**Class- X Session- 2022-23**

**Subject- Mathematics (Basic)**

**Sample Question Paper - 10**

**with Solution**

**Time Allowed: 3 Hrs.** **Maximum Marks : 80**

**General Instructions:**

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

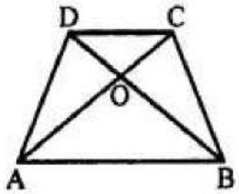
## Section A

1. The degree of the polynomial $5x^3 - 3x^2 - x + \sqrt{2}$ is **[1]**

a) 2 b) 3

c) 1 d) 0

2. In the given figure, ABCD is a trapezium whose diagonals AC and BD intersect at O such that OA = (3x - 1) cm, OB = (2x +1) cm, OC = (5x - 3) cm and OD = (6x - 5) cm. Then, x = ? **[1]**




a) 4 b) 2

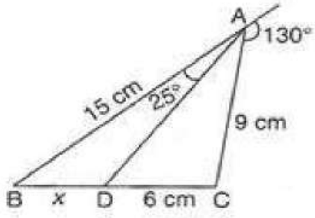
c) 3.5 d) 3

3. The system of equations x - 4y = 8, 3x - 12y = 24 **[1]**

a) has infinitely many solutions b) may or may not have a solution

c) has no solution d) has a unique solution

4. Find the value of a given figure. **[1]**




a) 12 cm.
b) 6 cm.
c) 10 cm.
d) 15 cm.

5. For what value of k, do the equations **[1]**
$3x - y + 8 = 0$
and $6x - ky = -16$
represent coincident lines?

a) $-2$
b) 2
c) $-\frac{1}{2}$
d) $\frac{1}{2}$

6. If $\sec \theta = \frac{25}{7}$ then $\sin \theta = ?$ **[1]**

a) $\frac{24}{7}$
b) $\frac{24}{25}$
c) $\frac{7}{24}$
d) none of these

7. In a lottery, there are 6 prizes and 24 blanks. What is the probability of not getting a prize? **[1]**

a) None of these
b) $\frac{3}{4}$
c) $\frac{3}{5}$
d) $\frac{4}{5}$

8. If the bisector of an angle of a triangle bisects the opposite side then the triangle is **[1]**

a) scalene
b) isosceles
c) equilateral
d) right-angled

9. In a data, if $l = 60$, $h = 15$, $f_1 = 16$, $f_0 = 6$, $f_2 = 6$, then the mode is **[1]**

a) 67.5
b) 72
c) 60
d) 62

10. $x^2 - 6ax = -6a^2$ discriminant of the given equation is ________ **[1]**

a) $12a^2$
b) $4a^2$

c) $6a^2$

d) $2a^2$

11. The abscissa of any point on the y-axis is **[1]**

a) 0

b) 1

c) y

d) -1

12. In the following distribution : **[1]**

| **Monthly income** | **Number of families** |
|---|---|
| More than 10000 | 100 |
| More than 13000 | 85 |
| More than 16000 | 69 |
| More than 19000 | 50 |
| More than 22000 | 33 |
| More than 25000 | 15 |

the number of families having income range (in ₹) 16000 – 19000 is

a) 15

b) 17

c) 16

d) 19

13. If sec $\theta$ + tan $\theta$ = x, then sec $\theta$ = **[1]**

a) $\frac{x^2+1}{x}$

b) $\frac{x^2-1}{2x}$

c) $\frac{x^2-1}{x}$

d) $\frac{x^2+1}{2x}$

14. The sum of the exponents of the prime factors in the prime factorisation of 196, is **[1]**

a) 2

b) 1

c) 4

d) 6

15. The length of the tangent from an external point P to a circle of radius 5 cm is 10 cm. The distance of the point from the centre of the circle is **[1]**

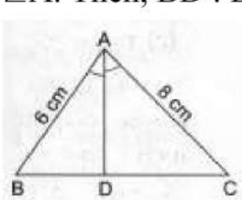
a) 12 cm

b) $\sqrt{125}$

c) $\sqrt{104}$ cm

d) 8 cm

16. In a $\triangle$ ABC it is given that AB = 6 cm, AC = 8 cm and AD is the bisector of $\angle$A. Then, BD : DC = ? **[1]**



a) 3 : 4

b) 9 :16

c) $\sqrt{3}$ : 2

d) 4 : 3

17. $3x^2 + 2x - 1 = 0$ have [1]

a) Real and Distinct roots

b) Real roots

c) real and equal root

d) No Real roots

18. **Assertion:** If one zero of polynominal $p(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of other, then k = 2. [1]
**Reason:** If $(x - \alpha)$ is a factor of p(x), then $p(\alpha) = 0$ i.e, $\alpha$ is a zero of p(x).

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

19. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 $cm^2$. [1]
**Reason (R):** Total surface area of a cuboid is 2(lb + bh + hl)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. The ________ is the line drawn from the eye of an observer to the point in the object viewed by the observer. [1]

a) Horizontal line

b) line of sight

c) None of these

d) Vertical line

## Section B

21. Find the centroid of the triangle whose vertices are given below : (4, -8), (-9, 7), (8, 13) [2]

OR

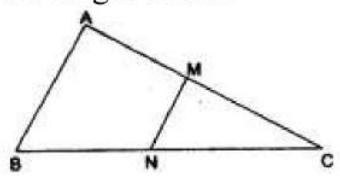
Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).

22. Solve the quadratic equation by factorization: [2]
$16x - \frac{10}{x} = 27$

23. Find the HCF of the following polynomials: $18\left(x^3 - x^2 + x - 1\right), 12\left(x^4 - 1\right)$ [2]

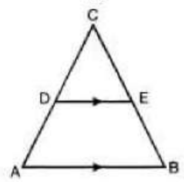
24. In $\triangle ABC$, right angled at A, if AB = 5, AC = 12 and BC = 13, find sin B, cos C and tan B. **[2]**

25. In the given figure, $MN \| AB$, BC = 7.5 cm, AM = 4 cm and MC = 2 cm. Find the length of BN. **[2]**



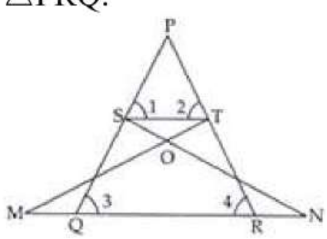

OR

In the given figure, $\angle A = \angle B$ and AD = BE. Show that DE $\|$ AB.




## Section C

26. In the given figure, if $\angle 1 = \angle 2$ and $\triangle NSQ \cong \triangle MTR$. Prove that $\triangle PTS \sim \triangle PRQ$. **[3]**




27. Solve: $x^2 - 2ax - (4b^2 - a^2) = 0$ **[3]**

28. Define HCF of two positive integers and find the HCF of the pair of numbers: 75 and 243. **[3]**

29. From the top of a building 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60° respectively. Find **[3]**
   i. The horizontal distance between the building and the lamp post.
   ii. The height of the lamp post. use( $\sqrt{3} = 1.732$)

OR

A man rowing a boat away from a lighthouse 150 m high takes 2 minutes to change the angle of elevation of the top of lighthouse from 45° to 30°. Find the speed of the boat. (Use $\sqrt{3} = 1.732$)

30. Find the median marks for the following distribution: **[3]**

| Classes | Number of students |
|---|---|
| 0 - 10 | 2 |
| 10 - 20 | 12 |

| Classes | Number of students |
|---|---|
| 20 - 30 | 22 |
| 30 - 40 | 8 |
| 40 - 50 | 6 |

31. Three vertices of a parallelogram are (a + b, a - b), (2a + b, 2a - b), (a - b , a + b). Find the fourth vertex. **[3]**

OR

Show that the points A (1, 7), B (4, 2), C (-1, -1) and D (-4, 4) are the vertices of a square.

**Section D**

32. A is a point at a distance 13 cm from the centre 'O' of a circle of radius 5 cm. AP and AQ are the tangents to circle at P and Q. If a tangent BC is drawn at point R lying on minor arc PQ to intersect AP at B AQ at C. Find the perimeter of ΔABC. **[5]**

33. One says, "Give me a hundred, friend! I shall then become twice as rich as you"." The other replies, If you give me ten, I shall be six times as rich as you." Tell me what is the amount of their (respective) capital. **[5]**
[Hint: x + 100 = 2(y-100), y + 10 = 6(x - 10)]

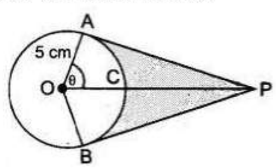
OR

Form the pair of linear equations in the problem, and find its solution graphically. 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. find the cost of one pencil and that of one pen.

34. Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is $\frac{24}{7}\text{cm}^2$. Find the radius of each circle. **[5]**

OR

An elastic belt is placed round the rim of a pulley of radius 5 cm. One point on the belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from O. Find the length of the belt that is in contact with the rim of the pulley. Also, find the shaded area.




35. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting **[5]**
   i. a king of red suit
   ii. a face card
   iii. a red face card
   iv. a queen of black suit

v. a jack of hearts

vi. a spade.

## Section E

36. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find an increase in the production of TV every year.

(ii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find in which year production of TV is 1000.

(iii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the 10th year.

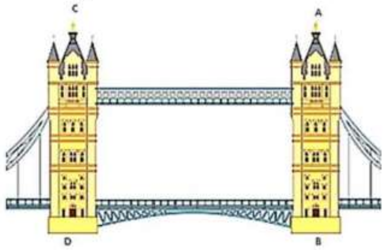
**OR**

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the total production in first 7 years.

37. **Read the text carefully and answer the questions:** **[4]**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was $60^{o}$ and $30^{o}$ respectively.




(i) Find the distances of the point from the base of the towers where Neeta was standing while measuring the height.

(ii) Neeta used some applications of trigonometry she learned in her class to find the height of the towers without actually measuring them. What would be the height of the towers she would have calculated?

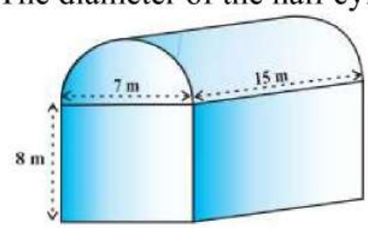
(iii) Find the distance between Neeta and top of tower AB?

**OR**

Find the distance between Neeta and top tower CD?

38. **Read the text carefully and answer the questions:** **[4]**

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m × 7 m × 8 m. The diameter of the half cylinder was 7 m and length was 15 m.




(i) Find the volume of the air that the shed can hold.

(ii) If the industry requires machinery which would occupy a total space of 300 $m^3$ and there are 20 workers each of whom would occupy 0.08 $m^3$ space on an average, how much air would be in the shed when it is working?

(iii) Find the surface area of the cuboidal part.

**OR**

Find the surface area of the cylindrical part.

# Solution

## Section A

1. **(b)** 3

   **Explanation:** The degree of the polynomial $5x^3 - 3x^2 - x + \sqrt{2}$ is 3. The degree of a polynomial is the highest power of that polynomial.

2. **(b)** 2

   **Explanation:** In the given figure,
   ABCD is a trapezium and its diagonals AC
   and BD intersect at O.
   and OA = (3x – 1) cm OB = (2x + 1) cm, OC and OD = (6x – 5) cm
   Now, $\frac{AO}{OC} = \frac{BO}{OD}$
   (Diagonals of a trapezium divides each other proportionally)
   $\Rightarrow \frac{3x-1}{5x-3} = \frac{2x+1}{6x-5}$
   $\Rightarrow (3x-1)(6x-5) = (2x+1)(5x-3)$
   $\Rightarrow 18x^2 - 10x^2 - 21x + 6x - 5x + 5 + 3 = 0$
   $\Rightarrow 8x^2 - 20x + 8 = 0$
   $\Rightarrow 2x^2 - 5x + 2 = 0$
   $\Rightarrow 2x^2 - x - 4x + 2 = 0$
   $\Rightarrow x(2x-1) - 2(2x-1) = 0$
   $\Rightarrow (2x-1)(x-2) = 0$
   Either 2x - 1 = 0, then x = $\frac{1}{2}$ but it does not satisfy
   or x - 2 = 0, then x = 2
   $\therefore$ x = 2

3. **(a)** has infinitely many solutions

   **Explanation:** Given: $a_1 = 1$, $a_2 = 3$, $b_1 = -4$, $b_2 = -12$, $c_1 = 8$ and $c_2 = 24$
   Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-4}{-12} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{8}{24} = \frac{1}{3}$
   $\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
   Therefore, the system has infinitely many solutions.

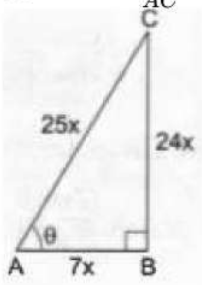
4. **(c)** 10 cm.

   **Explanation:** Here, $\angle CAD = 180^\circ - (130^\circ + 25^\circ) = 25^o$
   Now, since ∠CAD = ∠DAB, therefore, the AD is the bisector of ∠BAC.
   Since the internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
   $\therefore \frac{\text{BD}}{\text{DC}} = \frac{\text{AB}}{\text{AC}} \Rightarrow \frac{x}{6} = \frac{15}{9}$
   $\Rightarrow x = \frac{15 \times 6}{9}$ = 10 cm

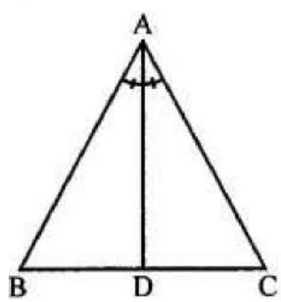
5. **(b)** 2

   **Explanation:** Condition for coincident lines is -
   $a_1/a_2 = b_1/b_2 = c_1/c_2$ ...(i)
   Given lines are,
   3x - y + 8 = 0
   and 6x - ky + 16 = 0;
   Comparing with the standard form, gives

$a_1 = 3, b_1 = -1, c_1 = 8;$
$a_2 = 6, b_2 = -k, c_2 = 16;$
and, from Eq. (i), $\frac{3}{6} = \frac{1}{k} = \frac{8}{16}$
$\frac{1}{k} = \frac{1}{2}$
So, k = 2

6. **(b)** $\frac{24}{25}$
**Explanation:** $\sec\theta = \frac{AC}{AB} = \frac{25}{7} = \frac{25x}{7x} \Rightarrow$ AC = 25x and AB = 7x
$\therefore BC^2 = AC^2 - AB^2 = 625x^2 - 49x^2 = 576x^2 \Rightarrow BC = 24x$
$\therefore \sin\theta = \frac{BC}{AC} = \frac{24x}{25x} = \frac{24}{25}$



7. **(d)** $\frac{4}{5}$
**Explanation:** Total number of tickets = 6 + 24 = 30.
Number of blanks = 24.
$\therefore$P{not getting a prize} = $\frac{24}{30} = \frac{4}{5}$

8. **(b)** isosceles
**Explanation:** If the bisector of angle of a triangle bisects the opposite side of a triangle.


$\because$ AD is the angle bisector of $\angle A$
$\therefore \frac{AB}{AC} = \frac{BD}{CD}$ (D bisects BC)
$\therefore$ AB = AC
$\therefore \triangle ABC$ is an isosceles triangle.

9. **(a)** 67.5
**Explanation:** Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$
$= 60 + \frac{16-6}{2\times16-6-6} \times 15$
$= 60 + \frac{10}{32-12} \times 15$
$= 60 + \frac{10}{20} \times 15$
= 60 + 7.5
= 67.5

10. **(a)** $12a^2$
**Explanation:** $x^2 - 6ax + 6a^2 = 0$

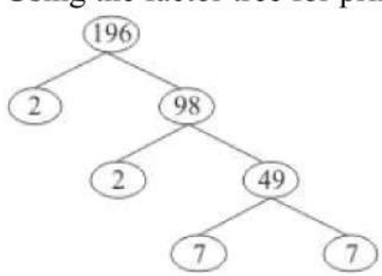
$D = b^2 - 4ac$

$D = (-6a)^2 - 4 \times 1 \times 6a^2$

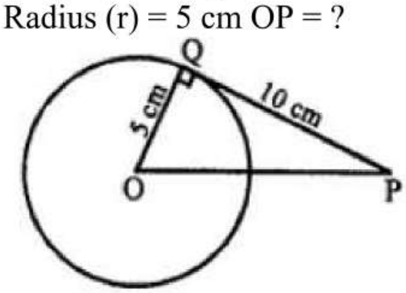
$D = 36a^2 - 24a^2$

$D = 12a^2$

11. **(a)** 0
    **Explanation:** Since coordinates of any point on y-axis is (0, y)
    Therefore, the abscissa is 0.
12. **(d)** 19
    **Explanation:** Between 16000 and 19000 we need to subtract the frequencies of these classes to get the desired result i.e., 69 - 50 = 19 .
13. **(d)** $\frac{x^2+1}{2x}$
    **Explanation:** Given, $\sec\theta + \tan\theta = x$
    We know that, $\sec^2\theta - \tan^2\theta = 1$
    $\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
    $\Rightarrow x(\sec\theta - \tan\theta) = 1$
    $\sec\theta - \tan\theta = \frac{1}{x}$
    Now $\sec\theta + \tan\theta = x$
    Adding we get,
    $2\sec\theta = \frac{1}{x} + x = \frac{1+x^2}{x}$
    $\sec\theta = \frac{1+x^2}{2x}$
14. **(c)** 4
    **Explanation:**
    Using the factor tree for prime factorisation, we have:




    Therefore,
    $196 = 2 \times 2 \times 7 \times 7$
    $196 = 2^2 \times 7^2$
    The exponents of 2 and 7 are 2 and 2 respectively.
    Thus the sum of the exponents is 4.
15. **(b)** $\sqrt{125}$
    **Explanation:**
    Length of a tangent to the circle from an external point = 10 cm
    Radius (r) = 5 cm OP = ?




    OQ is radius and QP is tangent

OQ ⊥ QP
In right △OPQ,
$OP^2 = OQ^2 + QP^2$ (Pythagoras Theorem) = $(5)^2 + (10)^2$ = 25 + 100 = 125
OP = $\sqrt{125}$ cm

16. **(a)** 3 : 4
**Explanation:** $\frac{BD}{DC} = \frac{AB}{AC} = \frac{6}{8} = \frac{3}{4}$ [by angle-bisector theorem]

17. **(a)** Real and Distinct roots
**Explanation:** D = $b^2$ - 4ac
D = $2^2$ - 4 × 3 × (-1)
D = 4 + 12
D = 16
D > 0.
Hence Real and distinct roots.

18. **(b)** Assertion and reason both are correct statements but reason is not correct explanation for assertion.
**Explanation:** Let $\alpha$, $\frac{1}{\alpha}$ = be the zeros of p(x), then
$\alpha \cdot \frac{1}{\alpha} = \frac{4k}{k^2+4}$
$1 = \frac{4k}{k^2+4}$
$k^2$ - 4k + 4 = 0
$(k - 2)^2 = 0$
k = 2
Assertion is true Since, Reason is not correct for Assertion.

19. **(d)** A is false but R is true.
**Explanation:** A is false but R is true.

20. **(b)** line of sight
**Explanation:** The line of sight is the imaginary line drawn from the eye of an observer to the point in the object viewed by the observer. The angle between the line of sight and the ground is called angle of elevation

## Section B

21. The given vertices of triangle are (4, –8), (–9, 7) and (8, 13).
Let (x, y) be the coordinates of the centroid. Then
$x = \frac{x_1+x_2+x_3}{3} = \frac{4+(-9)+8}{3}$
$= \frac{12-9}{3} = \frac{3}{3} = 1$
$y = \frac{y_1+y_2+y_3}{3} = \frac{(-8)+7+13}{3}$
$= \frac{20-8}{3} = \frac{12}{3} = 4$
∴ The coordinates of the centroid are (1, 4).

OR

We want to find coordinates of point A. AB is the diameter and coordinates of centerare (2, -3) and, coordinates of point B are (1, 4).
Let coordinates of point A are (x, y). Using section formula, we get
$2 = \frac{x+1}{2}$
⇒ 4 = x + 1
⇒ x = 3

Using section formula, we get

$-3 = \frac{4+y}{2}$

$\Rightarrow 4 + y = -6$

$\Rightarrow y = -6 - 4 = -10$

Therefore, Coordinates of point A are (3, −10).

22. We have, $16x - \frac{10}{x} = 27$

$\Rightarrow \frac{16x^2-10}{x} = 27$

$\Rightarrow 16x^2 - 10 = 27x$

$\Rightarrow 16x^2 - 27x - 10 = 0$

$\Rightarrow 16x^2 - 32x + 5x - 10 = 0$

$\Rightarrow 16x(x - 2) + 5(x - 2) = 0$

$\Rightarrow (16x + 5)(x - 2) = 0$

$\Rightarrow (16x + 5) = 0$ or $(x - 2) = 0$

$\Rightarrow x = -\frac{5}{16}$ or, $x = 2$

Hence, the roots of given quadratic equation are 2 and $-\frac{5}{16}$

23. Let $p(x) = 18\left(x^3 - x^2 + x - 1\right)$

$Q(x) = 12\,(x^4 - 1)$

Now $p(x) = 18\left(x^3 - x^2 + x - 1\right)$ }

$= 2.3.3\left[x^2(x-1) + 1(x-1)\right]$

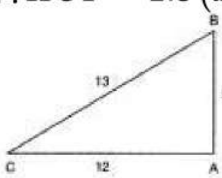
$= 2.3.3(x-1)\left(x^2+1\right)$

$Q(x) = 12\left(x^4 - 1\right) = 2.2.3\left[\left(x^2\right)^2 - 1^2\right]$

$= 2.2.3\left(x^2+1\right)\left(x^2-1\right)$

$= 2.2.3\left(x^2+1\right)(x+1)(x-1)$ using identity $a^2 - b^2 = (a+b)(a-b)$

$\therefore HCF = 2.3\left(x^2+1\right)(x-1)$

24. 


With reference to $\angle B$, we have given that

In $\triangle ABC$

$Base = AB = 5$, $Perpendicular = AC = 12$ and, $Hypotenuse = BC = 13.$

$\therefore \quad \sin B = \frac{AC}{BC} = \frac{12}{13}$

and, $\tan B = \frac{AC}{AB} = \frac{12}{5}$

With reference to $\angle C$, we have

$Base = AC = 12$, $Perpendicular = AB = 5$ and,

$Hypotenuse = BC = 13$

$\therefore \quad \cos C = \frac{AC}{BC} = \frac{12}{13}$

25. According to quesiton it is given that In $\triangle ABC$,

$MN \| AB$

Therefore by Thale's theroem

$\frac{MC}{AC} = \frac{NC}{BC}$

$\Rightarrow \frac{MC}{AM+MC} = \frac{NC}{BC}$

$\Rightarrow \frac{2}{4+2} = \frac{x}{7.5}$ (when NC = x cm)
$\Rightarrow \quad x = \frac{2\times7.5}{6}$
$= \frac{15}{6}$ = 2.5
⇒ NC = 2.5 cm
Hence, BN = BC - NC
= (7.5 - 2.5) cm
= 5 cm

OR

In △CAB, ∠A = ∠B(Given)
∴ AC = CB (By isosceles triangle property)
But, AD = BE (Given).....(i)
⇒ AC- CD = CB - BE
∴ CD = CE .......(ii)
Dividing equation (ii) by (i),
$\frac{CD}{AD} = \frac{CE}{BE}$
By converse of BPT,
DE ∥ AB.
∴ If ∠A = ∠B and AD = BE then, DE ∥ AB.

**Section C**

26. Proof: △NSQ ≅ △MTR [Given]
$\therefore SQ = TR \ldots\ldots(1).$
$\angle 1 = \angle 2$ [Given]
$\therefore PT = PS \ldots\ldots(2)$
On dividing (2) by (1) we get
$\Rightarrow \frac{PT}{TR} = \frac{PS}{SQ}$
∴ ST ∥ QR [By converse of BPT]
In PST△ and △PQR
ST ∥ QR [Proved above]
$\angle 1 = \angle 3$
$\angle 2 = \angle 4$
∴ △PTS ~ △PRQ [By AA similarity criterion]
Hence, proved.

27. $x^2 - 2ax - (4b^2 - a^2) = 0$
$\Rightarrow x^2 - 2ax + (a^2 - 4b^2) = 0$
$\Rightarrow x^2 - 2ax + (a-2b)(a+2b) = 0$
$\Rightarrow x^2 - (a-2b)x - (a+2b)x + (a-2b)(a+2b) = 0 \quad [2ax = (a-2b)x + (a+2b)x]$
$\Rightarrow x[x-(a-2b)] - (a+2b)[x-(a-2b)] = 0$
⇒ [x - (a - 2b)] [x - (a + 2b)] = 0
⇒ x - (a - 2b) = 0 or x - (a + 2b) = 0
⇒ x = a -2b or x = a + 2b

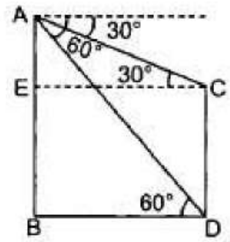
28. **HCF (highest common factor) :** The largest positive integer that divides given two positive integers is called the Highest Common Factor of these positive integers.
We need to find H.C.F. of 75 and 243.
By applying Euclid's Division lemma
243 = 75 × 3 + 18.
Since remainder ≠ 0, apply division lemma on 75 and remainder 18

$75 = 18 \times 4 + 3.$

Since remainder $\neq 0$, apply division lemma on divisor 18 and remainder 3

$18 = 3 \times 6 + 0.$

Therefore, H.C.F. of 75 and 243 = 3

29. 


Let AB = 60 m is height of building and CD is lamp post.

i. In $\Delta$ABD ,

$\frac{AB}{BD} = \tan 60^\circ$

$\Rightarrow \frac{60}{BD} = \sqrt{3} \Rightarrow \frac{60}{\sqrt{3}} = BD$

$\Rightarrow BD = \frac{60 \times \sqrt{3}}{3} = 20\sqrt{3}\text{m}$

$\Rightarrow \text{BD} = 20 \times 1.732 = 34.64\text{m}$

ii. In $\Delta$ AEC ,

$\frac{\text{AE}}{\text{EC}} = \tan 30^\circ$

$\Rightarrow \frac{\text{AE}}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ AE = 20 m

and

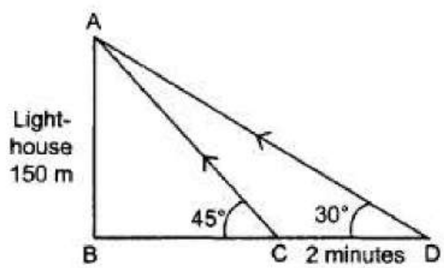
$EB = AB - AE = 60 - 20 = 40m$

Also ,

$EB = CD$

$\Rightarrow CD = 40m$

OR




AB = 150 m

Initially boat is at C and after 2 minutes it reaches at D.

In right $\triangle$ABC,

$\frac{\text{AB}}{\text{BC}} = \tan 45^\circ$

$\Rightarrow \frac{150}{\text{BC}} = 1 \Rightarrow \text{BC} = 150\text{m}$

In right $\triangle$ABD, $\frac{\text{AB}}{\text{BD}} = \tan 30^\circ$

$\Rightarrow \frac{150}{\text{BD}} = \frac{1}{\sqrt{3}} \Rightarrow \text{BD} = 150\sqrt{3}$

Distance covered in 2 minutes$= BD - BC = 150\sqrt{3} - 150 = 150(\sqrt{3} - 1)m$

$\therefore$ speed = $\frac{\text{Distance covered}}{\text{time taken}} = \frac{150(\sqrt{3}-1)}{2}$

$= 75 \times (1.732 - 1)$

$= 54.9\ m/min$

30.

| Classes | Number of students | Cumulative Frequency |
|---|---|---|
| 0 - 10 | 2 | 2 |
| 10 - 20 | 12 | 14 |
| 20 - 30 | 22 | 36 |
| 30 - 40 | 8 | 44 |
| 40 - 50 | 6 | 50 |

n = 50,

$\frac{n}{2} = \frac{50}{2} = 25$

Median Class = 20 - 30

l = 20, f = 22, c.f. = 14,h = 10

Median $= l + \frac{\left(\frac{n}{2} - c.f.\right)}{f} \times h$

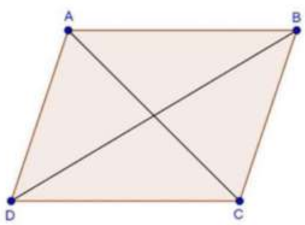
$= 20 + \frac{(25-14)}{22} \times 10$

$= 20 + \frac{11}{22} \times 10$

= 20 + 5

= 25

31.




Let A(a + b, a - b), B(2a + b, 2a - b), C(a - b, a + b) and D(x, y) be the given points.

Since, the diagonals of a parallelogram bisect each other.

∴ Coordinates of the mid-point of AC = Coordinates of the mid-point of BD

$\left(\frac{a+b+a-b}{2}, \frac{a-b+a+b}{2}\right) = \left(\frac{2a+b+x}{2}, \frac{2a-b+y}{2}\right)$

$\Rightarrow \quad (a, a) = \left(\frac{2a+b+x}{2}, \frac{2a-b+y}{2}\right)$

$\Rightarrow \quad \frac{2a+b+x}{2} = a$ and $\frac{2a-b+y}{2} = a$

⇒ 2a + b + x = 2a ⇒ 2a - b + y = 2a

⇒ x = -b ⇒y = b

Hence, the fourth vertex is (-b, b).

OR

Let A(1, 7), B(4, 2), C(-1, -1) and D(-4, 4) be the given points. One way of showing that ABCD is a square is to use the property that all its sides should be equal both its diagonals should also be equal. Now,

$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$

$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$

$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$

$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$

$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

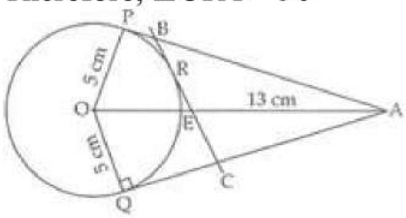
## Section D

32. OA = 13 cm

OP = OQ = 5 cm

OP and PA are radius and tangent respectively at contact point P.

Therefore, $\angle OPA = 90°$




In right angled $\Delta OPA$ by Pythagoras theorem

$PA^2 = OA^2 - OP^2 = 13^2 - 5^2 = 169 - 25 = 144$

$\Rightarrow PA = 12$ cm

Points A, B and C are exterior to the circle and tangents drawn from an external point to a circle are equal so

PA = QA

BP = BR

CR = CQ

Perimeter of $\triangle ABC$ = AB + BC + AC

= AB + BR + RC + AC [From figure]

= AB + BP + CQ + AC = AP + AQ

= AP + AP = 2AP = $2 \times 12$ = 24 cm

So, the perimeter of $\triangle ABC$ = 24 cm.

33. Let the amount of their respective capitals be x and y.

$\therefore$ According to the given condition,

x + 100 = 2(y - 100)

or, x + 100 = 2y - 200

or, x - 2y= - 300 ........(i)

and 6(x - 10) = y + 10

or, 6x - 60 = y + 10

$\Rightarrow$ 6x-y=70 .......(ii)

On multiplying eqn. (ii) by 2 and subtracting from eqn. (1),

$$\therefore \begin{array}{rcl} x - 2y &=& -300 \\ 12x - 2y &=& +140 \\ - \quad + && - \\ \hline -11x &=& -440 \end{array}$$

On substituting x = 40 in eqn. (1),

40 - 2y = - 300

or, 2y = 340

$\therefore$ y = 170

Hence, the amount of their respective capitals are ₹40 and ₹170.

OR

Let the cost of one pencil and a pen be Rs. x and Rs. y respectively. Then the pair of linear equations formed is

5x + 7y = 50 ...(1)

and 7x + 5y = 46 ...(2)

let us draw the graphs of equations (1) and (2) by finding two solutions for each of the equations.

These two solutions of the equations (1) and (2) are given below in table 1 and table 2 respectively.

For equation (1) 5x + 7y = 50

⇒ 7y = 50 - 5x ⇒ $y = \frac{50-5x}{7}$

Table 1 of solutions

| | | |
|---|---|---|
| x | 3 | -4 |
| y | 5 | 10 |

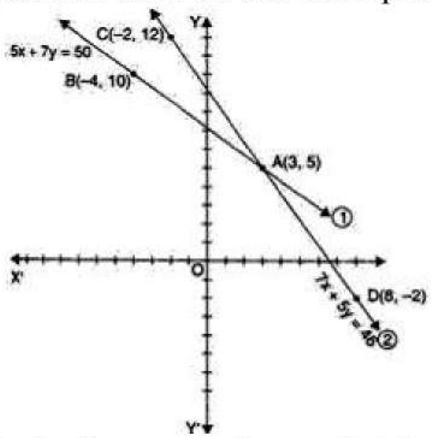
For equation (2) 7x + 5y = 46 ⇒ 5y = 46 - 7x ⇒ $y = \frac{46-7x}{5}$

Table 2 of solutions

| | | |
|---|---|---|
| x | -2 | 8 |
| y | 12 | -2 |

We plot the points A(3, 5) and B(-4, 10) on a graph paper and join these points to form the line AB representing the equation (1) as shown in the figure,

Also, we plot the points C(-2, 12) and D(8, -2) on the same graph paper and join these points to form the line CD representing the equation (2) as shown in the same figure.




In the figure, we observe that the two lines intersect at the point A(3, 5).

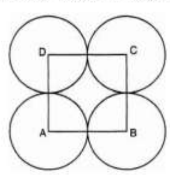
So, x = 3 and y = 5 is the required solution of the pair of linear equations for used, i.e., the cost of one pencil and a pen is Rs. 3 and Rs. 5 respectively.

Verification : Substituting x = 3 and y = 5 in (1) and (2), We find that both the equations are satisfied as shown below:

5x + 3y = 5(3) + 7(5) = 15 + 35 = 50

7x + 5y = 7(3) + 5(5) = 21 + 25 = 46

This verifies the solution.

34. 


Let $r$ cm be the radius of each circle.

Area of square - Area of 4 sectors = $\frac{24}{7}\text{cm}^2$

(side)$^2$ - 4 $\left[\frac{\theta}{360}\pi r^2\right] = \frac{24}{7}\text{cm}^2$

or, $(2r)^2 - 4\left(\frac{90^\circ}{360^\circ} \times \pi r^2\right) = \frac{24}{7}$

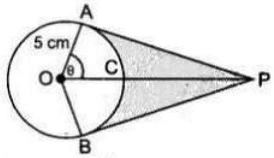
or, $(2r)^2 - 4\left(\frac{1}{4^\circ} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - \left(\pi r^2\right) = \frac{24}{7}$

or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$

or, $\frac{28r^2-22r^2}{7}=\frac{24}{7}$
or, $6r^2=24$
or, $r^2=4$
or, $r=\pm 2$
or, Radius of each circle is 2 cm (r cannot be negative)

OR




$\cos\theta=\frac{1}{2}$ or, $\theta=60^\circ$
Reflex $\angle AOB=120^\circ$
$\therefore$ ADB = $\frac{2\times 3.14\times 5\times 240}{360}$ = 20.93 cm
Hence length of elastic in contact = 20.93 cm
Now, AP = $5\sqrt{3}$cm
a (△OAP) = $\frac{1}{2}\times$ base $\times$ height = $\frac{1}{2}\times 5\times 5\sqrt{3}$ $=\frac{25\sqrt{3}}{2}$
Area (△OAP + △OBP) = 2$\times$ $\frac{25\sqrt{3}}{2}$ $=25\sqrt{3}$ = 43.25 cm$^2$
Area of sector OACB = $\frac{\theta}{360}\times\pi r^2$
$=\frac{25\times 3.14\times 120}{360}$ = 26.16 cm$^2$
Shaded Area = 43.25 - 26.16 = 17.09 cm$^2$

35. In a pack of 52 cards, Total number of outcomes = 52

i. Let $E_1$ be the event of getting a king of red suit.
Number of favorable outcomes = 2
Then, P(getting a king of red suit) = P($E_1$)=$\frac{\text{Number of outcomes favorable to } E_1}{\text{Number of all possible outcomes}}=\frac{2}{52}=\frac{1}{26}$
Thus, the probability of getting a king of red suit is $\frac{1}{26}$.

ii. Let $E_2$ be the event of getting a face card.
Number of favorable outcomes = 12
Then, P(getting a face card) = P($E_2$) =$\frac{\text{Number of outcomes favorable to } E_2}{\text{Number of all possible outcomes}}=\frac{12}{52}=\frac{3}{13}$
Thus, the probability of getting a face card is $\frac{3}{13}$.

iii. Let $E_3$ be the event of getting red face card.
Number of favorable outcomes = 6
Then, P(getting a red face card) = P($E_3$) = $\frac{\text{Number of outcomes favorable to } E_3}{\text{Number of all possible outcomes}}=\frac{6}{52}=\frac{3}{26}$
Thus, the probability of getting a red face card is $\frac{3}{26}$.

iv. Let $E_4$ be the event of getting a queen of black suit.
Number of favorable outcomes = 2
Then, P(getting a queen of black suit) = P($E_4$) =
$\frac{\text{Number of outcomes favorable to } E_4}{\text{Number of all possible outcomes}}=\frac{2}{52}=\frac{1}{26}$
Therefore, the probability of getting a queen of black suit is $\frac{1}{26}$.

v. Let $E_5$ be the event of getting a jack of hearts.
Number of favorable outcomes = 1

Then, P(getting a jack of hearts) = P($E_5$) = $\frac{\text{Number of outcomes favorable to } E_5}{\text{Number of all possible outcomes}} = \frac{1}{52}$

Then, the probability of getting a jack of hearts is $\frac{1}{52}$.

vi. Let $E_6$ be the event of getting a spade.

Number of favorable outcomes = 13

Then, P(getting a spade) = P($E_6$) = $\frac{\text{Number of outcomes favorable to } E_6}{\text{Number of all possible outcomes}} = \frac{13}{52} = \frac{1}{4}$

Therefore, the probability of getting a spade is $\frac{1}{4}$

## Section E

36. **Read the text carefully and answer the questions:**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We have, $a_3 = 600$ and

$a_3 = 600$

$\Rightarrow 600 = a + 2d$

$\Rightarrow a = 600 - 2d$ ...(i)

$\Rightarrow a_7 = 700$

$\Rightarrow a_7 = 700$

$\Rightarrow 700 = a + 6d$

$\Rightarrow a = 700 - 6d$ ...(ii)

From (i) and (ii)

$600 - 2d = 700 - 6d$

$\Rightarrow 4d = 100$

$\Rightarrow d = 25$

(ii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We know that first term = a = 550 and common difference = d = 25

$a_n = 1000$

$\Rightarrow 1000 = a + (n - 1)d$

$\Rightarrow 1000 = 550 + 25n - 25$

$\Rightarrow 1000 - 550 + 25 = 25n$

$\Rightarrow 475 = 25n$

$\Rightarrow n = \frac{475}{25} = 19$

(iii)Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

The production in the 10th term is given by $a_{10}$. Therefore, production in the 10th year = $a_{10}$ = a + 9d = 550 + 9 × 25 = 775. So, production in 10th year is of 775 TV sets.

OR

Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$

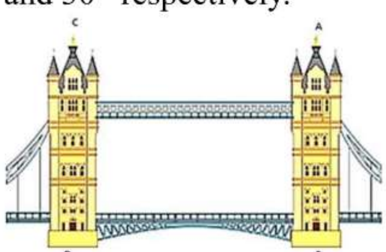
$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$

$\Rightarrow S_7 = 4375$

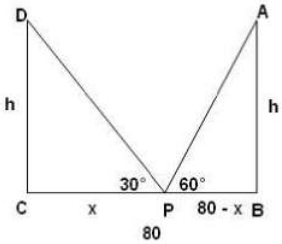
37. **Read the text carefully and answer the questions:**

Tower Bridge is a Grade I listed combined bascule and suspension bridge in London, built between 1886 and 1894, designed by Horace Jones and engineered by John Wolfe Barry. The bridge is 800 feet (240 m) in length and consists of two bridge towers connected at the upper level by two horizontal walkways, and a central pair of bascules that can open to allow shipping.

In this bridge, two towers of equal heights are standing opposite each other on either side of the road, which is 80 m wide. During summer holidays, Neeta visited the tower bridge. She stood at some point on the road between these towers. From that point between the towers on the road, the angles of elevation of the top of the towers was $60^{o}$ and $30^{o}$ respectively.



(i)




Suppose AB and CD are the two towers of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m, therefore BP = (80 – x).

Also, ∠APB = $60^{o}$ and ∠DPC = $30^{o}$

In right angled triangle DCP,

$\tan 30^{o} = \frac{CD}{CP}$

$\Rightarrow \frac{h}{x} = \frac{1}{\sqrt{3}}$

$\Rightarrow h = \frac{x}{\sqrt{3}}$ .......(i)

In right angled triangle ABP,

$\tan 60^{o} = \frac{AB}{AP}$

$\Rightarrow \frac{h}{80-x} = \sqrt{3}$

$\Rightarrow h = \sqrt{3}(80 - x)$

$\Rightarrow \frac{x}{\sqrt{3}} = \sqrt{3}(80 - x)$

$\Rightarrow$ x = 3(80 – x)

$\Rightarrow$ x = 240 – 3x

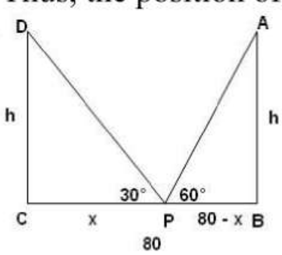
$\Rightarrow$ x + 3x = 240

$\Rightarrow$ 4x = 240

$\Rightarrow$ x = 60

Thus, the position of the point P is 60 m from C.

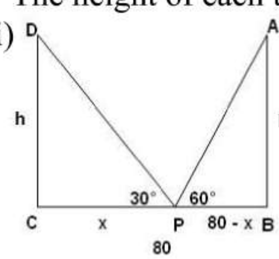
(ii)




Height of the tower, h = $\frac{x}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

The height of each tower is $20\sqrt{3}$ m.

(iii)




The distance between Neeta and top of tower AB.

In △ABP

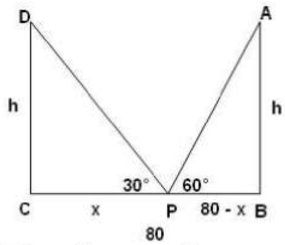
$\sin 60^{o} = \frac{AB}{AP}$

$\Rightarrow AP = \frac{AB}{\sin 60^{0}}$

$\Rightarrow AP = \frac{20\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AP = 40 m

OR




The distance between Neeta and top of tower CD.

In $\triangle$CDP

$\sin 30^{0} = \frac{CD}{PD}$

$\Rightarrow PD = \frac{CD}{\sin 30^{0}}$

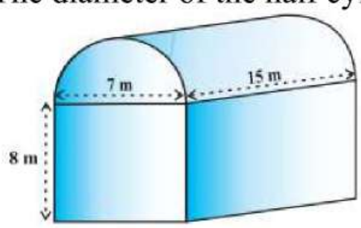
$\Rightarrow PD = \frac{20\sqrt{3}}{\frac{1}{2}} = 40\sqrt{3}$

$\Rightarrow PD = 40\sqrt{3}$

38. **Read the text carefully and answer the questions:**

Shanta runs an industry in a shed which was in the shape of a cuboid surmounted by half cylinder. The dimensions of the base were 15 m × 7 m × 8 m.

The diameter of the half cylinder was 7 m and length was 15 m.




(i) Total volume = volume of cuboid + $\frac{1}{2}$ × volume of cylinder.

For cuboidal part we have

length = 15 m, breadth = 7 m and height = 8 m

$\therefore$ Volume of cuboidal part = l × b × h $= 15 \times 7 \times 8\text{m}^2 = 840\text{m}^3$

Clearly,

r = Radius of half-cylinder = $\frac{1}{2}$ (Width of the cuboid) = $\frac{7}{2}$m

and, h = Height (length) of half-cylinder = Length of cuboid = 15 m

$\therefore$ Volume of half-cylinder = $\frac{1}{2}\pi r^2 h = \frac{1}{2} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 15\text{m}^3 = \frac{1155}{4}\text{m}^3 = 288.75\text{m}^3$

Thus the volume of the air that the shed can hold = (840 + 288.75) $m^3$ = 1128.75 $m^3$

(ii) Total space occupied by 20 workers = $20 \times 0.08\text{m}^3$ = = 1.6 $m^3$

Total space occupied by the machinery = 300 $m^3$

$\therefore$ Volume of the air inside the shed when there are machine and workers inside it

= (1128.75 - 1.6 - 300) $m^3$

= (1128.75 - 301.6) $m^3$ = 827.15 $m^3$

Hence, volume of air when there are machinery and workers is 827.15

(iii) Given for the cuboidal part

length L = 15 m, Width B = 7 m, Height = 8 m

Surface area of the cuboidal part

= 2(LB + BH + HL)

= 2(15 × 7 + 7 × 8 + 8 × 15)

= 2(105 + 56 + 120) = 2 × 281 = 562 $m^2$

OR

For the cylindrical part r = 3.5 cm and l = 15 m

Thus the surface area of the cylindrical part

$= \frac{1}{2}(2\pi rl) = 3.14 \times 3.5 \times 15$

= 164.85 $m^2$