Class- X Session- 2022-23

Subject- Mathematics (Basic)

Sample Question Paper - 6

with Solution

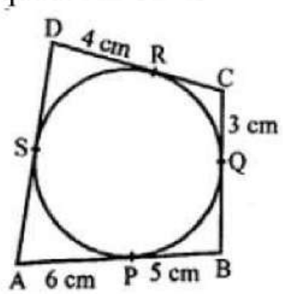
Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. In the given figure, quad. ABCD is circumscribed, touching the circle at P, Q, R and S. If AP = 6 cm, BP = 5 cm, CQ = 3 cm and DR = 4 cm, then perimeter of quad. ABCD is **[1]**



a) 36 cm b) 32 cm

c) 27 cm d) 18 cm

2. The distance of a point from the y-axis is called **[1]**

a) origin b) None of these

c) abscissa d) ordinate

3. The distance of a point from the x-axis is called **[1]**

a) None of these b) origin

c) abscissa d) ordinate

4. Which of the following can't be the probability of an event? **[1]**

a) $\frac{1}{3}$

b) 5%

c) $\frac{17}{16}$

d) 0.1

5. The centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by **[1]**

a) $\left(\frac{x_1+x_2+x_3}{6}, \frac{y_1+y_2+y_3}{6}\right)$

b) $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$

c) $\left(\frac{x_1+x_2+x_3}{4}, \frac{y_1+y_2+y_3}{4}\right)$

d) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

6. The probability of getting 2 heads, when two coins are tossed, is **[1]**

a) $\frac{1}{4}$

b) 1

c) $\frac{1}{2}$

d) $\frac{3}{4}$

7. If a cone is cut into two parts by a horizontal plane passing through the mid-point of its axis, the ratio of the volumes of the upper part and the cone is **[1]**

a) 1 : 2

b) 1 : 4

c) 1 : 6

d) 1 : 8

8. The probability of guessing the correct answer to a certain test questions is $\frac{x}{12}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then x = **[1]**

a) 6

b) 4

c) 2

d) 3

9. If the lines given by 3x + 2ky = 2 and 2x + 5y + 1 = 0 are parallel, then the value of k is **[1]**

a) $\frac{-5}{4}$

b) $\frac{3}{2}$

c) $\frac{15}{4}$

d) $\frac{2}{5}$

10. A quadratic equation whose one root is 3 is **[1]**

a) $x^2 - 5x + 6 = 0$

b) $x^2 - 6x - 6 = 0$

c) $x^2 - 5x - 6 = 0$

d) $x^2 + 6x - 5 = 0$

11. $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$ **[1]**

a) $\cos 60^o$

b) $\sin 60^o$

c) $\sin 30^\circ$

d)

$\tan 60^{o}$

12. (2+ $\sqrt{5}$) is [1]

a) an irrational number

b) not real number

c) a rational number

d) an integer

13. Point $P\left(\frac{a}{8}, 4\right)$ is the mid-point of the line segment joining the points A(- 5, 2) and B(4, 6). The value of **a** is: [1]

a) -4

b) 4

c) -8

d) -2

14. A boy is flying a kite, the string of the kite makes an angle of 30^{o} with the ground. If the height of the kite is 18 m, then the length of the string is [1]

a) 18 m

b) 36 m

c) $18\sqrt{3}m$

d) $36\sqrt{3}m$

15. If the product of the roots of the equation $x^2 - 3x + k = 10$ is -2 then the value of k is [1]

a) -8

b) 12

c) -2

d) 8

16. If $9^{x + 2} = 240 + 9^{x}$, then the value of x is [1]

a) 0.5

b) 0.1

c) 0.3

d) 0.2

17. The marks obtained by 9 students in Mathematics are 59, 46, 31, 23, 27, 40, 52, 35 and 29. The mean of the data is [1]

a) 30

b) 41

c) 23

d) 38

18. **Assertion (A):** $\sqrt{a}$ is an irrational number, where a is a prime number.
Reason (R): Square root of any prime number is an irrational number. [1]

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

19. **Assertion (A):** In a triangle PQR, X and Y are points on sides PQ and PR [1]

respectively, such that XY || QR, then $\frac{PX}{XQ} = \frac{PY}{YR}$.

Reason (R): Basic proportionality theorem.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. The graphic representation of the equations x + 2y = 3 and 2x + 4y + 7 = 0 gives a pair of **[1]**

a) parallel lines

b) none of these

c) coincident lines

d) intersecting lines

Section B

21. Form the pair of linear equations for the problem and find its solution by substitution method: **[2]**

Five year hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages ?

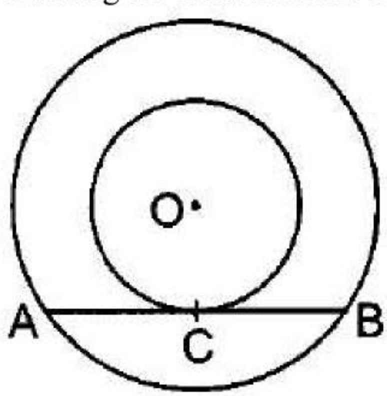
OR

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

22. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag. **[2]**

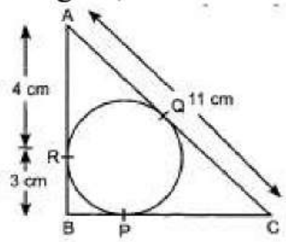
23. Find the relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5). **[2]**

24. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following: In figure, O is the centre of the two concentric circles. AB is a chord of the larger circle touching the smaller circle at C. Prove that AC = BC. **[2]**



OR

In figure, $\triangle$ABC is circumscribing a circle. Find the length of BC.



25. Find a quadratic polynomial, the sum and product of whose zeroes are -3 and 2 respectively. **[2]**

Section C

26. In $\triangle PQR$, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sinP, cosP and tanP. **[3]**

27. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction.After how many minutes will they meet again at the starting point? **[3]**

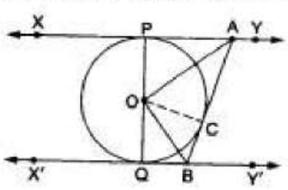
OR

There are 156, 208 and 260 students in groups A, B and C respectively. Buses are to be hired to take them for a field trip. Find the minimum number of buses to be hired, if the same number studets should be accommodated in each bus.

28. Solve 2x + 3y = 11 and 2x – 4y = – 24 and hence find the value of 'm' for which y = mx + 3. **[3]**

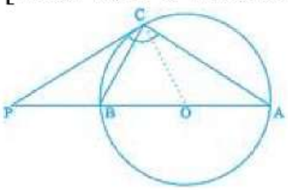
29. In triangles PQR and MST, $\angle$P = 55^{o}, $\angle$Q = 25^{o}, $\angle$M = 100^{o} and $\angle$S = 25^{o}. Is $\triangle$QPR ~ $\triangle$TSM? Why? **[3]**

30. In the given figure, XY and X 'Y ' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that $\angle AOB = 90°$. **[3]**



OR

The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle$PCA = 110^{o}, find $\angle$CBA.
[Hint: Join C with centre O].



31. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the jet plane. **[3]**

Section D

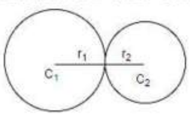
32. In a triangle $\triangle PQR$, N is a point on PR such that $QN \perp PR$. If $PN \times NR = QN^2$, then prove that $\angle PQR = 90°$. **[5]**

33. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two number **[5]**

OR

If roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal.

34. Two farmers have circular plots. The plots are watered with the same water source placed in the point common to both the plots as shown in the figure. The sum of their areas is 130π and the distance between their centres is 14 m.Find the radii of the circles.What value is depicted by the farmers? **[5]**



OR

A semicircular region and a square region have equal perimeters. The area of the square region exceeds that of the semicircular region by 4 cm^2. Find the perimeters and areas of the two regions.

35. The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories of India. Find the mean percentage of female teachers by assumed mean method. **[5]**

Percentage of female teachers	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of states/ U.T.	6	11	7	4	4	2	1

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Find the volume of the Hermika, if the side of cubical part is 10 m.

(ii) Find the volume of cylindrical base part whose diameter and height 48 m and 14 m.

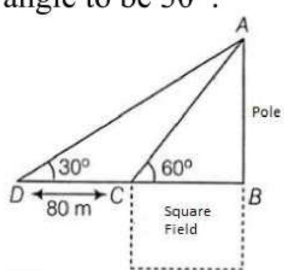
(iii) If the volume of each brick used is 0.01 m^3, then find the number of bricks used to make the cylindrical base.

OR

If the diameter of the Anda is 42 m, then find the volume of the Anda.

37. **Read the text carefully and answer the questions:** **[4]**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60^{o}. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30^{o}.



(i) Find the height of the pole too so that he can arrange a ladder accordingly to put an effigy on the pole.

(ii) Find the length of his square field so that he can buy material to do the fencing work accordingly.

(iii) Find the Distance from Farmer at position C and top of the pole?

OR

Find the Distance from Farmer at position D and top of the pole?

38. **Read the text carefully and answer the questions:** **[4]**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Find the amount saved by Sehaj on 10^{th} day.

(ii) Find the amount saved by Sehaj on 25^{th} day.

(iii) Find the total amount saved by Sehaj in 30 days.

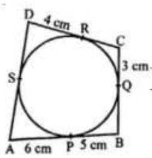
OR

Find in how many days Sehaj saves ₹1400.

Solution

Section A

1. **(a)** 36 cm

   Explanation: 

In the given figure, quad. ABCD is circumscribed touching the circle at P, Q, R and S
   AP = 6 cm, BP = 5 cm, CQ = 3 cm and DR = 4 cm.
   Now, we have to find the perimeter of the quad. ABCD.
   We know that tangents from an external point to the circle are equal.
   AP = AS = 6 cm
   BP = BQ = 5 cm
   CQ = CR = 3 cm
   DR = DS = 4 cm
   AB = AP + BP = 6 + 5 = 11 cm
   BC = BQ + CQ = 5 + 3 = 8 cm
   CD = CR + DR = 3 + 4 = 7 cm
   and DA = AS + DS = 6 + 4 = 10 cm
   Perimeter of the quad. ABCD
   = AB + BC + CD + DA
   = (11 + 8 + 7 + 10) cm
   = 36 cm
2. **(c)** abscissa

   Explanation: The distance of a point from the y-axis is the x (horizontal) coordinate of the point and is called abscissa.
3. **(d)** ordinate

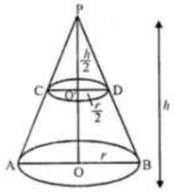
   Explanation: The distance of a point from the x-axis is the y (vertical) coordinate of the point and is called ordinate.
4. **(c)** $\frac{17}{16}$

   Explanation: $\frac{17}{16}$ cannot be the probability of an event because probability of an event cannot be greater than 1.
5. **(d)** $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

   Explanation: The centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
   It is the point of intersection of the three medians in the triangle. it is also called the centre of gravity of the triangle.
6. **(a)** $\frac{1}{4}$

   Explanation: All possible outcomes are HH, HT, TH, TT. Their number is 4.
   Getting 2 heads, means getting HH. Its number is 1.
   $\therefore$ P(getting 2 heads) = $\frac{1}{4}$
7. **(d)** 1 : 8

   Explanation: In the figure, C and D are the mid-points and CD || AB which divide the cone into two parts



Height OO' = $\frac{1}{2}$ OP and diameter CD = $\frac{1}{2}$ AB

Let h be the height and r be the radius of the cone, then

$\frac{h}{2}$ will be the height of the smaller cone and $\frac{r}{2}$ be its radius, then

Volume of bigger cone = $\frac{1}{3}\pi r^2 h$

and volume of smaller cone

$= \frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{h}{2}\right)$

$\therefore \frac{\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{h}{2}\right)}{\frac{1}{3}\pi r^2 h} = \frac{\frac{1}{3}\pi\frac{r^2}{4}\times\frac{h}{2}}{\frac{1}{3}\pi r^2 h}$

$= \frac{\frac{1}{8}\left(\frac{1}{3}\pi r^2 h\right)}{\frac{1}{3}\pi r^2 h} = \frac{1}{8}$

$\therefore$ Ratio = 1 : 8

8. **(b)** 4

Explanation: Probability of guessing the correct answer

$= \frac{x}{12}$

and probability of not guessing the correct

answer $= \frac{2}{3}$

$\frac{x}{12} + \frac{2}{3} = 1 \ \because (A + \bar{A} = 1)$

$\Rightarrow \frac{x}{12} = 1 - \frac{2}{3} = \frac{1}{3} \Rightarrow x = \frac{12}{3} = 4$

$\therefore$ x = 4

9. **(c)** $\frac{15}{4}$

Explanation: Condition for parallel lines is

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \ldots (i)$

Given lines,

3x + 2ky - 2 = 0 and

2x + 5y - 1 = 0;

Comparing with standard form,

Here, $a_1 = 3$, $b_1 = 2k$, $c_1 = -2$

and $a_2 = 2$, $b_2 = 5$, $c_2 = -1$

From Eq. (i),

$\frac{3}{2} = \frac{2k}{5}$

$k = \frac{15}{4}$

10. **(a)** $x^2 - 5x + 6 = 0$

Explanation: since 3 is the root of the equation, x = 3 must satisfy the equation.

Applying x = 3 in the equation $x^2 - 5x + 6 = 0$

gives, $(3)^2 - 5(3) + 6 = 0$

$\Rightarrow$ 9 - 15 + 6 =0

$\Rightarrow$ 15 - 15 = 0

$\Rightarrow 0 = 0$

$\Rightarrow$ L.H.S. = R.H.S.

Hence, $x^2 - 5x + 6 = 0$ is a required equation which has 3 as root.

11. **(b)** sin 60^o

Explanation: $\frac{2tan30^o}{1+tan^2 30^o} = \frac{2\times\frac{1}{\sqrt{3}}}{1+(\frac{1}{\sqrt{3}})^2}$

$\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$

= $\sin 60^o$

12. **(a)** an irrational number

Explanation: The sum of a rational and an irrational number is an irrational number hence it is an irrational number.

13. **(a)** -4

Explanation: We have given that the mid point of A(-5, 2), B(4, 6) is p = $(\frac{a}{8}, 4)$

the mid point of A(-5, 2), B(4, 6) = $(\frac{-1}{2}, 4)$

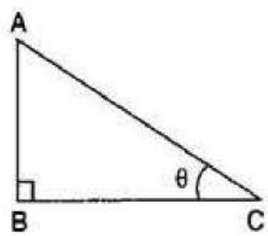
so $\frac{a}{8} = \frac{-1}{2}$

2a = -8

a = $\frac{-8}{2}$

a = -4

14. **(b)** 36 m

Explanation: 

Let Height of the kite AB = 18 m, length of the string = AC and angle of elevation = $\theta = 30^\circ$

$\therefore \sin 30^\circ = \frac{\text{AB}}{\text{AC}}$

$\Rightarrow \frac{1}{2} = \frac{18}{\text{AC}}$

$\Rightarrow$ AC = 36 m

15. **(d)** 8

Explanation: Given equation is $x^2 - 3x + (k - 10) = 0$.

Product of roots = (k - 10).

So, k - 10 = -2 $\Rightarrow$ k = 8.

16. **(a)** 0.5

Explanation: $9^{x+2} = 240 + 9^x$

$\Rightarrow 9^x \times 9^2 = 240 + 9^x$

$\Rightarrow 9^x (81 - 1) = 240$

$\Rightarrow 9^x = 3$

$\Rightarrow 9^x = 9^{1/2}$

$\Rightarrow x = \frac{1}{2} = 0.5$

17. **(d)** 38

Explanation: Given: 59, 46, 31, 23, 27, 40, 52, 35 and 29

Mean = $\frac{\text{Sum of all observations}}{\text{Number of observations}}$

$= \frac{59+46+31+23+27+40+52+35+29}{9}$

$= \frac{342}{9}$

= 38

18. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: As we know that square root of every prime number is an irrational number. So, both assertion and reason are correct and reason explains assertion.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Statement of Basic Proportionality Theorem (Thale's Theorem).

20. **(a)** parallel lines

Explanation: Given: Two equations, x + 2y = 3

⇒ x + 2y – 3 = 0 .... (i)

2x + 4y + 7 = 0 .... (ii)

We know that the general form for a pair of linear equations in 2 variables x and y is $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

Comparing with above equations,

we have $a_1 = 1$, $b_1 = 2$, $c_1 = -3$; $a_2 = 2$, $b_2 = 4$, $c_2 = 7$

$\frac{a_1}{a_2} = \frac{1}{2}; \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}; \frac{c_1}{c_2} = \frac{-3}{7}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

∴Both lines are parallel to each other.

Section B

21. Let x (in years) be the present age of Jacob's son and y (in years) be the present age of Jacob. 5 years hence, it has relation:

(y + 5) = 3(x + 5)

or, y + 5 = 3x + 15

3x + 15 - y - 5 = 0

or, 3 x - y + 10 = 0 ....(i)

5 years ago, it has relation

(y - 5) = 7(x - 5)

y - 5 = 7x - 35

or, 7x - 35 - y + 5 = 0

or, 7 x - y - 3 0 = 0 ....(ii)

From eqn. (i), y = 3x + 10 ....(iii)

On substituting the value of y in eqn. (ii), we get

7x-(3x + 10) - 30 = 0

7x - 3x - 10 - 30 = 0

or, 4x - 40 = 0

or, 4x = 40

x=10

On substituting x = 10 in eqn. (iii),

$y = 3 \times 10 + 10$

y = 30 + 10

∴ y = 40

Hence, the present age of Jacob = 40 years and son's age = 10 years

OR

Let the numerator and denominator of fraction be x and y respectively.

Then, the fraction is $\frac{x}{y}$.

As per first condition

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator.

x + y = 2x + 4

⇒ -x + y = 4........(i)

According to the second condition,

If the numerator and denominator are increased by 3, they are in the ratio 2 : 3.

$\frac{x+3}{y+3} = \frac{2}{3}$

⇒ 3x + 9 = 2y + 6

⇒3x - 2y = -3.......(ii)

Multiply (i) by -2, we get

-2x + 2y = 8 .... (iii)

Adding (ii) and (iii) , we get

and 3x - 2x = -3 + 8

⇒ x = 5

Substituting x = 5 in (i), we get

5 - y = 4

y = 9

Hence, the required fraction is $\frac{5}{9}$

22. Let there be x blue balls in the bag.

∴Total number of balls in the bag = $5 + x$

Now, P_1 = Probability of drawing a blue ball = $\frac{x}{5+x}$

And P_2 = Probability of drawing a blue ball = $\frac{5}{5+x}$

But according to question, $P_1 = 2P_2$

$\frac{x}{5+x} = 2 \times \frac{5}{5+x}$

$\frac{x}{5+x} \times \frac{5+x}{5} = 2$

x = 10

Hence, there are 10 blue balls in the bag.

23. Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5)

AP = BP (Given)

⇒ $AP^2 = BP^2$

⇒ $(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$

⇒ $x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$

⇒ 49 - 14x + 1 - 2y = 9 - 6x + 25 - 10y

⇒ - 14x + 6x - 2y + 10 y = 34 - 50

⇒ -8x + 8y = -16

⇒ x - y = 2

24. Construction : Draw OC

Proof : Line AB is tangent to smaller circle at point C.

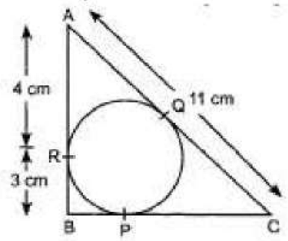
∴ segment $OC \perp AB$

AB is chord to larger circle and

as perpendicular drawn from centre to chord bisects the chord.
$\therefore AC = CB$

OR

Given,



$AR = 4\ cm.$
Also, $AR = AQ \Rightarrow AQ = 4cm$
Now, $QC = AC - AQ$
$= 11cm - 4cm = 7cm$ ...(i)
Also, $BP = BR$
$\therefore$ BP = 3 cm and PC = QC
$\therefore$ PC = 7 cm [From (i)]
$BC = BP + PC$
$= 3cm + 7cm$
$= 10cm$

25. The standard form of quadratic polynomial be given as: $ax^2 + bx + c$, and its zeroes will be α and β.
We have
$\alpha + \beta = -3 = \frac{-b}{a}$
and $\alpha\beta = 2 = \frac{c}{a}$
If a = 1, then b = 3 and c = 2. So, one quadratic polynomial which fits the given conditions is $x^2 + 3x + 2$

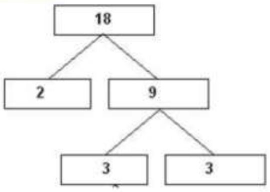
Section C

26.

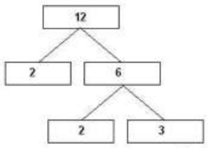
In $\triangle PQR$ by Pythagoras theorem
$PR^2 = PQ^2 + QR^2$
$\Rightarrow (25 - QR)^2 = 5^2 + QR^2 [\because PR + QR = 25 \text{ cm} \Rightarrow PR = 25 - \text{QR}]$
$625 - 50QR + QR^2 = 25 + QR^2$
$\Rightarrow 600 - 50QR = 0$
$\Rightarrow QR = \frac{600}{50} = 12$ cm
Now, PR + QR = 25 cm
$\Rightarrow$ PR = 25 - Q R = 25 - 12 = 13 cm
Hence, $\sin P = \frac{QR}{PR} = \frac{12}{13}, \cos P = \frac{PQ}{PR} = \frac{5}{13}$ and, $\tan P = \frac{QR}{PQ} = \frac{12}{5}$

27. By taking LCM of time taken (in minutes) by Sonia and Ravi, We can get the actual number of minutes after which they meet again at the starting point after both start at the same point and at the same time, and go in the same direction.

$18 = 2 \times 3 \times 3 = 2 \times 3^2$



$12 = 2 \times 2 \times 3 = 2^2 \times 3$



$\text{LCM }(18, 12) = 2^2 \times 3^2 = 36$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

OR

Given numbers are 156, 208 and 260.

Here, 260 > 208> 156

Let us find the HCF of 260 and 208,

By using Euclid's division lemma for 260 and 208,

we get

$260 = (208 \times 1) + 52$

Here , the remainder is 52, not zero.

On taking 208 as new dividend and 52 as new divisor and then apply Euclid's division lemma, we get

$208 = (52 \times 4) + 0$

Here, the remainder is zero and the divisor is 52.

So, HCF of 208 and 260 is 52.

Now, 156 >52

Let us find the HCf of 52 and 156. By using Euclid's division lemma , we get

$156 = (52 \times 3) + 0$

Here, the remiander is zero and the divisor is 52.

So, HCF of 52 amd 156 is 52.

Thus, HCf of 156, 208 and 260 is 52.

Hence, the minimum number of buses

$= \frac{156}{52} + \frac{208}{52} + \frac{260}{52} = \frac{156+208+260}{52} = \frac{624}{52} = 12$

The minimum number of buses is 12.

28. The given pair of linear equations

$2x + 3y = 11$ ...... (1)

$2x - 4y = -24$ .... (2)

From equation (1), $3y = 11 - 2x$

$\Rightarrow \quad y = \frac{11-2x}{3}$

Substituting this value of y in equation (2), we get

$2x - 4\left(\frac{11-2x}{3}\right) = -24$

$\Rightarrow 6x - 44 + 8x = -72$

$\Rightarrow 14x - 44 = -72$

$\Rightarrow 14x = 44 - 72$

$\Rightarrow 14x = -28$

$\Rightarrow x = -\frac{28}{14} = -2$

Substituting this value of x in equation (3), we get
$y=\frac{11-2(-2)}{3}=\frac{11+4}{3}=\frac{15}{3}=5$
Verification, Substituting x = -2 and y = 5, we find that both the equations (1) and (2) are satisfied as shown below:
$2x+3y=2(-2)+3(5)=-4+15=11$
$2x-4y=2(-2)-4(5)=-4-20=-24$
This verifies the solution,
Now, y = axe + 3
$\Rightarrow 5=m(-2)+3$
$\Rightarrow -2m=5-3$
$\Rightarrow -2m=2$
$\Rightarrow$ m $=\frac{2}{-2}=-1$

29. We know that, the sum of three angles of a triangle is 180^{o}.

In $\triangle$PQR, $\angle P+\angle Q+\angle R=180^{o}$

$\Rightarrow 55^{o}+25^{o}+\angle R=180^{o}$

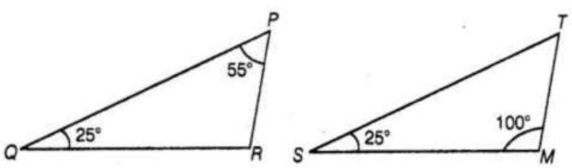
$\Rightarrow \angle R=180^{o}-(55^{o}+25^{o})=180^{o}-80^{o}=100^{o}$

In $\triangle$TSM, $\angle T+\angle S+\angle M=180^{o}$

$\Rightarrow \angle T+\angle 25^{o}+1000^{o}=180^{o}$

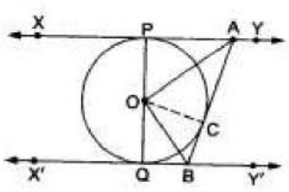
$\Rightarrow \angle T=180^\circ-(25^{o}+100^{o})$

$=180^\circ-125^{o}=55^{o}$



In $\triangle$PQR and $\triangle$TSM, we have
$\angle P=\angle T$, $\angle Q=\angle S$,
and $\angle R=\angle M$
Therefore, $\triangle PQR \sim \triangle TSM$ [since, all corresponding angles are equal]
Hence, $\triangle$QPR is not similar to $\triangle$TSM, since correct correspondence is P $\leftrightarrow$ T, Q < r $\leftrightarrow$ S and R $\leftrightarrow$ M.

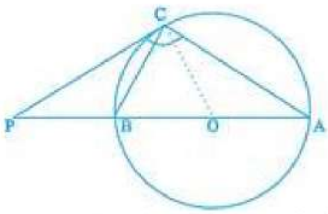
30. According to the question, XY and X 'Y ' are x two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X 'Y ' at B.



In quad. APQB, we have
$\angle APO=90^\circ$
and $\angle BQO=90^\circ$ [$\because$ tangent at any point is perpendicular to the radius through the point of contact]
Now, $\angle APO+\angle BQO+\angle QBC+\angle PAC=360^\circ$
$\Rightarrow \angle PAC+\angle QBC=360^\circ-(\angle APO+\angle BQO)=180^\circ$ ...(i)
We have,

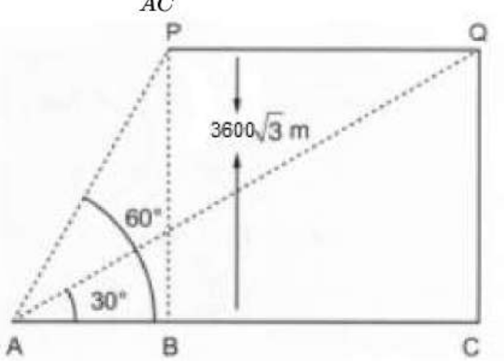
$\angle CAO = \frac{1}{2}\ \angle PAC$
and $\angle CBO = \frac{1}{2} \angle QBC$ [$\because$ tangents from an external point are equally inclined to the line segment joining the centre to that point]
$\therefore \angle \text{CAO} + \angle \text{CBO} = \frac{1}{2}\ (\angle PAC + \angle QBC) = \frac{1}{2} \times 180^\circ = 90^\circ$ .....(ii)
In $\triangle AOB$, we have
$\angle CAO + \angle AOB + \angle CBO = 180^\circ$
$\Rightarrow 90^\circ + \angle AOB = 180^\circ$
$\Rightarrow \quad \angle AOB = 90^\circ$

OR



Let D be the centre of the circle.
A, D, B, P all are on the same line and P and C are points on the tangent.
Now, $\angle$BCA is inscribed in a semi-circle, $\angle BCA = 90^\circ$
C is the point on the circle where the tangent touches the circle.
So, $\angle \text{DCP} = 90^\circ$
$\angle \text{PCA} = \angle \text{PCD} + \angle \text{DCA}$
$\Rightarrow 110^{\text{o}} = 90^{\text{o}} + \angle$DCA
$\Rightarrow \angle DCA = 20^\circ$
In $\triangle$ADC,
AD = DC .... (Radii of the same circle)
$\Rightarrow \angle DCA = \angle CAD = 20^\circ$
In $\triangle$ABC,
$\angle \text{BCA} = 90^\circ, \angle \text{CAB} = 20^\circ$
So, $\angle CBA = 70^\circ$

31. Let P and Q be the two positions of the plane and let A be the point of observation. Let ABC be the horizontal line through A. It is given that angles of elevation of the plane in two positions P and Q from point A are 60° and 30° respectively.
$\therefore \angle$PAB = 60^{o}, $\angle$QAB = 30^{o}. It is also given that PB = 3600 $\sqrt{3}$ metres
In $\triangle$ ABP, we have
tan $60^{\text{o}} = \frac{BP}{AB}$
$\Rightarrow \sqrt{3} = \frac{3600\sqrt{3}}{AB}$
$\Rightarrow$ AB = 3600 m
In $\triangle$ ACQ, we have
tan $30^{\text{o}} = \frac{CQ}{AC}$



$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{AC}$

$\Rightarrow$ AC = 3600 × 3 = 10800 m

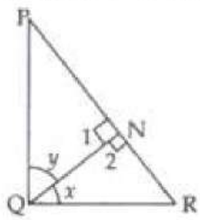
$\therefore$ PQ = BC = AC - AB = 10800 - 3600 = 7200 m

Thus, the plane travels 7200 m in 30 seconds.

Hence, Speed of plane = $\frac{7200}{30}$ = 240 m / sec = $\frac{240}{1000} \times 60 \times 60$ = 864 km / hr

Section D

32. Given: $\triangle$PQR in which QN $\perp$ PR and PN × NR = QN^2.



To Prove: $\angle PQR = 90^\circ$

Proof: **In** $\triangle$ QNP **and** $\triangle$ QNR

QN $\perp$ PR [Given]

$\therefore \angle 1 = \angle 2 = 90$

$QN^2 = PN \times NR\ (Given)$

$\mathrm{QN}^2 = \mathrm{NR} \times \mathrm{NP}$

$\Rightarrow \frac{\mathrm{QN}}{\mathrm{NR}} = \frac{\mathrm{NP}}{\mathrm{QN}}$

$\therefore \triangle \mathrm{PNQ} \sim \triangle \mathrm{RNQ}$

$\angle \mathrm{P} = \angle \mathrm{RQN} = \mathrm{x} \ldots\ldots (1)$

$\angle \mathrm{PQN} = \angle \mathrm{R} = \mathrm{y} \ldots\ldots. (2)$

In $\triangle$ PQR

$\angle \mathrm{P} + \angle \mathrm{PQR} + \angle \mathrm{R} = 180^\circ$ [Angle sum property of a triangle]

$\Rightarrow$ x+x+y+y=180°[Using (1) and (2)]

$\Rightarrow 2\mathrm{x} + 2\mathrm{y} = 180^\circ$

$\Rightarrow \mathrm{x} + \mathrm{y} = 90^\circ$

$\Rightarrow \angle \mathrm{PQR} = 90^\circ$

Hence proved

33. Let smaller number = x and let larger number = y

According to condition:

$y^2 - x^2 = 180$ .... (1)

Also, we are given that square of smaller number is 8 times the larger number.

$\Rightarrow x^2 = 8y$ .... (2)

Putting equation (2) in (1), we get

$y^2 - 8y = 180$

$\Rightarrow y^2 - 8y - 180 = 0$

Comparing equation $y^2 - 8y - 180 = 0$ with general form $ay^2 + by + c = 0$,

We get $a = 1$, $b = -8$ and $c = -180$

Using quadratic formula $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-180)}}{2 \times 1}$

$\Rightarrow y = \frac{8 \pm \sqrt{64 + 720}}{2}$

$\Rightarrow y=\frac{8\pm\sqrt{784}}{2}=\frac{8\pm 28}{2}$

$\Rightarrow y=\frac{8+28}{2},\frac{8-28}{2}$

$\Rightarrow y$ =18, -10

Using equation (2) to find smaller number: $x^2 = 8y$

$\Rightarrow x^2 = 8y = 8 \times 18 = 144$

$\Rightarrow x = \pm 12$

And, $x^2 = 8y = 8\times - 10 = -80$ {No real solution for x}

Therefore two numbers are (12,18) or (−12, 18)

OR

For equal roots of $x^2+2px+mn=0, 4p^2-4mn=0$

or, $p^2=mn$ ......(i)

For equal roots of

$x^2-2(m+n)x+\left(m^2+n^2+2p^2\right)=0$

$4(m+n)^2-4\left(m^2+n^2+2p^2\right)=0$

$m^2+n^2+2mn-m^2-n^2-2(mn)=0$ {From (i))

$\therefore$ If root of $x^2+2px+mn=0$ are equal then those of

$x^2-2a(m+n)x+\left(m^2+n^2+2p^2\right)=0$ are equal.

34. Let the radii of the two circular plots be r_1 and r_2, respectively.

Then, $r_1 + r_2 = 14$ [$\because$ Distance between the centres of two circular plots = 14 cm,given]....(i)

Also, Sum of Areas of the plots = 130π

$\therefore \pi r_1^2+\pi r_2^2=130\pi \Rightarrow r_1^2+r_2^2=130$ ...(ii)

Now, from equation (i) and equation (ii),

$\Rightarrow (14-r_2)^2+r_2^2=130$

$\Rightarrow 196-2r_2+2r_2^2=130$

$\Rightarrow 66-2r_2+2r_2^2=0$

Solving the quadratic equation we get,

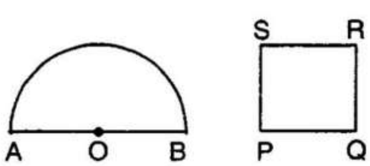
$r_2 = 3$ or $r_2 = 11$,

but from figure it is clear that, $r_1 > r_2$

$\therefore r_1 = 11$ cm and $r_2 = 3$ cm

The value depicted by the farmers are of cooperative nature and mutual understanding.

OR



Let radius of semicircular region be r units.

Perimeter = 2r + πr

Let side of square be x units

Perimeter = 4x units.

A.T.Q, 4x = 2r + πr $\Rightarrow x=\frac{2r+\pi r}{4}$

Area of semicircle = $\frac{1}{2}\pi r^2$

Area of square = x^2

A.T.Q, $x^2=\frac{1}{2}\pi r^2+4$

$\Rightarrow \left(\frac{2r+\pi r}{4}\right)^2 = \frac{1}{2}\pi r^2 + 4$
$\Rightarrow \frac{1}{16}\left(4r^2 + \pi^2 r^2 + 4\pi r^2\right) = \frac{1}{2}\pi r^2 + 4$
$\Rightarrow 4r^2 + \pi^2 r^2 + 4\pi r^2 = 8\pi r^2 + 64$
$\Rightarrow 4r^2 + \pi^2 r^2 - 4\pi r^2 = 64$
$\Rightarrow r^2\left(4 + \pi^2 - 4\pi\right) = 64$
$\Rightarrow r^2(\pi - 2)^2 = 64$
$\Rightarrow r = \sqrt{\frac{64}{(\pi-2)^2}}$
$\Rightarrow r = \frac{8}{\pi-2} = \frac{8}{\frac{22}{7}-2}$ = 7 cm

Perimeter of semicircle $= 2 \times 7 + \frac{22}{7} \times 7$ = 36 cm

Perimeter of square = 36 cm

Side of square = $\frac{36}{4}$= 9 cm

Area of square = 9× 9 = 81 cm^2

Area of semicircle = $\frac{\pi r^2}{2} = \frac{22}{2\times 7} \times 7 \times 7$ = 77 cm^2

35. Let, $a = 50$

C.I.	Number of states/ U.T. (f_i)	x_i	$d_i = x_i - 50$	$f_i d_i$
15 - 25	6	20	-30	-180
25 - 35	11	30	-20	-220
35 - 45	7	40	-10	-70
45 - 55	4	50	0	0
55 - 65	4	60	10	40
65 - 75	2	70	20	40
75 - 85	1	80	30	30

From table, $\Sigma f_i d_i = -360, \Sigma f_i = 36$

we know that, mean=$\overline{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$= 50 + \frac{-360}{35}$

$= 39.71$

Section E

36. **Read the text carefully and answer the questions:**

Ashish is a Class IX student. His class teacher Mrs Verma arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called

Hermika at the top. Path around Anda is known as Pradakshina Path.

(i) Volume of Hermika = side^3 = 10 × 10 × 10 = 1000 m^3

(ii) r = radius of cylinder = 24, h = height = 16

Volume of cylinder = $\pi r^2 h$

$\Rightarrow$ V = $\frac{22}{7} \times 24 \times 24 \times 14$ = 25344 m^3

(iii) Volume of brick = 0.01 m^3

$\Rightarrow$ n = Number of bricks used for making cylindrical base = $\frac{\text{Volume of cylinder}}{\text{Volume of one brick}}$

$\Rightarrow n = \frac{25344}{0.01}$ = 2534400

OR

Since Anda is hespherical in shape r = radius = 21

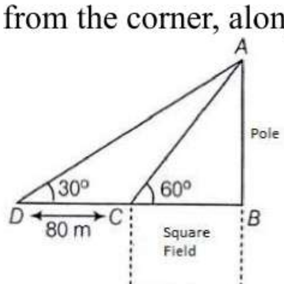
V = Volume of Anda = $\frac{2}{3} \times \pi \times r^3$

$\Rightarrow$ V = $\frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

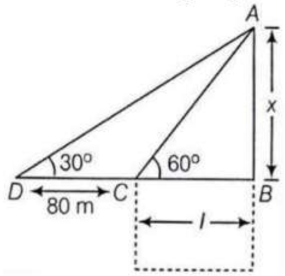
$\Rightarrow$ V = 44 × 21 × 21 = 19404 m^3

37. **Read the text carefully and answer the questions:**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is 60°. When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30°.



(i) The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

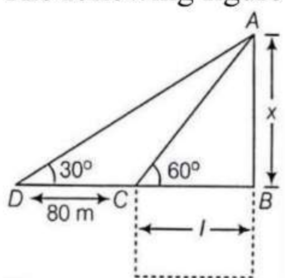
Now, l = 40 metres

We get,

$x = \sqrt{3}\, l = 40\sqrt{3} = 69.28$

Thus, height of the pole is 69.28 metres.

(ii) The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

In △ABC,

$\tan 60^{o} = \frac{x}{l}$

$\sqrt{3} = \frac{x}{l}$

$x = \sqrt{3}\, l$ ...(i)

Now, in △ABD,

$\tan 30^{o} = \frac{x}{80+l}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}l}{80+l}$ (From eq(i))

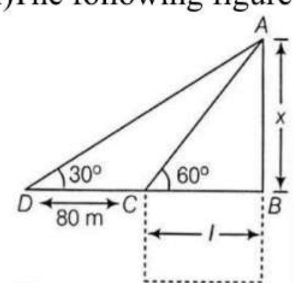
$80 + l = 3l$

$2l = 80$

$l = 40$

Thus, length of the field is 40 metres.

(iii)The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

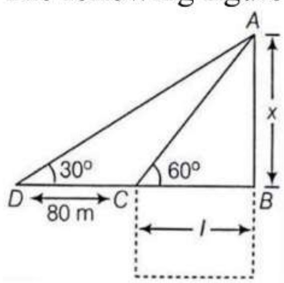
Distance from Farmer at position C and top of the pole is AC.

In △ABC

$\cos 60^{o} = \frac{CB}{AC}$
$\Rightarrow AC = \frac{CB}{\cos 60^{0}}$
$\Rightarrow AC = \frac{40}{\frac{1}{2}}$
$\Rightarrow$ AC = 80 m

OR

The following figure can be drawn from the question:



Here AB is the pole of height x metres and BC is one side of the square field of length l metres.
Distance from Farmer at position D and top of the pole is AD
In △ABC
$\cos 30^{o} = \frac{DB}{AD}$
$\Rightarrow AD = \frac{DB}{\cos 30^{0}}$
$\Rightarrow AD = \frac{120}{\frac{\sqrt{2}}{2}} = \frac{240}{\sqrt{3}}$
$\Rightarrow$ AC = 138.56 m

38. **Read the text carefully and answer the questions:**
Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Money saved on 1st day = ₹27.5
∵ Sehaj increases his saving by a fixed amount of ₹2.5
∴ His saving form an AP with a = 27.5 and d = 2.5
∴ Money saved on 10th day,
a_{10} = a + 9d = 27.5 + 9(2.5)
= 27.5 + 22.5 = ₹50
(ii) a_{25} = a = 24d
= 27.5 + 24(2.5)
= 27.5 + 60 = ₹ 87.5
(iii)Total amount saved by Sehaj in 30 days.
= $\frac{30}{2}[2 \times 27.5 + (30 - 1) \times 2.5]$

= 15(55 + 29(2.5)
= ₹1912.5

OR

Let S_n = 387.5, a = 27.5 and d = 2.5

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 387.5 = \frac{n}{2}[2 \times 27.5 + (n-1)2.5]$

$\Rightarrow 387.5 = \frac{n}{2}[55 + (n-1) \times 2.5]$

$\Rightarrow$ 775 = 55n + 2.5n^2 - 2.5n

$\Rightarrow$ 25n^2 + 525n = 7750 = 0

$\Rightarrow$ n^2 + 21n - 310 = 0

$\Rightarrow$ (n + 31)(n - 10) = 0

$\Rightarrow$ n = -31 reject n = 10 accept

So in 10 years Sehaj saves ₹ 387.5.