Class- X Session- 2022-23

Subject- Mathematics (Basic)

Sample Question Paper - 8

with Solution

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. From an external point Q, the length of the tangent to a circle is 5 cm and the distance of Q from the centre is 8 cm. The radius of the circle is: **[1]**

a) 3 cm b) 7 cm

c) 39 cm d) $\sqrt{39}$ cm

2. The point (-3, 5) lies in the ________ quadrant **[1]**

a) 4th b) 2nd

c) 3rd d) 1st

3. The distance of a point from the y-axis is called **[1]**

a) origin b) None of these

c) abscissa d) ordinate

4. An event is unlikely to happen. Its probability is closest to **[1]**

a) 0.00001 b) 0.0001

c) 0.1 d) 1

5. The pair of linear equations ax + by = c and px + qy = r has a unique solution then **[1]**

a) $aq \neq bp$ b) $aq = bp$

c) ap = bq

d) $ap \neq bq$

6. If the probability of an event is 'p', the probability of its complementary event will be **[1]**

a) p

b) p – 1

c) 1 – p

d) $1 - \frac{1}{p}$

7. If the distance between the points (4, p) and (1,0) is 5, then the value of p is **[1]**

a) 0

b) 4 only

c) -4 only

d) ± 4

8. If P(E) denotes the probability of an event E then **[1]**

a) $0 \leq P(E) \leq 1$

b) $-1 \leq P(E) \leq 1$

c) $P(E) < 0$

d) $P(E) > 0$

9. A solid is hemispherical at the bottom and conical above. If the surface areas of the two parts are equal, then the ratio of its radius and the height of its conical part is **[1]**

a) 1 : 1

b) $1 : \sqrt{3}$

c) $\sqrt{3} : 1$

d) 1 : 3

10. If 2 is a root of the equation $x^2 + bx + 12 = 0$ and the equation $x^2 + bx + q = 0$ has equal roots, then q = **[1]**

a) 8

b) -16

c) 16

d) -8

11. If p = -7 and q = 12 and $x^2 + px + q = 0$, Then the value of x is **[1]**

a) 3 and 4

b) 3 and -4

c) -3 and -4

d) -3 and 4

12. The HCF of 135 and 225 is: **[1]**

a) 5

b) 15

c) 45

d) 75

13. The distance of the point P(-6, 8) from the origin is **[1]**

a) $2\sqrt{7}$

b) 6

c) 8

d) 10

14. If $\tan\theta = \sqrt{3}$, then $\sec\theta =$ **[1]**

a) $\sqrt{\frac{3}{2}}$

b) 2

c) $\frac{2}{\sqrt{3}}$

d) $\frac{1}{\sqrt{3}}$

15. Consider the following frequency distribution: **[1]**

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

The upper limit of the median class is

a) 18.5

b) 17.5

c) 18

d) 17

16. A river is 60 m wide. A tree of unknown height is on one bank. The angle of elevation of the top of the tree from the point exactly opposite to the foot of the tree, on the other bank, is 30°. The height of the tree is **[1]**

a) $30\sqrt{3}$ m

b) $10\sqrt{3}$ m

c) $20\sqrt{3}$ m

d) $60\sqrt{3}$ m

17. A pair of linear equations which has a unique solution x = 2, y = –3 is **[1]**

a) $x - 4y - 14 = 0$
$5x - y + 13 = 0$

b) $2x - y = 1$
$3x + 2y = 0$

c) $x + y = -1$
$2x - 3y = -5$

d) $2x + 5y = -11$
$4x + 10y = -22$

18. If $m^2 - 1$ is divisible by 8, then m is **[1]**

a) an odd integer

b) a natural number

c) an even integer

d) a whole number

19. **Assertion (A):** D and E are points on the sides AB and AC respectively of a $\triangle ABC$ such that $DE \| BC$ then the value of x is 4, when AD = x cm, DB = (x - 2) cm, AE = (x + 2) cm and EC = (x - 1) cm. **[1]**

Reason (R): If a line is parallel to one side of a triangle then it divides the other two sides in the same ratio.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** 2 is a rational number. **[1]**

Reason (R): The square roots of all positive integers are irrationals.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the pair of linear equations are consistent, or inconsistent: $\frac{4}{3}x$ + 2y = 8; 2x + 3y = 12. **[2]**

OR

Ratio between the girls and boys in a class of 40 students is 2:3. Five new students joined the class. How many of them must be boys so that the ratio between girls and boys becomes 4: 5?

22. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is:(i) red? (ii) not red? **[2]**

23. Find the area of the rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order. **[2]**
[Hint: Area of a rhombus $= \frac{1}{2}$ (product of its diagonals)]

24. A point P is 25 cm away from the centre of a circle and the length of tangent drawn from P to the circle is 24 cm. Find the radius of the circle. **[2]**

OR

PQ is a tangent drawn from a point P to a circle of centre O and QOR is a diameter of the circle such that ∠POR = 110°, Find ∠OPQ.

25. Find the zeroes of a quadratic polynomial given as $t^2 - 15$ and verify the relationship between the zeroes and the coefficients. **[2]**

Section C

26. The cost of 2 kg of apples and 1 kg of grapes in a day was found to be Rs.160. After a month, the cost of 4 kg of apples and 2 kg of grapes in Rs.300. Represent the situation algebraically and geometrically. **[3]**

27. In a right triangle ABC, right angled at C, if tan A = 1, then verify that 2sin A cos A = 1 **[3]**

28. Prove that $\sqrt{p}+\sqrt{q}$ is irrational, where p, q are primes. **[3]**

OR

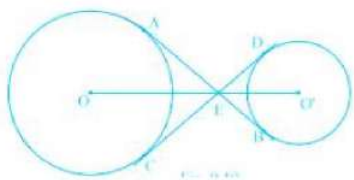
Find the LCM of the following polynomials:a^8-b^8 and $\left(a^4-b^4\right)(a+b)$

29. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower. **[3]**

30. A ladder rests against a wall at an angle a to the horizontal. Its foot is pulled away from the wall through a distance a, so that it slides a distance b down the wall making an angle $\beta$with the horizontal. Show that **[3]**

$\frac{a}{b}=\frac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$

31. The common tangents AB and CD to two circles with centres O and O' intersect at E between their centres. Prove that the points O, E and O' are collinear. **[3]**



OR

A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

Section D

32. Solve the quadratic equation by factorization: **[5]**

$\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}; x\neq 0,-1,2$

OR

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of field.

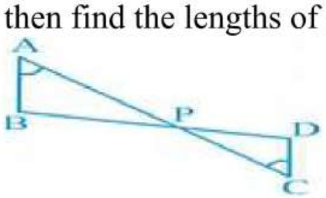
33. A chord of a circle of radius 10cm subtends a right angle at the center. Find the area of the corresponding: (Use π = 3.14) **[5]**

i. minor sector
ii. major sector
iii. minor segment
iv. major segment

OR

Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles is $\frac{24}{7}\text{cm}^2$. Find the radius of each circle.

34. In the given figure, if ∠A = ∠C, AB = 6cm, BP = 15 cm, AP = 12cm and CP = 4, then find the lengths of PD and CD. **[5]**



35. The following is the cummulative frequency distribution (of less than type) of 1000 persons each of age 20 years and above. Determine the mean age. **[5]**

Age below (in years)	30	40	50	60	70	80
Number of persons	100	220	350	750	950	1000

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Find the amount saved by Sehaj on 10^{th} day.

(ii) Find the amount saved by Sehaj on 25^{th} day.

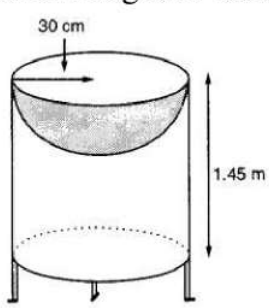
(iii) Find the total amount saved by Sehaj in 30 days.

OR

Find in how many days Sehaj saves ₹1400.

37. **Read the text carefully and answer the questions:** **[4]**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Find the curved surface area of the hemisphere.

(ii) Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)

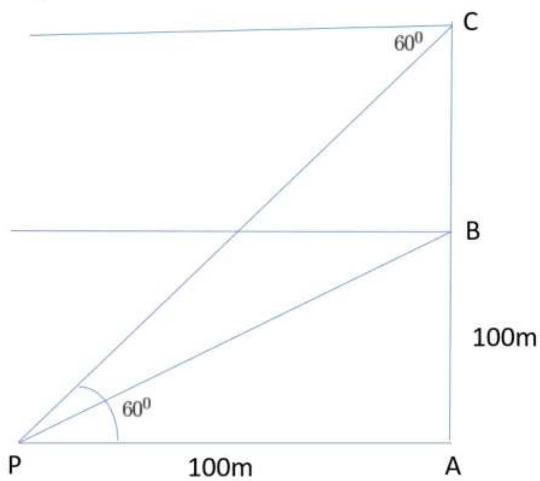
(iii) What is total cost for making the bird bath?

OR

Mayank and his brother thought of increasing the radius of hemisphere to 35 cm with same material so that birds get more space, then what is the new height of cylinder?

38. **Read the text carefully and answer the questions:** **[4]**

A hot air balloon is rising vertically from a point A on the ground which is at distance of 100m from a car parked at a point P on the ground. Amar, who is riding the balloon, observes that it took him 15 seconds to reach a point B which he estimated to be equal to the horizontal distance of his starting point from the car parked at P.



(i) Find the angle of depression from the balloon at a point B to the car at point P.

(ii) Find the speed of the balloon?

(iii) After certain time Amar observes that the angle of depression is 60^{o}. Find the vertical distance travelled by the balloon during this time.

OR

Find the total time taken by the balloon to reach the point C from ground?

Solution

Section A

1. **(d)** $\sqrt{39}$ cm

   Explanation: $\sqrt{39}$ cm
2. **(b)** 2nd

   Explanation: Since x-coordinate is negative and y-coordinate is positive.
   Therefore, the point (-3, 5) lies in II quadrant.
3. **(c)** abscissa

   Explanation: The distance of a point from the y-axis is the x (horizontal) coordinate of the point and is called abscissa.
4. **(a)** 0.00001

   Explanation: An event is unlikely to happen. Its probability is very very close to zero but not zero, So it is equal to 0.00001
5. **(a)** aq $\neq$ bp

   Explanation: Given: $a_1 = a$, $a_2 = p$, $b_1 = b$, $b_2 = q$, $c_1 = c$ and $c_2 = r$.
   Since, the pair of given linear equations has a unique solution.
   $$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{a}{p} \neq \frac{b}{q}$$
   $\Rightarrow$ aq $\neq$ bp
6. **(c)** $1 - p$

   Explanation: If the probability of an event is p, the probability of its complementary event will be 1 - p. because we know that the sum of probability of an event and its complementary event is always 1.
   Hence, p + 1 - p = 1
7. **(d)** ± 4

   Explanation: Distance between (4, p) and (1, 0) = 5
   $$\Rightarrow \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} = 5$$
   $$\Rightarrow \sqrt{(1-4)^2 + (0-p)^2} = 5$$
   $$\sqrt{(-3)^2 + (-p)^2} = 5$$
   Squaring, both sides
   $(-3)^2 + (-p)^2 = (5)^2 \Rightarrow 9 + p^2 = 25$
   $\Rightarrow p^2 = 25 - 9 = 16$
   $\therefore p = \pm\sqrt{16} = \pm 4$
8. **(a)** $0 \leq P(E) \leq 1$

   Explanation: The probability of any event is always positive. It could be at the least equal to zero but not less than that. The probability of sure event at the maximum could be equal to 1 so probability lies between 0 and 1 both included.
9. **(b)** $1 : \sqrt{3}$

   Explanation: Surface area of hemispherical part = surface area of conical part

$\Rightarrow 2\pi r^2 = \pi rl \Rightarrow 2r = l$

$\Rightarrow 2r = \sqrt{r^2 + h^2} \Rightarrow 4r^2 = r^2 + h^2$

$\Rightarrow 3r^2 = h^2 \Rightarrow \frac{r^2}{h^2} = \frac{1}{3}$

$\Rightarrow \frac{r}{h} = \frac{1}{\sqrt{3}}$

$\therefore$ Roots = $1 : \sqrt{3}$

10. **(c)** 16

Explanation: $x^2 + bx + 12 = 0$

$\because$ 2 is its root, then 2 will satisfy it

$\therefore (2)^2 + b \times 2 + 12 \Rightarrow 4 + 2b + 12 = 0$

$\Rightarrow 2b + 16 = 0 \Rightarrow b = \frac{-16}{2} = -8$

Now equation

$x^2 + bx + q = 0$, has equal rooots then

$D = 0 \Rightarrow b^2 - 4q = 0$

$\Rightarrow (-8)^2 - 4q = 0 \Rightarrow 64 = 4q$

$\Rightarrow q = 16$

11. **(a)** 3 and 4

Explanation: Putting the values of p and q in given equation, we get

$x^2 + (-7)x + 12 = 0$

$\Rightarrow x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 4x - 3x + 12 = 0$

$\Rightarrow x(x - 4) - 3(x - 4) = 0$

$\Rightarrow (x - 3)(x - 4) = 0$

$\Rightarrow x - 3 = 0$ and $x - 4 = 0$

$\Rightarrow x = 3$ and $x = 4$

12. **(c)** 45

Explanation: We have,

$135 = 3 \times 45$

$= 3 \times 3 \times 15$

$= 3 \times 3 \times 3 \times 5$

$= 3^3 \times 5$

Now, for 225 will be

$225 = 3 \times 75$

$= 3 \times 3 \times 5 \times 5$

$= 3^2 \times 5^2$

The HCF will be $3^2 \times 5 = 45$

13. **(d)** 10

Explanation: The distance of the point P(-6,8) from the origin (0, 0)

$= \sqrt{(-6)^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$

14. **(b)** 2

Explanation: Since $\sec\theta = \sqrt{1 + \tan^2\theta}$

$\therefore \sec\theta = \sqrt{1 + \left(\sqrt{3}\right)^2}$

$= \sqrt{1 + 3} = \sqrt{4} = 2$

15. **(b)** 17.5

Explanation: Given, classes are not continuous, so we make continuous by subtracting 0.5 from lower limit and adding 0.5 to upper limit of each class.

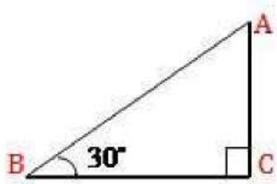
Class	Frequency	Cumulative frequency
-0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Here, $\frac{N}{2} = \frac{57}{2} = 28.5$, which lies in the interval 11.5 - 17.5.

Hence, the upper limit is 17.5.

16. **(c)** $20\sqrt{3}$ m

Explanation: Let BC= 60 m be the width of the river



and angle of elevation = 30 °

To find: Height of the tree AC

$\therefore \tan 30^\circ = \frac{AC}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AC}{60}$

$\Rightarrow AC = \frac{60}{\sqrt{3}}$

$= \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}\text{m}$

Therefore, the height of the tree is $20\sqrt{3}$m.

17. **(d)** $2x + 5y = -11$

$4x + 10y = -22$

Explanation: If x = 2 and y = - 3 is a unique solution of any pair of equation, then these values must satisfy that pair of equations.

Putting the values in the equations for every option and checking it -

LHS = 2x + 5y = 2(2+(- 3) = 4 + (- 15) = - 11 = RHS

and

LHS = 4x + 10y = 4(2) +10(- 3) = 8 + (- 30) = - 22 = RHS

It satisfies the pair of linear equation and hence is the unique solution for the equation.

18. **(a)** an odd integer

Explanation: Let $a = m^2 - 1$

Here m can be ever or odd.

Case I: m = Even i.e., m = 2k, where k is an integer,

$\Rightarrow a = (2k)^2 - 1$

$\Rightarrow a = 4k^2 - 1$

At k = -1, $= 4(-1)^2 - 1 = 4 - 1 = 3$, which is not divisible by 8.

At k = 0, $a = 4(0)^2 - 1 = 0 - 1 = -1$, which is not divisible by 8, which is not.

Case II: m = Odd i.e., m = 2k + 1, where k is an odd integer.

$\Rightarrow a = 2k + 1$

$\Rightarrow a = (2k + 1)^2 - 1$

$\Rightarrow a\ 4k^2 + 4k + 1 - 1$

$\Rightarrow a = 4k^2 + 4k$

$\Rightarrow a = 4k(k + 1)$

At k = -1, a = 4(-1)(-1 + 1) = 0 which is divisible by 8.

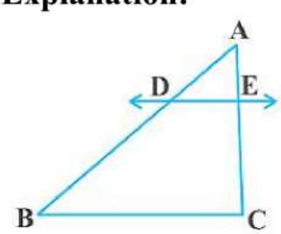
At k = 0, a = 4(0)(0 + 1) = 4 which is divisible by 8.

At k = 1, a = 4(1)(1 + 1) = 8 which is divisible by 8.

Hence, we can conclude from the above two cases, if m is odd, then $m^2 - 1$ is divisible by 8.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:



We know that if a line is parallel to one side of a triangle then it divides the other two sides in the same ratio. This is Basic Proportionality theorem.

So, Reason is correct.

By Basic Proportionality theorem, we have $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow x(x - 1) = (x - 2)(x + 2)$

$\Rightarrow\ x^2 - x = x^2 - 4$

$\Rightarrow\ x = 4$ cm

So, Assertion is correct.

20. **(c)** A is true but R is false.

Explanation: Here reason is not true. $\sqrt{4} = \pm 2$, which is not an irrational number.

Section B

21. Given equations are:

$\frac{4}{3}x + 2y = 8;\ 2x + 3y = 12$

Compare equation $\frac{4}{3}x + 2y = 8$ with $a_1x + b_1y + c_1 = 0$ and $2x + 3y = 12$

with $a_2x + b_2y + c_2 = 0$, We get, $a_1 = \frac{4}{3}$, $a_1 = \frac{4}{3}$, $b_1 = 2$, $c_1 = -8$, $a_2 = 2$, $b_2 = 3$, $c_2 =$ -12

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{2}{3} \text{ and } \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the lines have infinitely many solutions.

Hence, they are consistent.

OR

Let the number of girls = 2x and number of boys = 3x

$\therefore\ 2x + 3x = 40 \Rightarrow x = 8$

$\therefore$ Number of girls = 2(8) = 16

and number of boys = 3(8) = 24

Let out of 5 students, y are boys.

$\therefore$ Number of girls = 5 - y

ATQ.

$$\frac{16+5-y}{24+y} = \frac{4}{5}$$

$5(21 - y) = 4(24 + y)$

$\Rightarrow\ 105 - 5y = 96 + 4y$

$9y = 105 - 96$

$9y = 9 \Rightarrow y = 1$

22. There are 3 + 5 = 8 balls in a bag. Out of these 8 balls, one can be chosen in 8 ways.

$\therefore$ Total number of elementary events = 8

$$\textit{Proabibilty of the event} = \frac{\textit{Number of favourble outcomes}}{\textit{Total number of possible outcomes}}$$

i. Since the bag contains 3 red balls, therefore, one red ball can be drawn in 3 ways.

$\therefore$ Favourable number of elementary events = 3

Hence P (getting a red ball) = $\frac{3}{8}$

ii. Since the bag contains 5 black balls along with 3 red balls, therefore one black (not red) ball can be drawn in 5 ways.

$\therefore$ Favourable number of elementary events = 5

Hence P (getting "not a red ball") = $\frac{5}{8}$

23. Let A (3, 0), B (4, 5), C (-1, 4) and D (-2, -1)

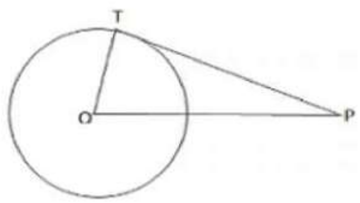
$AC = \sqrt{(-1-3)^2 + (4-0)^2} = 4\sqrt{2}$

$BD = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{36+36} = 6\sqrt{2}$

Area of rhombus = $\frac{1}{2}d_1 \times d_2$

$= \frac{1}{2}AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ Sq. unit.

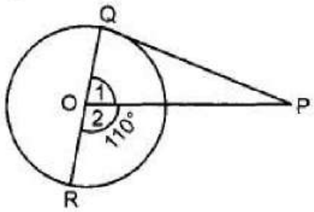
24. 

Given: OP = 25cm.
Let TP be the tangent, so that TP = 24cm
Join OT where OT is radius.
Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.
$\therefore OT \perp PT$
In $\triangle OTP$,
By Pythagoras theorem, $OT^2 + TP^2 = OP^2$
$OT^2 + 24^2 = 25^2$
$OT^2 = 625 - 576$
$OT^2 = 49$
OT = 7
The radius of the circle will be 7cm.

OR

Given PQ is a tangent to the circle with centre O from a point P.

QOR is a diameter of the circle and $\angle POR = 110^o$



$\angle POR = 110°$
QR is the diameter of the circle.

$\Rightarrow \angle 1 + \angle 2 = 180°$ [Linear pair axiom]

$\Rightarrow \angle 1 + 110° = 180°$

$\Rightarrow \angle 1 = 70°$

$\angle OQP = 90°$

In $\triangle OPQ$

$\angle 1 + \angle OQP + \angle QPO = 180°$

$\Rightarrow 70° + 90° + \angle QPO = 180°$

$\Rightarrow \angle OPQ = 180° - 160°$

$\Rightarrow \angle OPQ = 20°$

25. We have quadratic polynomial as $t^2 - 15$

$= t^2 - (\sqrt{15})^2$

$= (t - \sqrt{15})(t + \sqrt{15})$ [As, $x^2 - y^2 = (x - y)(x + y)$]

The value of $t^2 - 15$ is zero when $(t - \sqrt{15}) = 0$ or $(t + \sqrt{15}) = 0$,

i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes $= \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{coefficient of } t)}{\text{coefficient of } t^2}$

Product of zeroes $= (\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{constant term}}{\text{coefficient of } t^2}$

Hence verified.

Section C

26. Let the cost of 1 kg of apples be Rs. x and of 1 kg of grapes be Rs. y.

Then the algebraic representation is given by the following

Equations:

$2x + y = 160$ ...(1)

$4x + 2y = 300$

$\Rightarrow 2x + y = 150$ ...(2)

To represent these equation graphically, we find two solutions for each equations are given below:

For Equation (1) $2x + y = 160$

$\Rightarrow y = 160 - 2x$

Table (1) of solutions

x	50	40
y	60	80

For Equation (2) $2x + y = 150$

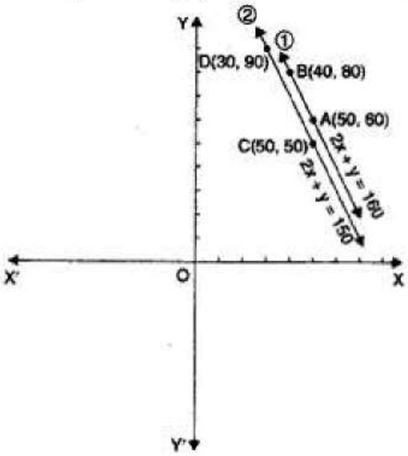
$\Rightarrow y = 150 - 2x$

Table 2 of solutions

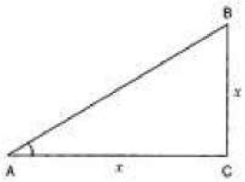
x	50	30
y	50	90

We plot the points A(50, 60) and B(40, 80) corresponding to the solutions in table 1 on a graph paper for get the line.

AB representing the equation (1) and the points C(50, 50) and D(30, 90) corresponding the equation (2) as shown in figure given below.



We observe in figure that, the two lines do not intersect anywhere i.e., they are parallel.

27. 

In $\triangle ABC$,

$\tan A = 1$

$\Rightarrow \quad \frac{BC}{AC} = 1$

$\Rightarrow$ BC = x and AC = x

Using Pythagoras theorem,

$\Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow \quad AB^2 = x^2 + x^2$

$\Rightarrow \quad AB = \sqrt{2}x$

$\therefore \quad \sin A = \frac{BC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$ and $\cos A = \frac{AC}{\sqrt{2}x} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

$2 \sin A \cos A = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

28. Suppose $\sqrt{p}$ be rational $\Rightarrow$ it can be written in the form of $\frac{a}{b}$.

$\sqrt{p} = \frac{a}{b}$(where a and b are co-prime)

On squaring both sides, we get

$p = \frac{a^2}{b^2}$

a^2 has a factor p.

$pb^2 = a^2$ ....(i)

a also has a factor p.

So $a = pc$

$pb^2 = a^2$

$a^2 = p^2c^2$

Put the value of a^2 in equation (i),

$pb^2 = p^2c^2$

b^2 has a factor p,

$\therefore$ b has a factor p.

a and b have common factor p.

But we assume that a and b are co-prime

$\therefore$ our assumption is wrong.

$\sqrt{p}$ must be an irrational number, (p is a prime number.)

$\sqrt{q}$ is also an irrational number (q is a prime number.)

Sum of two irrational numbers is irrational

$\therefore$ $\sqrt{p} + \sqrt{q}$ is irrational number.

OR

$P(x) = a^8 - b^8 = \left(a^4 + b^4\right)\left(a^4 - b^4\right)$

$= \left(a^4 + b^4\right)\left(a^2 + b^2\right)\left(a^2 - b^2\right)$

$= \left(a^4 + b^4\right)\left(a^2 + b^2\right)(a + b)(a - b)$

$Q(x) = (a + b)\left(a^4 - b^4\right)$

$= (a + b)\left(a^2 + b^2\right)\left(a^2 - b^2\right)$

$= (a + b)\left(a^2 + b^2\right)(a + b)(a - b)$ {Using Identity $a^2 - b^2 = (a + b)(a - b)$}

Common factors: $\left(a^2 + b^2\right)$, $(a - b)$, $(a + b)$

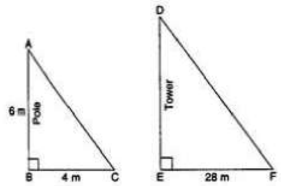
Uncommon factors: $\left(a^4 + b^4\right)$, $(a + b)$

$\therefore$ *LCM* of $P(x)$ and $Q(x)$

$= \left(a^2 + b^2\right)(a - b)(a + b) \times \left(a^4 + b^4\right)(a + b)$

$= \left(a^4 + b^4\right)\left(a^2 + b^2\right)(a + b)^2(a - b)$

29. Let AB denoted the vertical pole of length 6m.BC is the shadow of the pole on the ground BC = 4m.
Let DE denote the tower.
EF is shadow of the tower on the ground.
EF = 28 m.
Let the height of the tower be h m.



In $\triangle ABC$ and $\triangle DEF$,

$\angle B = \angle E$ ......[Each equal to 90° because pole and tower are standing vertical to the ground]
$\angle C = \angle F$ .....[Same elevation]
$\angle A = \angle D$ $\because$ shadows are cast at the same time
$\therefore$ $\triangle ABC$ and $\triangle DEF$,

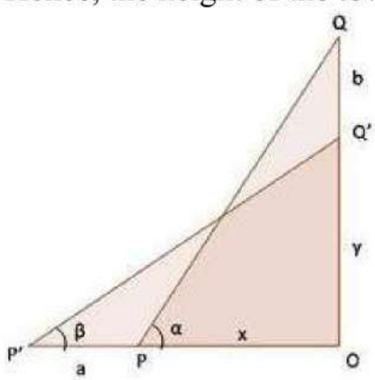
$\angle B = \angle E$ ....[Each equal to 90° because pole and tower are standing vertical to the ground.]
$\angle A = \angle D$ ($\because$ shadows are cast at the same time)
$\therefore$ $\triangle ABC \sim \triangle DEF$ ......(AA similarity criterion)

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$ ....[$\because$ corresponding sides of two similar triangles are proportional]

$\Rightarrow \frac{6}{h} = \frac{4}{28}$

$\Rightarrow h = \frac{6 \times 28}{4} \Rightarrow h = 42$

Hence, the height of the tower is 42 m

30. 

Let PQ be the ladder such that is top Q is on the wall OQ.
The ladder is pulled away from the wall through a distance a, so Q slides and takes position Q'.
Clearly, $PQ = P'Q'$.
In $\Delta' s\ POQ\ and\ P'OQ'$, we have

$\sin\alpha = \frac{OQ}{PQ}, \cos\alpha = \frac{OP}{PQ}, \sin\beta = \frac{OQ'}{P'Q'}, \cos\beta = \frac{OP'}{P'Q'}$

$\Rightarrow \sin\alpha = \frac{b+y}{PQ}, \cos\alpha\frac{x}{PQ}, \sin\beta = \frac{y}{PQ}, \cos\beta = \frac{a+x}{PQ}$

$\Rightarrow \sin\alpha - \sin\beta = \frac{b+y}{PQ} - \frac{y}{PQ}$ and

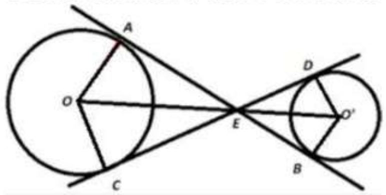
$\cos\beta - \cos\alpha = \frac{a+x}{PQ} - \frac{x}{PQ}$

$\Rightarrow \sin\alpha - \sin\beta = \dfrac{b}{PQ}$ and

$\cos\beta - \cos\alpha = \dfrac{a}{PQ}$

$\Rightarrow \dfrac{a}{b} = \dfrac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

31. Construction : Join OA and OC.



$\angle AEC = \angle DEB$ ....(vertically opposite angles)

In ΔOAE and ΔOCE,

OA = OC ...(Radii of the same circle)

OE = OE ...(Common side)

$\angle OAE = \angle OCE$ ....(each is 90°)

$\Rightarrow \Delta OAE \cong \Delta OCE$ ....(RHS congruence criterion)

$\Rightarrow \angle AEO = \angle CEO$ ....(cpct)

Similarly, for the circle with centre O',

$\angle DEO' = \angle BEO'$

Now, $\angle AEC = \angle DEB$

$\Rightarrow \frac{1}{2}\angle AEC = \frac{1}{2}\angle DEB$

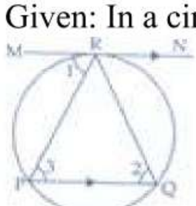
$\Rightarrow \angle AEO = \angle CEO = \angle DEO' = \angle BEO'$

Hence, all the fours angles are equal and bisected by OE and O'E.

So, O, E and O' are collinear.

OR

Given: In a circle a chord PQ and a tangent MRN at R such that QP || MRN



To prove: R bisects the arc PRQ.

Construction**:** Join RP and RQ.

Proof: Chord RP subtends ∠1 with tangent MN and ∠2 in alternates segment of circle so ∠1 = ∠2.

MRN || PQ

∴ ∠1 = ∠3 [Alternate interior angles]

⇒ ∠2 = ∠3

⇒ PR = RQ [Sides opp. to equal ∠s in Δ RPQ]

∵ Equal chords subtend equal arcs in a circle so

arc PR = arc RQ

or R bisect the arc PRQ. Hence, proved.

Section D

32. We have,

$$\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}$$

$$\frac{2x}{x+1}+\frac{3x}{2(x-2)}=\frac{23}{5}$$

Taking LCM

$$\frac{2\times 2x(x-2)+3x(x+1)}{2(x+1)(x-2)}=\frac{23}{5}$$

$$\frac{4x^2-8x+3x^2+3x}{2\left(x^2-x-2\right)}=\frac{23}{5}$$

$$\frac{7x^2-5x}{2\left(x^2-x-2\right)}=\frac{23}{5}$$

By cross multiplication,

$35x^2-25x=46x^2-46x-92$

$11x^2-21x-92=0$

$$\therefore \quad x=\frac{21\pm\sqrt{(-21)^2-4(11)(-92)}}{2\times 11}$$

$$=\frac{21\pm\sqrt{441+4048}}{22}$$

$$=\frac{21\pm\sqrt{4489}}{22}$$

$$=\frac{21\pm 67}{22}$$

$$x=\frac{21+67}{22} \text{ or } x=\frac{21-67}{22}$$

$$\therefore \quad x=4, \frac{-23}{11}$$

OR

Let the shorter side of the rectangular field be x metres.

Then, the longer side of the rectangular field = (x + 30) metres

Therefore, the diagonal of the rectangular field =

$=\sqrt{(Length\ of\ the\ shorter\ side)^2+(Length\ of\ the\ longer\ side)^2}$ [By Pythagoras theorem]

$=\sqrt{x^2+(x+30)^2}$ metres

According to the question, $\sqrt{x^2+(x+30)^2}=x+60$

Squaring both sides,we get

$x^2 + (x + 30)^2 = (x + 60)^2$

$\Rightarrow x^2 + x^2 + 60x + 900$

$= x^2 + 120x + 3600$

$\Rightarrow x^2 - 60x - 2700 = 0$

which is a quadratic equation in x.

Here, a = 1, b = -60, c = -2700

Using the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

we get $= \dfrac{-(-60) \pm \sqrt{(-60)^2 - 4(1)(-2700)}}{2(1)}$

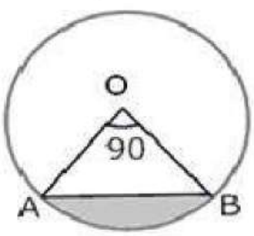
$= \dfrac{60 \pm \sqrt{3600 + 10800}}{2} = \dfrac{60 \pm \sqrt{14400}}{2}$

$= \dfrac{60 \pm 120}{2} = \dfrac{60 + 120}{2}, \dfrac{60 - 120}{2}$

$\Rightarrow x = 90, \ -30$

Since, x cannot be negative, being a dimension, the length of the shorter side of the rectangular field is 90 metres.

The length of the longer side = x + 30 = 90 + 30 = 120 metres

33. 

i. Area of minor sector = $\dfrac{\theta}{360}\pi r^2$

$= \dfrac{90}{360}(3.14)(10)^2$

$= \dfrac{1}{4} \times 3.14 \times 100$

$= \dfrac{314}{4}$

$= 78.50 = 78.5 \text{ cm}^2$

ii. Area of major sector = Area of circle - Area of minor sector

$= \pi(10)^2 - \dfrac{90}{360}\pi(10)^2 = 3.14\ (100) - \dfrac{1}{4}(3.14)\ (100)$

$= 314 - 78.50 = 235.5 \text{ cm}^2$

iii. We know that area of minor segment

= Area of minor sector OAB - Area of ΔOAB

$\because$ area of $\triangle$OAB = $\frac{1}{2}(OA)(OB)\sin\angle AOB$

$= \frac{1}{2}(OA)(OB)\left(\because \angle AOB = 90°\right)$

Area of sector = $\frac{\theta}{360}\pi r^2$

$= \frac{1}{4}$(3.14) (100) - 50 = 25(3.14) - 50 = 78.50 - 50 = 28.5 cm^2

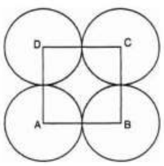
iv. Area of major segment = Area of the circle - Area of minor segment

$= \pi(10)^2$ - 28.5

= 100(3.14) - 28.5

= 314 - 28. 5 = 285.5 cm^2

OR



Let r cm be the radius of each circle.

Area of square - Area of 4 sectors = $\frac{24}{7}cm^2$

$(\text{side})^2 - 4\left[\frac{\theta}{360}\pi r^2\right] = \frac{24}{7}cm^2$

or, $(2r)^2 - 4\left(\frac{90°}{360°} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - 4\left(\frac{1}{4°} \times \pi r^2\right) = \frac{24}{7}$

or, $(2r)^2 - \left(\pi r^2\right) = \frac{24}{7}$

or, $4r^2 - \frac{22}{7}r^2 = \frac{24}{7}$

or, $\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$

or, $6r^2 = 24$

or, $r^2 = 4$

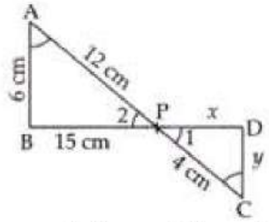
or, $r = \pm 2$

or, Radius of each circle is 2 cm (r cannot be negative)

34. Given that in fig, if $\angle$A = $\angle$C, AB = 6cm, BP = 15 cm, AP = 12cm and CP = 4, we have to find the lengths of PD and CD.

Now,In ΔABP and ΔCDP, we have,

$\angle A = \angle C$ [Given]
$\angle 2 = \angle 1$ [Vertically opposite angles]
$\therefore \Delta ABP \sim \Delta CDP$ [By AA similarity criterion]



$\Rightarrow \frac{AB}{CD} = \frac{AP}{CP} = \frac{BP}{DP}$ (Since corresponding sides of two similar triangles are proportional)

$\Rightarrow \frac{6}{y} = \frac{12}{4} = \frac{15}{x}$

$\Rightarrow \frac{6}{y} = \frac{12}{4}$

$\Rightarrow y = \frac{6 \times 4}{12} = 2cm$

and $\frac{15}{x} = \frac{12}{4}$

$\Rightarrow \frac{15}{x} = 3$

$\Rightarrow x = 5$ cm

Therefore, PD = 5 cm and CD = 2 cm.

35.

Class interval	**Frequency f_i**	**Mid-value X_i**	$u_i = \frac{x_i - A}{h} = \frac{x_i - 45}{10}$	**$f_i u_i$**
20-30	100	25	-2	-200
30-40	120	35	-1	-120
40-50	130	45=A	0	0
50-60	400	55	1	400
60-70	200	65	2	400
70-80	50	75	3	150
	$\Sigma f_i = 1000$			$\sum f_i u_i = 630$

A = 45, h = 10,
$\Sigma f_i = 1000$, $\sum f_i u_i = 630$

$$\text{Mean} = A + \left\{ h \times \frac{\sum f_i u_i}{\sum f_i} \right\}$$

$= 45 + \left\{10 \times \frac{630}{1000}\right\}$

$= 45 + 6.3$

$= 51.3$

Section E

36. **Read the text carefully and answer the questions:**

Sehaj Batra gets pocket money from his father every day. Out of pocket money, he saves money for poor people in his locality. On 1st day he saves ₹27.5 On each succeeding day he increases his saving by ₹2.5.

(i) Money saved on 1st day = ₹27.5

$\because$ Sehaj increases his saving by a fixed amount of ₹2.5

$\therefore$ His saving form an AP with a = 27.5 and d = 2.5

$\therefore$ Money saved on 10th day,

$a_{10} = a + 9d = 27.5 + 9(2.5)$

$= 27.5 + 22.5 =$ ₹50

(ii) $a_{25} = a = 24d$

$= 27.5 + 24(2.5)$

$= 27.5 + 60 =$ ₹ 87.5

(iii)Total amount saved by Sehaj in 30 days.

$= \frac{30}{2}[2 \times 27.5 + (30 - 1) \times 2.5]$

$= 15(55 + 29(2.5)$

$=$ ₹1912.5

OR

Let $S_n = 387.5$, a = 27.5 and d = 2.5

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow 387.5 = \frac{n}{2}[2 \times 27.5 + (n-1)2.5]$$

$$\Rightarrow 387.5 = \frac{n}{2}[55 + (n-1) \times 2.5]$$

$\Rightarrow 775 = 55n + 2.5n^2 - 2.5n$

$\Rightarrow 25n^2 + 525n = 7750 = 0$

$\Rightarrow n^2 + 21n - 310 = 0$

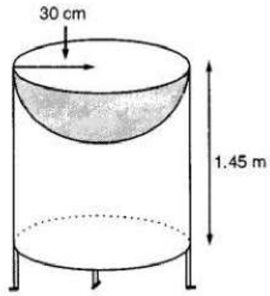
$\Rightarrow (n + 31)(n - 10) = 0$

$\Rightarrow n = -31$ reject $n = 10$ accept

So in 10 years Sehaj saves ₹ 387.5.

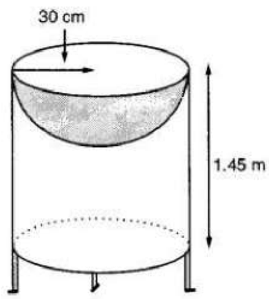
37. **Read the text carefully and answer the questions:**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.

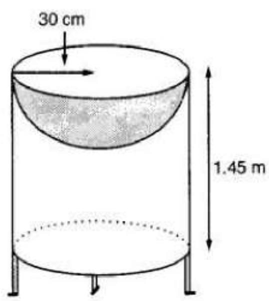


Curved surface area of the hemisphere = $2\pi r^2$

$= 2 \times 3.14 \times 30^2 = 0.56\ m^2$

(ii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



Let S be the total surface area of the birdbath.

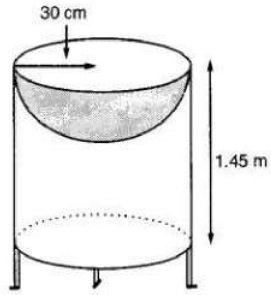
S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$\Rightarrow S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

$\Rightarrow S = 2 \times \frac{22}{7} \times 30(145 + 30) = 33000\ cm^2 = 3.3\ m^2$

(iii)Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



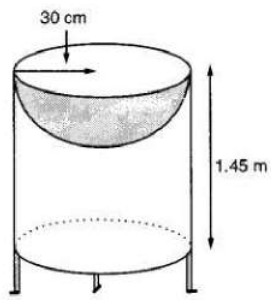
Total Cost of material = Total surface area $\times$ cost per sq m^2

= 3.3 $\times$ 40 = ₹132

OR

Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

Then, r = 30 cm and h = 1.45 m = 145 cm.



$r = 35 \text{ cm} = \frac{35}{100} \text{m}$

We know that S.A = 3.3 m^2

$S = 2\pi r(r + h)$

$$\Rightarrow 3.3 = 2 \times \frac{22}{7} \times \frac{35}{100}\left(\frac{35}{100} + h\right)$$

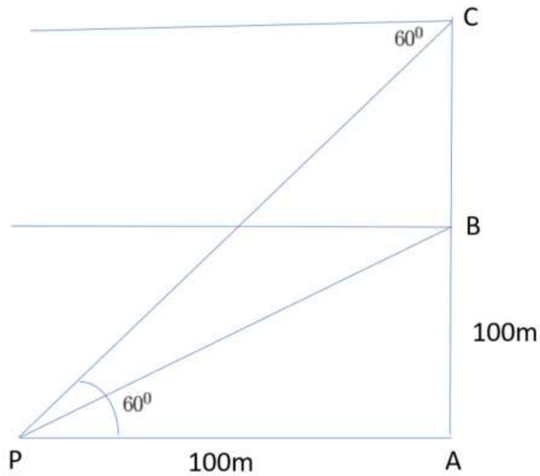
$$\Rightarrow 3.3 = \frac{22}{10}\left(\frac{35}{100} + h\right)$$

$$\Rightarrow \frac{33}{22} = \frac{35}{100} + h$$

$$\Rightarrow h = \frac{3}{2} - \frac{7}{20} = \frac{23}{20} = 1.15 \text{ m}$$

38. **Read the text carefully and answer the questions:**

A hot air balloon is rising vertically from a point A on the ground which is at distance of 100m from a car parked at a point P on the ground. Amar, who is riding the balloon, observes that it took him 15 seconds to reach a point B which he estimated to be equal to the horizontal distance of his starting point from the car parked at P.



(i) The angle of depression from the balloon at a point B to the car at point P.

In $\triangle$APB

$$\tan B = \frac{AB}{AP} = \frac{100}{100} = 1$$

$\Rightarrow \tan B = 1$

$\Rightarrow \tan B = \tan 45^{o}$

$\Rightarrow B = 45^{o}$

(ii) The speed of the balloon is

$$\text{Speed} = \frac{\text{Distan } ce}{\text{Time}}$$

$$\Rightarrow \text{Speed} = \frac{100}{15} = \frac{25}{3} = 6.6 \text{ m/sec}$$

(iii) The vertical distance travelled by the balloon when angle of depression is 60^{o}.

In $\triangle$APC

Let BC = x

$$\tan 60^\circ = \frac{AC}{AP} = \frac{AB + x}{100}$$

$$\Rightarrow \sqrt{3} = \frac{100 + x}{100}$$

$\Rightarrow 100\sqrt{3} - 100 = x$

$\Rightarrow x = 100(\sqrt{3} - 1)$

$\Rightarrow x = 73.21$ m

OR

The total time taken by the balloon to reach the point C from ground.

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\Rightarrow T = \frac{100(\sqrt{3}-1)}{\frac{25}{3}}$$

$$\Rightarrow T = 12(\sqrt{3} - 1) = 8.78 \text{ sec}$$