Class- X Session- 2022-23

Subject- Mathematics (Basic)

Sample Question Paper - 9

with Solution

Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. Which of the following is a true statement? **[1]**

a) $5x^3$ is a monomial

b) $x^2 + 5x - 3$ is a linear polynomial

c) x + 1 is a monomial

d) $x^2 + 4x - 1$ is a binomial

2. XY is drawn parallel to the base BC of a △ABC cutting AB at X and AC at Y. If AB = 4 BX and YC = 2cm, then AY = **[1]**

a) 8 cm

b) 4 cm

c) 6 cm

d) 2 cm

3. If 2x + 3y = 12 and 3x - 2y = 5 then **[1]**

a) x = 3, y = 2

b) x = 2, y = -3

c) x = 2, y = 3

d) x = 3, y = -2

4. If in △ABC and △PQR, we have $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then **[1]**

a) $\triangle BCA \sim \triangle PQR$

b) $\triangle PQR \sim \triangle ABC$

c) $\triangle QRP \sim \triangle ABC$

d) $\triangle CBA \sim \triangle PQR$

5. The pair of equations 5x – 15y = 8 and 3x – 9y = $\frac{24}{5}$ has **[1]**

a) infinitely many solutions

b) no solution

c) two solutions

d) one solution

6. If x tan 45° cos 60° = sin 60° cot 60°, then x is equal to **[1]**

a) $\frac{1}{2}$

b) 1

c) $\frac{1}{\sqrt{2}}$

d) $\sqrt{3}$

7. Which of the following cannot be the probability of an event? **[1]**

a) - 1.5

b) 0.7

c) $\frac{2}{3}$

d) 15%

8. Which of the following is a true statement? **[1]**

a) Two triangles are similar if their corresponding sides are proportional.

b) Two polygons are similar if their corresponding sides are proportional.

c) Two similar triangles are always congruent.

d) Two figures are similar if they have the same shape and size.

9. The mean of all the factors of 24 is **[1]**

a) 7.5

b) 7

c) 6.5

d) 24

10. The roots of the quadratic equation x^2 - 11x - 10 = 0 are **[1]**

a) None of these

b) not real roots

c) real and equal

d) real and distinct

11. $(2+\sqrt{2})$ is **[1]**

a) none of these

b) an integer

c) a rational number

d) an irrational number

12. For the following distribution: **[1]**

Class:	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The sum of the lower limits of the median class and modal class is:

a) 25

b) 30

c) 35

d) 15

13. If the distance between the points (4, p) and (1,0) is 5, then the value of p is **[1]**

a) 0
b) 4 only
c) -4 only
d) ± 4

14. If the angle of depression of a car from a 100 m high tower is 45°, then the distance of the car from the tower is **[1]**

a) 100 m
b) 200 m
c) $100\sqrt{3}$ m
d) $200\sqrt{3}$ m

15. If sin A = $\frac{1}{2}$, then the value of cot A is **[1]**

a) $\sqrt{3}$
b) $\frac{\sqrt{3}}{2}$
c) $\frac{1}{\sqrt{3}}$
d) 1

16. $\triangle ABC \sim \triangle DEF$ and their perimeters are 32 cm and 24 cm respectively. If AB = 10 cm then DE = ? **[1]**

a) 8 cm
b) 7.5 cm
c) $5\sqrt{3}$ cm
d) 15 cm

17. If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents to the circle drawn from A and B is **[1]**

a) 60°
b) 90°
c) 30°
d) 120°

18. **Assertion:** $(2-\sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2+\sqrt{3})$. **[1]**
Reason: Irrational zeros (roots) always occurs in pairs.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.
b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.
c) Assertion is correct statement but reason is wrong statement.
d) Assertion is wrong statement but reason is correct statement.

19. If the sum of the roots of a quadratic equation is 6 and their product is 6, the equation is **[1]**

a) $x^2 - 6x + 6 = 0$
b) $x^2 + 6x - 6 = 0$
c) $x^2 - 6x - 6 = 0$
d) $x^2 + 6x + 6 = 0$

20. **Assertion (A):** Two identical solid cubes of side 5 cm are joined end to end. The total surface area of the resulting cuboid is 350 cm^2. **[1]**

Reason (R): Total surface area of a cuboid is 2(lb + bh + hl)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the distance between the pair of points (a, b), (-a, -b) **[2]**

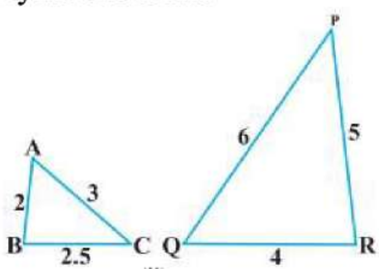
OR

Find the distance between the points:
P(a sin α, a cos α,) and Q(a cos α, - a sin α)

22. Find the roots of the quadratic equation $15x^2 - 10\sqrt{6}x + 10 = 0$. **[2]**

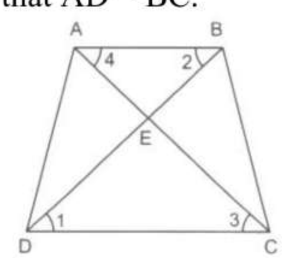
23. Prove that $4 + \sqrt{2}$ is irrational. **[2]**

24. State the pair of triangles in Fig, are similar. Write the similarity criterion used by you for answering the question & also write the pair of similar triangles in the symbolic form: **[2]**



OR

In Fig., ABCD is a trapezium with AB || DC. If $\triangle AED$ is similar to $\triangle BEC$, prove that AD = BC.



25. Evaluate $2\sin^2 30^\circ \tan 60^\circ - 3\cos^2 60^\circ \sec^2 30^\circ$. **[2]**

Section C

26. $\triangle ABC$ is a right triangle in which $\angle C = 90^\circ$ and $CD \perp AB$. If BC = a, CA = b, AB = c and CD = p then prove that. **[3]**

i. cp = ab

ii. $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

27. The sum of a natural number and its positive square root is 132. Find the number. **[3]**

28. Find a point which is equidistant from the points A (-5,4) and B (-1,6). How many such points are there? **[3]**

OR

Two vertices of an isosceles triangle are (2,0) and (2,5). Find the third vertex if the length of the equal sides is 3.

29. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30^{o}, than when it is 60^{o}. Find the height of the tower. **[3]**

OR

The angle of elevation of the top of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation of the top is 45°. Calculate the height of the tower.

30. Find the arithmetic mean of the following frequency distribution using step-deviation method: **[3]**

Age(in years)	18 - 24	24 - 30	30 - 36	36 - 42	42 - 48	48 - 54
Number of workers	6	8	12	8	4	2

31. If (x–3) is the HCF of $x^3 - 2x^2 + px + 6$ and $x^2 - 5x + q$, find 6p + 5q **[3]**

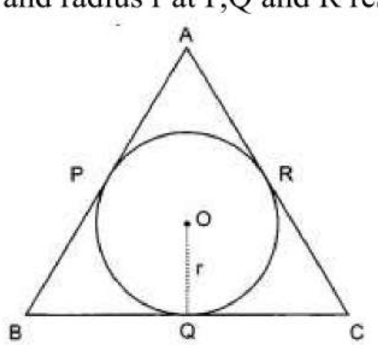
Section D

32. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I Shall be three times as old as you will be". (Isn't this interesting) Represent this situation Algebraically and graphically. **[5]**

OR

The sum of digits of a two digit number is 13. If the number is subtracted from the one obtained by interchanging the digits, the result is 45. What is the number?

33. In figure the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P,Q and R respectively. **[5]**



Prove that:

i. AB + CQ = AC + BQ

ii. $Area(\Delta ABC) = \frac{1}{2}(\text{Perimeter of } \Delta ABC) \times r$

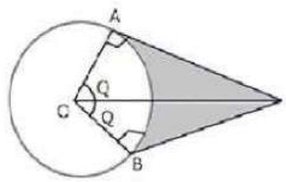
34. Cards marked with numbers 3, 4, 5..........., 50 are placed in a bag and mixed thoroughly. One card is drawn at random from the bag. Find the probability that number on the card drawn is : **[5]**

a. Divisible by 7.

b. A perfect square.

c. A multiple of 6.

35. An elastic belt is placed around therein of a pulley of radius 5cm. One point on the belt is pulled directly away from the center O of the pulley until it is at P, 10cm from O. Find the length of the best that is in contact with the rim of the pulley. Also, find the shaded area. **[5]**



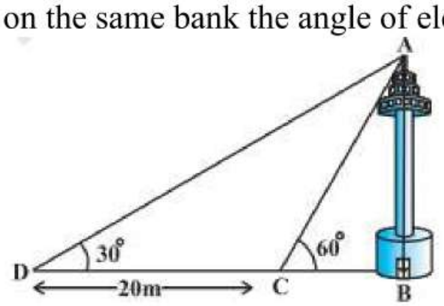
OR

Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use $\pi = 22/7$).

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30°.



(i) Find the width of the canal.

(ii) Find the height of tower.

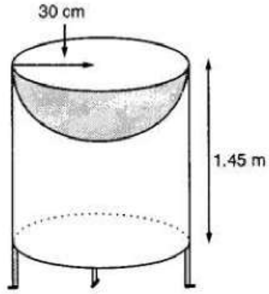
(iii) Find the distance between top of the tower and point D.

OR

Find the distance between top of tower and point C.

37. **Read the text carefully and answer the questions:** **[4]**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.



(i) Find the curved surface area of the hemisphere.

(ii) Find the total surface area of the bird-bath. (Take $\pi = \frac{22}{7}$)

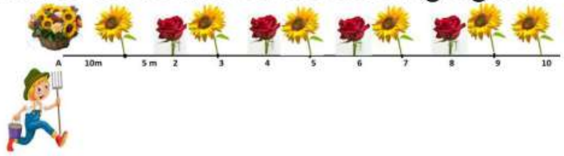
(iii) What is total cost for making the bird bath?

OR

Mayank and his brother thought of increasing the radius of hemisphere to 35 cm with same material so that birds get more space, then what is the new height of cylinder?

38. **Read the text carefully and answer the questions:** **[4]**

In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) Write the above information in the progression and find first term and common difference.

(ii) Find the distance covered by Dinesh to plant the first 5 plants and return to basket.

(iii) Find the distance covered by Dinesh to plant all 10 plants and return to basket.

OR

If the speed of Dinesh is 10 m/min and he takes 15 minutes to plant a flower plant then find the total time taken by Dinesh to plant 10 plants.

Solution

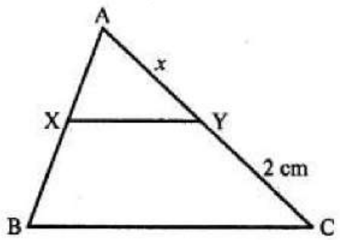
Section A

1. **(a)** $5x^3$ is a monomial

   Explanation: $5x^3$ is a monomial as it contains only one term.
2. **(c)** 6 cm

   Explanation: In ΔABC, $XY \parallel BC$

   $AB = 4BX$, $YC = 2$ cm



   $\therefore AB = 4BX \Rightarrow AX + BX = 4BX$

   $\Rightarrow AX = 4BX - BX = 3BX$

   Let AY = x

   $\because$ In ΔABC, $XY \parallel BC$

   $$\frac{AX}{BX} = \frac{AY}{CY} \Rightarrow \frac{3BX}{BX} = \frac{x}{2}$$

   $$\Rightarrow \frac{3}{1} = \frac{x}{2} \Rightarrow x = 3 \times 2 = 6$$

   $\therefore AY = 6$cm
3. **(a)** x = 3, y = 2

   Explanation: We have:

   $2x + 3y = 12$ …(i)

   $3x - 2y = 5$ …(ii)

   Now, by multiplying (i) by 2 and (ii) by 3 and then adding them we get:

   $4x + 9x = 24 + 15$

   $13x = 39$

   $$x = \frac{39}{13} = 3$$

   Now putting the value of x in (i), we get

   $2 \times 3 + 3y = 12$

   $$\therefore\ y = \frac{12-6}{3} = 2$$
4. **(c)** $\triangle QRP \sim \triangle ABC$

   Explanation: In $\triangle ABC$ and $\triangle PQR$,

   $$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$$

   Then, $\triangle ABC \sim \triangle QRP$

5. **(a)** infinitely many solutions

Explanation: Given: $a_1 = 5, a_2 = 3, b_1 = -15, b_2 = -9, c_1 = 8$ and $c_2 = \frac{24}{5}$ Here

$\frac{a_1}{a_2} = \frac{5}{3}, \frac{b_1}{b_2} = \frac{-15}{-9} = \frac{5}{3}, \frac{c_1}{c_2} = \frac{8}{\frac{24}{5}} = \frac{5}{3} \because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Since all have the same answer $\frac{5}{3}$.

Therefore, the pair of given linear equations has infinitely many solutions.

6. **(b)** 1

Explanation: We have, x tan 45° cos 60° = sin 60° cot 60°

$\Rightarrow x \times 1 \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} \Rightarrow \frac{x}{2} = \frac{1}{2}$

$\Rightarrow x = \frac{1}{2} \times 2 = 1$

7. **(a)** - 1.5

Explanation: – 1.5 cannot be the probability of an event because $0 \leqslant P(E) \leqslant 1$.
The probability of a sure event is 1 and the probability of an impossible event is 0.

8. **(a)** Two triangles are similar if their corresponding sides are proportional.

Explanation: Two similar figures are similar if they have same shape, not size in every case.

9. **(a)** 7.5

Explanation: All factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

$\therefore \text{ Mean} = \frac{\text{Sum of all factors of 24}}{\text{Number of factors of 24}}$

$= \frac{1+2+3+4+6+8+12+24}{8}$

$= \frac{60}{8}$

= 7.5

10. **(d)** real and distinct

Explanation: Here, a = 1, b = -11, c = -10

Then, $b^2 - 4ac = (-11)^2 - 4 \times 1 \times (-10)$

$\Rightarrow 121 + 40 = 161$

Since, $b^2 - 4ac > 0$

Therefore, The roots of the quadratic equation $x^2 - 11x - 10 = 0$ is real and distinct.

11. **(d)** an irrational number

Explanation: $(2 + \sqrt{2})$ is an irrational number.

If it is rational, then the difference of two rational is rational.

$\therefore (2 + \sqrt{2}) - 2 = \sqrt{2}$ = irrational, which is a contradiction.

Hence, $(2 + \sqrt{2})$, is an irrational number.

12. **(a)** 25

Explanation:

Class	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66

Here, $\frac{N}{2} = \frac{66}{2}$ = 33, which lies in the interval 10-15.

Therefore, lower limit of the median class is 10.

The highest frequency is 20, which lies in the interval = 15-20.

Therefore, lower limit of modal class is 15.

Hence, required sum is = 10 + 15 = 25

13. **(d)** ±4

Explanation: Distance between (4, p) and (1, 0) = 5

$\Rightarrow \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} = 5$

$\Rightarrow \sqrt{(1-4)^2 + (0-p)^2} = 5$

$\sqrt{(-3)^2 + (-p)^2} = 5$

Squaring, both sides

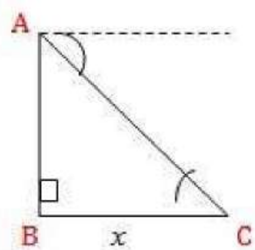
$(-3)^2 + (-p)^2 = (5)^2 \Rightarrow 9 + p^2 = 25$

$\Rightarrow p^2 = 25 - 9 = 16$

$\therefore p = \pm\sqrt{16} = \pm 4$

14. **(a)** 100 m

Explanation:



Let the distance of the car from the tower be $x$meters.

$\therefore \tan 45^\circ = \frac{AC}{BC}$

$\Rightarrow 1 = \frac{100}{x}$ m

$\Rightarrow x = 100$ m

Therefore, the distance of the car from the tower is 100 m.

15. **(a)** $\sqrt{3}$

Explanation: Given: $\sin A = \frac{1}{2}$ ...(i)

And we know that, $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}$ ...(ii)

We need to find the value of cos A.

$\cos A = \sqrt{1 - \sin^2 A}$ ...(iii)

Substituting eq. (i) in eq. (iii), we get

$\cos A = \sqrt{\left(1 - \frac{1}{4}\right)}$

$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Substituting values of sin A and cos A in eq. (ii), we get

$\cot A = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$

16. **(b)** 7.5 cm

Explanation: $\because \triangle ABC \sim \triangle DEF$

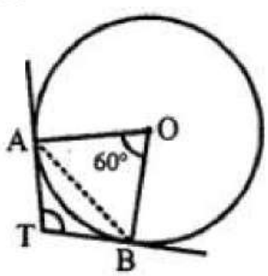
$\therefore \frac{\text{Perimeter }(\triangle ABC)}{\text{Perimeter }(\triangle DEF)} = \frac{AB}{DE}$

$\Rightarrow \frac{32}{24} = \frac{10}{DE}$

$\Rightarrow DE = \frac{10 \times 24}{32} = 7.5$ cm

17. **(d)** 120°

Explanation: A chord AB subtends an angle of 60° at the centre of a circle with centre O.



TA and TB are tangents drawn to the circle.

Then, ∠ATB = 180° – ∠AOB = 180° – 60° = 120°

18. **(a)** Assertion and reason both are correct statements and reason is correct explanation for assertion.

Explanation: As irrational roots/zeros always occurs in pairs, therefore, when one zero is $(2 - \sqrt{3})$ then other will be $(2 + \sqrt{3})$. So, both Assertion and Reason are correct and Reason explains Assertion.

19. **(a)** $x^2 - 6x + 6 = 0$

Explanation: Required equation is $x^2 - 6x + 6 = 0$.

20. **(d)** A is false but R is true.

Explanation: A is false but R is true.

Section B

21. (a, b), (-a, -b)

Required distance

$$= \sqrt{(-a-a)^2 + (-b-b)^2}$$

$$= \sqrt{(-2a)^2 + (-2b)^2}$$

$$= \sqrt{4a^2 + 4b^2}$$

$$= \sqrt{4(a^2 + b^2)}$$

$$= 2\sqrt{a^2 + b^2}$$

OR

The given points are P(a sin α, a cos α) and Q(a cos α, - a sin α)

$\left(x_1 = a\sin\alpha, y_1 = a\cos\alpha\right)$and $\left(x_2 = a\cos\alpha, y_2 = -a\sin\alpha\right)$

Therefore,by using distance formula,we have,

$$PQ = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

$$= \sqrt{(a\cos\alpha - a\sin\alpha)^2 + (-a\sin\alpha - a\cos\alpha)^2}$$

$$= \sqrt{a^2\cos^2\alpha + a^2\sin^2\alpha - 2a^2\cos\alpha\sin\alpha + a^2\cos^2\alpha + a^2\sin^2\alpha + 2a^2\cos\alpha\sin\alpha}$$

$$= \sqrt{a^2\cos^2\alpha + a^2\sin^2\alpha + a^2\cos^2\alpha + a^2\sin^2\alpha}$$

$$= \sqrt{a^2\left(\cos^2\alpha + \sin^2\alpha\right) + a^2\left(\cos^2\alpha + \sin^2\alpha\right)}$$

$$= \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a \text{ units}$$

22. $15x^2 - 10\sqrt{6}x + 10 = 0$.

$3x^2 - 2\sqrt{6}x + 2 = 0$

$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$$\therefore \quad x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$$

23. Let us assume that $4 + \sqrt{2}$ is rational. Then, there exist positive co-primes a and b such that

$$4 + \sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 4$$

$$\sqrt{2} = \frac{a-4b}{b}$$

As a - 4b and b are integers.

So, $\frac{a-4b}{b}$ is a rational number .

But $\sqrt{2}$ is not rational number .

Since a rational number cannot be equal to an irrational number . Our assumption that $4 + \sqrt{2}$ is a rational number is wrong .

Hence, $4 + \sqrt{2}$ is irrational.

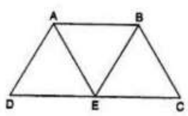
24. From the triangle, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ} = 0.5$

Hence the corresponding sides are propotional. Thus the corresponding angles will be equal. The triangles ABC and QRP are similar i.e, $\triangle ABC \sim \triangle QRP$ by SSS similarity.

OR

Given: ABCD is a trapezium with AB $\parallel$ DC.Also $\triangle$ AED $\sim$ $\triangle$BED,

To Prove:AD = BC



Proof:$\frac{ar(\triangle AEC)}{ar(\triangle BEC)} = 1$ ......Similar triangle between the same parallels

$\Rightarrow \left(\frac{AD}{BC}\right)^2 = 1$ ......The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\Rightarrow$ $AD^2 = BC^2$ $\Rightarrow$ AD = BC

25. We have,

$$\sin 30^\circ = \frac{1}{2},\ \tan 60^\circ = \sqrt{3},\ \cos 60^\circ = \frac{1}{2}\ and\ \sec 30^\circ = \frac{2}{\sqrt{3}}$$

therefore,

$$2\sin^2 30^\circ \tan 60^\circ - 3\cos^2 60^\circ \sec^2 30^\circ$$

$$= 2\left(\sin 30^\circ\right)^2 \tan 60^\circ - 3\left(\cos 60^\circ\right)^2\left(\sec 30^\circ\right)^2$$

$= 2 \times \left(\frac{1}{2}\right)^2 \times \sqrt{3} - 3 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2$

$= 2 \times \frac{1}{4} \times \sqrt{3} - 3 \times \frac{1}{4} \times \frac{4}{3} = \frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{3}-2}{2}$

Section C

26. i. We have

$\text{ar}(\Delta ABC) = \frac{1}{2} \times AB \times CD = \frac{1}{2}cp$

[taking AB as base]

and $\text{ar}(\Delta ABC) = \frac{1}{2} \times BC \times AC = \frac{1}{2}ab$

[taking BC as base].

$\therefore \quad \frac{1}{2}cp = \frac{1}{2}ab$

$\Rightarrow$ cp = ab.

Hence, cp = ab.

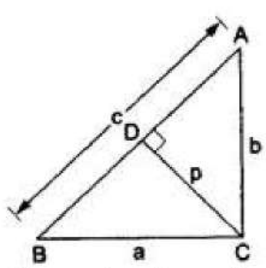
ii. cp = ab

$\Rightarrow \frac{1}{p} = \frac{c}{ab}$

$\therefore \quad \frac{1}{p^2} = \frac{c^2}{a^2b^2} = \frac{b^2+a^2}{a^2b^2} \left[\because AB^2 = AC^2 + BC^2\right]$

$= \left(\frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2}\right) = \left(\frac{1}{a^2} + \frac{1}{b^2}\right)$

Hence, $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.



27. Let the natural number be x.

Then, its positive square root will be $\sqrt{x}$

According to the question,

x + $\sqrt{x}$ = 132 ....(1) (sum of the number & it's square root is 132)

Let $\sqrt{x}$ =y $\Rightarrow x = y^2$

Hence, from (1), we have :-

$\Rightarrow$ $y^2 + y = 132$

$\Rightarrow y^2 + y - 132 = 0$

$\Rightarrow y^2 + 12y - 11y - 132 = 0$

$\Rightarrow y(y + 12) - 11(y + 12) = 0$

$\Rightarrow (y + 12)(y - 11) = 0$

$\Rightarrow y + 12 = 0$ or $y - 11 = 0$

$\Rightarrow y = -12$ or $y = 11$

Square root of a number cannot be negative, $y \neq -12$ (as $y = \sqrt{x}$)

Hence, $y = 11$

$\Rightarrow \sqrt{x} = 11 \Rightarrow x = 11^2$

$\Rightarrow x = 121$

Hence, the required natural number is $x = 121$.

28. Let P(x, y) be equidistant from the points A(-5, 4) and B(-1, 6).

Now,

AP = BP

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x + 5)^2 + (y - 4)^2 = (x + 1)^2 + (y - 6)^2$

$\Rightarrow x^2 + 25 + 10x + y^2 + 16 - 8y = x^2 + 1 + 2x + y^2 + 36 - 12y$

$\Rightarrow 10x + 41 - 8y = 2x + 37 - 12y$

$\Rightarrow 8x + 4y + 4 = 0$

$\Rightarrow 2x + y + 1 = 0$

Thus, all the points which lie on line $2x + y + 1 = 0$ are equidistant from A and B.

OR

Two vertices of an isosceles triangle are A(2, 0) and B(2, 5), Let C(X, y) be the third vertex.

$$AB = \sqrt{(2-2)^2 + (5-0)^2} = \sqrt{(0)^2 + (5)^2} = \sqrt{25} = 5$$

$$BC = \sqrt{(x-2)^2 + (y-5)^2} = \sqrt{x^2 + 4 - 4x + y^2 + 25 - 10y}$$

$$= \sqrt{x^2 - 4x + y^2 - 10y + 29}$$

$$AC = \sqrt{(x-2)^2 + (y-0)^2} = \sqrt{x^2 + 4 - 4x + y^2}$$

Also we are given that

AC = BC = 3

$\Rightarrow AC^2 = BC^2 = 9$

$\Rightarrow x^2 + 4 - 4x - y^2 = x^2 - 4x + y^2 - 10y + 29$

$\Rightarrow 10y = 25$

$$\Rightarrow \quad y = \frac{25}{10} = \frac{5}{2} = 2.5$$

$AC^2 = 9$

$x^2 + 4 - 4x + y^2 = 9$

$x^2 + 4 - 4x + (2.5)^2 = 9$

$x^2 + 4 - 4x + 6.25 = 9$

$x^2 - 4x + 1.25 = 0$

$D = (-4)^2 - 4 \times 1 \times 1.25$

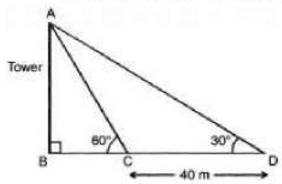
$D = 16 - 5$

$D = 11$

$$x = \frac{-(-4)+\sqrt{11}}{2 \times 1} = \frac{4+3.31}{2} = \frac{7.31}{2} = 3.65$$

$$\text{or, } x = \frac{-(-4)-\sqrt{11}}{2} = \frac{4-\sqrt{11}}{2} = \frac{4-3.31}{2} = 0.35$$

$\therefore$ The third vertex is (3.65, 2.5) or (0.35, 2.5)

29. 

In $\triangle ABC$, $\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \quad AB = \sqrt{3}BC$ .....(i)

In $\triangle ABD$,

$$\tan 30^\circ = \frac{AB}{BC+40}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BC+40} = \frac{\sqrt{3}BC}{BC+40}$$

3BC = BC + 40

BC = 20, Hence from (i) we get

$AB = 20\sqrt{3} = 20 \times 1.73 = 34.6$ meter

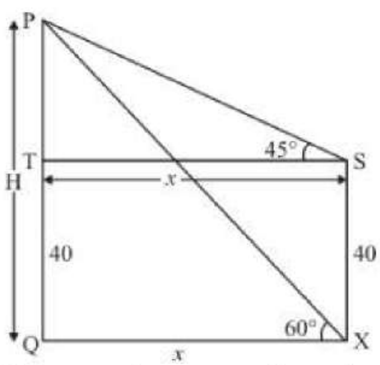
OR

Let PQ be the tower of height H m and an angle of elevation of the top of tower PQ from point X is 60°. The angle of elevation at 40 m vertical from point X is 45°.

Let PQ = H m and SX = 40m. Q$X = x$, $\angle PST = 45^o$, $\angle PXQ = 60^o$,

Here we have to find height of tower

The corresponding figure is as follows



We use trigonometric ratios.

In $\triangle$PST

$\Rightarrow \tan 45^o = \frac{h}{x}$

$\Rightarrow \ 1 = \frac{h}{x}$

$\Rightarrow$ x = h

Again in △PXQ

$\Rightarrow \ \tan 60^o = \frac{h+40}{x}$

$\Rightarrow \ \sqrt{3} = \frac{h+40}{x}$

$\Rightarrow$ h + 40 = $\sqrt{3}h$

$\Rightarrow$ h ($\sqrt{3} - 1$) = 40

$\Rightarrow \ h = \frac{40}{\sqrt{3}-1}$

$\Rightarrow$ h = 54.64

Therefore H = 54.64 + 40

$\Rightarrow$ H = 94.64

Hence the height of tower is 94.64 m.

30.

Class Interval	**Frequency(f_i)**	**Mid value x_i**	$u_i = \frac{x_i - A}{h} = \frac{x_i - 39}{6}$	$(f_i \times u_i)$
18 - 24	6	21	-3	-18
24 - 30	8	27	-2	-16
30 - 36	12	33	-1	-12
36 - 42	8	39 = A	0	0
42 - 48	4	45	1	4
48 - 54	2	51	2	4
	$\Sigma f_i = 40$			$\Sigma\left(f_i \times u_i\right)$= -38

Thus, A = 39, h = 6 $\sum f_i = 40$ and $\sum f_i u_i = -38$

$$\text{Mean} = A + \left\{h \times \frac{\sum f_i u_i}{\sum f_i}\right\}$$

$$= 39 + \left\{6 \times \frac{-38}{40}\right\}$$

= 39 - 5.7

= 33.3

31. Here, x - 3 is the HCF of

$x^3 - 2x^2 + px + 6$ and $x^2 - 5x + q$

SInce x - 3 is a common factor of given expression

$f(x) = x^3 - 2x^2 + px + 6$,then by factor theorem

f(x) = 0

$\Rightarrow 3^3 - 2 \times 3^2 + p \times 3 + 6 = 0$

$\Rightarrow 27 - 18 + 3p + 6 = 0 \Rightarrow 15 + 3p = 0$

$\Rightarrow 3p = -15 \Rightarrow p = \frac{-15}{3} = -5$

SInce x -3 is a factor of g(x) = $x^2 - 5x + q$,

then by factor theorem, g(3) = 0

$\Rightarrow 3^2 - 5 \times 3 + q = 0 \Rightarrow 9 - 15 + q = 0$

$\Rightarrow -6 + q = 0 \Rightarrow q = 6$

$\therefore 6p + 5q = 6 \times (-5) + 5 \times 6$

$= -30 + 30 = 0$

Hence 6p + 5q = 0

Section D

32. Let the present ages of Aftab and his daughter be x year and y year respectively. Then the algebraic representation is

Given by the following equations:

x - 7 = 7(y - 7)

$\Rightarrow$ x - 7y + 42 = 0 ...(1)

And x + 3 = 3(y + 3)

$\Rightarrow$ x - 3y - 6 = 0 ...(2)

To, represent this equation graphically, well find two solution for each equation, These solution are given below;

For Equation (1) x - 7y + 42 = 0

$\Rightarrow$ 7y = x + 42

$\Rightarrow y = \frac{x+42}{7}$

Table 1 of solutions

x	0	7
y	6	7

For Equation (2) x - 3y - 6 = 0

$\Rightarrow$ 3y = x - 6

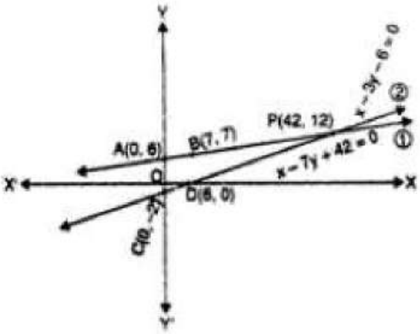
$\Rightarrow y = \frac{x-6}{3}$

Table 2 of solutions

x	0	6
y	-2	0

We plot the A(0, 6) and B(7, 7)

Corresponding to the solutions in table 1 on a graph paper to get the line AB representing the equation (1) and the points C(0, -2) and D(6, 0) corresponding to the solutions in table 2 on the same graph paper to get the line CD representing the equation (2), as shown in the figure



We observe in figure that the two lines representing the two equations are intersecting at the point P(42, 12).

OR

Let the digits at units and tens place of the given number be x and y respectively.
Thus, the number is $10y + x$.
The sum of the digits of the number is 13.
Thus, we have $x + y = 13$
According to the question,
After interchanging the digits, the number becomes $10x + y$.
The difference between the number obtained by interchanging the digits and the original number is 45.
Thus, we have $(10x + y) - (10y + x) = 45$
$\Rightarrow 10x + y - 10y - x = 45$
$\Rightarrow 9x - 9y = 45$
$\Rightarrow 9(x - y) = 45$
$\Rightarrow x - y = 5$
So, we have two equations
$x + y = 13$
$x - y = 5$
Here x and y are unknowns.
We have to solve the above equations for x and y.
Adding the two equations, we have
$(x + y) + (x - y) = 13 + 5$
$\Rightarrow x + y + x - y = 18$
$\Rightarrow 2x = 18$
$\Rightarrow x = 9$
Substituting the value of x in the first equation,
$\Rightarrow 9 + y = 13$
$\Rightarrow y = 13 - 9$
$\Rightarrow y = 4$
Hence, the number is $10 \times 4 + 9 = 49$

33. As we know that the lengths of tangents to a circle are equal which are drawn from an external point.
Therefore, AP = AR, BP = BQ and CQ = CR
Proof of Part (i):
AB + CQ = AP + PB + CQ
= AR + BQ + CQ [$\because$ AP = AR and PB = BQ]
= (AR + CR) + BQ [$\because$ CQ = CR]
= AC + BQ [$\because$ AR + CR = AC]

Hence proved.

Proof of Part (ii):

Area (ΔABC)= Area (ΔOBC)+ Area (ΔOAB)+ Area (ΔOAC)

$$= \frac{1}{2}(BC \times OQ) + \frac{1}{2}(AB \times OP) + \frac{1}{2}(AC \times OR)$$

$$= \frac{1}{2}(BC \times r) + \frac{1}{2}(AB \times r) + \frac{1}{2}(AC \times r)$$

$$= \frac{1}{2}(BC + AB + AC) \times r$$

$$= \frac{1}{2}(\text{ Perimeter of } \Delta ABC) \times r$$

Hence proved.

34. Total cards having number divisible by 7 are 7,14, 21, 28,35, 42,49

Hence m=7

So P(E)=$\frac{m}{n} = \frac{7}{48}$

Number of cards having a perfect square are = 4,9,16, 25,36,49

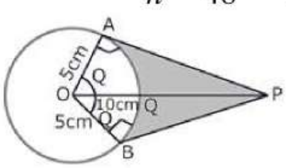
Hence m=6

So P(E) =$\frac{m}{m}\ \frac{6}{48} = \frac{1}{8}$

Number of multiples of 6 from 3 to 50 are = 6,12,18, 24,30,36,42,48

Hence m=8

So P(E) =$\frac{m}{n} = \frac{8}{48} = \frac{1}{6}$

35. 

$$\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \theta = 60^{o}$$

$$\Rightarrow \angle AOB = 2 \times \theta = 120^{o}$$

$$\therefore \text{ ARC AB} = \frac{120 \times 2 \times \pi \times 5}{360} cm = \frac{10\pi}{3} cm \left[\because l = \frac{\theta}{360} \times 2\pi r \right]$$

Length of the belt that is in contact with the rim of the pulley

= Circumference of the rim - length of arc AB

$$= 2\pi \times 5 \text{ cm} - \frac{10\pi}{3} \text{cm}$$

$$= \frac{20\pi}{3} \text{cm}$$

Now, the area of sector OAQB =

$\frac{120 \times \pi \times 5 \times 5}{360} cm^2 = \frac{25\pi}{3} cm^2 \left[\because Area = \frac{\theta}{360} \times \pi r^2\right]$

Area of quadrilateral OAPB = 2(Area of $\triangle$OAP) = $25\sqrt{3}$ cm^2

$\left[\because AP = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}\ cm\right]$

Hence, shaded area $= 25\sqrt{3} - \frac{25\pi}{3} = \frac{25}{3}[3\sqrt{3} - \pi]\ cm^2$

OR

We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

For the first triangle, we have a = 35, b = 53 and c = 66.

$\therefore \quad s = \frac{a+b+c}{2} = \frac{35+53+66}{2} = 77cm$

Let Δ_1 be the area of the first triangle. Then,

$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)} = \sqrt{77 \times 42 \times 24 \times 11}$

$\Rightarrow \quad \Delta_1 = \sqrt{7 \times 11 \times 7 \times 6 \times 6 \times 4 \times 11} =$

$\sqrt{7^2 \times 11^2 \times 6^2 \times 2^2} = 7 \times 11 \times 6 \times 2 = 924cm^2$ ...(i)

For the second triangle, we have a = 33,b = 56,c = 65

$\therefore \quad s = \frac{a+b+c}{2} = \frac{33+56+65}{2} = 77cm$

Let Δ_2 be the area of the second triangle. Then,

$\Delta_2 = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_2 = \sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow \quad \Delta_2 = \sqrt{77 \times 44 \times 21 \times 12} =$

$\sqrt{7 \times 11 \times 4 \times 11 \times 3 \times 7 \times 3 \times 4} = \sqrt{7^2 \times 11^2 \times 4^2 \times 3^2}$

$\Rightarrow \quad \Delta_2 = 7 \times 11 \times 4 \times 3 = 924cm^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2 = \Delta_1 + \Delta_2$

$\Rightarrow \quad \pi r^2 = 924 + 924$

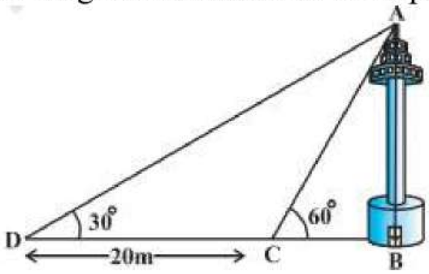
$\Rightarrow \quad \frac{22}{7} \times r^2 = 1848$

$\Rightarrow \quad r^2 = 1848 \times \frac{7}{22} = 3 \times 4 \times 7 \times 7 \Rightarrow r = \sqrt{3 \times 2^2 \times 7^2} = 2 \times 7 \times \sqrt{3} = 14\sqrt{3}cm$

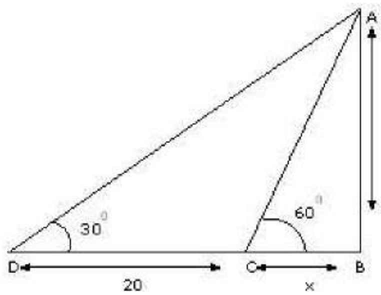
Section E

36. **Read the text carefully and answer the questions:**

A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30°.



(i)



Let 'h' (AB) be the height of tower and x be the width of the river.

In $\triangle ABC, \frac{h}{x} = \tan 60^{\circ}$

$\Rightarrow h = \sqrt{3}x$ ...(i)

In $\Delta ABD, \frac{h}{x+20} = \tan 30^{\circ}$

$\Rightarrow h = \frac{x+20}{\sqrt{3}}$ ...(ii)

Equating (i) and (ii),

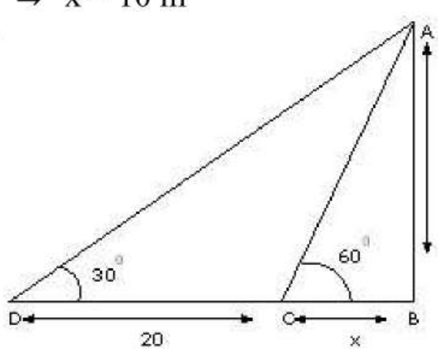
$\sqrt{3}x = \frac{x+20}{\sqrt{3}}$

$\Rightarrow$ 3x = x + 20

$\Rightarrow$ 2x = 20

$\Rightarrow$ x = 10 m

(ii)



Let 'h' (AB) be the height of tower and x be the width of the river.

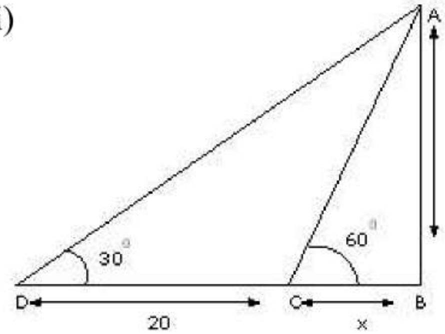
In $\triangle ABC, \frac{h}{x} = \tan 60^{\circ}$

$\Rightarrow h = \sqrt{3}x$ ...(i)

Put x = 10 in (i), $h = \sqrt{3}x$

$\Rightarrow h = 10\sqrt{3}$m

(iii)



In $\triangle$ABD

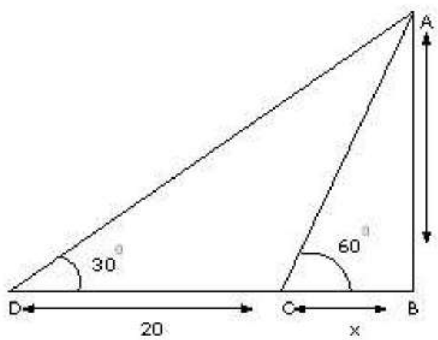
$\sin 30^{o} = \frac{AB}{AD}$

$\Rightarrow$ AD = $\frac{AB}{\sin 30^{\circ}}$

$\Rightarrow$ AD = $\frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}$ m

OR



In $\triangle$ABC

$\sin 60^{o} = \frac{AB}{AC}$

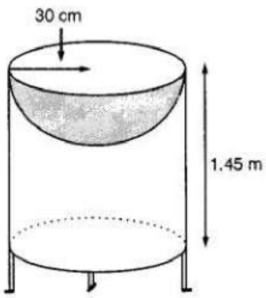
$\Rightarrow$ AC = $\frac{AB}{\sin 30^{0}}$

$\Rightarrow$ AC = $\frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

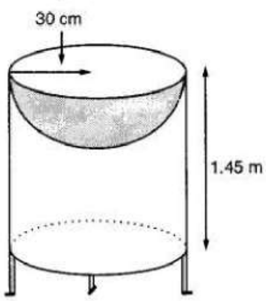
$\Rightarrow$ AC = 20 m

37. **Read the text carefully and answer the questions:**

Mayank a student of class 7^{th} loves watching and playing with birds of different kinds. One day he had an idea in his mind to make a bird-bath on his garden. His brother who is studying in class 10^{th} helped him to choose the material and shape of the birdbath. They made it in the shape of a cylinder with a hemispherical depression at one end as shown in the Figure below. They opted for the height of the hollow cylinder as 1.45 m and its radius is 30 cm. The cost of material used for making bird bath is ₹40 per square meter.

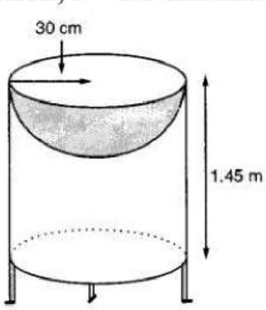


(i) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.
Then, r = 30 cm and h = 1.45 m = 145 cm.



Curved surface area of the hemisphere = $2\pi r^2$
$= 2 \times 3.14 \times 30^2 = 0.56\ m^2$

(ii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.
Then, r = 30 cm and h = 1.45 m = 145 cm.



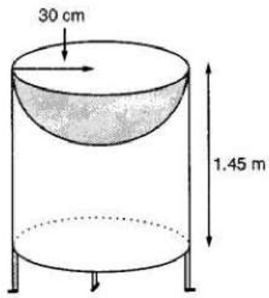
Let S be the total surface area of the birdbath.
S = Curved surface area of the cylinder + Curved surface area of the hemisphere

$$\Rightarrow S = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$$

$$\Rightarrow S = 2 \times \frac{22}{7} \times 30(145 + 30) = 33000\ cm^2 = 3.3\ m^2$$

(iii) Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.

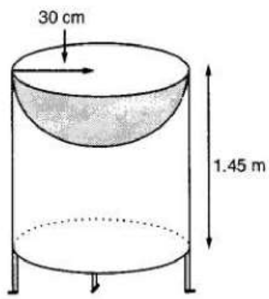
Then, r = 30 cm and h = 1.45 m = 145 cm.



Total Cost of material = Total surface area $\times$ cost per sq m^2
= 3.3 $\times$ 40 = ₹132

OR

Let r be the common radius of the cylinder and hemisphere and h be the height of the hollow cylinder.
Then, r = 30 cm and h = 1.45 m = 145 cm.



$r = 35 \text{ cm} = \frac{35}{100} \text{m}$

We know that S.A = 3.3 m^2
$S = 2\pi r(r + h)$

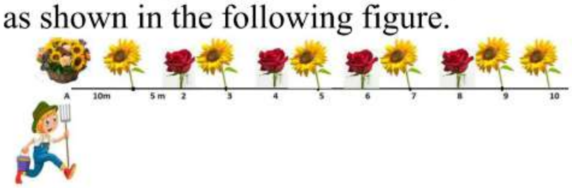
$$\Rightarrow 3.3 = 2 \times \frac{22}{7} \times \frac{35}{100}\left(\frac{35}{100} + h\right)$$

$$\Rightarrow 3.3 = \frac{22}{10}\left(\frac{35}{100} + h\right)$$

$$\Rightarrow \frac{33}{22} = \frac{35}{100} + h$$

$$\Rightarrow h = \frac{3}{2} - \frac{7}{20} = \frac{23}{20} = 1.15 \text{ m}$$

38. **Read the text carefully and answer the questions:**
In a school garden, Dinesh was given two types of plants viz. sunflower and rose flower as shown in the following figure.



The distance between two plants is to be 5m, a basket filled with plants is kept at point A which is 10 m from the first plant. Dinesh has to take one plant from the basket and then he will have to plant it in a row as shown in the figure and then he has to return to the basket to collect another plant. He continues in the same way until all the flower plants in the basket. Dinesh has to plant ten numbers of flower plants.

(i) The distance covered by Dinesh to pick up the first flower plant and the second flower plant,
$= 2 \times 10 + 2 \times (10 + 5) = 20 + 30$
therefore, the distance covered for planting the first 5 plants
$= 20 + 30 + 40 + \dots$ 5 terms
This is in AP where the first term a = 20
and common difference d = 30 - 20 = 10

(ii) We know that a = 20, d = 10 and number of terms = n = 5 so,
$$S_n = \frac{n}{2}[2a + (n - 1)d]$$
So, the sum of 5 terms
$$S_5 = \frac{5}{2}[2 \times 20 + 4 \times 10] = \frac{5}{2} \times 80 = 200 \text{ m}$$
Hence, Dinesh will cover 200 m to plant the first 5 plants.

(iii) As a = 20, d = 10 and here n = 10
$$\text{So, } S_{10} = \frac{10}{2}[2 \times 20 + 9 \times 10] = 5 \times 130 = 650 \text{ m}$$
So, hence Ramesh will cover 650 m to plant all 10 plants.

OR

Total distance covered by Ramesh 650 m
$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{650}{10} = 65 \text{ minutes}$$
Time taken to plant all 10 plants = $15 \times 10 = 150$ minutes
Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes