpha\beta$

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

19. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 300 cm^2. **[1]**
Reason (R): Total surface area of a cuboid is 2(lb + bh + lh)

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. The roots of a quadratic equation are 5 and -2. Then, the equation is **[1]**

a) $x^2 - 3x + 10 = 0$ b) $x^2 - 3x - 10 = 0$

c) $x^2 + 3x + 10 = 0$ d) $x^2 + 3x - 10 = 0$

Section B

21. The line joining the points (2, -1) and (5, -6) is bisected at P. If P lies on the line 2x + 4y + k = 0. find the value of k. **[2]**

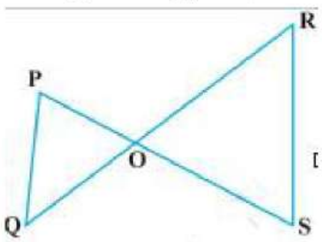
22. Determine whether the given values are solutions of the given equation or not: **[2]**
$x^2 - 3\sqrt{3}x + 6 = 0$, $x = \sqrt{3}$, $x = -2\sqrt{3}$

23. Find H.C.F. and L.C.M. of 56 and 112 by prime factorisation method. **[2]**

24. In a ΔABC, D and E are points on the sides AB and AC respectively such that $DE \| BC$. If $\frac{AD}{DB} = \frac{2}{3}$ and AC =18 cm, find AE. **[2]**

OR

In Fig. if PQ || RS, then prove that $\triangle POQ \sim \triangle SOR$.



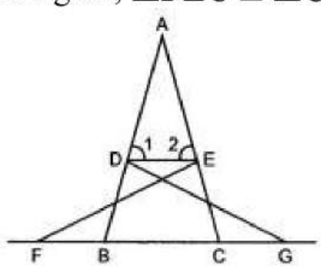
25. Prove that: $\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$ **[2]**

OR

If cosecθ - sinθ = m and secθ - cosθ = n, prove that $(m^2n)^{2/3} + (mn^2)^{2/3} = 1$

Section C

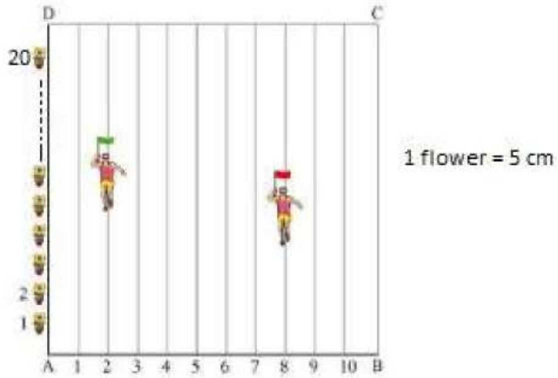
26. In figure, $\triangle FEC \cong \triangle GBD$ and $\angle 1 = \angle 2$. Prove that $\triangle ADE \sim \triangle ABC$. **[3]**



27. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects. **[3]**

28. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. Niharika runs the distance AD on the 2nd line and posts a green flag. Preet runs the **[3]**

distance AD on the eighth line and posts a red flag.



i. Calculate the distance Niharika and Preet posted the green flag and reg flag respectively.
ii. What is the distance between both the flags?
iii. If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

OR

Find the distance between the pair of points $(a \sin \alpha, -b \cos \alpha)$ and $(-a \cos \alpha, b \sin \alpha)$.

29. If a tower 30m high, casts a shadow $10\sqrt{3}m$ long on the ground, then what is the angle of elevation of the sun? **[3]**

OR

The angle of elevation of the top of a tower from a point A on the ground is 30^{o}. On moving a distance of 20 metre towards the foot of the tower to a point B the angle of elevation increases to 60^{o}. Find the height of the tower and the distance of the tower from the point A.

30. The weights of tea in 70 packets are shown in the following table: **[3]**

Weight(in grams)	200 - 201	201 - 202	202 - 203	203 - 204	204 - 205	205 - 206
Number of packets	13	27	18	10	1	1

Find the mean weight of packets using step-deviation method.

31. Prove that $15 + 17\sqrt{3}$ is an irrational number. **[3]**

Section D

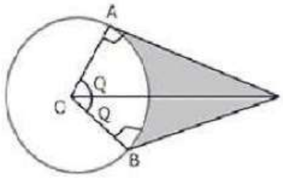
32. AB is a diameter and AC is chord of a circle with centre O such that ∠BAC = 30°. The tangent at C intersects extended AB at a point D. Prove that BC = BD. **[5]**

33. The path of a train A is given by the equation x + 2y - 4 = 0 and the path of another train B is given by the equation 2x + 4y - 12 = 0. Represent this situation graphically. **[5]**

OR

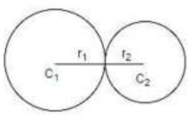
Solve the system of equations x - 2y + 2 = 0, 2x + y - 6 = 0 graphically and find the vertices and area of the triangle formed by these lines and the x-axis.

34. An elastic belt is placed around therein of a pulley of radius 5cm. One point on the belt is pulled directly away from the center O of the pulley until it is at P, 10cm from O. Find the length of the best that is in contact with the rim of the pulley. Also, find the shaded area. **[5]**



OR

Two farmers have circular plots. The plots are watered with the same water source placed in the point common to both the plots as shown in the figure. The sum of their areas is 130π and the distance between their centres is 14 m.Find the radii of the circles.What value is depicted by the farmers?



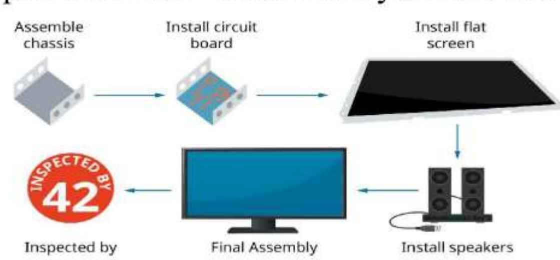
35. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting **[5]**

i. a king of red colour.
ii. the queen of diamonds.
iii. an ace.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a laptop manufacturer. The company works for many branded laptop companies and also provides them with spare parts. Elpis Technology produced 6000 units in 3rd year and 7000 units in the 7th year.



Assuming that production increases uniformly by a fixed number every year.

(i) Find the production in the 1st year.

(ii) Find the production in the 5th year.

OR

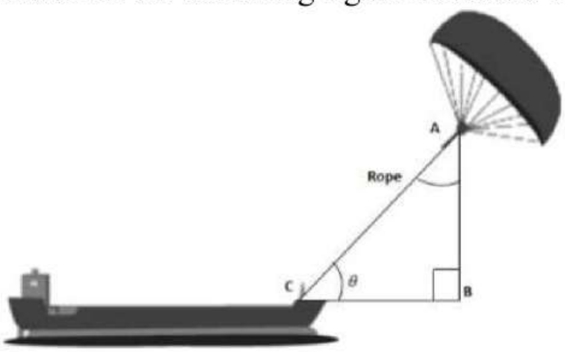
Find in which year 10000 units are produced?

(iii) Find the total production in 7 years.

37. **Read the text carefully and answer the questions:** **[4]**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(i) In the given figure, if $\sin\theta = \cos(\theta - 30^\circ)$, where θ and $\theta - 30^\circ$ are acute angles, then find the value of θ.

(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200m?

(iii) In the given figure, if $\sin\theta = \cos(3\theta - 30^\circ)$, where θ and $3\theta - 30^\circ$ are acute angles, then find the value of θ.

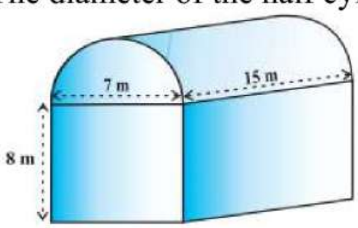
OR

What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 150m?

38. **Read the text carefully and answer the questions:** **[4]**

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m × 7 m × 8 m.

The diameter of the half cylinder was 7 m and length was 15 m.



(i) Find the volume of the air that the shed can hold.

(ii) If the industry requires machinery which would occupy a total space of 300 m^3 and there are 20 workers each of whom would occupy 0.08 m^3 space on an average, how much air would be in the shed when it is working?

OR

Find the surface area of the cylindrical part.

(iii) Find the surface area of the cuboidal part.

Solution

Section A

1. **(a)** -3

   Explanation: $\alpha+\beta=-6$ and $\alpha\beta=2$

   $\therefore \quad \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=\frac{(\alpha+\beta)}{\alpha\beta}=\frac{-6}{2}=-3$
2. **(d)** BC.DE = AB.EF

   Explanation: If $\triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$ then,

   $\Rightarrow \frac{\mathrm{BC}}{\mathrm{EF}}=\frac{\mathrm{AB}}{\mathrm{DE}}$ (corresponding sides are in proportion)

   Here according to the given condition, BC.DE = AB.EF
3. **(d)** intersecting or coincident

   Explanation: If a pair of linear equations in two variables is consistent, then its solution exists.

   $\therefore$ The lines represented by the equations are either intersecting or coincident.
4. **(a)** 320 m^2

   Explanation: Let the width be x

   then length be x + 4

   According to the question,

   l + b = 36

   x + (x + 4) = 36

   2x + 4 = 36

   2x = 36-4

   2x = 32

   x = 16.

   Hence, The length of the garden will be 20 m and width will be 16 m.

   Area = length $\times$ breath = 20 $\times$ 16 = 320 m^2
5. **(a)** $\frac{3}{8}$

   Explanation: Total outcomes = {HHH, TTT, HHT, HTH, HTT, THH, THT, TTH} = 8

   Number of possible outcomes = 3

   $\therefore$ Required Probability = $\frac{3}{8}$
6. **(a)** 1

   Explanation: Given: $\frac{\tan 30^{\circ}}{\cot 60^{\circ}}$

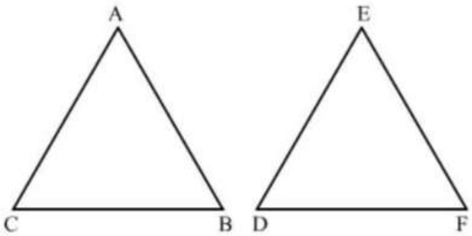
   $=\frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}}=1$
7. **(b)** $\angle$B = $\angle$D

   Explanation:

   Given: In $\triangle$ABC and $\triangle$DEF, $\frac{AB}{DE}=\frac{BC}{FD}$.

   We know that if in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.

   Then, $\angle$B = $\angle$D



Hence, △ABC is similar to △DEF, we should have ∠B = ∠D.

8. **(b)** 9/2

Explanation: In ΔABC and ΔADE,

∠ADE = ∠ABC [Given]

∠A = ∠A [Common]

$\therefore \Delta ABC \sim \Delta ADE$ [AA Similarity]

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{2}{3} = \frac{3}{EC}$

⇒ EC = 9/2 cm

9. **(a)** $\overline{x}$ + a

Explanation: Let terms be $x_1, x_2, x_3, ..., x_n$

$\therefore$ Mean $(\overline{x}) = \frac{x_1+x_2+x_3+...........+x_n}{n}$

New observations are $x_1 + a, x_2 + a, x_3 + a, ..., x_n + a$

$\therefore$ New Mean $= \frac{x_1+a+x_2+a+x_3+a+...........+x_n+a}{n}$

$= \frac{x_1+x_2+x_3+...........+x_n+na}{n}$

$= \overline{x} + a$

10. **(b)** a = c

Explanation: Since, If $ax^2 + bx + c = 0$ has equal roots, then

$b^2 - 4ac = 0$

$\Rightarrow (a + c)^2 - 4ac = 0$ ... [Given: b = a + c]

$\Rightarrow a^2 + c^2 + 2ac - 4ac = 0$

$\Rightarrow a^2 + c^2 - 2ac = 0$

$\Rightarrow (a - c)^2 = 0$

⇒ a - c = 0

⇒ a = c

11. **(d)** (3, 0)

Explanation: Let the required point be P(x, 0) then,

$AP^2 = BP^2 \Rightarrow (x-7)^2 + (0-6)^2 - (x+3)^2 + (0-4)^2$

$\Rightarrow x^2 - 14x + 85 = x^2 + 6x + 25$

$-20x - 60 = x = 3$

12. **(a)** an irrational number

Explanation: Let 2 - $\sqrt{3}$ be rational number

2 - $\sqrt{3} = \frac{p}{q}$ where p and q are composite numbers

$\sqrt{3} = \frac{p}{q} + 2$

$\sqrt{3} = \frac{(p+2q)}{q}$

since p, q are integers, so $\frac{(p+2q)}{q}$ is rational

$\therefore \sqrt{3}$ is an irrational number it shows our supposition was wrong

hence 2-$\sqrt{3}$ is an irrational number.

13. **(a)** 0.5

Explanation: Given data = 30, 34, 35, 36, 37, 38, 39, 40

Here n = 8 which is even

$\therefore$ Median = $\frac{1}{2}\left[\frac{n}{2}\text{th} + \left(\frac{n}{2}+1\right)\text{th}\right]$ term = $\frac{1}{2}$(4th + 5th term)

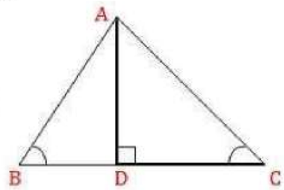
= $\frac{1}{2}(36 + 37) = \frac{73}{2}$ = 36.5

After removing 35, then n = 7

$\therefore$ New median = $\frac{7+1}{2}$th term = 4th term = 37

$\therefore$ Increase in median = 37 - 36.5 = 0.5

14. **(d)** $20(\sqrt{3}+3)m$

Explanation: 

Let the height of the tower = AD = 60 m and angles of elevation of the top of the tower of two men are 60°and 45°respectively.

To find: Distance between two men = BC

In triangle ABD,

$\tan 60^\circ = \frac{60}{\text{BD}} \Rightarrow \sqrt{3} = \frac{60}{\text{BD}}$

$\Rightarrow \text{BD} = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ = $20\sqrt{3}$m

In triangle ADC,

$\tan 45^\circ = \frac{60}{\text{DC}}$

$\Rightarrow 1 = \frac{60}{\text{DC}}$

$\Rightarrow \text{DC} = 60$ m

$\therefore$ BC = BD + DC = $20\sqrt{3} + 60$

= $20\left(\sqrt{3}+3\right)$ m

15. **(c)** $\frac{83}{8}$

Explanation: $\cos^2 30^\circ \cos^2 45^\circ + 4\sec^2 60^\circ + \frac{1}{2}\cos^2 90^\circ - 2\tan^2 60^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + \left(4 \times 2^2\right) + \left(\frac{1}{2} \times 0^2\right) - 2 \times (\sqrt{3})^2$

$= \left(\frac{3}{4} \times \frac{1}{2}\right) + 16 + 0 - 6 = \frac{3}{8} + 10 = \frac{83}{8}$

16. **(b)** 5.4 cm.

Explanation: Given: $\triangle\text{PQR} \sim \triangle\text{XYZ}$

$\therefore \frac{\text{Perimeter of } \Delta\text{PQR}}{\text{Perimeter of } \Delta\text{XYZ}} = \frac{\text{QR}}{\text{YZ}}$

$\Rightarrow \frac{30}{18} = \frac{9}{\text{YZ}}$

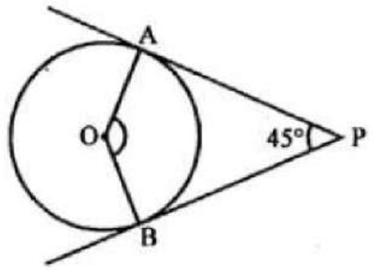
$\Rightarrow$YZ = 5.4 cm

17. **(a)** 135°

Explanation:

In the given figure, PA and PB are two tangents drawn from an external point P which inclined at an angle of 45°.

OA and OB are radii of the circle.



To find $\angle AOB$

$\triangle OBP$ is a cyclic quadrilateral

$\angle AOB + \angle APB = 180°$

$\Rightarrow \angle AOB + 45° = 180°$

$\Rightarrow \angle AOB = 180° - 45° = 135°$

18. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: $\alpha = 2$ and $\beta = 3 \Rightarrow \alpha + \beta = 5$ and $\alpha\beta = 6$

So, polynomial is $x^2 - 5x + 6$.

19. **(d)** A is false but R is true.

Explanation: A is false but R is true.

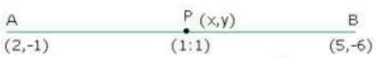
20. **(b)** $x^2 - 3x - 10 = 0$

Explanation: Sum of the roots = 5 + (-2) = 3, product of roots = 5 × (-2) = -10.

$\therefore x^2$ - (sum of roots) x + product of roots = 0.

Hence, $x^2 - 3x - 10 = 0$.

Section B

21. 

Coordinate of $P = \left(\frac{2+5}{2}, \frac{-1-6}{2}\right) = \left(\frac{7}{2}, \frac{-7}{2}\right)$

P lies on equation $2x + 4y + k = 0$

$\therefore 2\left(\frac{7}{2}\right) + 4\left(\frac{-7}{2}\right) + k = 0$

$\Rightarrow 7 - 14 + k = 0$

$\Rightarrow k = 7$

22. Substituting $x = \sqrt{3}$ in the LHS of the given equation, we get

$(\sqrt{3})^2 - 3\sqrt{3} \times \sqrt{3} + 6$

$= 3 - 3\times3 + 6$

$= 9 - 9$

$= 0 =$ RHS

$\therefore$ For $x = \sqrt{3},\ x^2 - 3\sqrt{3}x + 6 = 0$.

Hence, $x = \sqrt{3}$ is a solution of the given equation.

Now, substituting $x = -2\sqrt{3}$ in the LHS of the given equation, we get

$\left(-2\sqrt{3}\right)^2 - 3\sqrt{3} \times \left(-2\sqrt{3}\right) + 6$

$= 12 + (6 \times 3) + 6$

$= 12 + 18 + 6$

$= 36 \neq 0$ (RHS)

$\therefore$ For $x = -2\sqrt{3}, x^2 - 3\sqrt{3}x + 6 \neq 0$

So, $x = -2\sqrt{3}$ is not a solution of the given equation.

Thus, $x = \sqrt{3}$ is the only value satisfying the given equation. So it is one root of the given equation.

23. Using the factor tree we have,

$56 = 2^3 \times 7$ and $112 = 2^4 \times 7$

Hence HCF is $2^3 \times 7 = 56$ and

LCM is $2^4 \times 7 = 112$

24. We have,

$\frac{AD}{DB} = \frac{2}{3}$ and $DE \mathbb{I} BC$

Therefore, by basic proportionality theorem, we have,

$\frac{AD}{DB} = \frac{AE}{EC}$

Taking reciprocal on both sides, we get

$\frac{DB}{AD} = \frac{EC}{AE}$

$\Rightarrow \quad \frac{3}{2} - \frac{EC}{M}$

Adding 1 on both sdies, we get

$\frac{3}{2} + 1 = \frac{EC}{AE} + 1$

$\Rightarrow \quad \frac{3+2}{2} - \frac{EC+AE}{AE}$

$\Rightarrow \quad \frac{5}{2} = \frac{AC}{AE}$ [$\because$ AE + EC = AC]

$\Rightarrow \quad \frac{5}{2} = \frac{18}{AE}$ [$\because$ AC = 18]

$\Rightarrow \quad AE = \frac{18 \times 2}{5}$

$\Rightarrow \quad AE = \frac{36}{5} = 7.2\text{cm}$

OR

From the given figure we have,

PQ || RS (Given)

So, ∠P = ∠S (Alternate angles)

and ∠Q = ∠R (Alternate angles)

Also, ∠POQ = ∠SOR (Vertically opposite angles)

Therefore, $\triangle POQ \sim \triangle SOR$ (AAA similarity criterion)

25. We have,

$\frac{\sin\theta}{\cot\theta + \cos ec\theta} = 2 + \frac{\sin\theta}{\cot\theta - \cos ec\theta}$ or, $\frac{\sin\theta}{\cot\theta + \cos ec\theta} - \frac{\sin\theta}{\cot\theta - \cos ec\theta} = 2$

Now,

$\text{LHS} = \frac{\sin\theta}{\cot\theta + \cos ec\theta} - \frac{\sin\theta}{\cot\theta - \cos ec\theta}$

$\Rightarrow \quad \text{LHS} = \frac{\sin\theta}{\cos ec\theta + \cot\theta} + \frac{\sin\theta}{\cos ec\theta - \cot\theta}$

$\Rightarrow \quad \text{LHS} = \sin\theta \left\{ \frac{1}{\cos ec\theta + \cot\theta} + \frac{1}{\cos ec\theta - \cot\theta} \right\}$

$\Rightarrow \quad \text{LHS} = \sin\theta \left\{ \frac{\cos ec\theta - \cot\theta + \cos ec\theta + \cot\theta}{\cos ec^2\theta - \cot^2\theta} \right\} = \sin\theta \left(\frac{2\cos ec\theta}{1} \right)$

$\Rightarrow \quad \text{LHS} = \sin\theta(2\cos ec\theta) = 2\sin\theta \times \frac{1}{\sin\theta} = 2 = \text{RHS}$

OR

We have,

cosecθ - sinθ = m and secθ - cosθ = n

$\Rightarrow \quad \frac{1}{\sin\theta} - \sin\theta = m$ and $\frac{1}{\cos\theta} - \cos\theta \ = \text{n}$

$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = m$ and $\frac{1-\cos^2\theta}{\cos\theta} \ = \text{n}$

$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = m$ and $\frac{\sin^2\theta}{\cos\theta} \ = \text{n}$

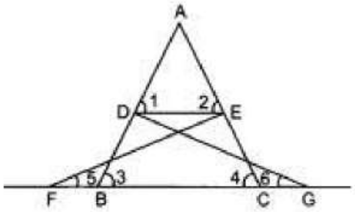
$\therefore \quad \left(m^2 n\right)^{2/3} + \left(mn^2\right)^{2/3} = \left(\frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos\theta} \right)^{2/3} + \left(\frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^4\theta}{\cos^2\theta} \right)^{2/3}$

$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3} = \cos^2\theta + \sin^2\theta = 1$

Hence, $(m^2n)^{2/3} + (mn^2)^{2/3} = 1$

Section C

26. Given: $\triangle FEC \cong \triangle GBD$ and $\angle 1 = \angle 2$



To prove: $\triangle ADE \sim \triangle ABC$

Proof: In ΔADE, $\angle 1 = \angle 2$ (Given)

$\Rightarrow$ AE = AD ....(i) (sides opposite to equal angles are equal)

Also, $\Delta FEC \cong \Delta GBD$ (Given)

$\Rightarrow$ BD = EC (by CPCT) ....(ii)

$\angle 3 = \angle 4$ [By CPCT]

Also AE + EC = AD + BD

AC = AB ....(iii)

Dividing (i) and (iii), we get

$\frac{AD}{AB} = \frac{AE}{AC}$ and $\angle A = \angle A$ (common)

$\therefore \quad \triangle ADE \sim \triangle ABC$ (SAS similarity)

27. Let Shefali's marks in Mathematics = x

Let Shefali's marks in English = 30 − x

If, she had got 2 marks more in Mathematics, her marks would be = x + 2

If, she had got 3 marks less in English, her marks in English would be = 30 – x − 3 = 27 − x

According to given condition:

$\Rightarrow (x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

Comparing quadratic equation $x^2 - 25x + 156 = 0$ with general form $ax^2 + bx + c = 0$,

We get a = 1, b = −25 and c = 156

Applying Quadratic Formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{25\pm\sqrt{(-25)^2-4(1)(156)}}{2\times1}$

$\Rightarrow \frac{25\pm\sqrt{625-624}}{2}$

$\Rightarrow x = \frac{25\pm\sqrt{1}}{2}$

$\Rightarrow x = \frac{25+1}{2}, \frac{25-1}{2}$

$\Rightarrow$ x = 13, 12

Therefore, Shefali's marks in Mathematics = 13 or 12

Shefali's marks in English = 30 – x = 30 – 13 = 17

Or Shefali's marks in English = 30 – x = 30 – 12 = 18

Therefore, her marks in Mathematics and English are (13, 17) or (12, 18).

28. i. It can be observed that Niharika posted the green flag at 5th position flower of the distance AD i.e., 5×5 = 25m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted a red flag at the distance of 4th flower position of AD i.e., 4×5

= 20m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

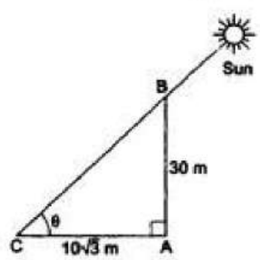
ii. According to distance formula,
Distance between these flags by using the distance formula, D
$= [(8 - 2)^2 + (25 - 20)^2]^{1/2} = (36 + 25)^{1/2} = \sqrt{61}m$

iii. The point at which Rashmi should post her blue flat is the mid-point of the line joining these points. Let this point be A(x, y)
Now by midpoint formula,
$(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$
$x = \frac{2+8}{2} = 5$
$y = \frac{25+20}{2} = 22.5$
Hence, A(x, y) = (5, 22.5)
Therefore, Rashmi should post her blue flag at 22.5 m on 5th line.

OR

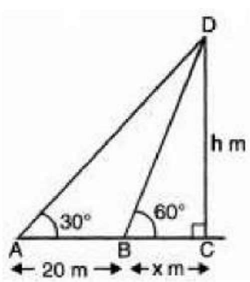
Required distance $= \sqrt{(asin\alpha + acos\alpha)^2 + (-bcos\alpha - b\sin\alpha)^2}$
$= \sqrt{a^2sin^2\alpha + a^2cos^2\alpha + 2a^2sin\alpha cos\alpha + b^2cos^2\alpha + b^2sin\alpha + 2b^2sin\alpha cos\alpha}$
$= \sqrt{a^2(sin^2\alpha + cos^2\alpha) + b^2(sin^2\alpha + cos^2\alpha) + (2a^2 + 2b^2)sin\alpha cos\alpha}$
$= \sqrt{a^2 + b^2 + 2(a^2 + b^2)sin\alpha cos\alpha}$
$= \sqrt{(a^2 + b^2)[1 + 2sin\alpha cos\alpha]}$
$= \sqrt{(a^2 + b^2)[sin^2\alpha + cos^2\alpha + 2sin\alpha cos\alpha]}$
$= \sqrt{(a^2 + b^2)[sin\alpha + cos\alpha]^2}$
$= (sin\alpha + cos\alpha)\sqrt{a^2 + b^2}$

29. Let AB be the pole and let AC be its shadow.



Let the angle of elevation of the sun be $\theta°$.
Then, $\angle ACB = \theta, \angle CAB = 90°$.
$AB = 30m$ and AC = $10\sqrt{3}m$.
From right ΔCAB, we have
$\tan\theta = \frac{AB}{AC} = \frac{30}{10\sqrt{3}} = \sqrt{3}$
$\Rightarrow \theta = 60°$

OR



Let height of tower be h m and distance BC be x m
In △ DBC, $\frac{h}{x} = \tan 60°$
$\Rightarrow \quad h = \sqrt{3}x$ .....(i)

$\frac{h}{x+20}=\tan 30^{\circ}=\frac{1}{\sqrt{3}}$

$\Rightarrow \quad \sqrt{3h}=x+20$ ....(ii)

Substituting the value of h from eq. (i) in eq. (ii), we get

3x = x + 20

3x - x = 20

Or 2x = 20

$\Rightarrow$ x = 10 m ...(iii)

Again$h=\sqrt{3x}$

or, $h=\sqrt{3}\times 10=10\sqrt{3}$

$=10\times 1\cdot 732$

= 17.32 m

[from (i) and (in)]

Hence, height of tower is 17.32 m and distance of tower from point A is 30 m

30. Calculation of mean by using step-deviation method.

Class Interval	**Frequency(f_i)**	**Mid value x_i**	$u_i=\frac{x_i-A}{h}=\frac{x_i-203.5}{1}$	$(f_i\times u_i)$
200 - 201	13	200.5	-3	-39
201 - 202	27	201.5	-2	-54
202 - 203	18	202.5	-1	-18
203 - 204	10	203.5 = A	0	0
204 - 205	1	204.5	1	1
205 - 206	1	205.5	2	2
	$\Sigma f_i=70$			$\Sigma(f_i\times u_i)$ = -108

$A=203.5, h=1$

Mean = $A+\left\{h\times\frac{\sum f_i u_i}{\sum f_i}\right\}$

$=203.5+\left\{1\times\frac{-108}{70}\right\}$

$=203.5-1.54=201.96$

mean weight of packets is 201.96

31. Suppose $\sqrt{3}=\frac{a}{b}$, where a and b are co-prime integers, $b\neq 0$

Squaring both sides,

$\Rightarrow 3=\frac{a^2}{b^2}$

Multiplying with b on both sides,

$\Rightarrow 3b=\frac{a^2}{b}$

LHS = $3\times b=Integer$

RHS = $\frac{a^2}{b}=\frac{\text{Integer}}{\text{Integer}}=Rational\ Number$

$\Rightarrow$ LHS $\neq$ RHS

$\therefore$ Our supposition is wrong.

$\Rightarrow \sqrt{3}$ is irrational.

Suppose $15+17\sqrt{3}$ is a rational number.

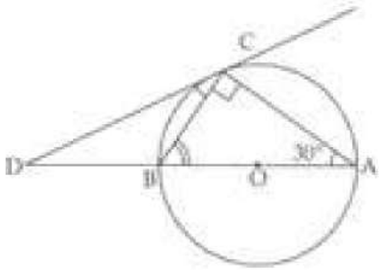
$\therefore 15+17\sqrt{3}=\frac{a}{b}$, where a and b are co-prime, $b\neq 0$

$\Rightarrow \quad 17\sqrt{3}=\frac{a}{b}-15$

$\sqrt{3}=\frac{a-15b}{17b}$
$\frac{a-15b}{17b}$ is rational number,
$\sqrt{3}$ is irrational.
$\therefore \quad \sqrt{3}\neq\frac{a-15b}{17b}$
$\therefore$ Our supposition is wrong.
$\Rightarrow \quad 15+17\sqrt{3}$ is irrational.

Section D

32. **Given:** A circle with centre O. A tangent CD at C.
Diameter AB is produced to D.
BC and AC chords are joined, ∠BAC = 30°



To prove: BC = BD
Proof: DC is tangent at C and, CB is chord at C.
Therefore, ∠DCB = ∠BAC [∠s in alternate segment of a circle]
⇒ ∠DCB = 30° …(i) [∵ ∠BAC = 30° (Given)]
AOB is diameter. [Given]
Therefore, ∠BCA = 90° [Angle in s semi circle]
Therefore, ∠ABC = 180° - 90° - 30° = 60°
In ΔBDC,
Exterior ∠B = ∠D + ∠BCD
⇒ 60° = ∠D + 30°
⇒ ∠D = 30° …(ii)
Therefore, ∠DCB = ∠D = 30° [From (i), (ii)]
⇒ BD = BC [∵ Sides opposite to equal angles are equal in a triangle]
Hence, proved.

33. We have,
$x+2y-4=0$ .....(i)
Putting $y=0$, we get
$x+0-4=0$
$\Rightarrow x=4$
Putting x = 0, we get
$0+2y-4=0$
$\Rightarrow y=2$
Thus, two solutions of equation $x+2y-4=0$ are:

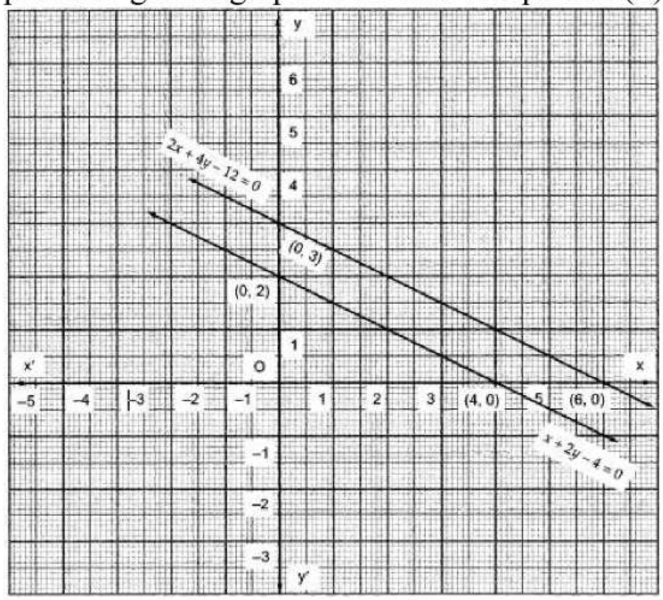
x	4	0
y	0	2

We have,
$2x+4y-12=0$ .....(ii)
Putting $x=0$, we get
$0+4y-12=0$
$\Rightarrow y=3$
Putting $y=0$, we get

$2x + 0 - 12 = 0$
$\Rightarrow$ x = 6
Thus, two solutions of equation $2x + 4y - 12 = 0$ are:

x	0	6
y	3	0

Now, we plot the points A (4, 0) and B (0, 2) and draw a line passing through these two points to get the graph of the line of equation (i).
We also plot the points P (0, 3) and Q (6, 0) and draw a line passing through these two points to get the graph of the line of equation (ii).



We observe that the lines are parallel and they do not intersect anywhere.

OR

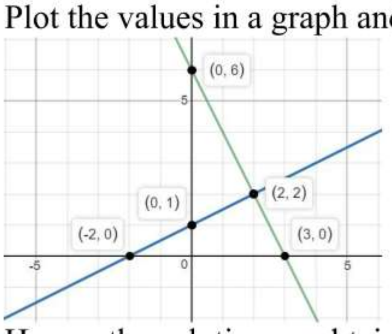
We can rewrite the equations as:
x - 2y = - 2 and 2x + y = 6
For equation, x - 2y = - 2
First, take x = 0 and find the value of y.
Then, take y = 0 and find the value of x.

x	0	-2
y	1	0

Now similarly solve for equation, 2x + y = 6

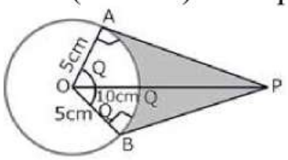
x	0	3
y	6	0

Plot the values in a graph and find the intersecting point for the solution.



Hence, the solution so obtained from the graph is (2, 2), which is the intersecting point of the two lines.
The vertices of the formed triangle by these lines and the x - axis in the graph are A(2, 2),

B(-2, 0) and C(3, 0).
Clearly, from the graph we can identify base and height of the triangle.
Now, we know
Area of Triangle = $\frac{1}{2}$ × base × height
Thus, Area(△ABC) = $\frac{1}{2}$ × 5 × 2 [∴ Base = BO + OC = 2 + 3 = 5 units and height = 2 units]
Area(△ABC) = 2 sq. units

34. 

$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$

$\Rightarrow \theta = 60^{\circ}$

$\Rightarrow \angle AOB = 2 \times \theta = 120^{\circ}$

$\therefore$ ARC AB = $\frac{120\times\ 2\times\pi\times5}{360} cm = \frac{10\pi}{3} cm\ \left[\because l = \frac{\theta}{360} \times 2\pi r\right]$

Length of the belt that is in contact with the rim of the pulley
= Circumference of the rim - length of arc AB
= $2\pi \times 5$ cm - $\frac{10\pi}{3}$cm
= $\frac{20\pi}{3}$cm

Now, the area of sector OAQB = $\frac{120\times\pi\times5\times5}{360} cm^2 = \frac{25\pi}{3} cm^2\ \left[\because Area = \frac{\theta}{360} \times \pi r^2\right]$

Area of quadrilateral OAPB = 2(Area of △OAP) = $25\sqrt{3}$ cm^2

$\left[\because AP = \sqrt{100\ - 25} = \sqrt{75}\ = 5\sqrt{3}\ cm\right]$

Hence, shaded area = $25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3}\ - \pi]\ cm^2$

OR

Let the radii of the two circlular plots be r_1 and r_2, respectively.
Then, $r_1 + r_2 = 14$ [∵ Distance between the centres of two circlular plots = 14 cm,given].... (i)
Also, Sum of Areas of the plots = 130π

$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi \Rightarrow r_1^2 + r_2^2 = 130$ ...(ii)

Now, from equation (i) and equation (ii),

$\Rightarrow (14 - r_2)^2 + r_2^2 = 130$

$\Rightarrow 196 - 2r_2 + 2r_2^2 = 130$

$\Rightarrow 66 - 2r_2 + 2r_2^2 = 0$

Solving the quadratic equation we get,
$r_2 = 3$ or $r_2 = 11$,
but from figure it is clear that, $r_1 > r_2$
$\therefore r_1 = 11$ cm and $r_2 = 3$ cm
The value depicted by the farmers are of cooperative nature and mutual understanding.

35. Total number of cards in one deck of cards = 52
∴ Total number of outcomes n = 52
i. Let E_1= Event of getting a king of red color. So number of outcomes favourable to E_1 and m = 2 [∵ there are two kings of red color in a deck]
So, P(E_1) = $\frac{m}{n} = \frac{2}{52} = \frac{1}{26}$

ii. Let E_2 = Event of getting the queen of the diamond

$\therefore$ Numbers of outcomes favourable to E_2 = 1 [$\because$ there is only one queen of diamond in a deck]

Hence, $P(E_2) = \frac{m}{n} = \frac{1}{52}$

iii. Let E_3 = Event of getting an ace.

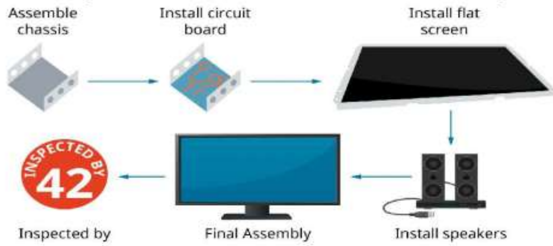
$\therefore$ Number of outcomes favourable to E_3 = 4

Hence, $P(E_3) = \frac{4}{52} = \frac{1}{13}$

Section E

36. **Read the text carefully and answer the questions:**

Elpis Technology is a laptop manufacturer. The company works for many branded laptop companies and also provides them with spare parts. Elpis Technology produced 6000 units in 3rd year and 7000 units in the 7th year.



Assuming that production increases uniformly by a fixed number every year.

(i) Let production in a 1st year be 'a' unit and increase in production (every year) be 'd' units.

Increase in production is constant, therefore unit produced every year forms an AP.

Now, $a_3 = 6000$

$a + 2d = 6000 \Rightarrow a = 6000 - 2d$ ...(i)

and $a_7 = 7000 \Rightarrow a + 6d = 7000$

$\Rightarrow (6000 - 2d) + 6d = 7000 \Rightarrow 4d = 1000$ [using eq. (i)]

$\Rightarrow d = 250$

When d = 250, eq. (i) becomes

$a = 6000 - 2(250) = 5500$

$\therefore$ Production in 1st year = 5500

(ii) Production in fifth year

$a_5 = a + 4d = 5500 + 4(250) = 6500$

OR

$a_n = 1000$ units

$a_n = 1000$

$\Rightarrow 10000 = a + (n - 1)d$

$\Rightarrow 1000 = 5500 + 250n - 250$

$\Rightarrow 10000 - 5500 + 250 = 250n$

$\Rightarrow 4750 = 250n$

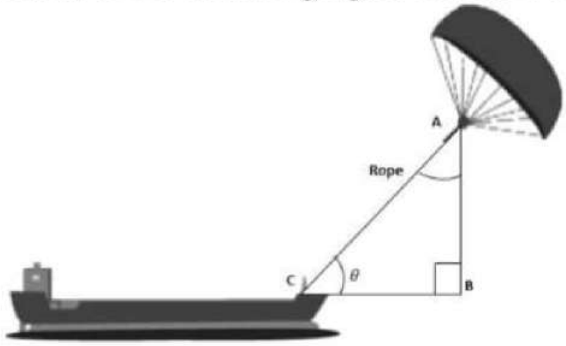
$\Rightarrow n = \frac{4750}{250} = 19$

(iii) Total production in 7 years = $\frac{7}{2}(5500 + 7000) = 43750$

37. **Read the text carefully and answer the questions:**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a

height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.
Based on the following figure related to sky sailing, answer the following questions:



(i) $\sin\theta = \cos(\theta - 30^{o})$

$\cos(90^{o} - \theta) = \cos(\theta - 30^{o})$

$\Rightarrow 90^\circ - \theta = \theta - 30^\circ$

$\Rightarrow \theta = 60^\circ$

(ii) $\frac{AB}{AC} = \sin 60^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin60^o} = \frac{200}{\frac{\sqrt{3}}{2}} = \frac{200\times2}{\sqrt{3}}$ = 230.94 m

(iii) $\sin\theta = \cos(3\theta - 30^{o})$

$\cos(90^{o} - \theta) = \cos(3\theta - 30^{o})$

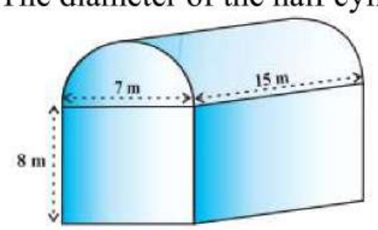
$\Rightarrow 90^\circ - \theta = 3\theta - 30^\circ \Rightarrow \theta = 30^\circ$

OR

$\frac{AB}{AC} = \sin 30^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin30^o} = \frac{150}{\frac{1}{2}}$ = 150 × 2 = 300 m

38. **Read the text carefully and answer the questions:**
Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m × 7 m × 8 m.
The diameter of the half cylinder was 7 m and length was 15 m.



(i) Total volume = volume of cuboid + $\frac{1}{2}$ × volume of cylinder.
For cuboidal part we have
length = 15 m, breadth = 7 m and height = 8 m
$\therefore$ Volume of cuboidal part = l × b × h = $15 \times 7 \times 8 m^2 = 840 m^3$
Clearly,
r = Radius of half-cylinder = $\frac{1}{2}$ (Width of the cuboid) = $\frac{7}{2}$m
and, h = Height (length) of half-cylinder = Length of cuboid = 15 m

$\therefore$ Volume of half-cylinder = $\frac{1}{2}\pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15\text{m}^3 \ = \frac{1155}{4}\text{m}^3 = 288.75\text{m}^3$

Thus the volume of the air that the shed can hold = (840 + 288.75) m^3 = 1128.75 m^3

(ii) Total space occupied by 20 workers = $20 \times 0.08\text{m}^3$ = = 1.6 m^3

Total space occupied by the machinery = 300 m^3

$\therefore$ Volume of the air inside the shed when there are machine and workers inside it

= (1128.75 - 1.6 - 300) m^3

= (1128.75 - 301.6) m^3 = 827.15 m^3

Hence, volume of air when there are machinery and workers is 827.15

OR

For the cylindrical part r = 3.5 cm and l = 15 m

Thus the surface area of the cylindrical part

$= \frac{1}{2}(2\pi rl) = 3.14 \times 3.5 \times 15$

= 164.85 m^2

(iii) Given for the cuboidal part

length L = 15 m, Width B = 7 m, Height = 8 m

Surface area of the cuboidal part

= 2(LB + BH + HL)

= 2(15 × 7 + 7 × 8 + 8 × 15)

= 2(105 + 56 + 120) = 2 × 281 = 562 m^2