

## TASK

Run intervention analysis on the price of stock M: model a function of the price as ARIMA with outliers and interventions.

## SOLUTION

The document below is an abridged version of the solution provided to the client. The SAS output associated with the whole study is huge. Here we display only selected output for illustrative purposes. The objective is to give an idea of the types of analysis that this project required.

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# ARIMA ANALYSIS WITH INTERVENTIONS / OUTLIERS

## Analysis

Our study is done in several consecutive steps. Please see the SAS output, which is attached in blocks. If you need to learn the intervention analysis methodology, check out *William W.S. Wei, "Time Series Analysis, Univariate and Multivariate Methods"*.

**STEP 1:** We analyze ACF / PACF / IACF of Price, daily differences of price (Diff) and daily log-returns (LogReturn). We also run augmented Dickey-Fuller tests for the three time series. The conclusions are: Price is clearly non-stationary, while Diff and LogReturn seem to be stationary. We choose Diff for subsequent ARMA modeling, as that may lead to a relatively simple and nice ARIMA model for Price.

**STEP 2:** We experiment with ARMA models for Diff until the residuals exhibit the properties of white noise. Also, we use Akaike information criterion to identify the best structure (it is displayed in tables "Minimum Information Criterion"). An MA(2) model for Diff seems to be the best fit.

**STEP 3:** We identify 5 additive outliers (AO type in the language of the book). We add them to the ARMA model as additive shifts. The meaning of them can be additive interventions, related to stricter regulations etc. In particular, the shift that happened on April 9, 2008 is especially big. It may be related to FDA introducing new rules requiring additional tests for diabetes drugs (which are produce of company M). The estimates of the magnitudes of the additive shifts are contained in the output.

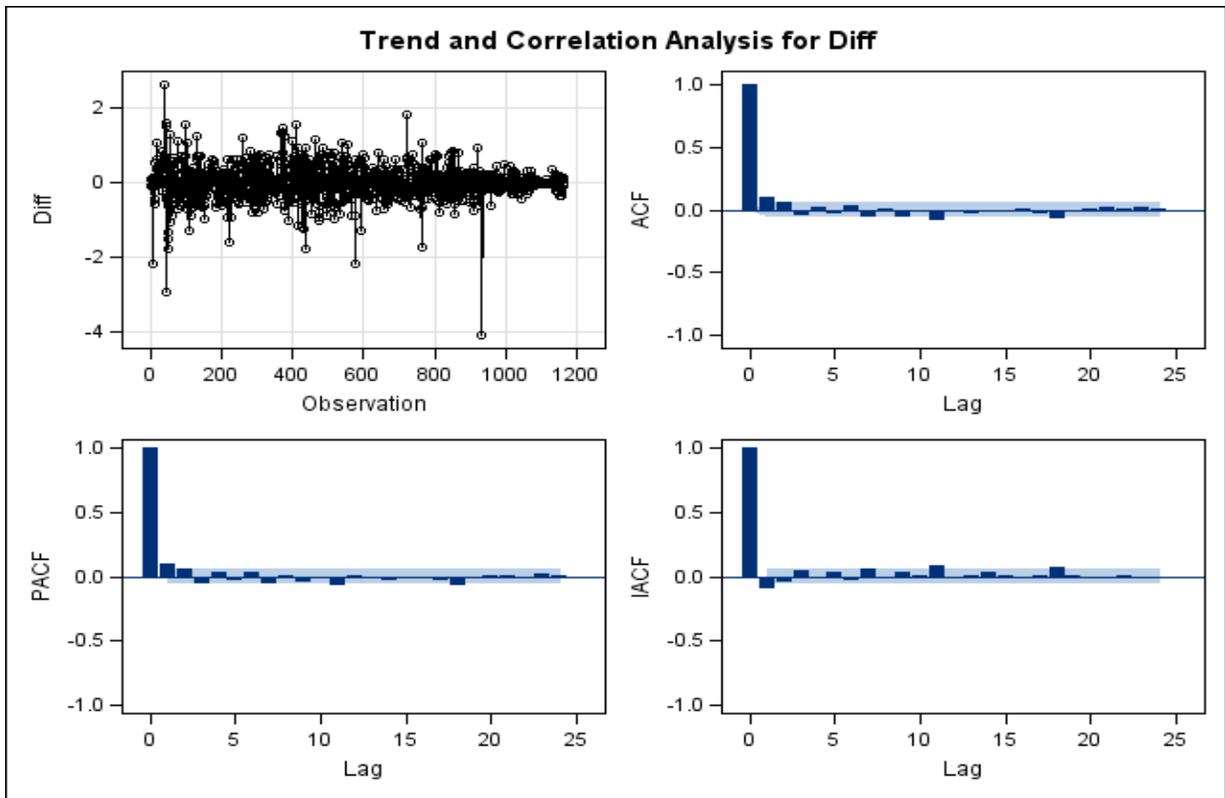
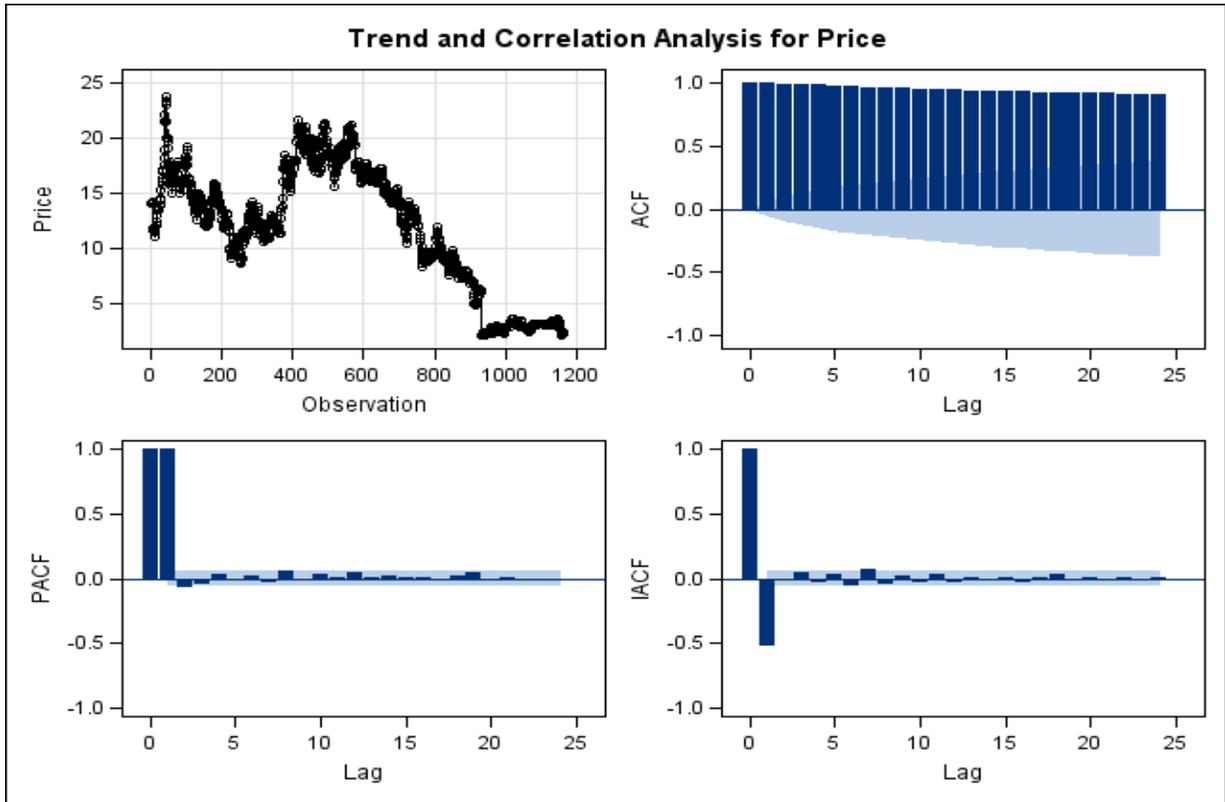
**STEP 4:** Without the shifts, Diff may be described by an ARMA model. However, the structure of the model may be slightly different from that identified in step 2. Now outliers / shifts are not obscuring the true correlation picture. So we perform model identification again, making sure the new residuals exhibit the properties of white noise. The new optimal model for Diff turns out to be seasonal ARMA((11, 14, 18), 1) + AO-type shifts with pulse functions (see the output). Therefore the optimal model for Price is

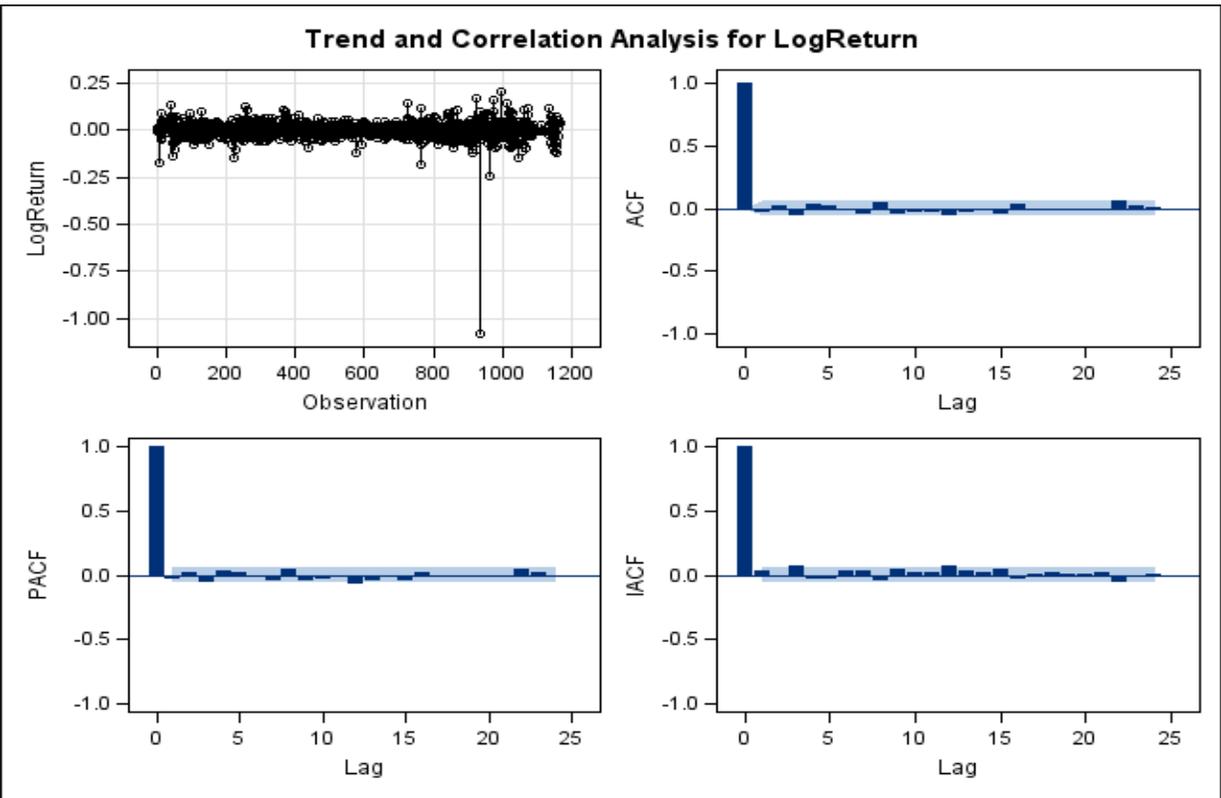
ARIMA((11, 14, 18), 1, 1) + AO-type shifts with step functions

**STEP 5:** We perform forecasts for Diff based on this model.

# Selected SAS Output

## STEP 1





## STEP 2

The ARIMA Procedure

### ARIMA Estimation Optimization Summary

Estimation Method	Conditional Least Squares
Parameters Estimated	3
Termination Criteria	Maximum Relative Change in Estimates
Iteration Stopping Value	0.001
Criteria Value	2.86E-15
Maximum Absolute Value of Gradient	0.004723
R-Square Change from Last Iteration	0.000026
Objective Function	Sum of Squared Residuals
Objective Function Value	227.9931
Marquardt's Lambda Coefficient	1E12
Numerical Derivative Perturbation Delta	0.001
Iterations	3
Warning Message	Estimates may not have converged.

### Conditional Least Squares Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag
MU	-0.0099615	0.01519	-0.66	0.5122	0
MA1,1	-0.09742	0.02929	-3.33	0.0009	1
MA1,2	-0.07156	0.02929	-2.44	0.0147	2

Constant Estimate	-0.00996
Variance Estimate	0.196546
Std Error Estimate	0.443335
AIC	1411.409
SBC	1426.585
Number of Residuals	1163

\* AIC and SBC do not include log determinant.

Correlations of Parameter Estimates

Parameter	MU	MA1, 1	MA1, 2
MU	1.000	-0.000	-0.000
MA1, 1	-0.000	1.000	0.090
MA1, 2	-0.000	0.090	1.000

The ARIMA Procedure

Autocorrelation Check of Residuals

To Lag	Chi - Square	DF	Pr > Chi Sq	-----Autocorrelations-----					
6	6.22	4	0.1831	-0.003	-0.002	-0.040	0.031	-0.036	0.039
12	19.73	10	0.0319	-0.051	0.009	-0.045	-0.005	-0.082	0.011
18	26.59	16	0.0463	-0.017	-0.023	-0.017	0.014	-0.025	-0.062
24	28.02	22	0.1751	-0.017	0.012	0.016	0.005	0.018	0.013
30	34.80	28	0.1758	-0.015	-0.047	0.034	-0.019	-0.029	-0.029
36	38.47	34	0.2742	0.001	0.001	0.022	-0.045	0.010	0.021
42	44.95	40	0.2723	0.067	-0.015	0.005	-0.006	0.012	0.020
48	51.06	46	0.2815	0.036	0.015	-0.027	-0.031	-0.026	0.034

Model for variable Diff

Estimated Mean -0.00996

Moving Average Factors

Factor 1: 1 + 0.09742 B\*\*(1) + 0.07156 B\*\*(2)

**STEP 3**

The ARIMA Procedure

Outlier Detection Summary

Maximum number searched	5
Number found	5
Significance used	0.05

Outlier Details

Obs	Time ID	Type	Estimate	Chi - Square	Approx Prob> Chi Sq
933	09-APR-2008	Additive	-4.11506	198.47	<.0001
45	28-SEP-2004	Additive	-2.91894	100.09	<.0001
40	21-SEP-2004	Additive	2.39188	67.29	<.0001
8	05-AUG-2004	Additive	-2.15508	54.69	<.0001
575	02-NOV-2006	Additive	-2.11761	52.81	<.0001

## STEP 4

### The ARIMA Procedure

#### Conditional Least Squares Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag	Variable	Shift
MU	-0.0022725	0.01349	-0.17	0.8663	0	Price	0
MA1, 1	-0.11343	0.02952	-3.84	0.0001	1	Price	0
MA1, 2	-0.02943	0.02948	-1.00	0.3184	2	Price	0
NUM1	-4.12707	0.39953	-10.33	<.0001	0	outl933	0
NUM2	-3.04425	0.40034	-7.60	<.0001	0	outl45	0
NUM3	2.46446	0.40011	6.16	<.0001	0	outl40	0
NUM4	-2.17135	0.39953	-5.43	<.0001	0	outl8	0
NUM5	-2.07302	0.40044	-5.18	<.0001	0	outl575	0

Constant Estimate      -0.00227  
 Variance Estimate      0.161618  
 Std Error Estimate      0.402017  
 AIC                      1188.833  
 SBC                      1229.303  
 Number of Residuals      1163

\* AIC and SBC do not include log determinant.

#### Correlations of Parameter Estimates

Variable Parameter		Price MU	Price MA1, 1	Price MA1, 2	outl933 NUM1
Price MU		1.000	0.001	0.000	-0.026
Price MA1, 1		0.001	1.000	0.112	0.006
Price MA1, 2		0.000	0.112	1.000	0.004
outl933 NUM1		-0.026	0.006	0.004	1.000
outl45 NUM2		-0.025	-0.027	-0.061	0.000
outl40 NUM3		-0.025	0.044	0.036	0.001
outl8 NUM4		-0.026	-0.002	0.004	0.001
outl575 NUM5		-0.026	-0.067	0.000	0.000

#### Correlations of Parameter Estimates

Variable Parameter		outl45 NUM2	outl40 NUM3	outl8 NUM4	outl575 NUM5
Price MU		-0.025	-0.025	-0.026	-0.026
Price MA1, 1		-0.027	0.044	-0.002	-0.067
Price MA1, 2		-0.061	0.036	0.004	0.000
outl933 NUM1		0.000	0.001	0.001	0.000
outl45 NUM2		1.000	-0.002	0.000	0.002
outl40 NUM3		-0.002	1.000	0.001	-0.002
outl8 NUM4		0.000	0.001	1.000	0.001
outl575 NUM5		0.002	-0.002	0.001	1.000

### The ARIMA Procedure

#### Autocorrelation Check of Residuals

To Lag	Chi-Square	DF	Pr > Chi Sq	-----Autocorrelations-----					
6	3.26	4	0.5159	-0.001	-0.002	-0.021	0.037	-0.031	-0.002
12	14.83	10	0.1385	-0.014	0.010	-0.032	0.001	-0.090	0.020
18	24.50	16	0.0792	-0.013	-0.056	-0.006	0.017	-0.025	-0.062
24	26.58	22	0.2276	0.001	0.031	0.021	0.016	0.010	0.004
30	33.85	28	0.2060	-0.020	-0.048	0.027	-0.021	-0.045	0.018
36	35.60	34	0.3931	0.001	0.003	0.029	-0.021	-0.012	0.005
42	41.31	40	0.4131	0.062	-0.007	-0.011	0.010	0.004	0.024
48	46.37	46	0.4571	0.028	0.025	-0.026	-0.023	-0.027	0.030

Model for variable Price

Estimated Intercept -0.00227  
Period(s) of Differencing 1

Moving Average Factors

Factor 1:  $1 + 0.11343 B^{(1)} + 0.02943 B^{(2)}$

Input Number 1

Input Variable outI933  
Overall Regression Factor -4.12707

Input Number 2

Input Variable outI45  
Overall Regression Factor -3.04425

Input Number 3

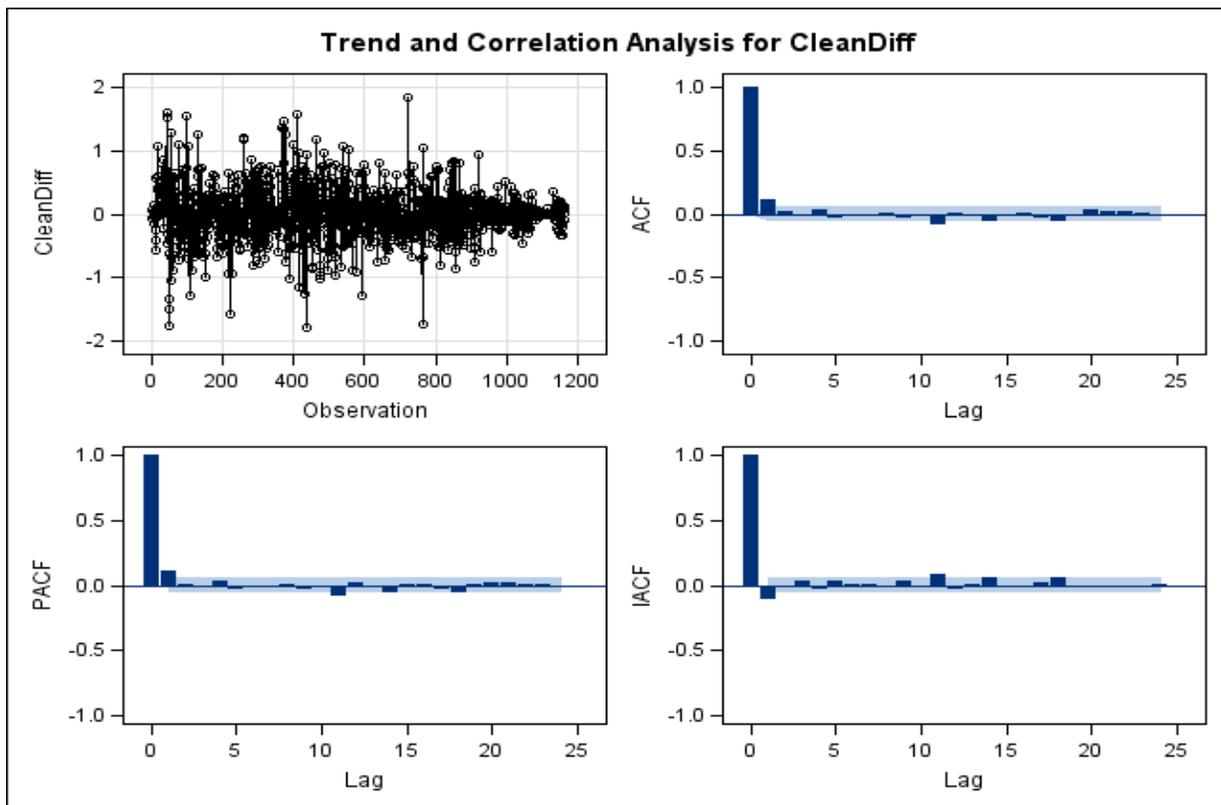
Input Variable outI40  
Overall Regression Factor 2.464455

Input Number 4

Input Variable outI8  
Overall Regression Factor -2.17135

Input Number 5

Input Variable outI575  
Overall Regression Factor -2.07302



The ARIMA Procedure

Name of Variable = CleanDiff

Mean of Working Series -0.00232  
 Standard Deviation 0.403345  
 Number of Observations 1163

Autocorrelation Check for White Noise

To Lag	Chi - Square	DF	Pr > Chi Sq	-----Autocorrelations-----					
6	18.37	6	0.0054	0.114	0.025	-0.018	0.030	-0.028	-0.007
12	29.41	12	0.0034	-0.014	0.005	-0.034	-0.012	-0.089	0.007
18	40.00	18	0.0021	-0.020	-0.058	-0.012	0.010	-0.029	-0.064
24	42.71	24	0.0107	-0.003	0.032	0.026	0.020	0.012	0.001

The ARIMA Procedure

Conditional Least Squares Estimation

Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag
MU	-0.0023017	0.01073	-0.21	0.8302	0
MA1, 1	-0.10862	0.02923	-3.72	0.0002	1
AR1, 1	-0.09327	0.02920	-3.19	0.0014	11
AR1, 2	-0.05561	0.02922	-1.90	0.0573	14
AR1, 3	-0.06110	0.02923	-2.09	0.0368	18

Constant Estimate -0.00278  
 Variance Estimate 0.158877  
 Std Error Estimate 0.398594  
 AIC 1165.96  
 SBC 1191.253  
 Number of Residuals 1163

\* AIC and SBC do not include log determinant.

Correlations of Parameter Estimates

Parameter	MU	MA1, 1	AR1, 1	AR1, 2	AR1, 3
MU	1.000	0.000	-0.000	-0.001	-0.002
MA1, 1	0.000	1.000	0.007	-0.016	-0.026
AR1, 1	-0.000	0.007	1.000	0.025	0.014
AR1, 2	-0.001	-0.016	0.025	1.000	-0.035
AR1, 3	-0.002	-0.026	0.014	-0.035	1.000

Autocorrelation Check of Residuals

To Lag	Chi - Square	DF	Pr > Chi Sq	-----Autocorrelations-----					
6	4.55	2	0.1029	0.003	0.027	-0.030	0.033	-0.034	-0.004
12	6.73	8	0.5661	-0.019	0.005	-0.032	0.005	-0.004	0.020
18	8.48	14	0.8626	-0.014	-0.004	-0.011	0.016	-0.029	-0.005
24	10.19	20	0.9647	0.007	0.028	0.018	0.014	0.010	0.001
30	19.10	26	0.8324	-0.029	-0.040	0.021	-0.028	-0.056	0.024
36	20.86	32	0.9347	-0.000	-0.001	0.029	-0.020	-0.014	-0.004
42	26.42	38	0.9215	0.061	-0.006	-0.012	0.005	0.008	0.025
48	32.00	44	0.9108	0.023	0.027	-0.028	-0.024	-0.029	0.034

The ARIMA Procedure

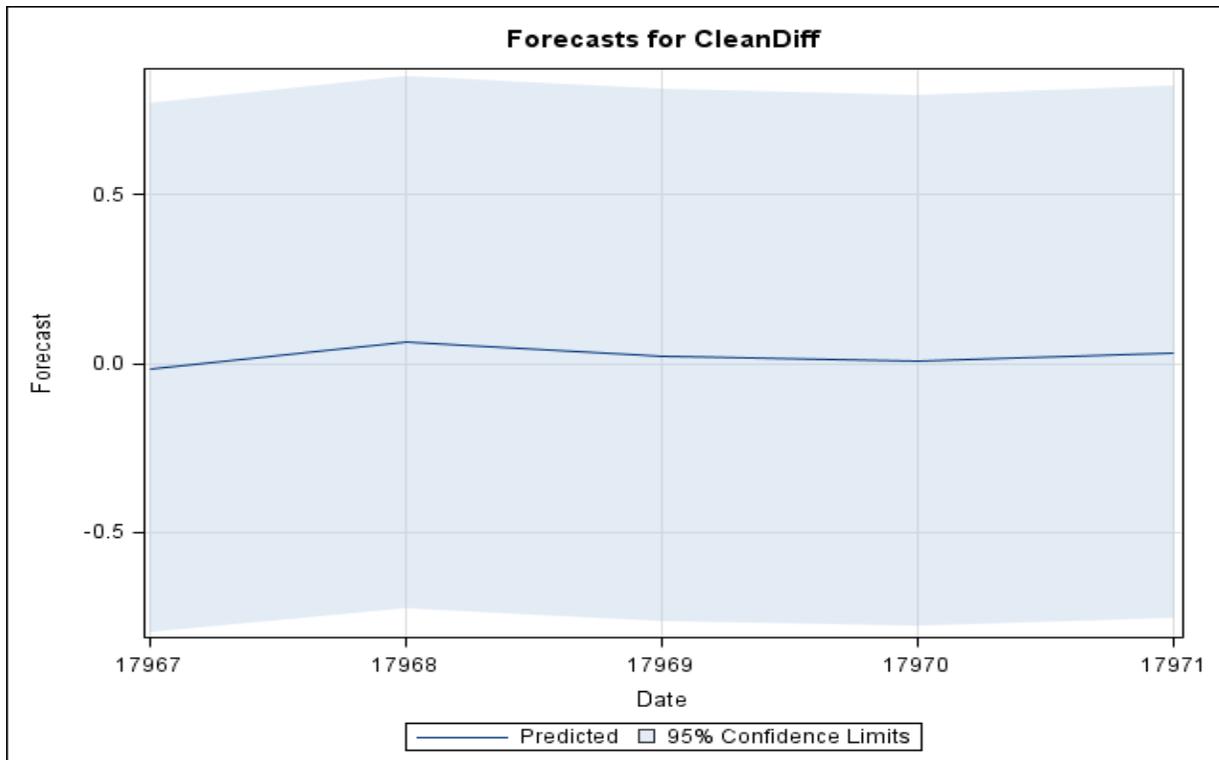
Autoregressive Factors

Factor 1:  $1 + 0.09327 B^{**}(11) + 0.05561 B^{**}(14) + 0.0611 B^{**}(18)$

Moving Average Factors

Factor 1:  $1 + 0.10862 B^{**}(1)$

## STEP 5



The ARIMA Procedure

Forecasts for variable CleanDiff

Obs	Forecast	Std Error	95% Confidence Limits	
1165	-0.0154	0.3986	-0.7966	0.7658
1166	0.0622	0.4009	-0.7236	0.8481
1167	0.0228	0.4009	-0.7630	0.8086
1168	0.0064	0.4009	-0.7794	0.7923
1169	0.0317	0.4009	-0.7541	0.8175

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