> # in developing expressions for x[r,c], I have found that for example x[1,2] = $z^2 + 40$ has one parameter, namely z.

> # the expression for x[a,1] has two parameters - 'a' and 'z'.

> #these expressions cause h(x[r,c]) to factor algebraically and be composite for all integer z.

>

>

>

> $with(CurveFitting):$

> $Interactive(\,)$

$$\frac{1021}{25}\,x0^2 - \frac{5102}{25}\,x0 - \frac{19}{25} \quad (1)$$

>

> $restart$

> $h := n^2 + n + 41;$

$$h := n^2 + n + 41 \quad (2)$$

> # I label y[b,1] as such because later I use variable 'a' as a prameter and still want to reference letter b. but you can read y[a,1] as a function of 'a' and 'z'.

> $y[b, 1] := a\cdot z^2 + (a-1)\cdot z + 41\cdot a - 1;$

$$y_{b,\,1} := a\,z^2 + (a-1)\,z + 41\,a - 1 \quad (3)$$

> $x[b, 1] := factor(subs(n = y[b, 1], h));$

$$x_{b,\,1} := \left(41 + z + z^2\right)\left(a^2 z^2 + z\,a^2 - 2\,z\,a - a + 41\,a^2 + 1\right) \quad (4)$$

> $y[b, b-1] := a\cdot(a-1)\cdot z^2 + \left(a^2 - 3\cdot a + 1\right)\cdot z + \left(41\cdot a^2 - 42\,a + 1\right);$

$$y_{b,\,b-1} := a\,(a-1)\,z^2 + \left(a^2 - 3\,a + 1\right)z + 41\,a^2 - 42\,a + 1 \quad (5)$$

> # y big 1 sub a minus 1

> $x[b, b-1] := factor(subs(n = y[b, b-1], h));$

$$x_{b,\,b-1} := \left(a^2 z^2 + z\,a^2 - 2\,z\,a - a + 41\,a^2 + 1\right)\left(a^2 z^2 - 2\,a\,z^2 + z^2 + 3\,z + 43 - 4\,z\,a - 83\,a + z\,a^2 + 41\,a^2\right) \quad (6)$$

> #don't require a odd for y[1,a-1]... I think y[1,a-1] works for all integers a.

> $y[b, 2] := 2\cdot a\cdot z^2 + z + \frac{(163\cdot a - 1)}{2}$

$$y_{b,\,2} := 2\,a\,z^2 + z + \frac{163}{2}\,a - \frac{1}{2} \quad (7)$$

> $x[b, 2] := factor(subs(n = y[b, 2], h));$

$$x_{b,\,2} := \frac{1}{4}\left(163 + 4\,z^2\right)\left(4\,a^2 z^2 + 4\,z\,a + 1 + 163\,a^2\right) \quad (8)$$

> $y[b, b-2] := a\cdot(a-2)\cdot z^2 + (a-1)\cdot z + \frac{\left(163\,a^2 - 2\cdot 163\cdot a - 1\right)}{4};$

$$y_{b,\,b-2} := a\,(a-2)\,z^2 + (a-1)\,z + \frac{163}{4}\,a^2 - \frac{163}{2}\,a - \frac{1}{4} \quad (9)$$

> $x[b, b-2] := factor(subs(n = y[b, b-2], h));$

$$x_{b,\,b-2} := \frac{1}{16}\left(4\,a^2 z^2 + 4\,z\,a + 1 + 163\,a^2\right)\left(4\,a^2 z^2 - 16\,a\,z^2 + 16\,z^2 - 8\,z + 653 + 4\,z\,a \right. \quad (10)$$

$- 652\, a + 163\, a^2)$

> # need a odd for y[a,2] and y[a,a-2]

> $y[b, 3, 1] := 3 \cdot a \cdot z^2 + (a+1) \cdot z + \frac{(367 \cdot a - 1)}{3}$

$$y_{b,3,1} := 3\, a\, z^2 + (a+1)\, z + \frac{367}{3}\, a - \frac{1}{3} \quad \textbf{(11)}$$

> $x[b, 3, 1] := factor(subs(n = y[b, 3, 1], h));$

$$x_{b,3,1} := \frac{1}{9}\, (9\, z^2 + 3\, z + 367)\, (9\, a^2\, z^2 + 3\, z\, a^2 + 6\, z\, a + 1 + a + 367\, a^2) \quad \textbf{(12)}$$

> #need a=1 mod 3 for $y[a, 3, 1]$

>

>

> #a:=4;

> $y[b, b-3, 1] := a \cdot (a-3) \cdot z^2 + \frac{(a^2 - a - 3)}{3} \cdot z + \frac{(367\, a^2 - 1100 \cdot a - 5)}{9}$

$$y_{b,b-3,1} := a\, (a-3)\, z^2 + \frac{1}{3}\, (a^2 - a - 3)\, z + \frac{367}{9}\, a^2 - \frac{1100}{9}\, a - \frac{5}{9} \quad \textbf{(13)}$$

> $x[b, b-3, 1] := factor(subs(n = y[b, b-3, 1], h));$

$$x_{b,b-3,1} := \frac{1}{81}\, (9\, a^2\, z^2 - 54\, a\, z^2 + 81\, z^2 + 9\, z + 3301 - 12\, z\, a - 2201\, a + 3\, z\, a^2 + 367\, a^2)\, (9\, a^2\, z^2 + 3\, z\, a^2 + 6\, z\, a + 1 + a + 367\, a^2) \quad \textbf{(14)}$$

> # require a=1 mod 3 for y[b,b-3,1]

> $y[b, 3, 2] := 3 \cdot a \cdot z^2 + (a-1) \cdot z + \frac{(367 \cdot a - 2)}{3}$

$$y_{b,3,2} := 3\, a\, z^2 + (a-1)\, z + \frac{367}{3}\, a - \frac{2}{3} \quad \textbf{(15)}$$

> $x[b, 3, 2] := factor(subs(n = y[b, 3, 2], h));$

$$x_{b,3,2} := \frac{1}{9}\, (9\, z^2 + 3\, z + 367)\, (9\, a^2\, z^2 + 3\, z\, a^2 - 6\, z\, a + 1 - a + 367\, a^2) \quad \textbf{(16)}$$

> # require a=2 mod 3 for y[b,3,2], this is why the 3rd subscript is a 2.

> $y[b, b-3, 2] := a \cdot (a-3) \cdot z^2 + \frac{(a^2 - 5 \cdot a + 3)}{3} \cdot z + \frac{(367 \cdot a^2 - 1102 \cdot a - 2)}{9};$

$$y_{b,b-3,2} := a\, (a-3)\, z^2 + \frac{1}{3}\, (a^2 - 5\, a + 3)\, z + \frac{367}{9}\, a^2 - \frac{1102}{9}\, a - \frac{2}{9} \quad \textbf{(17)}$$

> $x[b, b-3, 2] := factor(subs(n = y[b, b-3, 2], h));$

$$x_{b,b-3,2} := \frac{1}{81}\, (9\, a^2\, z^2 - 54\, a\, z^2 + 81\, z^2 + 45\, z + 3307 - 24\, z\, a - 2203\, a + 3\, z\, a^2 + 367\, a^2)\, (9\, a^2\, z^2 + 3\, z\, a^2 - 6\, z\, a + 1 - a + 367\, a^2) \quad \textbf{(18)}$$

> #again a=2 mod 3 for y[b,3,2] and y[b,b-3,2]

> $y[b, 4, 3] := 4 \cdot a \cdot z^2 + (2 \cdot a - 1) \cdot z + \frac{(653 \cdot a - 3)}{4};$

$$y_{b,4,3} := 4\, a\, z^2 + (2\, a - 1)\, z + \frac{653}{4}\, a - \frac{3}{4} \quad \textbf{(19)}$$

> $x[b,4,3] := factor(subs(n=y[b,4,3],h));$

$$x_{b,4,3} := \frac{1}{16}\left(16z^2+8z+653\right)\left(16a^2z^2+8za^2-8za+1-2a+653a^2\right) \quad \textbf{(20)}$$

> # y[b,4,3] requires a=3 mod 4

> $y[b,b-4,3] := a\cdot(a-4)\cdot z^2+\frac{(a^2-5a+2)}{2}\cdot z+\frac{(653a^2-2614\cdot a-3)}{16};$

$$y_{b,b-4,3} := a\,(a-4)\,z^2+\frac{1}{2}\left(a^2-5a+2\right)z+\frac{653}{16}a^2-\frac{1307}{8}a-\frac{3}{16} \quad \textbf{(21)}$$

> $x[b,b-4,3] := factor(subs(n=y[b,b-4,3],h));$

$$x_{b,b-4,3} := \frac{1}{256}\left(16a^2z^2-128az^2+256z^2+160z+10457-72za-5226a+8za^2+653a^2\right)\left(16a^2z^2+1-8za-2a+8za^2+653a^2\right) \quad \textbf{(22)}$$

> #y[b,b-4,3] requires a=3 mod 4

> $y[b,4,1] := 4\cdot a\cdot z^2+(2\cdot a+1)\cdot z+\frac{(653\cdot a-1)}{4};$

$$y_{b,4,1} := 4az^2+(2a+1)z+\frac{653}{4}a-\frac{1}{4} \quad \textbf{(23)}$$

> $x[b,4,1] := factor(subs(n=y[b,4,1],h));$

$$x_{b,4,1} := \frac{1}{16}\left(16z^2+8z+653\right)\left(16a^2z^2+8za+8za^2+1+2a+653a^2\right) \quad \textbf{(24)}$$

> #y[b,4,1] requires a=1 mod 4

>

> $y[b,b-4,1] := a\cdot(a-4)\cdot z^2+\frac{(a^2-3\text{a-}2)}{2}\cdot z+\frac{(653\cdot a^2-2610\cdot a-11)}{16};$

$$y_{b,b-4,1} := a\,(a-4)\,z^2+\frac{1}{2}\left(a^2-3a-2\right)z+\frac{653}{16}a^2-\frac{1305}{8}a-\frac{11}{16} \quad \textbf{(25)}$$

> $x[b,b-4,1] := factor(subs(n=y[b,b-4,1],h));$

$$x_{b,b-4,1} := \frac{1}{256}\left(16a^2z^2+1+8za+2a+8za^2+653a^2\right)\left(16a^2z^2-128az^2+256z^2+96z+10441-56za-5222a+8za^2+653a^2\right) \quad \textbf{(26)}$$

> #y[b,b-4,1] requires a=1 mod 4

> $y[b,5,1] := 5\cdot a\cdot z^2+(3\cdot a+1)\cdot z+\frac{(1021\cdot a-1)}{5};$

$$y_{b,5,1} := 5az^2+(3a+1)z+\frac{1021}{5}a-\frac{1}{5} \quad \textbf{(27)}$$

> $x[b,5,1] := factor(subs(n=y[b,5,1],h));$

$$x_{b,5,1} := \frac{1}{25}\left(15z+1021+25z^2\right)\left(25a^2z^2+15za^2+10za+3a+1+1021a^2\right) \quad \textbf{(28)}$$

> #y[b,5,1] requires a=1 mod 5

> $y[b,5,2] := 5\cdot a\cdot z^2+(a-1)\cdot z+\frac{(1019a-3)}{5};$

$$y_{b,5,2} := 5az^2+(a-1)z+\frac{1019}{5}a-\frac{3}{5} \quad \textbf{(29)}$$

> $x[b,5,2] := factor(subs(n=y[b,5,2],h));$

$$x_{b,5,2} := \frac{1}{25}\left(1019 + 25 z^2 + 5 z\right)\left(25 a^2 z^2 - 10 z a + 5 z a^2 - a + 1019 a^2 + 1\right) \quad \mathbf{(30)}$$

> *#y[b,5,2] requires a=2 mod 5*

> $y[b, 5, 3] := 5 \cdot a \cdot z^2 + (a+1) \cdot z + \frac{(1019 \cdot a - 2)}{5};$

$$y_{b,5,3} := 5 a z^2 + (a+1) z + \frac{1019}{5} a - \frac{2}{5} \quad \mathbf{(31)}$$

> $x[b, 5, 3] := factor(subs(n = y[b, 5, 3], h));$

$$x_{b,5,3} := \frac{1}{25}\left(1019 + 5 z + 25 z^2\right)\left(25 a^2 z^2 + 10 z a + 5 z a^2 + a + 1 + 1019 a^2\right) \quad \mathbf{(32)}$$

> *#y[b,5,3] requires a=3 mod 5*

> $y[b, 5, 4] := 5 \cdot a \cdot z^2 + (3 \cdot a - 1) \cdot z + \frac{(1021 \cdot a - 4)}{5};$

$$y_{b,5,4} := 5 a z^2 + (3 a - 1) z + \frac{1021}{5} a - \frac{4}{5} \quad \mathbf{(33)}$$

> $x[b, 5, 4] := factor(subs(n = y[b, 5, 4], h));$

$$x_{b,5,4} := \frac{1}{25}\left(25 z^2 + 15 z + 1021\right)\left(25 a^2 z^2 - 10 z a + 15 z a^2 + 1 - 3 a + 1021 a^2\right) \quad \mathbf{(34)}$$

> *#y[b,5,4] requires a=4 mod 5*

> $y[b, b-5, 1] := a \cdot (a-5) \cdot z^2 + \frac{(3 \cdot a^2 - 13 \cdot a - 5)}{5} \cdot z + \frac{(1021 a^2 - 5102 a - 19)}{25};$

$$y_{b,b-5,1} := a (a-5) z^2 + \frac{1}{5}\left(3 a^2 - 13 a - 5\right) z + \frac{1021}{25} a^2 - \frac{5102}{25} a - \frac{19}{25} \quad \mathbf{(35)}$$

> $x[b, b-5, 1] := factor(subs(n = y[b, b-5, 1], h));$

$$x_{b,b-5,1} := \frac{1}{625}\left(25 a^2 z^2 - 250 a z^2 + 625 z^2 + 325 z + 25511 - 140 z a - 10207 a + 15 z a^2 + 1021 a^2\right)\left(25 a^2 z^2 + 1 + 10 z a + 3 a + 15 z a^2 + 1021 a^2\right) \quad \mathbf{(36)}$$

>

>

>

>

```
> for a from 1 to 50 do
xs[a, 1] := x[b, 1];
ys[a, 1] := y[b, 1];
end do;
```

$$xs_{1,\,1} := \left(41 + z + z^2\right)\left(z^2 - z + 41\right)$$

$$ys_{1,\,1} := z^2 + 40$$

$$xs_{2,\,1} := \left(41 + z + z^2\right)\left(163 + 4\,z^2\right)$$

$$ys_{2,\,1} := 2\,z^2 + z + 81$$

$$xs_{3,\,1} := \left(41 + z + z^2\right)\left(9\,z^2 + 3\,z + 367\right)$$

$$ys_{3,\,1} := 3\,z^2 + 2\,z + 122$$

$$xs_{4,\,1} := \left(41 + z + z^2\right)\left(16\,z^2 + 8\,z + 653\right)$$

$$ys_{4,\,1} := 4\,z^2 + 3\,z + 163$$

$$xs_{5,\,1} := \left(41 + z + z^2\right)\left(25\,z^2 + 15\,z + 1021\right)$$

$$ys_{5,\,1} := 5\,z^2 + 4\,z + 204$$

$$xs_{6,\,1} := \left(41 + z + z^2\right)\left(36\,z^2 + 24\,z + 1471\right)$$

$$ys_{6,\,1} := 6\,z^2 + 5\,z + 245$$

$$xs_{7,\,1} := \left(41 + z + z^2\right)\left(49\,z^2 + 35\,z + 2003\right)$$

$$ys_{7,\,1} := 7\,z^2 + 6\,z + 286$$

$$xs_{8,\,1} := \left(41 + z + z^2\right)\left(64\,z^2 + 48\,z + 2617\right)$$

$$ys_{8,\,1} := 8\,z^2 + 7\,z + 327$$

$$xs_{9,\,1} := \left(41 + z + z^2\right)\left(81\,z^2 + 63\,z + 3313\right)$$

$$ys_{9,\,1} := 9\,z^2 + 8\,z + 368$$

$$xs_{10,\,1} := \left(41 + z + z^2\right)\left(100\,z^2 + 80\,z + 4091\right)$$

$$ys_{10,\,1} := 10\,z^2 + 9\,z + 409$$

$$xs_{11,\,1} := \left(41 + z + z^2\right)\left(121\,z^2 + 99\,z + 4951\right)$$

$$ys_{11,\,1} := 11\,z^2 + 10\,z + 450$$

$$xs_{12,\,1} := \left(41 + z + z^2\right)\left(144\,z^2 + 120\,z + 5893\right)$$

$$ys_{12,\,1} := 12\,z^2 + 11\,z + 491$$

$$xs_{13,\,1} := \left(41 + z + z^2\right)\left(169\,z^2 + 143\,z + 6917\right)$$

$$ys_{13,\,1} := 13\,z^2 + 12\,z + 532$$

$$xs_{14,\,1} := \left(41 + z + z^2\right)\left(196\,z^2 + 168\,z + 8023\right)$$

$$ys_{14,\,1} := 14\,z^2 + 13\,z + 573$$
$$xs_{15,\,1} := (41 + z + z^2)\,(225\,z^2 + 195\,z + 9211)$$
$$ys_{15,\,1} := 15\,z^2 + 14\,z + 614$$
$$xs_{16,\,1} := (41 + z + z^2)\,(256\,z^2 + 224\,z + 10481)$$
$$ys_{16,\,1} := 16\,z^2 + 15\,z + 655$$
$$xs_{17,\,1} := (41 + z + z^2)\,(289\,z^2 + 255\,z + 11833)$$
$$ys_{17,\,1} := 17\,z^2 + 16\,z + 696$$
$$xs_{18,\,1} := (41 + z + z^2)\,(324\,z^2 + 288\,z + 13267)$$
$$ys_{18,\,1} := 18\,z^2 + 17\,z + 737$$
$$xs_{19,\,1} := (41 + z + z^2)\,(361\,z^2 + 323\,z + 14783)$$
$$ys_{19,\,1} := 19\,z^2 + 18\,z + 778$$
$$xs_{20,\,1} := (41 + z + z^2)\,(400\,z^2 + 360\,z + 16381)$$
$$ys_{20,\,1} := 20\,z^2 + 19\,z + 819$$
$$xs_{21,\,1} := (41 + z + z^2)\,(441\,z^2 + 399\,z + 18061)$$
$$ys_{21,\,1} := 21\,z^2 + 20\,z + 860$$
$$xs_{22,\,1} := (41 + z + z^2)\,(484\,z^2 + 440\,z + 19823)$$
$$ys_{22,\,1} := 22\,z^2 + 21\,z + 901$$
$$xs_{23,\,1} := (41 + z + z^2)\,(529\,z^2 + 483\,z + 21667)$$
$$ys_{23,\,1} := 23\,z^2 + 22\,z + 942$$
$$xs_{24,\,1} := (41 + z + z^2)\,(576\,z^2 + 528\,z + 23593)$$
$$ys_{24,\,1} := 24\,z^2 + 23\,z + 983$$
$$xs_{25,\,1} := (41 + z + z^2)\,(625\,z^2 + 575\,z + 25601)$$
$$ys_{25,\,1} := 25\,z^2 + 24\,z + 1024$$
$$xs_{26,\,1} := (41 + z + z^2)\,(676\,z^2 + 624\,z + 27691)$$
$$ys_{26,\,1} := 26\,z^2 + 25\,z + 1065$$
$$xs_{27,\,1} := (41 + z + z^2)\,(729\,z^2 + 675\,z + 29863)$$
$$ys_{27,\,1} := 27\,z^2 + 26\,z + 1106$$
$$xs_{28,\,1} := (41 + z + z^2)\,(784\,z^2 + 728\,z + 32117)$$
$$ys_{28,\,1} := 28\,z^2 + 27\,z + 1147$$

$$xs_{29,\,1} := (41 + z + z^2)\,(841\,z^2 + 783\,z + 34453)$$

$$ys_{29,\,1} := 29\,z^2 + 28\,z + 1188$$

$$xs_{30,\,1} := (41 + z + z^2)\,(900\,z^2 + 840\,z + 36871)$$

$$ys_{30,\,1} := 30\,z^2 + 29\,z + 1229$$

$$xs_{31,\,1} := (41 + z + z^2)\,(961\,z^2 + 899\,z + 39371)$$

$$ys_{31,\,1} := 31\,z^2 + 30\,z + 1270$$

$$xs_{32,\,1} := (41 + z + z^2)\,(1024\,z^2 + 960\,z + 41953)$$

$$ys_{32,\,1} := 32\,z^2 + 31\,z + 1311$$

$$xs_{33,\,1} := (41 + z + z^2)\,(1089\,z^2 + 1023\,z + 44617)$$

$$ys_{33,\,1} := 33\,z^2 + 32\,z + 1352$$

$$xs_{34,\,1} := (41 + z + z^2)\,(1156\,z^2 + 1088\,z + 47363)$$

$$ys_{34,\,1} := 34\,z^2 + 33\,z + 1393$$

$$xs_{35,\,1} := (41 + z + z^2)\,(1225\,z^2 + 1155\,z + 50191)$$

$$ys_{35,\,1} := 35\,z^2 + 34\,z + 1434$$

$$xs_{36,\,1} := (41 + z + z^2)\,(1296\,z^2 + 1224\,z + 53101)$$

$$ys_{36,\,1} := 36\,z^2 + 35\,z + 1475$$

$$xs_{37,\,1} := (41 + z + z^2)\,(1369\,z^2 + 1295\,z + 56093)$$

$$ys_{37,\,1} := 37\,z^2 + 36\,z + 1516$$

$$xs_{38,\,1} := (41 + z + z^2)\,(1444\,z^2 + 1368\,z + 59167)$$

$$ys_{38,\,1} := 38\,z^2 + 37\,z + 1557$$

$$xs_{39,\,1} := (41 + z + z^2)\,(1521\,z^2 + 1443\,z + 62323)$$

$$ys_{39,\,1} := 39\,z^2 + 38\,z + 1598$$

$$xs_{40,\,1} := (41 + z + z^2)\,(1600\,z^2 + 1520\,z + 65561)$$

$$ys_{40,\,1} := 40\,z^2 + 39\,z + 1639$$

$$xs_{41,\,1} := (41 + z + z^2)\,(1681\,z^2 + 1599\,z + 68881)$$

$$ys_{41,\,1} := 41\,z^2 + 40\,z + 1680$$

$$xs_{42,\,1} := (41 + z + z^2)\,(1764\,z^2 + 1680\,z + 72283)$$

$$ys_{42,\,1} := 42\,z^2 + 41\,z + 1721$$

$$xs_{43,\,1} := (41 + z + z^2)\,(1849\,z^2 + 1763\,z + 75767)$$

$$ys_{43,\,1} := 43\,z^2 + 42\,z + 1762$$
$$xs_{44,\,1} := \left(41 + z + z^2\right)\left(1936\,z^2 + 1848\,z + 79333\right)$$
$$ys_{44,\,1} := 44\,z^2 + 43\,z + 1803$$
$$xs_{45,\,1} := \left(41 + z + z^2\right)\left(2025\,z^2 + 1935\,z + 82981\right)$$
$$ys_{45,\,1} := 45\,z^2 + 44\,z + 1844$$
$$xs_{46,\,1} := \left(41 + z + z^2\right)\left(2116\,z^2 + 2024\,z + 86711\right)$$
$$ys_{46,\,1} := 46\,z^2 + 45\,z + 1885$$
$$xs_{47,\,1} := \left(41 + z + z^2\right)\left(2209\,z^2 + 2115\,z + 90523\right)$$
$$ys_{47,\,1} := 47\,z^2 + 46\,z + 1926$$
$$xs_{48,\,1} := \left(41 + z + z^2\right)\left(2304\,z^2 + 2208\,z + 94417\right)$$
$$ys_{48,\,1} := 48\,z^2 + 47\,z + 1967$$
$$xs_{49,\,1} := \left(41 + z + z^2\right)\left(2401\,z^2 + 2303\,z + 98393\right)$$
$$ys_{49,\,1} := 49\,z^2 + 48\,z + 2008$$
$$xs_{50,\,1} := \left(41 + z + z^2\right)\left(2500\,z^2 + 2400\,z + 102451\right)$$
$$ys_{50,\,1} := 50\,z^2 + 49\,z + 2049 \qquad \textbf{(37)}$$

>

>

>

```
> for a from 2 to 50 do
  ys[a, a − 1] := y[b, b − 1];
  xs[a, a − 1] := simplify(x[b, b − 1]);
  end do;
```

$$ys_{2,\,1} := 2\,z^2 - z + 81$$

$$xs_{2,\,1} := \left(163 + 4\,z^2\right)\,\left(z^2 - z + 41\right)$$

$$ys_{3,\,2} := 6\,z^2 + z + 244$$

$$xs_{3,\,2} := \left(9\,z^2 + 3\,z + 367\right)\,\left(163 + 4\,z^2\right)$$

$$ys_{4,\,3} := 12\,z^2 + 5\,z + 489$$

$$xs_{4,\,3} := \left(16\,z^2 + 8\,z + 653\right)\,\left(9\,z^2 + 3\,z + 367\right)$$

$$ys_{5,\,4} := 20\,z^2 + 11\,z + 816$$

$$xs_{5,\,4} := \left(25\,z^2 + 15\,z + 1021\right)\,\left(16\,z^2 + 8\,z + 653\right)$$

$$ys_{6,\,5} := 30\,z^2 + 19\,z + 1225$$

$$xs_{6,\,5} := \left(36\,z^2 + 24\,z + 1471\right)\,\left(25\,z^2 + 15\,z + 1021\right)$$

$$ys_{7,\,6} := 42\,z^2 + 29\,z + 1716$$

$$xs_{7,\,6} := \left(49\,z^2 + 35\,z + 2003\right)\,\left(36\,z^2 + 24\,z + 1471\right)$$

$$ys_{8,\,7} := 56\,z^2 + 41\,z + 2289$$

$$xs_{8,\,7} := \left(64\,z^2 + 48\,z + 2617\right)\,\left(49\,z^2 + 35\,z + 2003\right)$$

$$ys_{9,\,8} := 72\,z^2 + 55\,z + 2944$$

$$xs_{9,\,8} := \left(81\,z^2 + 63\,z + 3313\right)\,\left(64\,z^2 + 48\,z + 2617\right)$$

$$ys_{10,\,9} := 90\,z^2 + 71\,z + 3681$$

$$xs_{10,\,9} := \left(100\,z^2 + 80\,z + 4091\right)\,\left(81\,z^2 + 63\,z + 3313\right)$$

$$ys_{11,\,10} := 110\,z^2 + 89\,z + 4500$$

$$xs_{11,\,10} := \left(121\,z^2 + 99\,z + 4951\right)\,\left(100\,z^2 + 80\,z + 4091\right)$$

$$ys_{12,\,11} := 132\,z^2 + 109\,z + 5401$$

$$xs_{12,\,11} := \left(144\,z^2 + 120\,z + 5893\right)\,\left(121\,z^2 + 99\,z + 4951\right)$$

$$ys_{13,\,12} := 156\,z^2 + 131\,z + 6384$$

$$xs_{13,\,12} := \left(169\,z^2 + 143\,z + 6917\right)\,\left(144\,z^2 + 120\,z + 5893\right)$$

$$ys_{14,\,13} := 182\,z^2 + 155\,z + 7449$$

$$xs_{14,\,13} := \left(196\,z^2 + 168\,z + 8023\right)\,\left(169\,z^2 + 143\,z + 6917\right)$$

$$ys_{15,\,14} := 210\,z^2 + 181\,z + 8596$$

$$xs_{15,\,14} := (225\,z^2 + 195\,z + 9211)\,(196\,z^2 + 168\,z + 8023)$$
$$ys_{16,\,15} := 240\,z^2 + 209\,z + 9825$$
$$xs_{16,\,15} := (256\,z^2 + 224\,z + 10481)\,(225\,z^2 + 195\,z + 9211)$$
$$ys_{17,\,16} := 272\,z^2 + 239\,z + 11136$$
$$xs_{17,\,16} := (289\,z^2 + 255\,z + 11833)\,(256\,z^2 + 224\,z + 10481)$$
$$ys_{18,\,17} := 306\,z^2 + 271\,z + 12529$$
$$xs_{18,\,17} := (324\,z^2 + 288\,z + 13267)\,(289\,z^2 + 255\,z + 11833)$$
$$ys_{19,\,18} := 342\,z^2 + 305\,z + 14004$$
$$xs_{19,\,18} := (361\,z^2 + 323\,z + 14783)\,(324\,z^2 + 288\,z + 13267)$$
$$ys_{20,\,19} := 380\,z^2 + 341\,z + 15561$$
$$xs_{20,\,19} := (400\,z^2 + 360\,z + 16381)\,(361\,z^2 + 323\,z + 14783)$$
$$ys_{21,\,20} := 420\,z^2 + 379\,z + 17200$$
$$xs_{21,\,20} := (441\,z^2 + 399\,z + 18061)\,(400\,z^2 + 360\,z + 16381)$$
$$ys_{22,\,21} := 462\,z^2 + 419\,z + 18921$$
$$xs_{22,\,21} := (484\,z^2 + 440\,z + 19823)\,(441\,z^2 + 399\,z + 18061)$$
$$ys_{23,\,22} := 506\,z^2 + 461\,z + 20724$$
$$xs_{23,\,22} := (529\,z^2 + 483\,z + 21667)\,(484\,z^2 + 440\,z + 19823)$$
$$ys_{24,\,23} := 552\,z^2 + 505\,z + 22609$$
$$xs_{24,\,23} := (576\,z^2 + 528\,z + 23593)\,(529\,z^2 + 483\,z + 21667)$$
$$ys_{25,\,24} := 600\,z^2 + 551\,z + 24576$$
$$xs_{25,\,24} := (625\,z^2 + 575\,z + 25601)\,(576\,z^2 + 528\,z + 23593)$$
$$ys_{26,\,25} := 650\,z^2 + 599\,z + 26625$$
$$xs_{26,\,25} := (676\,z^2 + 624\,z + 27691)\,(625\,z^2 + 575\,z + 25601)$$
$$ys_{27,\,26} := 702\,z^2 + 649\,z + 28756$$
$$xs_{27,\,26} := (729\,z^2 + 675\,z + 29863)\,(676\,z^2 + 624\,z + 27691)$$
$$ys_{28,\,27} := 756\,z^2 + 701\,z + 30969$$
$$xs_{28,\,27} := (784\,z^2 + 728\,z + 32117)\,(729\,z^2 + 675\,z + 29863)$$
$$ys_{29,\,28} := 812\,z^2 + 755\,z + 33264$$
$$xs_{29,\,28} := (841\,z^2 + 783\,z + 34453)\,(784\,z^2 + 728\,z + 32117)$$

$$ys_{30,\,29} := 870\,z^2 + 811\,z + 35641$$
$$xs_{30,\,29} := (900\,z^2 + 840\,z + 36871)\,(841\,z^2 + 783\,z + 34453)$$
$$ys_{31,\,30} := 930\,z^2 + 869\,z + 38100$$
$$xs_{31,\,30} := (961\,z^2 + 899\,z + 39371)\,(900\,z^2 + 840\,z + 36871)$$
$$ys_{32,\,31} := 992\,z^2 + 929\,z + 40641$$
$$xs_{32,\,31} := (1024\,z^2 + 960\,z + 41953)\,(961\,z^2 + 899\,z + 39371)$$
$$ys_{33,\,32} := 1056\,z^2 + 991\,z + 43264$$
$$xs_{33,\,32} := (1089\,z^2 + 1023\,z + 44617)\,(1024\,z^2 + 960\,z + 41953)$$
$$ys_{34,\,33} := 1122\,z^2 + 1055\,z + 45969$$
$$xs_{34,\,33} := (1156\,z^2 + 1088\,z + 47363)\,(1089\,z^2 + 1023\,z + 44617)$$
$$ys_{35,\,34} := 1190\,z^2 + 1121\,z + 48756$$
$$xs_{35,\,34} := (1225\,z^2 + 1155\,z + 50191)\,(1156\,z^2 + 1088\,z + 47363)$$
$$ys_{36,\,35} := 1260\,z^2 + 1189\,z + 51625$$
$$xs_{36,\,35} := (1296\,z^2 + 1224\,z + 53101)\,(1225\,z^2 + 1155\,z + 50191)$$
$$ys_{37,\,36} := 1332\,z^2 + 1259\,z + 54576$$
$$xs_{37,\,36} := (1369\,z^2 + 1295\,z + 56093)\,(1296\,z^2 + 1224\,z + 53101)$$
$$ys_{38,\,37} := 1406\,z^2 + 1331\,z + 57609$$
$$xs_{38,\,37} := (1444\,z^2 + 1368\,z + 59167)\,(1369\,z^2 + 1295\,z + 56093)$$
$$ys_{39,\,38} := 1482\,z^2 + 1405\,z + 60724$$
$$xs_{39,\,38} := (1521\,z^2 + 1443\,z + 62323)\,(1444\,z^2 + 1368\,z + 59167)$$
$$ys_{40,\,39} := 1560\,z^2 + 1481\,z + 63921$$
$$xs_{40,\,39} := (1600\,z^2 + 1520\,z + 65561)\,(1521\,z^2 + 1443\,z + 62323)$$
$$ys_{41,\,40} := 1640\,z^2 + 1559\,z + 67200$$
$$xs_{41,\,40} := (1681\,z^2 + 1599\,z + 68881)\,(1600\,z^2 + 1520\,z + 65561)$$
$$ys_{42,\,41} := 1722\,z^2 + 1639\,z + 70561$$
$$xs_{42,\,41} := (1764\,z^2 + 1680\,z + 72283)\,(1681\,z^2 + 1599\,z + 68881)$$
$$ys_{43,\,42} := 1806\,z^2 + 1721\,z + 74004$$
$$xs_{43,\,42} := (1849\,z^2 + 1763\,z + 75767)\,(1764\,z^2 + 1680\,z + 72283)$$
$$ys_{44,\,43} := 1892\,z^2 + 1805\,z + 77529$$

$$xs_{44,\,43} := (1936\,z^2 + 1848\,z + 79333)\,(1849\,z^2 + 1763\,z + 75767)$$

$$ys_{45,\,44} := 1980\,z^2 + 1891\,z + 81136$$

$$xs_{45,\,44} := (2025\,z^2 + 1935\,z + 82981)\,(1936\,z^2 + 1848\,z + 79333)$$

$$ys_{46,\,45} := 2070\,z^2 + 1979\,z + 84825$$

$$xs_{46,\,45} := (2116\,z^2 + 2024\,z + 86711)\,(2025\,z^2 + 1935\,z + 82981)$$

$$ys_{47,\,46} := 2162\,z^2 + 2069\,z + 88596$$

$$xs_{47,\,46} := (2209\,z^2 + 2115\,z + 90523)\,(2116\,z^2 + 2024\,z + 86711)$$

$$ys_{48,\,47} := 2256\,z^2 + 2161\,z + 92449$$

$$xs_{48,\,47} := (2304\,z^2 + 2208\,z + 94417)\,(2209\,z^2 + 2115\,z + 90523)$$

$$ys_{49,\,48} := 2352\,z^2 + 2255\,z + 96384$$

$$xs_{49,\,48} := (2401\,z^2 + 2303\,z + 98393)\,(2304\,z^2 + 2208\,z + 94417)$$

$$ys_{50,\,49} := 2450\,z^2 + 2351\,z + 100401$$

$$xs_{50,\,49} := (2500\,z^2 + 2400\,z + 102451)\,(2401\,z^2 + 2303\,z + 98393) \qquad \textbf{(38)}$$

>

>

```
> for a from 1 to 50 by 2 do
  y[a, 2] := y[b, 2];
  x[a, 2] := simplify(x[b, 2]);
  end do;
```

$$y_{1,\,2} := 2\,z^2 + z + 81$$

$$x_{1,\,2} := \left(41 + z + z^2\right)\,\left(163 + 4\,z^2\right)$$

$$y_{3,\,2} := 6\,z^2 + z + 244$$

$$x_{3,\,2} := \left(9\,z^2 + 3\,z + 367\right)\,\left(163 + 4\,z^2\right)$$

$$y_{5,\,2} := 10\,z^2 + z + 407$$

$$x_{5,\,2} := \left(163 + 4\,z^2\right)\,\left(25\,z^2 + 5\,z + 1019\right)$$

$$y_{7,\,2} := 14\,z^2 + z + 570$$

$$x_{7,\,2} := \left(163 + 4\,z^2\right)\,\left(49\,z^2 + 7\,z + 1997\right)$$

$$y_{9,\,2} := 18\,z^2 + z + 733$$

$$x_{9,\,2} := \left(163 + 4\,z^2\right)\,\left(81\,z^2 + 9\,z + 3301\right)$$

$$y_{11,\,2} := 22\,z^2 + z + 896$$

$$x_{11,\,2} := \left(163 + 4\,z^2\right)\,\left(121\,z^2 + 11\,z + 4931\right)$$

$$y_{13,\,2} := 26\,z^2 + z + 1059$$

$$x_{13,\,2} := \left(163 + 4\,z^2\right)\,\left(169\,z^2 + 13\,z + 6887\right)$$

$$y_{15,\,2} := 30\,z^2 + z + 1222$$

$$x_{15,\,2} := \left(163 + 4\,z^2\right)\,\left(225\,z^2 + 15\,z + 9169\right)$$

$$y_{17,\,2} := 34\,z^2 + z + 1385$$

$$x_{17,\,2} := \left(163 + 4\,z^2\right)\,\left(289\,z^2 + 17\,z + 11777\right)$$

$$y_{19,\,2} := 38\,z^2 + z + 1548$$

$$x_{19,\,2} := \left(163 + 4\,z^2\right)\,\left(361\,z^2 + 19\,z + 14711\right)$$

$$y_{21,\,2} := 42\,z^2 + z + 1711$$

$$x_{21,\,2} := \left(163 + 4\,z^2\right)\,\left(441\,z^2 + 21\,z + 17971\right)$$

$$y_{23,\,2} := 46\,z^2 + z + 1874$$

$$x_{23,\,2} := \left(163 + 4\,z^2\right)\,\left(529\,z^2 + 23\,z + 21557\right)$$

$$y_{25,\,2} := 50\,z^2 + z + 2037$$

$$x_{25,\,2} := \left(163 + 4\,z^2\right)\,\left(625\,z^2 + 25\,z + 25469\right)$$

$$y_{27,\,2} := 54\,z^2 + z + 2200$$

$$x_{27,\,2} := \left(163 + 4\,z^2\right)\,\left(729\,z^2 + 27\,z + 29707\right)$$

$$y_{29,\,2} := 58\,z^2 + z + 2363$$

$$x_{29,\,2} := \left(163 + 4\,z^2\right)\,\left(841\,z^2 + 29\,z + 34271\right)$$

$$y_{31,\,2} := 62\,z^2 + z + 2526$$

$$x_{31,\,2} := \left(163 + 4\,z^2\right)\,\left(961\,z^2 + 31\,z + 39161\right)$$

$$y_{33,\,2} := 66\,z^2 + z + 2689$$

$$x_{33,\,2} := \left(163 + 4\,z^2\right)\,\left(1089\,z^2 + 33\,z + 44377\right)$$

$$y_{35,\,2} := 70\,z^2 + z + 2852$$

$$x_{35,\,2} := \left(163 + 4\,z^2\right)\,\left(1225\,z^2 + 35\,z + 49919\right)$$

$$y_{37,\,2} := 74\,z^2 + z + 3015$$

$$x_{37,\,2} := \left(163 + 4\,z^2\right)\,\left(1369\,z^2 + 37\,z + 55787\right)$$

$$y_{39,\,2} := 78\,z^2 + z + 3178$$

$$x_{39,\,2} := \left(163 + 4\,z^2\right)\,\left(1521\,z^2 + 39\,z + 61981\right)$$

$$y_{41,\,2} := 82\,z^2 + z + 3341$$

$$x_{41,\,2} := \left(163 + 4\,z^2\right)\,\left(1681\,z^2 + 41\,z + 68501\right)$$

$$y_{43,\,2} := 86\,z^2 + z + 3504$$

$$x_{43,\,2} := \left(163 + 4\,z^2\right)\,\left(1849\,z^2 + 43\,z + 75347\right)$$

$$y_{45,\,2} := 90\,z^2 + z + 3667$$

$$x_{45,\,2} := \left(163 + 4\,z^2\right)\,\left(2025\,z^2 + 45\,z + 82519\right)$$

$$y_{47,\,2} := 94\,z^2 + z + 3830$$

$$x_{47,\,2} := \left(163 + 4\,z^2\right)\,\left(2209\,z^2 + 47\,z + 90017\right)$$

$$y_{49,\,2} := 98\,z^2 + z + 3993$$

$$x_{49,\,2} := \left(163 + 4\,z^2\right)\,\left(2401\,z^2 + 49\,z + 97841\right) \quad \textbf{(39)}$$

>

>

>

```
> for a from 3 to 50 by 2 do
  y[a, a-2] := y[b, b - 2];
  x[a, a-2] := simplify(x[b, b - 2]);
  end do;
```

$$y_{3,\,1} := 3\,z^2 + 2\,z + 122$$

$$x_{3,\,1} := \left(41 + z + z^2\right)\,\left(9\,z^2 + 3\,z + 367\right)$$

$$y_{5,\,3} := 15\,z^2 + 4\,z + 611$$

$$x_{5,\,3} := \left(25\,z^2 + 5\,z + 1019\right)\,\left(9\,z^2 + 3\,z + 367\right)$$

$$y_{7,\,5} := 35\,z^2 + 6\,z + 1426$$

$$x_{7,\,5} := \left(49\,z^2 + 7\,z + 1997\right)\,\left(25\,z^2 + 5\,z + 1019\right)$$

$$y_{9,\,7} := 63\,z^2 + 8\,z + 2567$$

$$x_{9,\,7} := \left(81\,z^2 + 9\,z + 3301\right)\,\left(49\,z^2 + 7\,z + 1997\right)$$

$$y_{11,\,9} := 99\,z^2 + 10\,z + 4034$$

$$x_{11,\,9} := \left(121\,z^2 + 11\,z + 4931\right)\,\left(81\,z^2 + 9\,z + 3301\right)$$

$$y_{13,\,11} := 143\,z^2 + 12\,z + 5827$$

$$x_{13,\,11} := \left(169\,z^2 + 13\,z + 6887\right)\,\left(121\,z^2 + 11\,z + 4931\right)$$

$$y_{15,\,13} := 195\,z^2 + 14\,z + 7946$$

$$x_{15,\,13} := \left(225\,z^2 + 15\,z + 9169\right)\,\left(169\,z^2 + 13\,z + 6887\right)$$

$$y_{17,\,15} := 255\,z^2 + 16\,z + 10391$$

$$x_{17,\,15} := \left(289\,z^2 + 17\,z + 11777\right)\,\left(225\,z^2 + 15\,z + 9169\right)$$

$$y_{19,\,17} := 323\,z^2 + 18\,z + 13162$$

$$x_{19,\,17} := \left(361\,z^2 + 19\,z + 14711\right)\,\left(289\,z^2 + 17\,z + 11777\right)$$

$$y_{21,\,19} := 399\,z^2 + 20\,z + 16259$$

$$x_{21,\,19} := \left(441\,z^2 + 21\,z + 17971\right)\,\left(361\,z^2 + 19\,z + 14711\right)$$

$$y_{23,\,21} := 483\,z^2 + 22\,z + 19682$$

$$x_{23,\,21} := \left(529\,z^2 + 23\,z + 21557\right)\,\left(441\,z^2 + 21\,z + 17971\right)$$

$$y_{25,\,23} := 575\,z^2 + 24\,z + 23431$$

$$x_{25,\,23} := \left(625\,z^2 + 25\,z + 25469\right)\,\left(529\,z^2 + 23\,z + 21557\right)$$

$$y_{27,\,25} := 675\,z^2 + 26\,z + 27506$$

$$x_{27,\,25} := \left(729\,z^2 + 27\,z + 29707\right)\,\left(625\,z^2 + 25\,z + 25469\right)$$

$$y_{29,\,27} := 783\,z^2 + 28\,z + 31907$$

$$x_{29,\,27} := \left(841\,z^2 + 29\,z + 34271\right)\left(729\,z^2 + 27\,z + 29707\right)$$
$$y_{31,\,29} := 899\,z^2 + 30\,z + 36634$$
$$x_{31,\,29} := \left(961\,z^2 + 31\,z + 39161\right)\left(841\,z^2 + 29\,z + 34271\right)$$
$$y_{33,\,31} := 1023\,z^2 + 32\,z + 41687$$
$$x_{33,\,31} := \left(1089\,z^2 + 33\,z + 44377\right)\left(961\,z^2 + 31\,z + 39161\right)$$
$$y_{35,\,33} := 1155\,z^2 + 34\,z + 47066$$
$$x_{35,\,33} := \left(1225\,z^2 + 35\,z + 49919\right)\left(1089\,z^2 + 33\,z + 44377\right)$$
$$y_{37,\,35} := 1295\,z^2 + 36\,z + 52771$$
$$x_{37,\,35} := \left(1369\,z^2 + 37\,z + 55787\right)\left(1225\,z^2 + 35\,z + 49919\right)$$
$$y_{39,\,37} := 1443\,z^2 + 38\,z + 58802$$
$$x_{39,\,37} := \left(1521\,z^2 + 39\,z + 61981\right)\left(1369\,z^2 + 37\,z + 55787\right)$$
$$y_{41,\,39} := 1599\,z^2 + 40\,z + 65159$$
$$x_{41,\,39} := \left(1681\,z^2 + 41\,z + 68501\right)\left(1521\,z^2 + 39\,z + 61981\right)$$
$$y_{43,\,41} := 1763\,z^2 + 42\,z + 71842$$
$$x_{43,\,41} := \left(1849\,z^2 + 43\,z + 75347\right)\left(1681\,z^2 + 41\,z + 68501\right)$$
$$y_{45,\,43} := 1935\,z^2 + 44\,z + 78851$$
$$x_{45,\,43} := \left(2025\,z^2 + 45\,z + 82519\right)\left(1849\,z^2 + 43\,z + 75347\right)$$
$$y_{47,\,45} := 2115\,z^2 + 46\,z + 86186$$
$$x_{47,\,45} := \left(2209\,z^2 + 47\,z + 90017\right)\left(2025\,z^2 + 45\,z + 82519\right)$$
$$y_{49,\,47} := 2303\,z^2 + 48\,z + 93847$$
$$x_{49,\,47} := \left(2401\,z^2 + 49\,z + 97841\right)\left(2209\,z^2 + 47\,z + 90017\right) \quad \textbf{(40)}$$

>

>

>

> **for** *a* **from** 4 **to** 50 **by** 3 **do**
$y[a, 3] := y[b, 3, 1];$
$x[a, 3] := simplify(x[b, 3, 1]);$
end do;

$$y_{4, 3} := 12 z^2 + 5 z + 489$$
$$x_{4, 3} := (9 z^2 + 3 z + 367) (16 z^2 + 8 z + 653)$$
$$y_{7, 3} := 21 z^2 + 8 z + 856$$
$$x_{7, 3} := (9 z^2 + 3 z + 367) (49 z^2 + 21 z + 1999)$$
$$y_{10, 3} := 30 z^2 + 11 z + 1223$$
$$x_{10, 3} := (9 z^2 + 3 z + 367) (100 z^2 + 40 z + 4079)$$
$$y_{13, 3} := 39 z^2 + 14 z + 1590$$
$$x_{13, 3} := (9 z^2 + 3 z + 367) (169 z^2 + 65 z + 6893)$$
$$y_{16, 3} := 48 z^2 + 17 z + 1957$$
$$x_{16, 3} := (9 z^2 + 3 z + 367) (256 z^2 + 96 z + 10441)$$
$$y_{19, 3} := 57 z^2 + 20 z + 2324$$
$$x_{19, 3} := (9 z^2 + 3 z + 367) (361 z^2 + 133 z + 14723)$$
$$y_{22, 3} := 66 z^2 + 23 z + 2691$$
$$x_{22, 3} := (9 z^2 + 3 z + 367) (484 z^2 + 176 z + 19739)$$
$$y_{25, 3} := 75 z^2 + 26 z + 3058$$
$$x_{25, 3} := (9 z^2 + 3 z + 367) (625 z^2 + 225 z + 25489)$$
$$y_{28, 3} := 84 z^2 + 29 z + 3425$$
$$x_{28, 3} := (9 z^2 + 3 z + 367) (784 z^2 + 280 z + 31973)$$
$$y_{31, 3} := 93 z^2 + 32 z + 3792$$
$$x_{31, 3} := (9 z^2 + 3 z + 367) (961 z^2 + 341 z + 39191)$$
$$y_{34, 3} := 102 z^2 + 35 z + 4159$$
$$x_{34, 3} := (9 z^2 + 3 z + 367) (1156 z^2 + 408 z + 47143)$$
$$y_{37, 3} := 111 z^2 + 38 z + 4526$$
$$x_{37, 3} := (9 z^2 + 3 z + 367) (1369 z^2 + 481 z + 55829)$$
$$y_{40, 3} := 120 z^2 + 41 z + 4893$$
$$x_{40, 3} := (9 z^2 + 3 z + 367) (1600 z^2 + 560 z + 65249)$$
$$y_{43, 3} := 129 z^2 + 44 z + 5260$$

$$x_{43,\,3} := \left(9\,z^2+3\,z+367\right)\left(1849\,z^2+645\,z+75403\right)$$

$$y_{46,\,3} := 138\,z^2+47\,z+5627$$

$$x_{46,\,3} := \left(9\,z^2+3\,z+367\right)\left(2116\,z^2+736\,z+86291\right)$$

$$y_{49,\,3} := 147\,z^2+50\,z+5994$$

$$x_{49,\,3} := \left(9\,z^2+3\,z+367\right)\left(2401\,z^2+833\,z+97913\right) \tag{41}$$

>

>

```
> for a from 4 to 50 by 3 do
  ys[a, a − 3] := y[b, b − 3, 1];
  xs[a, a − 3] := simplify(x[b, b − 3, 1]);
end do;
```

$$ys_{4,\,1} := 4\,z^2 + 3\,z + 163$$

$$xs_{4,\,1} := \left(41 + z^2 + z\right)\left(16\,z^2 + 8\,z + 653\right)$$

$$ys_{7,\,4} := 28\,z^2 + 13\,z + 1142$$

$$xs_{7,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(49\,z^2 + 21\,z + 1999\right)$$

$$ys_{10,\,7} := 70\,z^2 + 29\,z + 2855$$

$$xs_{10,\,7} := \left(49\,z^2 + 21\,z + 1999\right)\left(100\,z^2 + 40\,z + 4079\right)$$

$$ys_{13,\,10} := 130\,z^2 + 51\,z + 5302$$

$$xs_{13,\,10} := \left(100\,z^2 + 40\,z + 4079\right)\left(169\,z^2 + 65\,z + 6893\right)$$

$$ys_{16,\,13} := 208\,z^2 + 79\,z + 8483$$

$$xs_{16,\,13} := \left(169\,z^2 + 65\,z + 6893\right)\left(256\,z^2 + 96\,z + 10441\right)$$

$$ys_{19,\,16} := 304\,z^2 + 113\,z + 12398$$

$$xs_{19,\,16} := \left(256\,z^2 + 96\,z + 10441\right)\left(361\,z^2 + 133\,z + 14723\right)$$

$$ys_{22,\,19} := 418\,z^2 + 153\,z + 17047$$

$$xs_{22,\,19} := \left(361\,z^2 + 133\,z + 14723\right)\left(484\,z^2 + 176\,z + 19739\right)$$

$$ys_{25,\,22} := 550\,z^2 + 199\,z + 22430$$

$$xs_{25,\,22} := \left(484\,z^2 + 176\,z + 19739\right)\left(625\,z^2 + 225\,z + 25489\right)$$

$$ys_{28,\,25} := 700\,z^2 + 251\,z + 28547$$

$$xs_{28,\,25} := \left(625\,z^2 + 225\,z + 25489\right)\left(784\,z^2 + 280\,z + 31973\right)$$

$$ys_{31,\,28} := 868\,z^2 + 309\,z + 35398$$

$$xs_{31,\,28} := \left(784\,z^2 + 280\,z + 31973\right)\left(961\,z^2 + 341\,z + 39191\right)$$

$$ys_{34,\,31} := 1054\,z^2 + 373\,z + 42983$$

$$xs_{34,\,31} := \left(961\,z^2 + 341\,z + 39191\right)\left(1156\,z^2 + 408\,z + 47143\right)$$

$$ys_{37,\,34} := 1258\,z^2 + 443\,z + 51302$$

$$xs_{37,\,34} := \left(1156\,z^2 + 408\,z + 47143\right)\left(1369\,z^2 + 481\,z + 55829\right)$$

$$ys_{40,\,37} := 1480\,z^2 + 519\,z + 60355$$

$$xs_{40,\,37} := \left(1369\,z^2 + 481\,z + 55829\right)\left(1600\,z^2 + 560\,z + 65249\right)$$

$$ys_{43,\,40} := 1720\,z^2 + 601\,z + 70142$$

$$xs_{43,\,40} := (1600\,z^2 + 560\,z + 65249)\,(1849\,z^2 + 645\,z + 75403)$$
$$ys_{46,\,43} := 1978\,z^2 + 689\,z + 80663$$
$$xs_{46,\,43} := (1849\,z^2 + 645\,z + 75403)\,(2116\,z^2 + 736\,z + 86291)$$
$$ys_{49,\,46} := 2254\,z^2 + 783\,z + 91918$$
$$xs_{49,\,46} := (2116\,z^2 + 736\,z + 86291)\,(2401\,z^2 + 833\,z + 97913) \tag{42}$$

>

>

```
> for a from 5 to 50 by 3 do
  ys[a, 3] := y[b, 3, 2];
  xs[a, 3] := simplify(x[b, 3, 2]);
end do;
```

$$ys_{5,\,3} := 15\,z^2 + 4\,z + 611$$

$$xs_{5,\,3} := \left(25\,z^2 + 5\,z + 1019\right)\left(9\,z^2 + 3\,z + 367\right)$$

$$ys_{8,\,3} := 24\,z^2 + 7\,z + 978$$

$$xs_{8,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(64\,z^2 + 16\,z + 2609\right)$$

$$ys_{11,\,3} := 33\,z^2 + 10\,z + 1345$$

$$xs_{11,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(121\,z^2 + 33\,z + 4933\right)$$

$$ys_{14,\,3} := 42\,z^2 + 13\,z + 1712$$

$$xs_{14,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(196\,z^2 + 56\,z + 7991\right)$$

$$ys_{17,\,3} := 51\,z^2 + 16\,z + 2079$$

$$xs_{17,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(289\,z^2 + 85\,z + 11783\right)$$

$$ys_{20,\,3} := 60\,z^2 + 19\,z + 2446$$

$$xs_{20,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(400\,z^2 + 120\,z + 16309\right)$$

$$ys_{23,\,3} := 69\,z^2 + 22\,z + 2813$$

$$xs_{23,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(529\,z^2 + 161\,z + 21569\right)$$

$$ys_{26,\,3} := 78\,z^2 + 25\,z + 3180$$

$$xs_{26,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(676\,z^2 + 208\,z + 27563\right)$$

$$ys_{29,\,3} := 87\,z^2 + 28\,z + 3547$$

$$xs_{29,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(841\,z^2 + 261\,z + 34291\right)$$

$$ys_{32,\,3} := 96\,z^2 + 31\,z + 3914$$

$$xs_{32,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(1024\,z^2 + 320\,z + 41753\right)$$

$$ys_{35,\,3} := 105\,z^2 + 34\,z + 4281$$

$$xs_{35,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(1225\,z^2 + 385\,z + 49949\right)$$

$$ys_{38,\,3} := 114\,z^2 + 37\,z + 4648$$

$$xs_{38,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(1444\,z^2 + 456\,z + 58879\right)$$

$$ys_{41,\,3} := 123\,z^2 + 40\,z + 5015$$

$$xs_{41,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(1681\,z^2 + 533\,z + 68543\right)$$

$$ys_{44,\,3} := 132\,z^2 + 43\,z + 5382$$

$$xs_{44,\,3} := (9\,z^2 + 3\,z + 367)\,(1936\,z^2 + 616\,z + 78941)$$

$$ys_{47,\,3} := 141\,z^2 + 46\,z + 5749$$

$$xs_{47,\,3} := (9\,z^2 + 3\,z + 367)\,(2209\,z^2 + 705\,z + 90073)$$

$$ys_{50,\,3} := 150\,z^2 + 49\,z + 6116$$

$$xs_{50,\,3} := (9\,z^2 + 3\,z + 367)\,(2500\,z^2 + 800\,z + 101939) \quad \textbf{(43)}$$

>

> **for** *a* **from** 5 **to** 50 **by** 3 **do**
> $ys[a, a-3] := y[b, b-3, 2]$;
> $xs[a, a-3] := simplify(x[b, b-3, 2])$;
> **end do**;

$$ys_{5,\,2} := 10\,z^2 + z + 407$$

$$xs_{5,\,2} := (163 + 4\,z^2)\,(25\,z^2 + 5\,z + 1019)$$

$$ys_{8,\,5} := 40\,z^2 + 9\,z + 1630$$

$$xs_{8,\,5} := (64\,z^2 + 16\,z + 2609)\,(25\,z^2 + 5\,z + 1019)$$

$$ys_{11,\,8} := 88\,z^2 + 23\,z + 3587$$

$$xs_{11,\,8} := (64\,z^2 + 16\,z + 2609)\,(121\,z^2 + 33\,z + 4933)$$

$$ys_{14,\,11} := 154\,z^2 + 43\,z + 6278$$

$$xs_{14,\,11} := (121\,z^2 + 33\,z + 4933)\,(196\,z^2 + 56\,z + 7991)$$

$$ys_{17,\,14} := 238\,z^2 + 69\,z + 9703$$

$$xs_{17,\,14} := (196\,z^2 + 56\,z + 7991)\,(289\,z^2 + 85\,z + 11783)$$

$$ys_{20,\,17} := 340\,z^2 + 101\,z + 13862$$

$$xs_{20,\,17} := (289\,z^2 + 85\,z + 11783)\,(400\,z^2 + 120\,z + 16309)$$

$$ys_{23,\,20} := 460\,z^2 + 139\,z + 18755$$

$$xs_{23,\,20} := (400\,z^2 + 120\,z + 16309)\,(529\,z^2 + 161\,z + 21569)$$

$$ys_{26,\,23} := 598\,z^2 + 183\,z + 24382$$

$$xs_{26,\,23} := (529\,z^2 + 161\,z + 21569)\,(676\,z^2 + 208\,z + 27563)$$

$$ys_{29,\,26} := 754\,z^2 + 233\,z + 30743$$

$$xs_{29,\,26} := (676\,z^2 + 208\,z + 27563)\,(841\,z^2 + 261\,z + 34291)$$

$$ys_{32,\,29} := 928\,z^2 + 289\,z + 37838$$

$$xs_{32,\,29} := (841\,z^2 + 261\,z + 34291)\,(1024\,z^2 + 320\,z + 41753)$$

$$ys_{35,\,32} := 1120\,z^2 + 351\,z + 45667$$

$$xs_{35,\,32} := (1024\,z^2 + 320\,z + 41753)\,(1225\,z^2 + 385\,z + 49949)$$

$$ys_{38,\,35} := 1330\,z^2 + 419\,z + 54230$$

$$xs_{38,\,35} := (1225\,z^2 + 385\,z + 49949)\,(1444\,z^2 + 456\,z + 58879)$$

$$ys_{41,\,38} := 1558\,z^2 + 493\,z + 63527$$

$$xs_{41,\,38} := (1444\,z^2 + 456\,z + 58879)\,(1681\,z^2 + 533\,z + 68543)$$

$$ys_{44,\,41} := 1804\,z^2 + 573\,z + 73558$$

$$xs_{44,\,41} := \left(1681\,z^2 + 533\,z + 68543\right)\left(1936\,z^2 + 616\,z + 78941\right)$$

$$ys_{47,\,44} := 2068\,z^2 + 659\,z + 84323$$

$$xs_{47,\,44} := \left(1936\,z^2 + 616\,z + 78941\right)\left(2209\,z^2 + 705\,z + 90073\right)$$

$$ys_{50,\,47} := 2350\,z^2 + 751\,z + 95822$$

$$xs_{50,\,47} := \left(2209\,z^2 + 705\,z + 90073\right)\left(2500\,z^2 + 800\,z + 101939\right) \tag{44}$$

>

```
> for a from 7 to 50 by 4 do
  ys[a, 4] := y[b, 4, 3];
  xs[a, 4] := simplify(x[b, 4, 3]);
end do;
```

$$ys_{7,\,4} := 28\,z^2 + 13\,z + 1142$$

$$xs_{7,\,4} := (49\,z^2 + 21\,z + 1999)\,(16\,z^2 + 8\,z + 653)$$

$$ys_{11,\,4} := 44\,z^2 + 21\,z + 1795$$

$$xs_{11,\,4} := (16\,z^2 + 8\,z + 653)\,(121\,z^2 + 55\,z + 4937)$$

$$ys_{15,\,4} := 60\,z^2 + 29\,z + 2448$$

$$xs_{15,\,4} := (16\,z^2 + 8\,z + 653)\,(225\,z^2 + 105\,z + 9181)$$

$$ys_{19,\,4} := 76\,z^2 + 37\,z + 3101$$

$$xs_{19,\,4} := (16\,z^2 + 8\,z + 653)\,(361\,z^2 + 171\,z + 14731)$$

$$ys_{23,\,4} := 92\,z^2 + 45\,z + 3754$$

$$xs_{23,\,4} := (16\,z^2 + 8\,z + 653)\,(529\,z^2 + 253\,z + 21587)$$

$$ys_{27,\,4} := 108\,z^2 + 53\,z + 4407$$

$$xs_{27,\,4} := (16\,z^2 + 8\,z + 653)\,(729\,z^2 + 351\,z + 29749)$$

$$ys_{31,\,4} := 124\,z^2 + 61\,z + 5060$$

$$xs_{31,\,4} := (16\,z^2 + 8\,z + 653)\,(961\,z^2 + 465\,z + 39217)$$

$$ys_{35,\,4} := 140\,z^2 + 69\,z + 5713$$

$$xs_{35,\,4} := (16\,z^2 + 8\,z + 653)\,(1225\,z^2 + 595\,z + 49991)$$

$$ys_{39,\,4} := 156\,z^2 + 77\,z + 6366$$

$$xs_{39,\,4} := (16\,z^2 + 8\,z + 653)\,(1521\,z^2 + 741\,z + 62071)$$

$$ys_{43,\,4} := 172\,z^2 + 85\,z + 7019$$

$$xs_{43,\,4} := (16\,z^2 + 8\,z + 653)\,(1849\,z^2 + 903\,z + 75457)$$

$$ys_{47,\,4} := 188\,z^2 + 93\,z + 7672$$

$$xs_{47,\,4} := (16\,z^2 + 8\,z + 653)\,(2209\,z^2 + 1081\,z + 90149) \qquad \textbf{(45)}$$

```
>
```

```
> for a from 7 to 50 by 4 do
  ys[a, a-4] := y[b, b-4, 3];
  xs[a, a-4] := simplify(x[b, b-4, 3]);
end do;
```

$$ys_{7,\,3} := 21\,z^2 + 8\,z + 856$$
$$xs_{7,\,3} := \left(9\,z^2 + 3\,z + 367\right)\left(49\,z^2 + 21\,z + 1999\right)$$
$$ys_{11,\,7} := 77\,z^2 + 34\,z + 3141$$
$$xs_{11,\,7} := \left(49\,z^2 + 21\,z + 1999\right)\left(121\,z^2 + 4937 + 55\,z\right)$$
$$ys_{15,\,11} := 165\,z^2 + 76\,z + 6732$$
$$xs_{15,\,11} := \left(121\,z^2 + 4937 + 55\,z\right)\left(225\,z^2 + 9181 + 105\,z\right)$$
$$ys_{19,\,15} := 285\,z^2 + 134\,z + 11629$$
$$xs_{19,\,15} := \left(225\,z^2 + 9181 + 105\,z\right)\left(361\,z^2 + 14731 + 171\,z\right)$$
$$ys_{23,\,19} := 437\,z^2 + 208\,z + 17832$$
$$xs_{23,\,19} := \left(361\,z^2 + 14731 + 171\,z\right)\left(529\,z^2 + 21587 + 253\,z\right)$$
$$ys_{27,\,23} := 621\,z^2 + 298\,z + 25341$$
$$xs_{27,\,23} := \left(529\,z^2 + 21587 + 253\,z\right)\left(729\,z^2 + 29749 + 351\,z\right)$$
$$ys_{31,\,27} := 837\,z^2 + 404\,z + 34156$$
$$xs_{31,\,27} := \left(729\,z^2 + 29749 + 351\,z\right)\left(961\,z^2 + 39217 + 465\,z\right)$$
$$ys_{35,\,31} := 1085\,z^2 + 526\,z + 44277$$
$$xs_{35,\,31} := \left(961\,z^2 + 39217 + 465\,z\right)\left(1225\,z^2 + 49991 + 595\,z\right)$$
$$ys_{39,\,35} := 1365\,z^2 + 664\,z + 55704$$
$$xs_{39,\,35} := \left(1225\,z^2 + 49991 + 595\,z\right)\left(1521\,z^2 + 62071 + 741\,z\right)$$
$$ys_{43,\,39} := 1677\,z^2 + 818\,z + 68437$$
$$xs_{43,\,39} := \left(1521\,z^2 + 62071 + 741\,z\right)\left(1849\,z^2 + 75457 + 903\,z\right)$$
$$ys_{47,\,43} := 2021\,z^2 + 988\,z + 82476$$
$$xs_{47,\,43} := \left(1849\,z^2 + 75457 + 903\,z\right)\left(2209\,z^2 + 90149 + 1081\,z\right) \quad \textbf{(46)}$$

```
>
```

```
> for a from 5 to 50 by 4 do
  ys[a, 4] := y[b, 4, 1];
  xs[a, 4] := simplify(x[b, 4, 1]);
end do;
```

$$ys_{5,\,4} := 20\,z^2 + 11\,z + 816$$

$$xs_{5,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(25\,z^2 + 15\,z + 1021\right)$$

$$ys_{9,\,4} := 36\,z^2 + 19\,z + 1469$$

$$xs_{9,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(81\,z^2 + 45\,z + 3307\right)$$

$$ys_{13,\,4} := 52\,z^2 + 27\,z + 2122$$

$$xs_{13,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(169\,z^2 + 91\,z + 6899\right)$$

$$ys_{17,\,4} := 68\,z^2 + 35\,z + 2775$$

$$xs_{17,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(289\,z^2 + 153\,z + 11797\right)$$

$$ys_{21,\,4} := 84\,z^2 + 43\,z + 3428$$

$$xs_{21,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(441\,z^2 + 231\,z + 18001\right)$$

$$ys_{25,\,4} := 100\,z^2 + 51\,z + 4081$$

$$xs_{25,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(625\,z^2 + 325\,z + 25511\right)$$

$$ys_{29,\,4} := 116\,z^2 + 59\,z + 4734$$

$$xs_{29,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(841\,z^2 + 435\,z + 34327\right)$$

$$ys_{33,\,4} := 132\,z^2 + 67\,z + 5387$$

$$xs_{33,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(1089\,z^2 + 561\,z + 44449\right)$$

$$ys_{37,\,4} := 148\,z^2 + 75\,z + 6040$$

$$xs_{37,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(1369\,z^2 + 703\,z + 55877\right)$$

$$ys_{41,\,4} := 164\,z^2 + 83\,z + 6693$$

$$xs_{41,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(1681\,z^2 + 861\,z + 68611\right)$$

$$ys_{45,\,4} := 180\,z^2 + 91\,z + 7346$$

$$xs_{45,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(2025\,z^2 + 1035\,z + 82651\right)$$

$$ys_{49,\,4} := 196\,z^2 + 99\,z + 7999$$

$$xs_{49,\,4} := \left(16\,z^2 + 8\,z + 653\right)\left(2401\,z^2 + 1225\,z + 97997\right) \quad \textbf{(47)}$$

```
>
```

```
> for a from 5 to 50 by 4 do
  ys[a, a-4] := y[b, b-4, 1];
  xs[a, a-4] := simplify(x[b, b-4, 1]);
end do;
```

$$ys_{5,\,1} := 5\,z^2 + 4\,z + 204$$

$$xs_{5,\,1} := \left(25\,z^2 + 1021 + 15\,z\right)\,\left(z^2 + z + 41\right)$$

$$ys_{9,\,5} := 45\,z^2 + 26\,z + 1837$$

$$xs_{9,\,5} := \left(81\,z^2 + 3307 + 45\,z\right)\,\left(25\,z^2 + 1021 + 15\,z\right)$$

$$ys_{13,\,9} := 117\,z^2 + 64\,z + 4776$$

$$xs_{13,\,9} := \left(169\,z^2 + 6899 + 91\,z\right)\,\left(81\,z^2 + 3307 + 45\,z\right)$$

$$ys_{17,\,13} := 221\,z^2 + 118\,z + 9021$$

$$xs_{17,\,13} := \left(289\,z^2 + 11797 + 153\,z\right)\,\left(169\,z^2 + 6899 + 91\,z\right)$$

$$ys_{21,\,17} := 357\,z^2 + 188\,z + 14572$$

$$xs_{21,\,17} := \left(441\,z^2 + 18001 + 231\,z\right)\,\left(289\,z^2 + 11797 + 153\,z\right)$$

$$ys_{25,\,21} := 525\,z^2 + 274\,z + 21429$$

$$xs_{25,\,21} := \left(625\,z^2 + 25511 + 325\,z\right)\,\left(441\,z^2 + 18001 + 231\,z\right)$$

$$ys_{29,\,25} := 725\,z^2 + 376\,z + 29592$$

$$xs_{29,\,25} := \left(841\,z^2 + 34327 + 435\,z\right)\,\left(625\,z^2 + 25511 + 325\,z\right)$$

$$ys_{33,\,29} := 957\,z^2 + 494\,z + 39061$$

$$xs_{33,\,29} := \left(1089\,z^2 + 44449 + 561\,z\right)\,\left(841\,z^2 + 34327 + 435\,z\right)$$

$$ys_{37,\,33} := 1221\,z^2 + 628\,z + 49836$$

$$xs_{37,\,33} := \left(1369\,z^2 + 55877 + 703\,z\right)\,\left(1089\,z^2 + 44449 + 561\,z\right)$$

$$ys_{41,\,37} := 1517\,z^2 + 778\,z + 61917$$

$$xs_{41,\,37} := \left(1681\,z^2 + 68611 + 861\,z\right)\,\left(1369\,z^2 + 55877 + 703\,z\right)$$

$$ys_{45,\,41} := 1845\,z^2 + 944\,z + 75304$$

$$xs_{45,\,41} := \left(2025\,z^2 + 82651 + 1035\,z\right)\,\left(1681\,z^2 + 68611 + 861\,z\right)$$

$$ys_{49,\,45} := 2205\,z^2 + 1126\,z + 89997$$

$$xs_{49,\,45} := \left(2401\,z^2 + 97997 + 1225\,z\right)\,\left(2025\,z^2 + 82651 + 1035\,z\right) \quad \textbf{(48)}$$

```
>
```

```
> for a from 6 to 50 by 5 do
  ys[a, 5] := y[b, 5, 1];
  xs[a, 5] := simplify(x[b, 5, 1]);
  end do;
```

$$ys_{6,\,5} := 30\,z^2 + 19\,z + 1225$$

$$xs_{6,\,5} := (15\,z + 1021 + 25\,z^2)\,(36\,z^2 + 24\,z + 1471)$$

$$ys_{11,\,5} := 55\,z^2 + 34\,z + 2246$$

$$xs_{11,\,5} := (15\,z + 1021 + 25\,z^2)\,(121\,z^2 + 77\,z + 4943)$$

$$ys_{16,\,5} := 80\,z^2 + 49\,z + 3267$$

$$xs_{16,\,5} := (15\,z + 1021 + 25\,z^2)\,(256\,z^2 + 160\,z + 10457)$$

$$ys_{21,\,5} := 105\,z^2 + 64\,z + 4288$$

$$xs_{21,\,5} := (15\,z + 1021 + 25\,z^2)\,(441\,z^2 + 273\,z + 18013)$$

$$ys_{26,\,5} := 130\,z^2 + 79\,z + 5309$$

$$xs_{26,\,5} := (15\,z + 1021 + 25\,z^2)\,(676\,z^2 + 416\,z + 27611)$$

$$ys_{31,\,5} := 155\,z^2 + 94\,z + 6330$$

$$xs_{31,\,5} := (15\,z + 1021 + 25\,z^2)\,(961\,z^2 + 589\,z + 39251)$$

$$ys_{36,\,5} := 180\,z^2 + 109\,z + 7351$$

$$xs_{36,\,5} := (15\,z + 1021 + 25\,z^2)\,(1296\,z^2 + 792\,z + 52933)$$

$$ys_{41,\,5} := 205\,z^2 + 124\,z + 8372$$

$$xs_{41,\,5} := (15\,z + 1021 + 25\,z^2)\,(1681\,z^2 + 1025\,z + 68657)$$

$$ys_{46,\,5} := 230\,z^2 + 139\,z + 9393$$

$$xs_{46,\,5} := (15\,z + 1021 + 25\,z^2)\,(2116\,z^2 + 1288\,z + 86423) \qquad \textbf{(49)}$$

```
>
```

```
> for a from 7 to 50 by 5 do
   ys[a, 5] := y[b, 5, 2];
   xs[a, 5] := simplify(x[b, 5, 2]);
  end do;
```

$$ys_{7,\,5} := 35\,z^2 + 6\,z + 1426$$

$$xs_{7,\,5} := \left(1019 + 25\,z^2 + 5\,z\right)\,\left(49\,z^2 + 7\,z + 1997\right)$$

$$ys_{12,\,5} := 60\,z^2 + 11\,z + 2445$$

$$xs_{12,\,5} := \left(1019 + 25\,z^2 + 5\,z\right)\,\left(144\,z^2 + 24\,z + 5869\right)$$

$$ys_{17,\,5} := 85\,z^2 + 16\,z + 3464$$

$$xs_{17,\,5} := \left(1019 + 25\,z^2 + 5\,z\right)\,\left(289\,z^2 + 51\,z + 11779\right)$$

$$ys_{22,\,5} := 110\,z^2 + 21\,z + 4483$$

$$xs_{22,\,5} := \left(1019 + 25\,z^2 + 5\,z\right)\,\left(484\,z^2 + 88\,z + 19727\right)$$

$$ys_{27,\,5} := 135\,z^2 + 26\,z + 5502$$

$$xs_{27,\,5} := \left(1019 + 25\,z^2 + 5\,z\right)\,\left(729\,z^2 + 135\,z + 29713\right)$$

$$ys_{32,\,5} := 160\,z^2 + 31\,z + 6521$$

$$xs_{32,\,5} := \left(1019 + 25\,z^2 + 5\,z\right)\,\left(1024\,z^2 + 192\,z + 41737\right)$$

$$ys_{37,\,5} := 185\,z^2 + 36\,z + 7540$$

$$xs_{37,\,5} := \left(1019 + 25\,z^2 + 5\,z\right)\,\left(1369\,z^2 + 259\,z + 55799\right)$$

$$ys_{42,\,5} := 210\,z^2 + 41\,z + 8559$$

$$xs_{42,\,5} := \left(1019 + 25\,z^2 + 5\,z\right)\,\left(1764\,z^2 + 336\,z + 71899\right)$$

$$ys_{47,\,5} := 235\,z^2 + 46\,z + 9578$$

$$xs_{47,\,5} := \left(1019 + 25\,z^2 + 5\,z\right)\,\left(2209\,z^2 + 423\,z + 90037\right) \quad \textbf{(50)}$$

```
>
```

```
> for a from 8 to 50 by 5 do
   ys[a, 5] := y[b, 5, 3];
   xs[a, 5] := simplify(x[b, 5, 3]);
  end do;
```

$$ys_{8,\,5} := 40\,z^2 + 9\,z + 1630$$

$$xs_{8,\,5} := (1019 + 5\,z + 25\,z^2)\,(64\,z^2 + 16\,z + 2609)$$

$$ys_{13,\,5} := 65\,z^2 + 14\,z + 2649$$

$$xs_{13,\,5} := (1019 + 5\,z + 25\,z^2)\,(169\,z^2 + 39\,z + 6889)$$

$$ys_{18,\,5} := 90\,z^2 + 19\,z + 3668$$

$$xs_{18,\,5} := (1019 + 5\,z + 25\,z^2)\,(324\,z^2 + 72\,z + 13207)$$

$$ys_{23,\,5} := 115\,z^2 + 24\,z + 4687$$

$$xs_{23,\,5} := (1019 + 5\,z + 25\,z^2)\,(529\,z^2 + 115\,z + 21563)$$

$$ys_{28,\,5} := 140\,z^2 + 29\,z + 5706$$

$$xs_{28,\,5} := (1019 + 5\,z + 25\,z^2)\,(784\,z^2 + 168\,z + 31957)$$

$$ys_{33,\,5} := 165\,z^2 + 34\,z + 6725$$

$$xs_{33,\,5} := (1019 + 5\,z + 25\,z^2)\,(1089\,z^2 + 231\,z + 44389)$$

$$ys_{38,\,5} := 190\,z^2 + 39\,z + 7744$$

$$xs_{38,\,5} := (1019 + 5\,z + 25\,z^2)\,(1444\,z^2 + 304\,z + 58859)$$

$$ys_{43,\,5} := 215\,z^2 + 44\,z + 8763$$

$$xs_{43,\,5} := (1019 + 5\,z + 25\,z^2)\,(1849\,z^2 + 387\,z + 75367)$$

$$ys_{48,\,5} := 240\,z^2 + 49\,z + 9782$$

$$xs_{48,\,5} := (1019 + 5\,z + 25\,z^2)\,(2304\,z^2 + 480\,z + 93913) \qquad \textbf{(51)}$$

```
>
```

```
> for a from 9 to 50 by 5 do
  ys[a, 5] := y[b, 5, 4];
  xs[a, 5] := simplify(x[b, 5, 4]);
  end do;
```

$$ys_{9,\,5} := 45\,z^2 + 26\,z + 1837$$

$$xs_{9,\,5} := (25\,z^2 + 15\,z + 1021)\,(81\,z^2 + 45\,z + 3307)$$

$$ys_{14,\,5} := 70\,z^2 + 41\,z + 2858$$

$$xs_{14,\,5} := (25\,z^2 + 15\,z + 1021)\,(196\,z^2 + 112\,z + 8003)$$

$$ys_{19,\,5} := 95\,z^2 + 56\,z + 3879$$

$$xs_{19,\,5} := (25\,z^2 + 15\,z + 1021)\,(361\,z^2 + 209\,z + 14741)$$

$$ys_{24,\,5} := 120\,z^2 + 71\,z + 4900$$

$$xs_{24,\,5} := (25\,z^2 + 15\,z + 1021)\,(576\,z^2 + 336\,z + 23521)$$

$$ys_{29,\,5} := 145\,z^2 + 86\,z + 5921$$

$$xs_{29,\,5} := (25\,z^2 + 15\,z + 1021)\,(841\,z^2 + 493\,z + 34343)$$

$$ys_{34,\,5} := 170\,z^2 + 101\,z + 6942$$

$$xs_{34,\,5} := (25\,z^2 + 15\,z + 1021)\,(1156\,z^2 + 680\,z + 47207)$$

$$ys_{39,\,5} := 195\,z^2 + 116\,z + 7963$$

$$xs_{39,\,5} := (25\,z^2 + 15\,z + 1021)\,(1521\,z^2 + 897\,z + 62113)$$

$$ys_{44,\,5} := 220\,z^2 + 131\,z + 8984$$

$$xs_{44,\,5} := (25\,z^2 + 15\,z + 1021)\,(1936\,z^2 + 1144\,z + 79061)$$

$$ys_{49,\,5} := 245\,z^2 + 146\,z + 10005$$

$$xs_{49,\,5} := (25\,z^2 + 15\,z + 1021)\,(2401\,z^2 + 1421\,z + 98051) \qquad \textbf{(52)}$$

```
>
```

```
> for a from 6 to 50 by 5 do
  ys[a, a-5] := y[b, b - 5, 1];
  xs[a, a-5] := simplify(x[b, b - 5, 1]);
end do;
```

$$ys_{6,\,1} := 6\,z^2 + 5\,z + 245$$

$$xs_{6,\,1} := (z^2 + z + 41)\,(36\,z^2 + 1471 + 24\,z)$$

$$ys_{11,\,6} := 66\,z^2 + 43\,z + 2696$$

$$xs_{11,\,6} := (36\,z^2 + 1471 + 24\,z)\,(121\,z^2 + 4943 + 77\,z)$$

$$ys_{16,\,11} := 176\,z^2 + 111\,z + 7189$$

$$xs_{16,\,11} := (121\,z^2 + 4943 + 77\,z)\,(256\,z^2 + 10457 + 160\,z)$$

$$ys_{21,\,16} := 336\,z^2 + 209\,z + 13724$$

$$xs_{21,\,16} := (256\,z^2 + 10457 + 160\,z)\,(441\,z^2 + 18013 + 273\,z)$$

$$ys_{26,\,21} := 546\,z^2 + 337\,z + 22301$$

$$xs_{26,\,21} := (441\,z^2 + 18013 + 273\,z)\,(676\,z^2 + 27611 + 416\,z)$$

$$ys_{31,\,26} := 806\,z^2 + 495\,z + 32920$$

$$xs_{31,\,26} := (676\,z^2 + 27611 + 416\,z)\,(961\,z^2 + 39251 + 589\,z)$$

$$ys_{36,\,31} := 1116\,z^2 + 683\,z + 45581$$

$$xs_{36,\,31} := (961\,z^2 + 39251 + 589\,z)\,(1296\,z^2 + 52933 + 792\,z)$$

$$ys_{41,\,36} := 1476\,z^2 + 901\,z + 60284$$

$$xs_{41,\,36} := (1296\,z^2 + 52933 + 792\,z)\,(1681\,z^2 + 68657 + 1025\,z)$$

$$ys_{46,\,41} := 1886\,z^2 + 1149\,z + 77029$$

$$xs_{46,\,41} := (1681\,z^2 + 68657 + 1025\,z)\,(2116\,z^2 + 86423 + 1288\,z) \qquad \textbf{(53)}$$

```
>
>
>
>
```

```
> # now to check the columns.
> r := 7 :
> for c from 1 to r − 1 do
  print( x[r, c],
  simplify( y[r, c]) );
   end do;
```

$$(41 + z + z^2)\ (49 z^2 + 35 z + 2003), 7 z^2 + 6 z + 286$$
$$(163 + 4 z^2)\ (49 z^2 + 7 z + 1997), 14 z^2 + z + 570$$
$$x_{7,\ 3}, y_{7,\ 3}$$
$$(9 z^2 + 3 z + 367)\ (49 z^2 + 21 z + 1999), 21 z^2 + 8 z + 856$$
$$(49 z^2 + 7 z + 1997)\ (25 z^2 + 5 z + 1019), 35 z^2 + 6 z + 1426$$
$$(49 z^2 + 35 z + 2003)\ (36 z^2 + 24 z + 1471), 42 z^2 + 29 z + 1716 \quad \textbf{(54)}$$

```
> # I want a general expression for x[r,c].  I think this expression will have 4 letters. a,b,c,z.  I'm
      not sure about this.  # the way I have things labeled 0<r<c, and gcd(r,c) = 1.
> # by Matt A. 5, 17, 2013
```