QUADRILATERALS

IMPORTANT POINTS

- A quadrilateral is a figure bounded by four line segments such that no three of them are parallel.
- Two sides of quadrilateral are consecutive or adjacent sides, if they have a common point (vertex).
- Two sides of a quadrilateral are opposite sides, if they have no common end-point (vertex).
- The consecutive angles of a quadrilateral are two angles which include a side in their intersection. In other words, two angles are consecutive, if they have a common arm.
- Two angles of a quadrilateral are said to be opposite angles if they do not have a common arm.
- The sum of the four angles of a quadrilateral is 360^o.

❖ EXAMPLES ❖

Ex.1 In a quadrilateral ABCD, the angles A, B, C and D are in the ratio 2 : 4 : 5 : 7. Find the measure of each angles of the quadrilateral.

Sol. We have $\angle A : \angle B : \angle C : \angle D = 2 : 4 : 5 : 7$. So, let $\angle A = 2x^o$, $\angle B = 4x^o$, $\angle C = 5x^o$, $\angle D = 7x^o$.

$\therefore \quad \angle A + \angle B + \angle C + \angle D = 360^o$

$\Rightarrow \quad 2x + 4x + 5x + 7x = 360^o$

$\Rightarrow \quad 18x = 360^o$

$\Rightarrow x = 20^o$

Thus, the angles are :

$\angle A = 40^o$, $\angle B = (4 \times 20)^o = 80^o$,

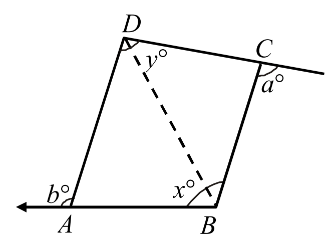
$\angle C = (5 \times 20)^o = 100^o$

and, $\angle D = (7x)^o = (7 \times 20)^o = 140^o$

Ex.2 The sides BA and DC of a quadrilateral ABCD are produced as shown in fig.

Prove that a + b = x + y.

Sol. Join BD. In ΔABD, we have



$\angle ABD + \angle ADB = b^o$ ....(i)

In ΔCBD, we have

$\angle CBD + \angle CDB = a^o$ ....(ii)

Adding (i) and (ii), we get

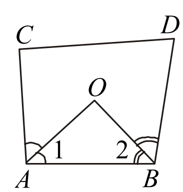
$(\angle ABD + \angle CBD) + (\angle ADB + \angle CDB) = a^o + b^o$

$\Rightarrow \quad x^o + y^o = a^o + b^o$

Hence, $\quad x + y = a + b$

Ex.3 In a quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively. Prove that $\angle AOB = \frac{1}{2}(\angle C + \angle D)$.

Sol. In ΔAOB, we have



$\angle AOB + \angle 1 + \angle 2 = 180^o$

$\Rightarrow \quad \angle AOB = 180^o - (\angle 1 + \angle 2)$

$\Rightarrow \quad \angle AOB = 180^o - \left(\frac{1}{2}\angle A + \frac{1}{2}\angle B\right)$

$\left[\because \angle 1 = \frac{1}{2}\angle A \text{ and } \angle 2 = \frac{1}{2}\angle B\right]$

$\Rightarrow \ \angle AOB = 180^{o} - \frac{1}{2}\ (\angle A + \angle B)$

$\Rightarrow \ \angle AOB = 180° - \frac{1}{2}\ [360^{o} - (\angle C + \angle D)]$

$[\Theta\ \angle A + \angle B + \angle C + \angle D = 360^{o}$

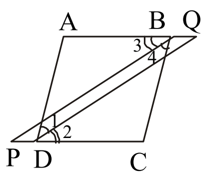
$\therefore \ \ \angle A + \angle B = 360^{o} - (\angle C + \angle D)]$

$\Rightarrow \ \angle AOB = 180^{o} - 180^{o} + \frac{1}{2}\ (\angle C + \angle D)$

$\Rightarrow \ \angle AOB = \frac{1}{2}\ (\angle C + \angle D)$

Ex.4 In figure bisectors of $\angle B$ and $\angle D$ of quadrilateral ABCD meet CD and AB produced at P and Q respectively. Prove that

$$\angle P + \angle Q = \frac{1}{2}\ (\angle ABC + \angle ADC)$$



Sol. In ΔPBC, we have

$\therefore \ \ \angle P + \angle 4 + \angle C = 180^{o}$

$\Rightarrow \ \angle P + \frac{1}{2}\angle B + \angle C = 180^{o}$ ...(i)

In ΔQAD, we have $\angle Q + \angle A + \angle 1 = 180^{o}$

$\Rightarrow \ \angle Q + \angle A + \frac{1}{2}\angle D = 180^{o}$ ...(ii)

Adding (i) and (ii), we get

$\angle P + \angle Q + \angle A + \angle C + \frac{1}{2}\angle B + \frac{1}{2}\angle D$

$= 180^{o} + 180^{o}$

$\Rightarrow \angle P + \angle Q + \angle A + \angle C + \frac{1}{2}\angle B + \frac{1}{2}\angle D = 360^{o}$

$\Rightarrow \ \angle P + \angle Q + \angle A + \angle C + \frac{1}{2}(\angle B + \angle D)$

$= \angle A + \angle B + \angle C + \angle D$

$[\therefore$ In a quadrilateral ABCD $\angle A + \angle B + \angle C$

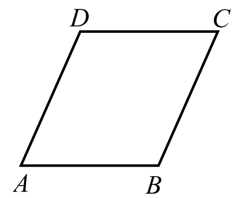
$+ \angle D = 360^{o}]$

$\Rightarrow \ \angle P + \angle Q = \frac{1}{2}(\angle B + \angle D)$

$\Rightarrow \ \angle P + \angle Q = \frac{1}{2}(\angle ABC + \angle ADC)$

Ex.5 In a parallelogram ABCD, prove that sum of any two consecutive angles is 180^{o}.

Sol. Since ABCD is a parallelogram. Therefore, AD || BC.



Now, AD || BC and transversal AB intersects them at A and B respectively.

$\therefore \ \ \angle A + \angle B = 180^{o}$

$[\Theta$ Sum of the interior angles on the same side of the transversal is $180^{o}]$

Similarly, we can prove that

$\angle B + \angle C = 180^{o}$, $\angle C + \angle D = 180^{o}$ and $\angle D + \angle A = 180^{o}$.

- A quadrilateral having exactly one pair of parallel sides, is called a trapezium.
- A trapezium is said to be an isoscels trapezium, if its non-parallel sides are equal.
- A quadrilateral is a parallelogram if its both pairs of opposite sides are parallel.
- A parallelogram having all sides equal is called a rhombus.
- A parallelogram whose each angle is a right angle, is called a rectangle.
- A square is a rectangle with a pair of adjacent sides equal.
- A quadrilateral is a kite if it has two pairs of equal adjacent sides and unequal opposite sides.
- A diagonal of a parallelogram divides it into two congruent triangles.
- In a parallelogram, opposite sides are equal.
- The opposite angles of a parallelogram are equal.
- The diagonals of a parallelogram bisect each other.
- In a parallelogram, the bisectors of any two consecutive angles intersect at right angle.
- If diagonal of a parallelogram bisects one of the angles of the parallelogram, it also bisects the second angle.
- The angle bisectors of a parallegram form a rectangle.

Ex.6 In a parallelogram ABCD, $\angle D = 115^o$, determine the measure of $\angle A$ and $\angle B$.

Sol. Since the sum of any two consecutive angles of a parallelogram is 180^o. Therefore,

$$\angle A + \angle D = 180^o \text{ and } \angle A + \angle B = 180^o$$

Now, $\angle A + \angle D = 180^o$

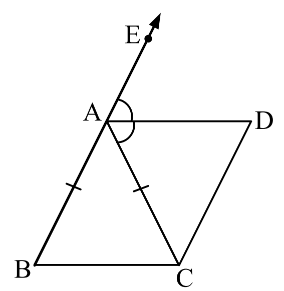
$\Rightarrow \angle A + 115^o = 180^o$ [Θ $\angle D = 115^o$ (given)]

$\Rightarrow \angle A = 65^o$ and $\angle A + \angle B = 180^o$

$\Rightarrow 65^o + \angle B = 180^o \Rightarrow \angle B = 115^o$

Thus, $\angle A = 65^o$ and $\angle B = 115^o$

Ex.7 In figure, AB = AC, $\angle EAD = \angle CAD$ and CD || AB. Show that ABCD is a parallelogram.



Sol. In ΔABC, AB = AC [Given]

$\Rightarrow \angle ABC = \angle ACB$ ....(1)

(Angles opposite the equal sides are equal)

$\angle EAD = \angle CAD$[Given] ....(2)

Now, $\angle EAC = \angle ABC + \angle ACB$

$\left(\begin{array}{l}\text{An exterior angle is equal to sum of} \\ \text{two interior opposite angles of a triangles}\end{array}\right)$

$\Rightarrow \angle EAD + \angle CAD = \angle ABC + \angle ACB$

$\Rightarrow \angle CAD + \angle CAD = \angle ACB + \angle ACB$

By (1) and (2)

$\Rightarrow 2\angle CAD = 2\angle ACB$

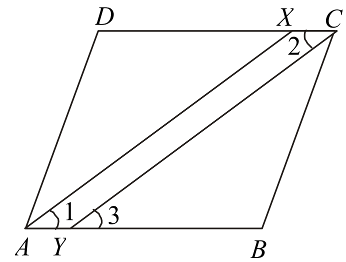
$\Rightarrow \angle CAD = \angle ACB$

$\Rightarrow$ BC | | AD

Also, CD | | AB [Given]

Thus, we have both pairs of opposite sides of quadrilateral ABCD parallel. Therefore, ABCD is a parallelogram.

Ex.8 ABCD is a parallelogram and line segments AX,CY are angle bisector of $\angle A$ and $\angle C$ respectively then show AX || CY.



Sol. Since opposite angles are equal in a parallelogram. Therefore, in parallelogram ABCD, we have $\angle A = \angle C$

$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$

$\Rightarrow \angle 1 = \angle 2$ ....(i)

[Θ AX and CY are bisectors of $\angle A$ and $\angle C$ respectively]

Now, AB || DC and the transversal CY intersects them.

$\therefore \angle 2 = \angle 3$ ....(ii)

[Θ Alternate interior angles are equal]

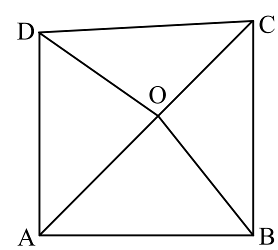
From (i) and (ii), we get

$\angle 1 = \angle 3$

Thus, transversal AB intersects AX and YC at A and Y such that $\angle 1 = \angle 3$ i.e. corresponding angles are equal.

$\therefore$ AX || CY

Ex.9 In the adjoining figure, a point O is taken inside an equilateral quad. ABCD such that OB = OD. Show that A, O and C are in the same straight line.



Sol. Given a quad. ABCD in which AB = BC = CD = DA and O is a point within it such that OB = OD.

To prove $\angle AOB + \angle COB = 180^o$

Proof In ΔOAB and OAD, we have

AB = AD (given), OA = OA

(common) and OB = OD (given)

$\therefore \quad \Delta OAB \cong \Delta OAD$

$\therefore \quad \angle AOB = \angle AOD \quad$ ….(i) (c.p.c.t.)

Similarly, $\Delta OBC \cong \Delta ODC$

$\therefore \ \angle COB = \angle COD \quad$ ….(ii)

Now, $\angle AOB + \angle COB + \angle COD + \angle AOD$

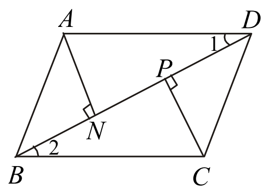
$= 360^o \quad$ [$\angle$ at a point]

$\Rightarrow 2(\angle AOB + \angle COB) = 360^o$

$\Rightarrow \angle AOB + \angle COB = 180^o$

Ex.10 In figure AN and CP are perpendiculars to the diagonal BD of a parallelogram ABCD. Prove that :

(i) $\Delta ADN \cong \Delta CBP \qquad$ (ii) AN = CP



Sol. Since ABCD is a parallelogram.

$\therefore \quad AD \parallel BC$

Now, AD || BC and transversal BD intersects them at B and D.

$\therefore \quad \angle 1 = \angle 2$

[$\because$ Alternate interior angles are equal]

Now, in Δs ADN and CBP, we have

$\angle 1 = \angle 2$

$\angle AND = \angle CPD$ and, AD = BC

[$\because$ Opposite sides of a $\|^{gm}$ are equal]

So, by AAS criterion of congruence

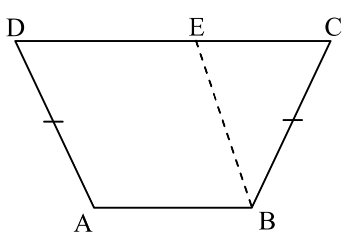
$\Delta ADN \cong \Delta CBP$

AN = CP

[$\because$Corresponding parts of congruent triangles are equal]

Ex.11 In figure, ABCD is a trapezium such that

AB || CD and AD = BC.



BE || AD and BE meets BC at E.

Show that (i) ABED is a parallelogram.

(ii) $\angle A + \angle C = \angle B + \angle D = 180^o$.

Sol. Here, $\quad AB \parallel CD \quad$ (Given)

$\Rightarrow \quad AB \parallel DE \quad$ ….(1)

Also, $\quad BE \parallel AD$ (Given) $\quad$ ….(2)

From (1) and (2),

ABED is a parallelogram

$\Rightarrow \quad AD = BE \quad$ ….(3)

Also, $\quad AD = BC$ (Given) $\quad$ ….(4)

From (3) and (4),

$BE = BC$

$\Rightarrow \quad \angle BEC = \angle BCE \quad$ ….(5)

Also, $\quad \angle BAD = \angle BED$

(opposite angles of parallelogram ABED)

i.e., $\quad \angle BED = \angle BAD \quad$ ….(6)

Now, $\angle BED + \angle BEC = 180^o$ (Linear pair of angles)

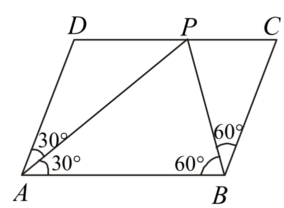
$\Rightarrow \quad \angle BAD + \angle BCE = 180^o$

By (5) and (6)

$\Rightarrow \quad \angle A + \angle C = 180^o$

Similarly, $\angle B + \angle D = 180^o$

Ex.12 In figure ABCD is a parallelogram and $\angle DAB = 60^o$. If the bisectors AP and BP of angles A and B respectively, meet at P on CD, prove that P is the mid-point of CD.



Sol. We have, $\angle DAB = 60^o$

$\angle A + \angle B = 180^o$

$\therefore \quad 60^o + \angle B = 180^o \Rightarrow \angle B = 120^o$

Now, AB || DC and transversal AP intersects them.

$\therefore \quad \angle PAB = \angle APD$

$\Rightarrow \quad \angle APD = 30^o \qquad$ [$\because \angle PAB = 30^o$]

Thus, in ΔAPD, we have

$\angle PAD = \angle APD$ [Each equal to 30º]

$\Rightarrow$ AD = PD .... (i)

[Θ Angles opposite to equal sides are equal] Since BP is the bisector of $\angle B$. Therefore,

$\angle ABP = \angle PBC = 60º$

Now, AB || DC and transversal BP intersects them.

$\therefore$ $\angle CPB = \angle ABP$

$\Rightarrow$ $\angle CPB = 60º$ [Θ $\angle ABP = 60º$]

Thus, in ΔCBP, we have

$\angle CBP = \angle CPB$ [Each equal to 60º]

$\Rightarrow$ CP = BC

Θ [Sides opp, to equal angles are equal]

$\Rightarrow$ CP = AD .... (ii)

[Θ ABCD is a $||^{gm}$ $\therefore$ AD = BC]

From (i) and (ii), we get

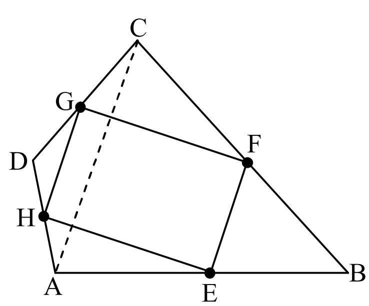
PD = CP

$\Rightarrow$ P is the mid point of CD.

- A quadrilateral is a parallelogam if its opposite sides are equal.
- A quadrilateral is a parallelogram if its opposite angles are equal.
- If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- A quadrilateral is a parallelogram, if its one pair of opposite sides are equal and parallel.

Ex.13 Prove that the line segments joining the mid-point of the sides of a quadrilateral forms a parallelogram.

Sol. Points E, F, G and H are the mid-points of the sides AB, BC, CD and DA respectively, of the quadrilateral ABCD. We have to prove that EFGH is a parallelogram.



Join the diagonal AC of the quadrilateral ABCD.

Now, in ΔABC, we have E and F mid-points of the sides BA and BC.

$\Rightarrow$ EF || AC

and $EF = \frac{1}{2} AC$ .... (1)

Similarly, from ΔADC, we have

GH || AC

and $GH = \frac{1}{2} AC$ ….(2)

Then from (1) and (2), we have

EF || GH

and EF = GH

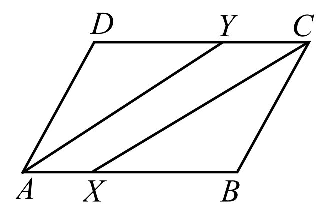
This proves that EFGH is a parallelogram.

Ex.14 In figure ABCD is a parallelogram and X, Y are the mid-points of sides AB and DC respectively. Show that AXCY is a parallelogram.

Sol. Since X and Y are the mid-points of AB and DC respectively. Therefore,

$AX = \frac{1}{2} AB$ and $CY = \frac{1}{2} DC$ ... (i)

But,AB = DC [Θ ABCD is a $||^{gm}$]



$\Rightarrow$ $\frac{1}{2} AB = \frac{1}{2} DC$

$\Rightarrow$ AX = CY .... (ii)

Also, AB || DC

$\Rightarrow$ AX || YC .... (iii)

Thus, in quadrilateral AXCY, we have

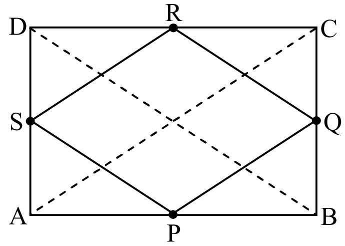
AX || YC and AX = YC

[From (ii) and (iii)]

Hence, quadrilateral AXCY is a parallelogram.

Ex.15 Prove that the line segments joining the mid-points of the sides of a rectangle forms a rhombus.

Sol. P, Q, R and S are the mid-points of the sides AB, BC, CD and DA of the rectangle ABCD.



Here, AC = BD (ΘΔABC ≅ ΔBAD)

Now, SR || AC and SR = $\frac{1}{2}$ AC

and PQ || AC and PQ = $\frac{1}{2}$ AC

⇒ SR || PQ and SR = PQ = $\frac{1}{2}$ AC

Similarly, PS || QR and PS = QR = $\frac{1}{2}$ BD

⇒ SR || PQ, PS || QR

and SR = PQ = PS = QR (ΘAC = BD)

PQRS is a rhombus.

Ex.16 In figure ABCD is a parallelogram and X and Y are points on the diagonal BD such that DX = BY. Prove that

(i) AXCY is a parallelogram

(ii) AX = CY, AY = CX

(iii) ΔAYB ≅ ΔCXD

Sol. Given : ABCD is a parallelogram. X and Y are points on the diagonal BD such that DX = BY

To Prove :

(i) AXCY is a parallelogram

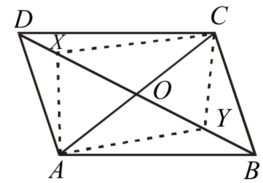
(ii) AX = CY, AY = CX

(iii) ΔAYB ≅ ΔCXD

Construction : join AC to meet BD at O.

Proof :

(i) We know that the diagonals of a parallelogram bisect each other. Therefore, AC and BD bisect each other at O.



∴ OB = OD

But,BY = DX

∴ OB – BY = OD – DX

⇒ OY = OX

Thus, in quadrilateral AXCY diagonals AC and XY are such that OX = OY and OA = OC i.e. the diagonals AC and XY bisect each other.

Hence, AXCY is a parallelogram.

(ii) Since AXCY is a parallelogram

∴ AX = CY and AY = CX

(iii) In triangles AYB and CXD, we have

AY = CX [From (ii)]

AB = CD

[Θ ABCD is a parallelogram]

BY = DX [Given]

So, by SSS-criterion of congruence, we have

ΔAYB ≅ ΔCXD

Ex.17 In fig. ABC is an isosceles triangle in which AB = AC. CP || AB and AP is the bisector of exterior ∠CAD of ΔABC. Prove that ∠PAC = ∠BCA and ABCP is a parallelogram.

Sol. Given : An isosceles ΔABC having AB = AC.AP is the bisector of ext ∠ CAD and CP || AB.

To Prove : ∠PAC = ∠BCA and ABCP

Proof : In ΔABC, we have

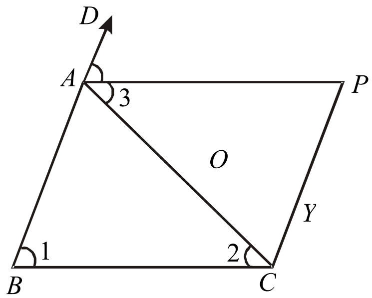
AB = AC [Given]

⇒ ∠1 = ∠2 .... (i)

[Θ Angles opposite to equal sides in a Δ are equal]

Now, in Δ ABC, we have

ext ∠CAD = ∠1 + ∠2



$\left[\begin{array}{l}\Theta \text{ An exterior angles is equal to the} \\ \text{sum of two opposite interior angles}\end{array}\right]$

$\Rightarrow$ ext $\angle CAD = 2\angle 2$ [Θ $\angle 1 = \angle 2$ (from (i))]

$\Rightarrow$ $2\angle 3 = 2\angle 2$

[Θ AP is the bisector of ext.$\angle CAD$ $\therefore \angle CAD = 2\angle 3$]

$\Rightarrow$ $\angle 3 = \angle 2$

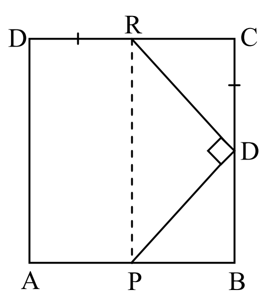
Thus, AC intersects lines AP and BC at A and C respectively such that $\angle 3 = \angle 2$ i.e., alternate interior angles are equal. Therefore,

AP || BC.

But,CP || AB [Gvien]

Thus, ABCP is a quadrilateral such that AP || BC and CP || AB. Hence, ABCP is a parallelogram.

Ex.18 In the given figure, ABCD is a square and $\angle PQR = 90^o$. If PB = QC = DR, prove that



(i) QB = RC, (ii) PQ = QR, (iii) $\angle QPR = 45^o$.

Sol. BC = DC, CQ = DR $\Rightarrow$ BC – CQ = ΔCDR

$\Rightarrow$ QB = RC

From ΔCQR, $\angle RQB = \angle QCR + \angle QRC$

$\Rightarrow \angle RQP + \angle PQB = 90^o + \angle QRC$

$\Rightarrow 90^o + \angle PQB = 90^o + \angle QRC$

Now, $\Delta RCQ \cong \Delta QBP$ and therefore,

QR = PQ

PQ = QR $\Rightarrow \angle QPR = \angle PRQ$

Bur, $\angle QPR + \angle PRQ = 90^o$.

So, $\angle QPR = 45^o$

- ❖ Each of the four angles of a rectangel is a right angle.
- ❖ Each of the four sides of a rhombus is of the same length.
- ❖ Each of the angles of a square is a right angle and each of the four sides is of the same length.
- ❖ The diagonals of a rectangle are of equal length.
- ❖ If the two diagonals of parallelogram are equal, it is a rectangle.
- ❖ The diagonals of a rhombus are perpendicular to each other.
- ❖ If the diagonals of a parallelogram are perpendicular, then it is a rhombus.
- ❖ The diagonals of a square are equal and perpendicular to each other.
- ❖ If the diagonals of a parallelogram are equal and intersect at right angles then the parallelogram is a square.

❖ EXA MPLES ❖

Ex.19 Prove that in a parallelogram

(i) opposite sides are equal

(ii) opposite angles are equal

(iii) each diagonal bisects the parallelogram

Sol. Given : A ||gm ABCD in which AB || DC and AD || BC.

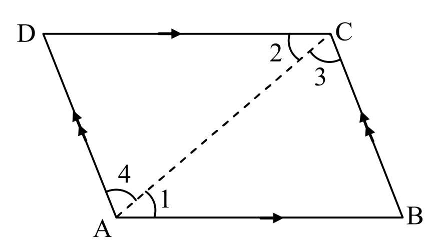
To prove (i) AB = CD and BC = AD;

(ii) $\angle B = \angle D$ and $\angle A = \angle C$,

(iii) $\Delta ABC = \Delta CDA$ and $\Delta ABD = \Delta CDB$

Construction join A and C.

In ΔABC and CDA, we have,



$\angle 1 = \angle 2$

[Alt. int. $\angle$, as AB || DC and CA cuts them]

$\angle 3 = \angle 4$

[Alt. int. $\angle$, as BC || AD and CA cuts them]

AC = CA (common)

$\therefore$ $\Delta ABC \cong \Delta CDA$ [AAS-criterial]

(i) $\Delta ABC \cong \Delta CDA$ (proved)

$\therefore$ AB = CD and BC = AD (c.p.c.t.)

(ii) $\Delta ABC \cong \Delta CDA$ (proved)

$\therefore$ $\angle B = \angle D$ (c.p.c.t.)

Also, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

$\angle 1 + \angle 4 = \angle 2 + \angle 3 \Rightarrow \angle A = \angle C$

Hence, $\angle B = \angle D$ and $\angle A = \angle C$

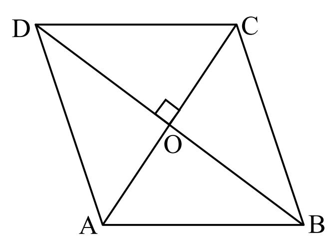
(iii) Since $\Delta ABC \cong \Delta CDA$ and congruent triangles are equal in area,

So we have $\Delta ABC = \Delta CDA$

Similarly, $\Delta ABD = \Delta CDB$

Ex.20 If the diagonals of a parallelogram are perpendicular to each other, prove that it is a rhombus.

Sol. Since the diagonals of a ||gm bisect each other,



we have, OA = OC and OB = OD.

Now, in ΔAOD and COD, we have

OA = OC, $\angle AOD = \angle COD = 90°$

and OD is common

$\therefore$ $\Delta AOD \cong \Delta COD$

$\therefore$ AD = CD (c.p.c.t.)

Now, AB = CD and AD = BC

(opp. sides of a ||gm)

and AD = CD (proved)

$\therefore$ AB = CD = AD = BC

Hence, ABCD is a rhombus.

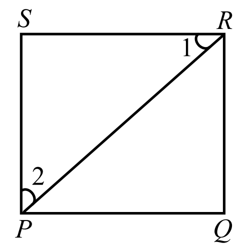
Ex.21 PQRS is a square. Determine $\angle SRP$.

Sol. PQRS is a square.

$\therefore$ PS = SR and $\angle PSR = 90°$

Now, in Δ PSR, we have

PS = SR



$\Rightarrow$ $\angle 1 = \angle 2$ $\left[\begin{array}{c}\Theta \text{ Angles opp. to} \\ \text{equal sides are equal}\end{array}\right]$

But, $\angle 1 + \angle 2 + \angle PSR = 180^o$

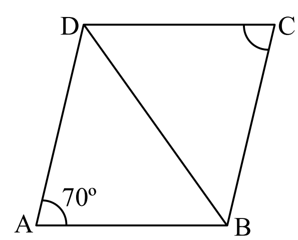
$\therefore$ $2\angle 1 + 90° = 180^o$ $[\Theta\ \angle PSR = 90°]$

$\Rightarrow$ $2\angle 1 = 90°$

$\Rightarrow$ $\angle 1 = 45°$

Ex.22 In the adjoining figure, ABCD is a rhombus. If $\angle A = 70^o$, find $\angle CDB$

Sol.



We have $\angle C = \angle A = 70^o$

(opposite $\angle$ of a ||gm)

Let $\angle CDB = x^o$

In ΔCDB, we have

CD = CB $\Rightarrow \angle CBD = \angle CDB = x^o$

$\therefore$ $\angle CDB + \angle CBD + \angle DCB = 180^o$

(angles of a triangle)

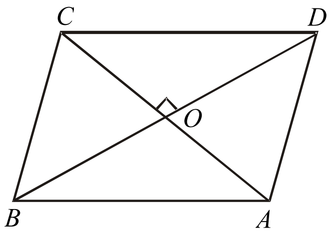
$\Rightarrow x^o + x^o + 70^o = 180^o$

$\Rightarrow 2x = 110$, i.e., $x = 55$

Hence, $\angle CDB = 55^o$

Ex.23 ABCD is a rhombus with $\angle ABC = 56°$. Determine $\angle ACD$.

Sol. ABCD is a parallelogram



$\Rightarrow$ $\angle ABC = \angle ADC$

$\Rightarrow$ $\angle ADC = 56°$ [Θ $\angle ABC = 56°$ (Given)]

$\Rightarrow$ $\angle ODC = 28°$ [Θ $\angle ODC = \frac{1}{2}\angle ADC$]

Now, ΔOCD we have,

$\angle OCD + \angle ODC + \angle COD = 180°$

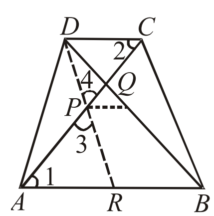
$\Rightarrow$ $\angle ODC + 28° + 90° = 180°$

$\Rightarrow$ $\angle OCD = 62° \Rightarrow \angle ACD = 62°$.

- The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
- The line drawn through the mid-point of one side of a triangle, parallel to another side, intersects the third side at its mid-point.

Ex.24 Prove that the line segment joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and is equal to half the difference of these sides.

Sol. Given : A trapezium ABCD in which AB || DC and P and Q are the mid-points of its diagonals AC and BD respectively.



To Prove : (i) PQ || AB or DC

(ii) $PQ = \frac{1}{2}(AB - DC)$

Construction : Join DP and produce DP to meet AB in R.

Proof : Since AB || DC and transversal AC cuts them at A and C respectively.

$\angle 1 = \angle 2$ .... (i)

[$\therefore$ Alternate angles are equal]

Now, in Δs APR and DPC, we have

$\angle 1 = \angle 2$ [From (i)]

AP = CP [Θ P is the mid-point of AC]

and, $\angle 3 = \angle 4$ [Vertically opposite angles]

So, by ASA criterion of congruence

$\Delta APR \cong \Delta DPC$

$\Rightarrow$ AR = DC and PR = DP ....(ii)

[Θ Corresponding parts of congruent triangles are equal]

In ΔDRB, P and Q are the mid-points of sides DR and DB respectively.

$\therefore$ PQ || RB

$\Rightarrow$ PQ || AB [Θ RB is a part of AB]

$\Rightarrow$ PQ || AB and DC [Θ AB || DC (Given)]

This proves (i).

Again, P and Q are the mid-points of sides DR and DB respectively in ΔDRB.

$\therefore$ $PQ = \frac{1}{2}RB \Rightarrow PQ = \frac{1}{2}(AB - AR)$

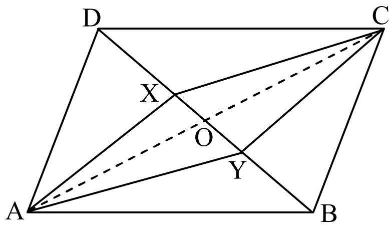
$\Rightarrow$ $PQ = \frac{1}{2}(AB - DC)$ [From (ii), AR = DC]

This proves (ii).

- A diagonal of a parallelogram divides it into two triangles of equal area.
- For each base of a parallelogram, the corresponding altitude is the line segment from a point on the base, perpendicular to the line containing the opposite side.
- Parallelograms on the same base and between the same parallels are equal in area.
- A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
- The area of a parallelogram is the product of its base and the corresponding altitude.
- Parallelograms on equal bases and between the same parallels are equal in area.

❖ EXAMPLES ❖

Ex.25 In the adjoining figure, ABCD is parallelogram and X, Y are the points on diagonal BD such that DX = BY. Prove that CXAY is a parallelogram.



Sol. Join AC, meeting BD at O.

Since the diagonals of a parallelogram bisect each other, we have OA = OC and OD = OB.

Now, OD = OB and DX = BY

$\Rightarrow$ OD – DX = OB – BY $\Rightarrow$ OX = OY

Now, OA = OC and OX = OY

$\therefore$ CXAY is a quadrilateral whose diagonals bisect each other.

$\therefore$ CXAY is a ||gm

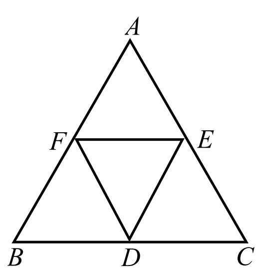
Ex.26 Prove that the four triangles formed by joining in pairs, the mid-points of three sides of a triangle, are concurrent to each other.

Sol. Given : A triangle ABC and D,E,F are the mid-points of sides BC, CA and AB respectively.

To Prove :

Δ AFE $\cong$ ΔFBD $\cong$ ΔEDC $\cong$ ΔDEF.

Proof : Since the segment joining the mid-points of the sides of a triangle is half of the third side. Therefore,



DE = $\frac{1}{2}$AB $\Rightarrow$ DE = AF = BF .... (i)

EF = $\frac{1}{2}$BC $\Rightarrow$ EF = BD = CD .... (ii)

DF = $\frac{1}{2}$AC $\Rightarrow$ DF = AE = EC ....(iii)

Now, in Δs DEF and AFE, we have

DE = AF [From (i)]

DF = AE [From (ii)]

and,EF = FE [Common]

So, by SSS criterion of congruence,

Δ DEF $\cong$ Δ AFE

Similarly, Δ DEF $\cong$ Δ FBD and Δ DEF $\cong$ Δ EDC

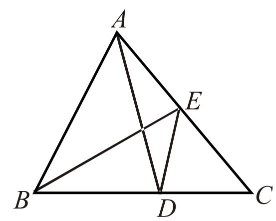
Hence, Δ AFE $\cong$ Δ FBD $\cong$ Δ EDC $\cong$ Δ DEF

Ex.27 In fig, AD is the median and DE || AB. Prove that BE is the median.

Sol. In order to prove that BE is the median, it is sufficient to show that E is the mid-point of AC.

Now, AD is the median in ΔABC

$\Rightarrow$ D is the mid-point of BC.



Since DE is a line drawn through the mid-point of side BC of ΔABC and is parallel to AB (given). Therefore, E is the mid-point of AC. Hence, BE is the median of ΔABC.

Ex.28 Let ABC be an isosceles triangle with AB = AC and let D,E,F be the mid-points of BC, CA and AB respectively. Show that AD $\perp$ FE and AD is bisected by FE.

Sol. Given : An isosceles triangle ABC with D, E and F as the mid-points of sides BC, CA and AB respectively such that AB = AC. AD intersects FE at O.

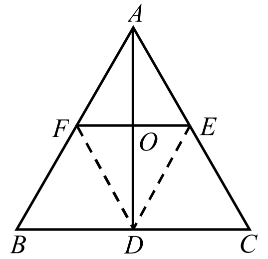
To Prove : AD $\perp$ FE and AD is bisected by FE.

Constructon : Join DE and DF.

Proof : Since the segment joining the mid-points of two sides of a triangle is parallel to third side and is half of it. Therefore,

DE || AB and DE = $\frac{1}{2}$ AB

Also, DF || AC and DF = $\frac{1}{2}$ AC



But,AB = AC [Given]

$\Rightarrow$ $\frac{1}{2}$ AB = $\frac{1}{2}$ AC

$\Rightarrow$ DE = DF .... (i)

Now, DE = $\frac{1}{2}$ AB $\Rightarrow$ DE = AF .... (ii)

and,DF = $\frac{1}{2}$ AC $\Rightarrow$ DF = AE ....(iii)

From (i), (ii) and (iii) we have

DE = AE = AF = DF

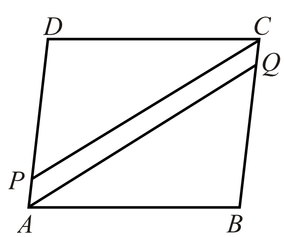
$\Rightarrow$ DEAF is a rhombus.

$\Rightarrow$ Diagonals AD and FE bisect each other at right angle.

AD $\perp$ FE and AD is bisected by FE.

Ex.29 ABCD is a parallelogram. P is a point on AD such that AP = $\frac{1}{3}$ AD and Q is a point on BC such that CQ = $\frac{1}{3}$ BP. Prove that AQCP is a parallelogram.

Sol. ABCD is a parallelogram.



$\Rightarrow$ AD = BC and AD || BC

$\Rightarrow$ $\frac{1}{3}$ AD = $\frac{1}{3}$ BC and AD || BC

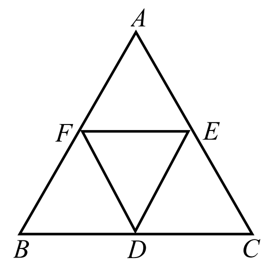
$\Rightarrow$ AP = CQ and AP || CQ

Thus, APCQ is a quadrilateral such that one pair of opposite side AP and CQ are parallel and equal.

Hence, APCQ is a parallelogram.

Ex.30 In fig. D,E and F are, respectively the mid-points of sides BC, CA and AB of an equilateral triangle ABC. Prove that DEF is also an equilateral triangle.

Sol. Since the segment joining the mid-points of two sides of a triangle is half of the third side. Therefore, D and E are mid-points of BC and AC respectively.



$\Rightarrow$ DE = $\frac{1}{2}$ AB .... (i)

E and F are the mid-points of AC and AB respectively.

$\therefore$ EF = $\frac{1}{2}$ BC .... (ii)

F and D are the mid-points AB and BC respectively.

$\Rightarrow$ FD = $\frac{1}{2}$ AC

Now, ΔABC is an equilateral triangle

$\Rightarrow$ AB = BC = CA

$\Rightarrow$ $\frac{1}{2}$ AB = $\frac{1}{2}$ BC = $\frac{1}{2}$ CA

$\Rightarrow$ DE = EF = FD

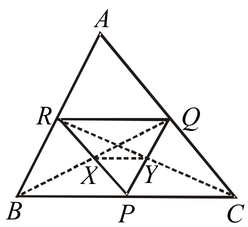
[Using (i), (ii) and (iii)]

Hence, ΔDEF is an equilateral triangle.

Ex.31 P,Q and R are, respectively, the mid-points of sides BC, CA and AB of a triangle ABC. PR and BQ meet at X. CR and PQ meet at Y. Prove that XY = $\frac{1}{4}BC$

Sol. Given : A ΔABC with P,Q and R as the mid-points of BC, CA and AB respectively. PR and BQ meet at X and CR and PQ meet at Y.

Construction : Join "X and Y.



Proof: Since the line segment joining the mid-points of two sides of a triangle is parallel to the third side and is half of it. Therefore, Q and R are mid-points of AC and AB respectively.

$\therefore$ RQ || BC and $RQ = \frac{1}{2}$ BC .... (i)

$$\left[\begin{array}{l}\Theta \text{ P is the mid} - \text{point} \\ \quad \text{of BC} \therefore \frac{1}{2}\text{BC} = \text{BP}\end{array}\right]$$

$\Rightarrow$ RQ || BP and RQ = BP

$\Rightarrow$ BPQR is a parallelogram.

Since the diagonals of a parallelogram bisect each other.

$\therefore$ X is the mid-point of PQ.

$$\left[\begin{array}{l}\Theta \text{ X is the point of intersection of} \\ \text{diagonals BQ and PR of } ||^{gm} \text{ BPQR}\end{array}\right]$$

Similarly, Y is the mid-point of PQ.

Now, consider ΔPQR. XY is the line segment joining the mid-points of sides PR and PQ.

$\therefore$ $XY = \frac{1}{2} RQ$ .... (i)

But $RQ = \frac{1}{2} BC$ [From (i)]

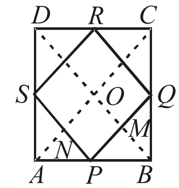
Hence, $XY = \frac{1}{4} BC.$

Ex.32 Show that the quadrilateral, formed by joining the mid-points of the sides of a square, is also a square.

Sol. Given : A square ABCD in which P, Q, R, S are the mid-points of sides AB, BC, CD, DA respectively. PQ, QR, RS and SP are joined.

To Prove : PQRS is a square.

Construction : Join AC and BD.



Proof : In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

$\therefore$ PQ || AC and $PQ = \frac{1}{2} AC$ .... (i)

In ΔADC, R and S are the mid-points of CD and AD respectively.

$\therefore$ RS || AC and $RS = \frac{1}{2} AC$ ....(ii)

From (i) and (ii), we have

PQ || RS and PQ = RS ....(iii)

Thus, in quadrilateral PQRS one pair of opposite sides are equal and parallel.

Hence, PQRS is a parallelogram.

Now, in Δs PBQ and RCQ, we have

PB = RC

$$\left[\begin{array}{l}\Theta \text{ ABCD, is a square} \therefore AB = BC = CD = DA \\ \Rightarrow \frac{1}{2}AB = \frac{1}{2}CD \text{ and } \frac{1}{2}AB = \frac{1}{2}BC\end{array}\right]$$

BQ = CQ [$\Rightarrow$ PB = CR and BQ = CQ]

and $\angle PBQ = \angle RCQ$ [Each equal to 90°]

So, by SAS criterion of congruence

$\Delta PBQ \cong \Delta RCQ$

$\Rightarrow$ PQ = QR ....(iv)

[Θ Corresponding parts of congruent Δs are equal]

From (iii) and (iv), we have

PQ = QR = RS

But,PQRS is a$||^{gm}$.

QR = PS

So, PQ = QR = RS = PS ....(v)

Now, PQ || AC [From (i)]

$\Rightarrow$ PM || NO ....(vi)

Since P and S are the mid-points of AB and AD respectively.

PS || BD

$\Rightarrow$ PM || MO ....(vii)

Thus, in quadrilateral PMON, we have

PM || NO [From (vi)]

PN || MO [From (vii)]

So, PMON is a parallelogram.

$\Rightarrow \angle MPN = \angle MON$

$\Rightarrow \angle MPN = \angle BOA$ [Θ $\angle MON = \angle BOA$]

$\Rightarrow \angle MPN = 90°$

$$z\begin{bmatrix} \Theta & \text{Diagonals of square are } \perp \\ \therefore & AC \perp BD \Rightarrow \angle BOA = 90° \end{bmatrix}$$

$\Rightarrow \angle QPS = 90°$

Thus, PQRS is a quadrilateral such that PQ = QR = RS = SP and $\angle$ QPS = 90°.

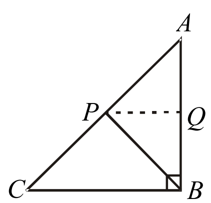
Hence, PQRS is a square.

Ex.33 ΔABC is a triangle right angled at B ; and P is the mid-point of AC. Prove that PB = PA = $\frac{1}{2}$ AC.

Sol. Given : ΔABC right angled at B, P is the mid-point of AC.

To Prove : PB = PA = $\frac{1}{2}$ AC.

Construction : Through P draw PQ || BC meeting AB at Q.



Proof : Since PQ || BC. Therefore,

$\angle$ AQP = $\angle$ ABC [Corresponding angles]

$\Rightarrow \angle AQP = 90°$
[Θ $\angle$ ABC = 90°]

But, $\angle$ AQP + $\angle$ BQP = 180°

[Θ $\angle$AQP & $\angle$BQP are angles of a linear pair]

$\therefore$ 90° + $\angle$ BQP = 180°

$\Rightarrow \angle BQP = 90°$

Thus, $\angle$ AQP = $\angle$ BQP = 90°

Now, in Δ ABC, P is the mid-point of AC and PQ || BC. Therefore, Q is the mid-point of AB i.e, AQ = BQ.

Consider now Δs APQ and BPQ.

we have, AQ = BC [Proved above]

$\angle$ AQP = $\angle$ BQP [From (i)]

and, PQ = PQ

So, by SAS cirterion of congruence

Δ APQ $\cong$ $\angle$ BPQ

$\Rightarrow$ PA = PB

Also,

PS = $\frac{1}{2}$ AC, since P is the mid-point of AC

Hence, PA = PB = $\frac{1}{2}$ AC.

Ex.34 Show that the quadrilateral formed by joining the mid-points of the consecutive sides of a rectangle is a rhombus.

Sol. Given : A rectangle ABCD in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.

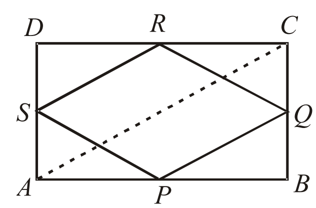
To Prove : PQRS is rhombus.

Construction : Join AC.

Proof : In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$ AC .... (i)

In Δ ADC, R and S are the mid-points of CD and AD respectively.



$\therefore$ SR || AC and SR = $\frac{1}{2}$ AC .... (ii)

From (i) and (ii), we get

PQ || SR and PQ = SR ....(iii)

$\Rightarrow$ PQRS is a parallelogram.

Now, ABCD is a rectangle.

$\Rightarrow$ AD = BC $\Rightarrow \frac{1}{2}$ AD = $\frac{1}{2}$ BC

$\Rightarrow$ AS = BQ ....(iv)

In Δs APS and BPQ , we have

AP = BP [$\therefore$ P is the mid-point of AB]

$\angle$ PAS = $\angle$ PBQ [Each equal to 90°]

and, AS = BQ [From (iv)]

So, by SAS criterion of congruence

$\Delta APS \cong \Delta BPQ$

PS = PQ ....(v)

[Θ Corresponding parts of congruent triangles are equal]

From (iii) and (v), we obtain that PQRS is a parallelogram such that PS = PQ i.e., two adjacent sides are equal.

Hence, PQRS is a rhombus.

IMPORTANT POINTS TO BE REMEMBERED

1. Sum of the angles of a quadrilateral is 360º.
2. A diagonal of a parallelogram divides it into two congruent triangles.
3. Two opposite angles of a parallelogram are equal.
4. The diagonals of a parallelogram bisect each other.
5. In a parallelogram, the bisectors of any two consecutive angles intersect at right angle.
6. If a diagonal of a parallelogram bisects one of the angles of the parallelogram it also bisects the second angle.
7. The angles bisectors of a parallelogram form a rectangle.
8. A quadrilateral is a parallelogram if its opposite sides are equal.
9. A quadrilateral is a parallelogram iff its opposite angles are equal.
10. The diagonals of a quadrilateral bisect each other, iff it is a parallelogram.
11. A quadrilateral is a parallelogram if its one pair of opposite sides are equal and parallel.
12. Each of the four angles of a rectangle is a right angle.
13. Each of the four sides of a rhombus of the same length.
14. The diagonals of a rectangle are of equal length.
15. Diagonals of a parallelogram are equal if and only if it is a rectangle.
16. The diagonals of a rhombus are perpendicular to each other.
17. Diagonals of a parallelogram are perpendicular if and only if it is a rhombus.
18. The diagonals of a square are equal and perpendicular to each other.
19. If the diagonals of a parallelogram are equal and intersect at right angle, then it is a square.
20. The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
21. A line through the mid-point of a side of a triangle parallel to another side bisects the third side.
22. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral, in order, is a parallelogram.

EXERCISE # 1

Q.1 The angle of a quadrilateral are respectively 100°, 98°, 92°. Find the fourth angle.

Q.2 Three angles of a quadrilateral are respectively equal to 110°, 50° and 40°. Find its fourth angles.

Q.3 In a quadrilateral ABCD, the angles A, B, C and D are in the ratio 1 : 2 : 4 : 5. Find the measure of each angles of the quadrilateral.

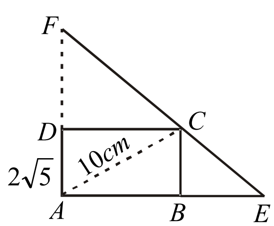
Q.4 In a quadrilateral ABCD, CO and DO are the bisectors of $\angle C$ and $\angle D$ respectively. Prove that $\angle COD = \frac{1}{2}(\angle A + \angle B)$.

Q.5 In fig. ABCD and PQRC are rectangles and Q is the mid-point of AC.

Prove that (i) DP = PC (ii) $PR = \frac{1}{2}AC$.

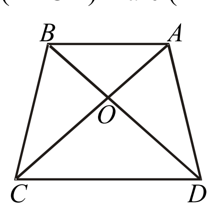
Q.6 BM and CN are perpendiculars to a line passing through the vertex A of a triangle ABC. If L is the mid-point of BC, prove that LM = LN.

Q.7 In the figure ABCD is a rectangle inscribed in a quadrant of a circle of radius 10 cm. If $AD = 2\sqrt{5}$ cm, find the area of the rectangle.



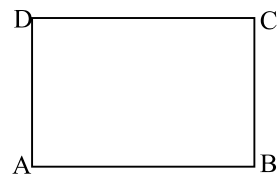
Q.8 In the following figure, ABCD is a trapezium in which AB || DC. Prove that

arc (ΔAOD) = arc (ΔBOC)



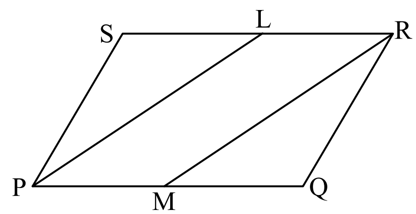
Q.9 Prove that area of rhombus = $\frac{1}{2}\times$ product of the diagonals.

Q.10 Show that each angle of a rectangle is a right angle.



Q.11 ABCD is a rhombus with $\angle ABC = 58^{o}$. Find $\angle ACD$.

Q.12 In the given figure, PQRS is a parallelogram in which PL and RM are bisectors of $\angle P$ and $\angle R$ respectively. Prove that PMRL is a parallelogram.



Q.13 The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60º. Find the angles of the parallelogram.

Q.14 PQ and RS are two equal and parallel line segments. Any point M not lying on PQ or RS is joined to Q and S and lines through P parallel to QM and through R parallel to SM meet at N. Prove that line segments MN and PQ are equal and parallel to each other.

Q.15 In ΔABC, P Q and R are mid-points of sides BC, CA and AB respectively. If AC = 21 cm, BC = 29 cm and AB = 30 cm, find the perimeter of the quadrilateral ARPQ.

➢ Fill in the Blanks

Q.16 The triangle formed by joining the mid-points of the sides of an isosceles triangle is.........

Q.17 The triangle formed by joining the mid-points of the sides of a right triangle is ....

Q.18 The figure formed by joining the mid-points of consecutive sides of a quadrilateral is.........

➢ True/False Type Questions

Q.19 In any quadrilateral, if a pair of opposite sides is equal, it is a parallelogram.

Q.20 If all the angles of a quadrilateral are equal, it is a parallelogram.

Q.21 If three sides of a quadrilateral are equal, it is a parallelogram.

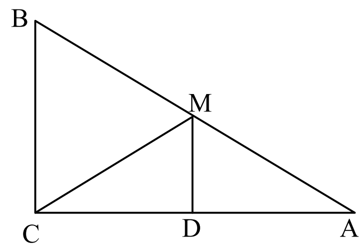
Q.22 If three angles of a quadrilateral are equal, it is a parallelogram.

Q.23 If all the sides of a quadrilateral are equal it is a parallelogram.

Q.24 A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

Q.25 In a parallelogram ABCD, AB = 10 cm and AD = 6 cm. The bisector of $\angle$A meets DC in E. AE and BC produced meet at F. Find the length of CF.

Q.26 ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that



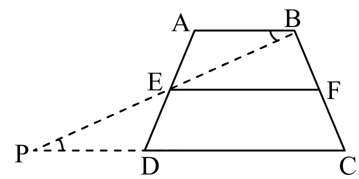
(i) D is the mid-point of AC

(ii) MD $\perp$ AC

(iii) CM = MA = $\frac{1}{2}$AB

Q.27 E, F are respectively the mid-points of non-parallel sides of a trapezium ABCD. Prove that

(i) EF || AB and (ii) EF = $\frac{1}{2}$(AB + CD)



Q.28 ABCD is || gm. P is a point on AD such that AP = $\frac{1}{3}$AD and Q is a point on BC such that CQ = $\frac{1}{3}$BC. Prove that the quadrilateral AQCP is a || gm.

ANSWER KEY

1. 70°	**2.** 160°	**3.** 30°, 60°, 120°,150°	**7.** 40 cm^2.	**14.** 51cm
16. Isosceles	**17.** Right triangle	**18.** Parallelogram	**19.** False	**20.** True
21. False	**22.** False	**23.** True	**25.** 4 cm.	

EXERCISE # 2

Q.1 In which quadrilateral is the lengths of diagonals equal ?

Q.2 If the diagonals of a quadrilateral bisect each other at right angles, then it is a :

Q.3 The length of the diagonals of a rhombus are 16 cm and 12 cm. The side of the rhombus is -

Q.4 The length of a side of a rhombus is 5 m and one of its diagonals is of length 8 m. Find the length of the other diagonal

Q.5 Find the angle where the bisectors of any two adjacent angles of a parallelogram intersect

Q.6 Give name of the figure formed by joining the mid points of the adjacent sides of a quadrilateral :

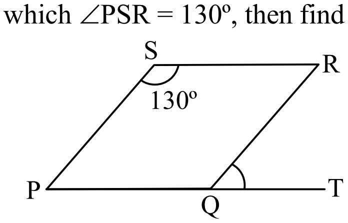
Q.7 Name the figure formed by joining the mid points of the adjacent sides of a rectangle

Q.8 Three angles of a quadrilateral are of magnitudes 80°, 95° and 120°. Find the magnitude of the fourth angle

Q.9 If ABCD is a rectangle, E, F are the mid points of BC and AD respectively and G is any point on EF, then prove that $\Delta\, GAB = \frac{1}{4}(ABCD)$

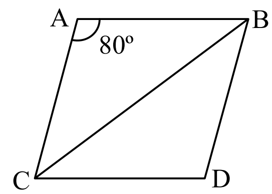
Q.10 Two consecutive angles of a parallelogram are in the ratio 1 : 3. Find the smaller angle

Q.11 In the given figure, PQRS is a parallelogram in which $\angle PSR = 130^o$, then find $\angle RQT$ -



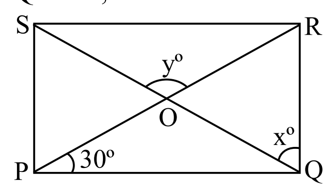
Q.12 If three angles of a quadrilateral are 100º, 75º and 105º, then find the measure of the fourth angle

Q.13 In the given figure, ABCD is a rhombus. If $\angle A = 80^o$, then find $\angle CDB$



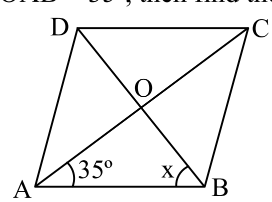
Q.14 The diagonals of a rhombus are 12 cm and 16 cm. Find the length of the side of the rhombus

Q.15 In the given figure, PQRS is a rectangle. If $\angle RPQ = 30^o$, then find the value of (x + y)

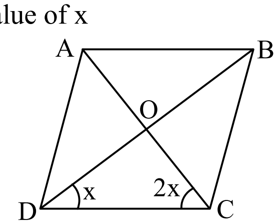


Q.16 If the length of the diagonal of a square is 8 cm. then find its area

Q.17 In the given figure, ABCD is a rhombus. If $\angle OAB = 35^o$, then find the value of x



Q.18 In the given figure, ABCD is a rhombus. Find the value of x



ANSWER KEY

1. Rectangle	**2.** Rhombus	**3.** 10 cm	**4.** 6 m	**5.** 90º
6. Parallelogram	**7.** rhombus	**8.** 65º	**10.** 45º	
11. 50º	**12.** 80º	**13.** 50º	**14.** 10 cm	**15.** 180º
16. 32 cm^2	**17.** 55º			