2

Water Pressure and Pressure Forces

2.1 The Free Surface of Water

When water fills a containing vessel, it automatically seeks a horizontal surface on which the pressure is constant everywhere. In practice, a free water surface is one that is not in contact with an overlying vessel cover. A free water surface may be subjected to atmospheric pressure (open vessel) or any other pressure that is exerted within the vessel (closed vessel).

2.2 Absolute and Gauge Pressures

A water surface in contact with the earth's atmosphere is subjected to atmospheric pressure, which is approximately equal to a 10.33-m-high column of water at sea level. In still water, any object located below the water surface is subjected to a pressure greater than atmospheric pressure. This additional pressure is often referred to as *hydrostatic pressure*. More precisely, it is the force per unit area acting in a normal direction on the surface of a body immersed in the fluid (in this case water).

To determine the variation of hydrostatic pressure between any two points in water (with a specific weight of γ), we may consider two arbitrary points A and B along an arbitrary x-axis, as shown in Figure 2.1. Consider that these points lie in the ends of a small prism of water having a cross-sectional area dA and a length L. P_A and P_B are the pressures at each

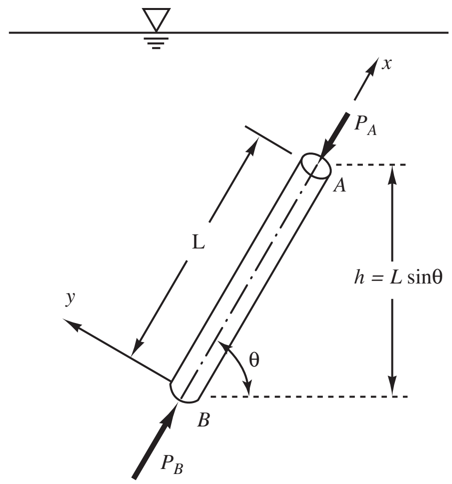


Figure 2.1 Hydrostatic pressure on a prism

end, where the cross-sectional areas are normal to the x-axis. Because the prism is at rest, all forces acting on it must be in equilibrium in all directions. For the force components in the x-direction, we may write

$$\Sigma F_x = P_A\, dA - P_B\, dA + \gamma L dA \sin\theta = 0$$

Note that $L \cdot \sin\theta = h$ is the vertical elevation difference between the two points. The above equation reduces to

$$P_B - P_A = \gamma h \tag{2.1}$$

Therefore, *the difference in pressure between any two points in still water is always equal to the product of the specific weight of water and the difference in elevation between the two points.*

If the two points are on the same elevation, $h = 0$ and $P_A = P_B$. In other words, *for water at rest, the pressure at all points in a horizontal plane is the same*. If the water body has a free surface that is exposed to *atmospheric pressure*, P_{atm}, we may position point A on the free surface and write

$$(P_B)_{abs} = \gamma h + P_A = \gamma h + P_{atm} \tag{2.2}$$

This pressure, $(P_B)_{abs}$, is commonly referred to as the *absolute pressure*.

Pressure gauges are usually designed to measure pressures above or below the atmospheric pressure. Pressure so measured, using atmospheric pressure as a base, is called *gauge pressure*, P. Absolute pressure is always equal to gauge pressure plus atmospheric pressure:

$$P = P_{abs} - P_{atm} \tag{2.3}$$

Figure 2.2 diagrammatically shows the relationship between the absolute and gauge pressure and two typical pressure-gauge dials. Comparing Equations 2.2 and 2.3, we have

$$P = \gamma h \tag{2.4}$$

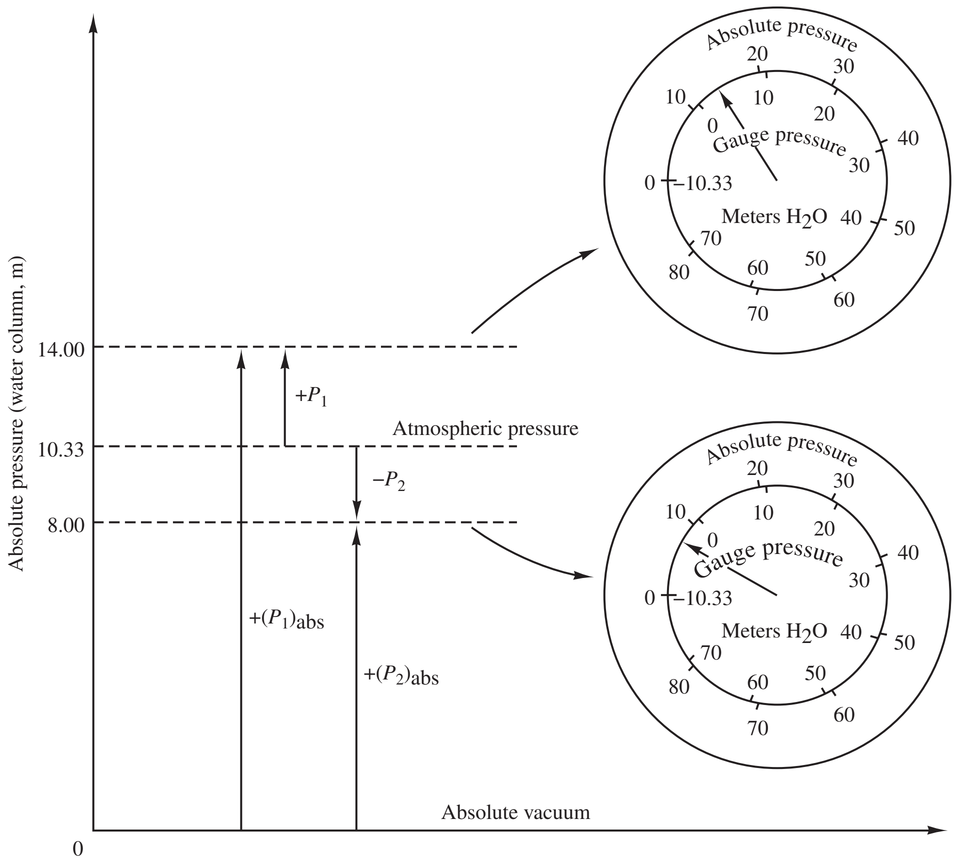


Figure 2.2 Absolute and gauge pressure

or

$$h = \frac{P}{\gamma} \tag{2.5}$$

Here the pressure is expressed in terms of the height of a water column h. In hydraulics it is known as the *pressure head*.

Equation 2.1 may thus be rewritten in a more general form as

$$\frac{P_B}{\gamma} - \frac{P_A}{\gamma} = \Delta h \tag{2.6}$$

meaning that the difference in pressure heads at two points in water at rest is always equal to the difference in elevation between the two points. From this relationship we can also see that any change in pressure at point B would cause an equal change at point A, because the difference in pressure head between the two points must remain the same value Δh. In other words, *a pressure applied at any point in a liquid at rest is transmitted equally and undiminished in all directions to every other point in the liquid*. This principle, also know as *Pascal's law*, has been made use of in the hydraulic jacks that lift heavy weights by applying relatively small forces.

Example 2.1

The diameters of cylindrical pistons *A* and *B* are 3 cm and 20 cm, respectively. The faces of the pistons are at the same elevation, and the intervening passages are filled with an incompressible hydraulic oil. A force *P* of 100 N is applied at the end of the lever, as shown in Figure 2.3. What weight *W* can the hydraulic jack support?

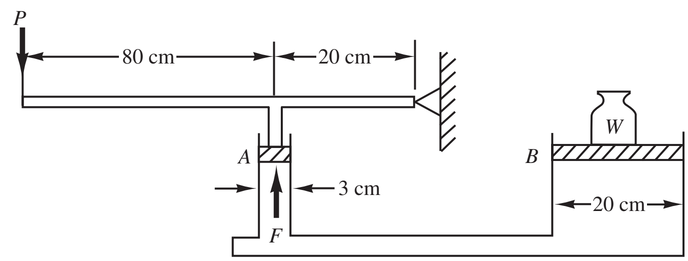


Figure 2.3 Hydraulic jack

Solution

Balancing the moments produced by *P* and *F* around the pin connection yields

$$(100\ \text{N})(100\ \text{cm}) = F(20\ \text{cm})$$

Thus,

$$F = 500\ \text{N}$$

From Pascal's law, the pressure P_A applied at *A* is the same as that of P_B applied at *B*. Therefore,

$$P_A = \frac{F}{[(\pi \cdot 3^2)/4]\ \text{cm}^2} \qquad P_B = \frac{W}{[(\pi \cdot 20^2)/4]\ \text{cm}^2}$$

$$\frac{500\ \text{N}}{7.07\ \text{cm}^2} = \frac{W}{314\ \text{cm}^2}$$

$$\therefore W = 500\ \text{N}\left(\frac{314\ \text{cm}^2}{7.07\ \text{cm}^2}\right) = 2.22 \times 10^4\ \text{N}$$

2.3 Surfaces of Equal Pressure

The hydrostatic pressure in a body of water varies with the vertical distance measured from the free water surface. In general, all points on a horizontal surface in a static body of water are subjected to the same hydrostatic pressure, according to Equation 2.4. For example, in Figure 2.4 (a), points 1, 2, 3, and 4 have equal pressure, and the horizontal surface that contains these four points is a *surface of equal pressure*. However, in Figure 2.4 (b), points 5 and 6 are on the same horizontal plane but the pressures are not equal. This is because the water in the two tanks is not connected and the overlying depths to the free surfaces are different. Applying Equation 2.4 would produce different pressures. Figure 2.4 (c) displays tanks filled with two immiscible liquids of different densities. (Note: Immiscible liquids do not readily mix under normal conditions.) The horizontal surface (7, 8) that passes through the interface of the two liquids is an

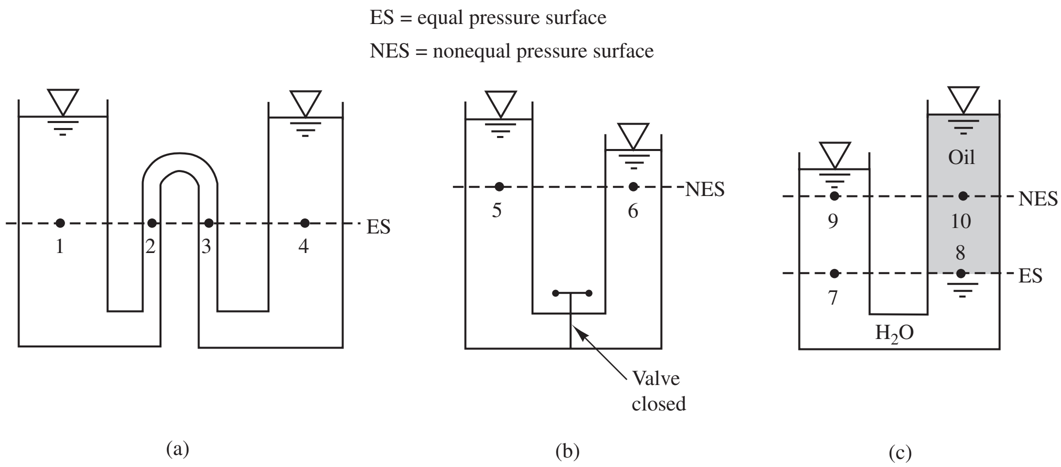


Figure 2.4 Hydraulic pressure in vessels

equal pressure surface. Applying Equation 2.4 at both points leads to the same pressure; we have the same fluid (water) at both locations (just below the interface at point 8), and both points are the same distance beneath the free water surface. However, points 9 and 10 are *not* on an equal pressure surface because they reside in different liquids. Verification would come from the application of Equation 2.4 using the different depths from the free surface to points 9 and 10 and the different specific weights of the fluids.

In summary, a surface of equal pressure requires that (1) the points on the surface be in the same liquid, (2) the points be at the same elevation (i.e., reside on a horizontal surface), and (3) the liquid containing the points be connected. The concept of equal pressure surface is a useful method in analyzing the strength or intensity of the hydrostatic pressure at various points in a container, as demonstrated in the following section.

2.4 Manometers

A *manometer* is a pressure-measurement device. It usually is a tube bent in the form of a "U" that contains a fluid of known specific gravity. The difference in elevations of the liquid surfaces under pressure indicates the difference in pressure at the two ends. Basically, there are two types of manometers:

1. An *open manometer* has one end open to atmospheric pressure and is capable of measuring the gauge pressure in a vessel.
2. A *differential manometer* has each end connected to a different pressure tap and is capable of measuring the pressure difference between the two taps.

The liquid used in a manometer is usually heavier than the fluids to be measured. It must form a distinct interface—that is, it must not mix with the adjacent liquids (i.e., immiscible liquids). The most frequently used manometer liquids are mercury (sp. gr. = 13.6), water (sp. gr. = 1.00), alcohol (sp. gr. = 0.9), and other commercial manometer oils of various specific gravities (e.g., from Meriam* Red Oil, sp. gr. = 0.827 to Meriam No. 3 Fluid, sp. gr. = 2.95).

* Meriam Process Technologies, Cleveland, Ohio 44102

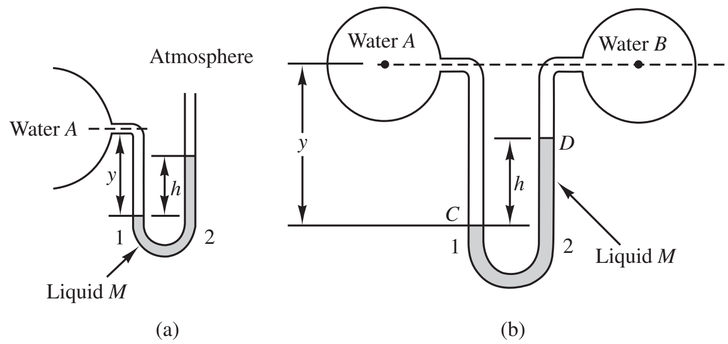


Figure 2.5 Types of manometers: (a) open manometer and (b) differential manometer

Figure 2.5 (a) shows a schematic of a typical open manometer; Figure 2.5 (b) shows a schematic of a typical differential manometer. It is obvious that the higher the pressure in vessel *A*, the larger the difference, *h*, in the surface elevations in the two legs of the manometer. A mathematical calculation of pressure in *A*, however, involves the densities of the fluids and the geometry involved in the entire measuring system.

A simple step-by-step procedure is suggested for pressure computation.

Step 1. Make a sketch of the manometer system, similar to that in Figure 2.5, and approximately to scale.

Step 2. Draw a horizontal line through the lower surface of the manometer liquid (point 1). The pressure at points 1 and 2 must be the same since the system is in static equilibrium.

Step 3. (a) For open manometers, the pressure on 2 is exerted by the weight of the liquid *M* column above 2; and the pressure on 1 is exerted by the weight of the column of water above 1 plus the pressure in vessel A. The pressures must be equal in value. This relation may be written as follows:

$$\gamma_M h = \gamma y + P_A \qquad \text{or} \qquad P_A = \gamma_M h - (\gamma y)$$

(b) For differential manometers, the pressure on 2 is exerted by the weight of the liquid *M* column above 2, the weight of the water column above *D*, and the pressure in vessel B, whereas the pressure on 1 is exerted by the weight of the water column above 1 plus the pressure in vessel A. This relationship may be expressed as:

$$\gamma_M h + \gamma\,(y - h) + P_B = \gamma y + P_A$$

or

$$\Delta P = P_A - P_B = h(\gamma_M - \gamma)$$

Either one of these equations can be used to solve for P_A. Of course, in the case of the differential manometer, P_B must be known. The same procedure can be applied to any complex geometry, as demonstrated in the following example.

Example 2.2

A mercury manometer (sp. gr. = 13.6) is used to measure the pressure difference in vessels *A* and *B*, as shown in Figure 2.6. Determine the pressure difference in pascals (N/m^2).

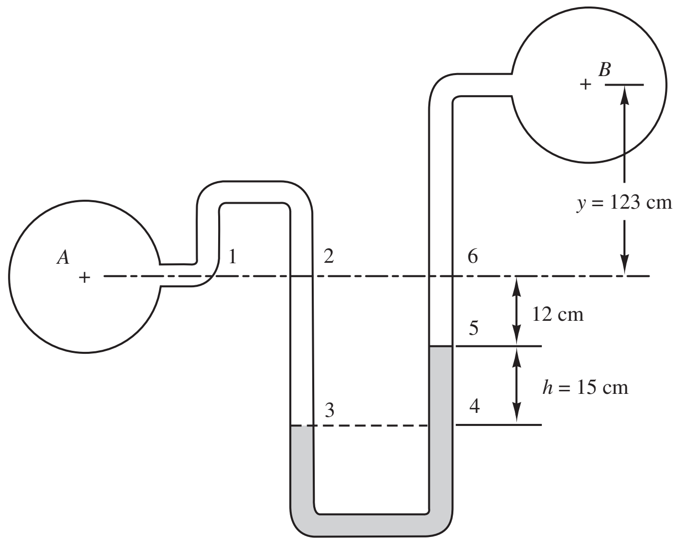


Figure 2.6

Solution

The sketch of the manometer system (step 1) is shown in Figure 2.6. Points 3 and 4 (P_3, P_4) are on a surface of equal pressure (step 2) and so are the vessel A and points 1 and 2 (P_1, P_2):

$$P_3 = P_4$$
$$P_A = P_1 = P_2$$

The pressures at points 3 and 4 are, respectively (step 3),

$$P_3 = P_2 + \gamma(27 \text{ cm}) = P_A + \gamma(27 \text{ cm})$$
$$P_4 = P_B + \gamma(135 \text{ cm}) + \gamma_M(15 \text{ cm})$$

Now

$$P_3 = P_A + \gamma(27 \text{ cm}) = P_4 = P_B + \gamma(135 \text{ cm}) + \gamma_M(15 \text{ cm})$$

and noting that $\gamma_M = \gamma$ (sp. gr.)

$$\Delta P = P_A - P_B = \gamma(135 \text{ cm} - 27 \text{ cm}) + \gamma_M(15 \text{ cm})$$
$$\Delta P = \gamma[108+(13.6)(15)]\text{cm} = (9790 \text{ N/m}^3)(3.12 \text{ m})$$
$$\Delta P = 30{,}500 \text{ N/m}^2 \text{ (pascals) or } 30.6 \text{ kilo-pascals}$$

The open manometer, or U-tube, requires readings of liquid levels at two points. In other words, any change in pressure in the vessel causes a drop of liquid surface at one end and a rise in the other. A *single-reading manometer* can be made by introducing a reservoir with a larger cross-sectional area than that of the tube into one leg of the manometer. A typical single-reading manometer is shown in Figure 2.7.

Because of the large area ratio between the reservoir and the tube, a small drop of surface elevation in the reservoir will cause an appreciable rise in the liquid column of the other leg. If there is an increase in pressure, ΔP_A will cause the liquid surface in the reservoir to drop by a small amount Δy. Then

$$A\Delta y = ah \tag{2.7}$$

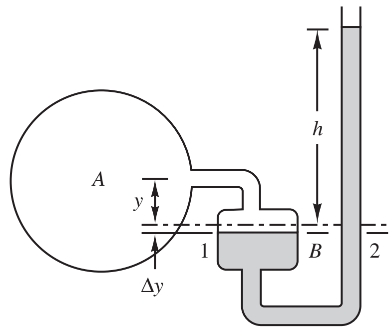


Figure 2.7 Single-reading manometer

where A and a are cross-sectional areas of the reservoir and the tube, respectively.

Applying step 2 to points 1 and 2, we may generally write

$$\gamma_A(y + \Delta y) + P_A = \gamma_B(h + \Delta y) \tag{2.8}$$

Simultaneous solution of Equations 2.7 and 2.8 give the value of P_A, the pressure in the vessel, in terms of h. All other quantities in Equations 2.7 and 2.8—A, a, y, γ_A, and γ_B—are quantities predetermined in the manometer design. A single reading of h will thus determine the pressure.

Because Δy can be made negligible by introducing a very large A/a ratio, the above relationship may be further simplified to

$$\gamma_A y + P_A = \gamma_B h \tag{2.9}$$

Thus, the height reading h is a measure of the pressure in the vessel.

The solution of practical hydraulic problems frequently requires the difference in pressure between two points in a pipe or a pipe system. For this purpose, differential manometers are frequently used. A typical differential manometer is shown in Figure 2.8.

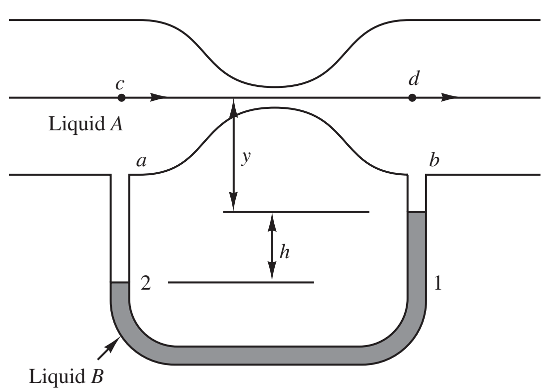


Figure 2.8 A differential manometer installed in a flow-measurement system

The same computation steps (steps 1, 2, and 3) suggested previously can be readily applied here, too. When the system is in static equilibrium, the pressure at the same elevation points, 1 and 2, must be equal. We may thus write

$$\gamma_A(y + h) + P_c = \gamma_B h + \gamma_A y + P_d$$

Hence, the pressure difference, ΔP, is expressed as

$$\Delta P = P_c - P_d = (\gamma_B - \gamma_A)h \tag{2.10}$$

2.5 Hydrostatic Forces on Flat Surfaces

Determining the total (or resultant) hydrostatic force on structures produced by hydrostatic pressure is often critical in engineering design and analysis. To determine the magnitude of this force, let's examine an arbitrary area *AB* (Figure 2.9) on the back face of a dam that inclines at an angle θ. Next, place the *x*-axis on the line where the surface of the water intersects with the dam surface (i.e., into the page) with the *y*-axis running downward along the surface or face of the dam. Figure 2.9 (a) shows a plan (front) view of the area and Figure 2.9 (b) shows the projection of *AB* on the dam surface.

We may assume that the plane surface *AB* is made up of an infinite number of horizontal strips, each having a width of *dy* and an area of *dA*. The hydrostatic pressure on each strip may be considered constant because the width of each strip is very small. For a strip at depth *h* below the free surface, the pressure is

$$P = \gamma h = \gamma y \sin\theta$$

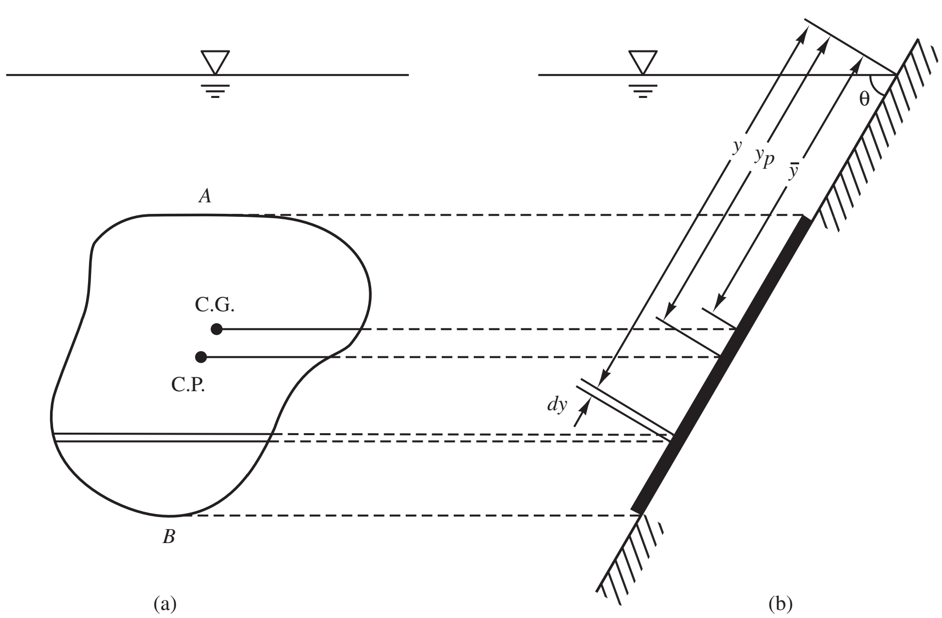


Figure 2.9 Hydrostatic pressure on a plane surface

The total pressure force on the strip is the pressure times the area

$$dF = \gamma y \sin\theta \, dA$$

The total pressure force (resultant force) over the entire *AB* plane surface is the sum of pressure on all the strips

$$F = \int_A dF = \int_A \gamma y \sin\theta \, dA = \gamma \sin\theta \int_A y dA$$
$$= \gamma \sin\theta \, A\bar{y} \tag{2.11}$$

where $\bar{y} = \int_A y dA / A$ is the distance measured from the *x*-axis to the centroid (or the center of gravity, C.G.) of the *AB* plane (Figure 2.9).

Substituting $\bar{h}$, the vertical distance of the centroid below the water surface, for $\bar{y} \sin\theta$, we have

$$F = \gamma \bar{h} A \tag{2.12}$$

This equation states that *the total hydrostatic pressure force on any submerged plane surface is equal to the product of the surface area and the pressure acting at the centroid of the plane surface.*

Pressure forces acting on a plane surface are distributed over every part of the surface. They are parallel and act in a direction normal to the surface. These parallel forces can be analytically replaced by a single *resultant force F* of the magnitude shown in Equation 2.12. The resultant force also acts normal to the surface. The point on the plane surface at which this resultant force acts is known as the *center of pressure* (C.P., Figure 2.9). Considering the plane surface as a free body, we see that the distributed forces can be replaced by the single resultant force at the pressure center without altering any reactions or moments in the system. Designating y_p as the distance measured from the *x*-axis to the center of pressure, we may thus write

$$F y_p = \int_A y dF$$

Hence,

$$y_p = \frac{\int_A y dF}{F} \tag{2.13}$$

Substituting the relationships $dF = \gamma y \sin\theta \, dA$ and $F = \gamma \sin\theta \, A\bar{y}$, we may write Equation 2.13 as

$$y_p = \frac{\int_A y^2 dA}{A\bar{y}} \tag{2.14}$$

in which $\int_A y^2 dA = I_x$ and $A\,\bar{y} = M_x$ are, respectively, the moment of inertia and the static moment of the plane surface *AB* with respect to the *x*-axis. Therefore,

$$y_p = \frac{I_x}{M_x} \tag{2.15}$$

With respect to the centroid of the plane, this may be written as

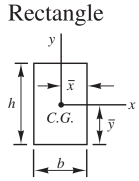
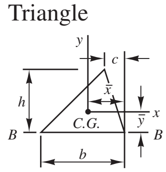
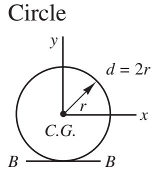
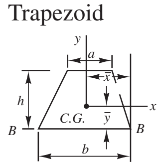
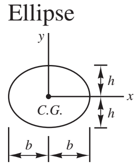
$$y_p = \frac{I_0 + A\bar{y}^2}{A\bar{y}} = \frac{I_0}{A\bar{y}} + \bar{y} \tag{2.16}$$

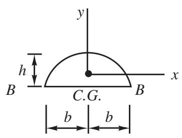
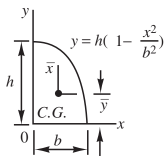
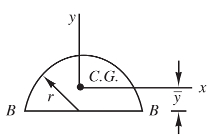
where I_0 is the moment of inertia of the plane with respect to its own centroid, A is the plane surface area, and $\bar{y}$ is the distance between the centroid and the x-axis.

The center of pressure of any submerged plane surface is always below the centroid of the surface area (i.e., $y_p > \bar{y}$). This must be true because all three variables in the first term on the right-hand side of Equation 2.16 are positive, making the term positive. That term is added to the centroidal distance ($\bar{y}$).

The centroid, area, and moment of inertia with respect to the centroid of certain common geometrical plane surfaces are given in Table 2.1.

TABLE 2.1 Surface Area, Centroid, and Moment of Inertia of Certain Simple Geometrical Plates

Shape	Area	Centroid	Moment of Inertia About the Neutral x-Axis
Rectangle 	bh	$\bar{x} = \frac{1}{2}b$ $\bar{y} = \frac{1}{2}h$	$I_0 = \frac{1}{12}bh^3$
Triangle 	$\frac{1}{2}bh$	$\bar{x} = \frac{b + c}{3}$ $\bar{y} = \frac{h}{3}$	$I_0 = \frac{1}{36}bh^3$
Circle 	$\frac{1}{4}\pi d^2$	$\bar{x} = \frac{1}{2}d$ $\bar{y} = \frac{1}{2}d$	$I_0 = \frac{1}{64}\pi d^4$
Trapezoid 	$\frac{h(a + b)}{2}$	$\bar{y} = \frac{h(2a + b)}{3(a + b)}$	$I_0 = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)}$
Ellipse 	πbh	$\bar{x} = b$ $\bar{y} = h$	$I_0 = \frac{\pi}{4}bh^3$

Shape	Area	Centroid	Moment of Inertia About the Neutral x-Axis
Semi-ellipse 	$\frac{\pi}{2}bh$	$\bar{x} = b$ $\bar{y} = \frac{4h}{3\pi}$	$I_0 = \frac{(9\pi^2 - 64)}{72\pi}bh^3$
Parabolic section 	$\frac{2}{3}bh$	$\bar{y} = \frac{2}{5}h$ $\bar{x} = \frac{3}{8}b$	$I_0 = \frac{8}{175}bh^3$
Semicircle 	$\frac{1}{2}\pi r^2$	$\bar{y} = \frac{4r}{3\pi}$	$I_0 = \frac{(9\pi^2 - 64)r^4}{72\pi}$

Example 2.3

A vertical trapezoidal gate with its upper edge located 5 m below the free surface of water is shown in Figure 2.10. Determine the total pressure force and the center of pressure on the gate.

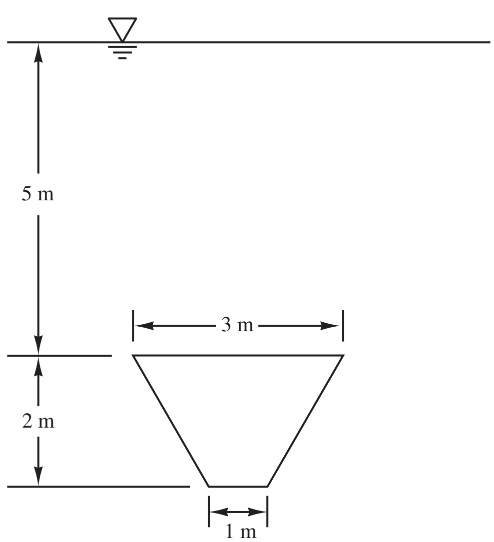


Figure 2.10

Solution

The total pressure force is determined using Equation 2.12 and Table 2.1.

$$F = \gamma \bar{h} A$$
$$= 9{,}790\left[5 + \frac{2[(2)(1) + 3]}{3(1 + 3)}\right]\left[\frac{2(3 + 1)}{2}\right]$$
$$= 2.28 \times 10^5 \text{N} = 228 \text{ kN}$$

The location of the center of pressure is

$$y_p = \frac{I_0}{A\bar{y}} + \bar{y}$$

where (from Table 2.1)

$$I_0 = \frac{2^3[1^2 + 4(1)(3) + 3^2]}{36(1 + 3)} = 1.22 \text{ m}^4$$
$$\bar{y} = 5.83 \text{ m}$$
$$A = 4.00 \text{ m}^2$$

Thus,

$$y_p = \frac{1.22}{4(5.83)} + 5.83 = 5.88 \text{ m}$$

below the water surface.

Example 2.4

An inverted semicircular gate (Figure 2.11) is installed at 45° with respect to the free water surface. The top of the gate is 5 ft below the water surface in the vertical direction. Determine the hydrostatic force and the center of pressure on the gate.

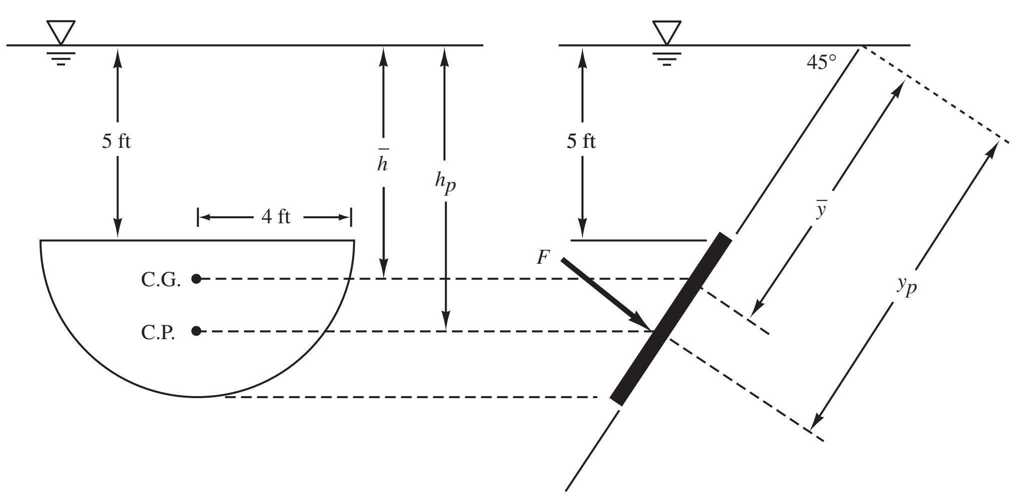


Figure 2.11

Solution

The total pressure force is

$$F = \gamma \bar{y} \sin \theta A$$

where

$$A = \frac{1}{2}[\pi (4)^2] = 25.1 \text{ ft}^2$$

and

$$\bar{y} = 5 \sec 45° + \frac{44}{3\pi} = 8.77 \text{ ft}$$

Therefore,

$$F = 62.3(\sin 45°)(8.77)(25.1) = 9{,}700 \text{ lbs}$$

This is the total hydrostatic force acting on the gate. The location of the center of pressure is

$$y_p = \frac{I_0}{A\bar{y}} + \bar{y}$$

where (from Table 2.1)

$$I_0 = \frac{(9\pi^2 - 64)}{72\pi} r^4 = 28.1 \text{ ft}^4$$

Therefore,

$$y_p = \frac{28.1}{25.1(8.77)} + 8.77 = 8.90 \text{ ft}$$

This is the inclined distance measured from the water surface to the center of pressure.

2.6 Hydrostatic Forces on Curved Surfaces

The hydrostatic force on a curved surface can be analyzed best by resolving the total pressure force on the surface into its horizontal and vertical components. (Remember that hydrostatic pressure acts normal to a submerged surface.) Figure 2.12 shows the curved wall of a container gate that has a unit width normal to the plane of the page.

Because the water body in the container is stationary, every part of the water body must be in equilibrium or each of the force components must satisfy the equilibrium conditions—that is, $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

In the free body diagram of the water contained in *ABA′*, equilibrium requires the horizontal pressure exerted on plane surface *A′B* (the vertical projection of *AB*) to be equal and opposite the horizontal pressure component F_H (the force that the gate wall exerts on the fluid). Likewise, the vertical component, F_V, must equal the total weight of the water body above gate *AB*. Hence, the horizontal and vertical pressure force on the gate may be expressed as

$$\Sigma F_x = F_{A'B} - F_H = 0$$
$$\therefore F_H = F_{A'B}$$
$$\Sigma F_y = F_V - (W_{AA'} + W_{ABA'}) = 0$$
$$\therefore F_V = W_{AA'} + W_{ABA'}$$

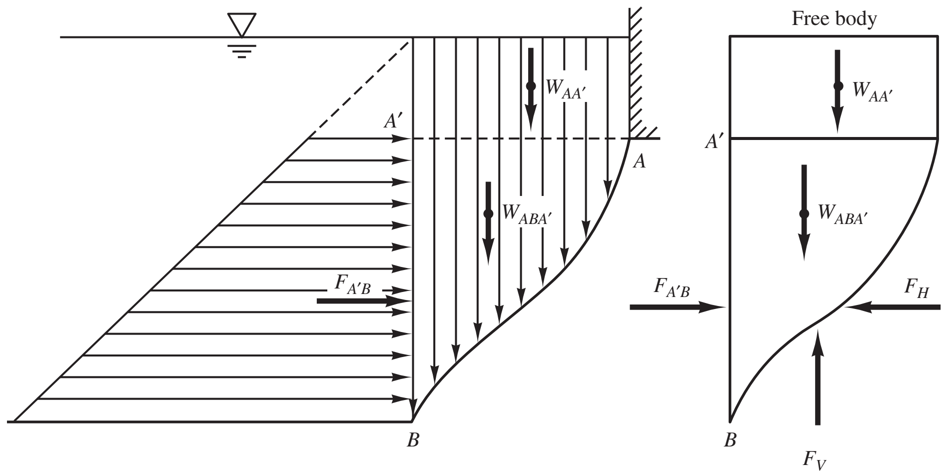


Figure 2.12 Hydrostatic pressure on a curved surface

Therefore, we may make the following statements.

1. *The horizontal component of the total hydrostatic pressure force on any surface is always equal to the total pressure on the vertical projection of the surface. The resultant force of the horizontal component can be located through the center of pressure of this projection.*
2. *The vertical component of the total hydrostatic pressure force on any surface is always equal to the weight of the entire water column above the surface extending vertically to the free surface. The resultant force of the vertical component can be located through the centroid of this column.*

Example 2.5

Determine the total hydrostatic pressure and the center of pressure on the 5-m-long, 2-m-high quadrant gate in Figure 2.13.

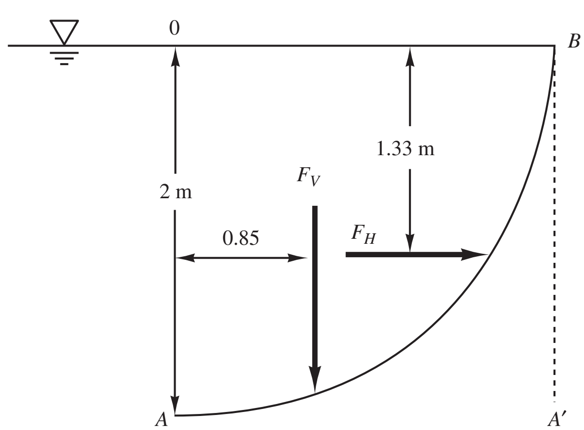


Figure 2.13

Solution

The horizontal component is equal to the hydrostatic pressure force on the projection plane *A'B*.

$$F_H = \gamma \bar{h} A = (9{,}790 \text{ N/m}^3)\left(\frac{1}{2}(2 \text{ m})\right)[(2 \text{ m})(5\text{m})] = 97{,}900 \text{ N}$$

The location of the horizontal component is $y_p = I_0/A\bar{y} + \bar{y}$, where $A = 10 \text{ m}^2$ (projected area) and $I_0 = [(5 \text{ m})(2 \text{ m})^3]/12 = 3.33 \text{ m}^4$, $y_p = (3.33 \text{ m}^4)/[(10 \text{ m}^2)(1 \text{ m})] + 1 = 1.33 \text{ m}$ below the free surface. The vertical component is equal to the weight of the water in the volume *AOB*. The direction of this pressure component is downward.

$$F_V = \gamma(Vol) = (9{,}790 \text{ N/m}^3)\left(\frac{1}{4}\pi(2 \text{ m})^2\right)(5 \text{ m}) = 154{,}000 \text{ N}$$

The pressure center is located at $4(2)/3\pi = 0.85 \text{ m}$ (Table 2.1), and the resultant force is

$$F = \sqrt{(97{,}900)^2 + (154{,}000)^2} = 182{,}000 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right) = \tan^{-1}\frac{154{,}000}{154{,}000} = 57.6°$$

Example 2.6

Determine the total hydrostatic pressure and the center of pressure on the semicylindrical gate shown in Figure 2.14.

Solution

The horizontal component of the hydrostatic pressure force on the projection plane *A'B'* per unit width can be expressed as

$$F_H = \gamma \bar{h} A = \gamma\left(\frac{H}{2}\right)(H) = \frac{1}{2}\gamma H^2$$

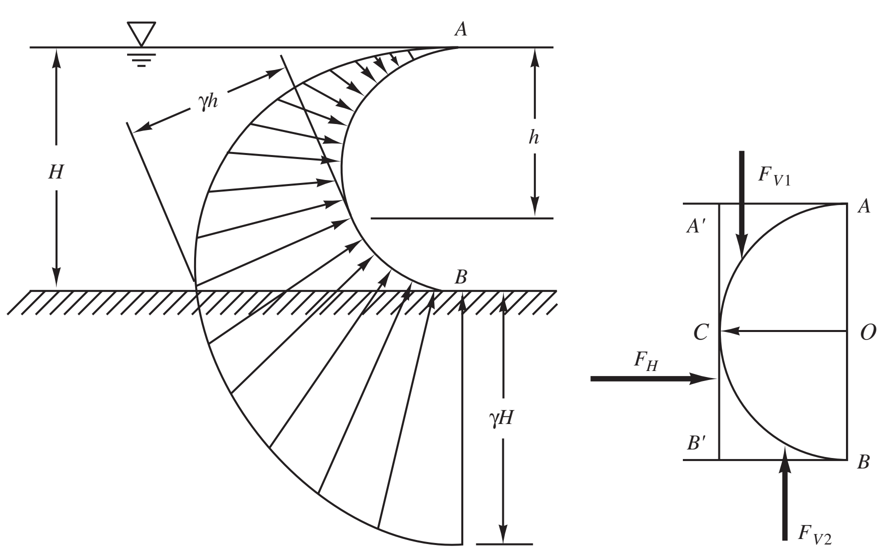


Figure 2.14

The pressure center of this component is located at a distance of $H/3$ from the bottom.

The vertical component can be determined as follows. The volume $AA'C$ over the upper half of the gate, AC, produces a downward vertical pressure force component:

$$F_{V_1} = -\gamma\left(\frac{H^2}{4} - \frac{\pi H^2}{16}\right)$$

The vertical pressure force component exerted by the water on the lower half of the gate, CB, is upward and equivalent to the weight of water replaced by the volume $AA'CB$:

$$F_{V_2} = \gamma\left(\frac{H^2}{4} + \frac{\pi H^2}{16}\right)$$

By combining these two components, one can see that the direction of the resultant vertical force is upward and equal to the weight of the water replaced by the volume ACB.

$$F_V = F_{V_1} + F_{V_2} = \gamma\left[-\left(\frac{H^2}{4} - \frac{\pi H^2}{16}\right) + \left(\frac{H^2}{4} + \frac{\pi H^2}{16}\right)\right] = \gamma\frac{\pi}{8}H^2$$

The resultant force is then

$$F = \gamma H^2\sqrt{\frac{1}{4} + \frac{\pi^2}{64}}$$

$$\theta = \tan^{-1}\frac{F_V}{F_H} = \tan^{-1}\left(\frac{\pi}{4}\right) = 38.1°$$

Because all pressure forces are concurrent at the center of the gate, point O, the resultant force must also act through point O.

2.7 Buoyancy

Archimedes discovered (~250 B.C.) that *the weight of a submerged body is reduced by an amount equal to the weight of the liquid displaced by the body*. *Archimedes' principle*, as we now call it, can be easily proven by using Equation 2.12.

Assume that a solid body of arbitrary shape, AB, is submerged in water as shown in Figure 2.15. A vertical plane MN may then be drawn through the body in the direction normal to the page. One observes that the horizontal pressure force components in the direction of the paper, F_H and F'_H, must be equal because they both are calculated using the same vertical projection area MN. The horizontal pressure force components in the direction normal to the page must also be equal for the same reason; they share the same projection in the plane of the page.

The vertical pressure-force component can be analyzed by taking a small vertical prism ab with a cross-sectional area dA. The vertical pressure force on top of the prism $(\gamma h_1 dA)$ acts downward. The vertical force on the bottom of the prism $(\gamma h_2 dA)$ acts upward. The difference gives the resultant vertical force component on the prism (*buoyancy force*)

$$F_V = \gamma h_2 dA - \gamma h_1 dA = \gamma(h_2 - h_1)dA \uparrow$$

which is exactly equal to the weight of the water column ab replaced by the prism. In other words, the weight of the submerged prism is reduced by an amount equal to the weight of the liquid replaced by the prism. A summation of the vertical forces on all the prisms that make up the entire submerged body AB gives the proof of Archimedes' principle.

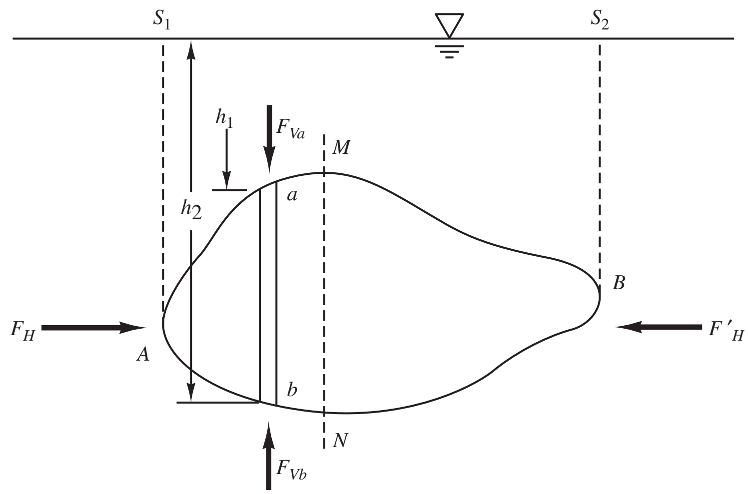


Figure 2.15 Buoyancy of a submerged body

Archimedes' principle may also be viewed as the difference of vertical pressure forces on the two surfaces *ANB* and *AMB*. The vertical pressure force on surface *ANB* is equal to the weight of the hypothetical water column (volume of S_1ANBS_2) acting upward; and the vertical pressure force on surface *AMB* is equal to the weight of the water column S_1AMBS_2 acting downward. Because the volume S_1ANBS_2 is larger than the volume S_1AMBS_2 by an amount exactly equal to the volume of the submerged body *AMBN*, the net difference is a force equal to the weight of the water that would be contained in the volume *AMBN* acting upward. This is the buoyancy force acting on the body.

A floating body is a body partially submerged resulting from a balance of the body weight and buoyancy force.

2.8 Flotation Stability

The stability of a floating body is determined by the relative positions of the *center of gravity* of body *G* and the *center of buoyancy B*, which is the center of gravity of the liquid volume replaced by the body, as show in Figure 2.16.

The body is in equilibrium if its center of gravity and its center of buoyancy lie on the same vertical line, as in Figure 2.16 (a). This equilibrium may be disturbed by a variety of causes (e.g., wind or wave action), and the floating body is made to heel or list through an angle θ as shown in Figure 2.16 (b). When the floating body is in the heeled position, the center of gravity of the body remains unchanged, but the center of buoyancy, which is now the center of gravity of area *a'cb'*, has been changed from *B* to *B'*. The buoyant force $\gamma \cdot Vol$, acting upward through *B'*, and the weight of the body *W*, acting downward through *G*, constitute a couple, $W \cdot X$, which resists further overturning and tends to restore the body to its original equilibrium position.

By extending the line of action of the buoyant force through the center of buoyancy *B'*, we see that the vertical line intersects the original axis of symmetry *c-t* at a point *M*. The point *M* is known as the *metacenter* of the floating body, and the distance between the center of gravity and the metacenter is known as the *metacentric height*. The metacentric height is a measure of the flotation stability of the body. When the angle of inclination is small, the position of *M* does not

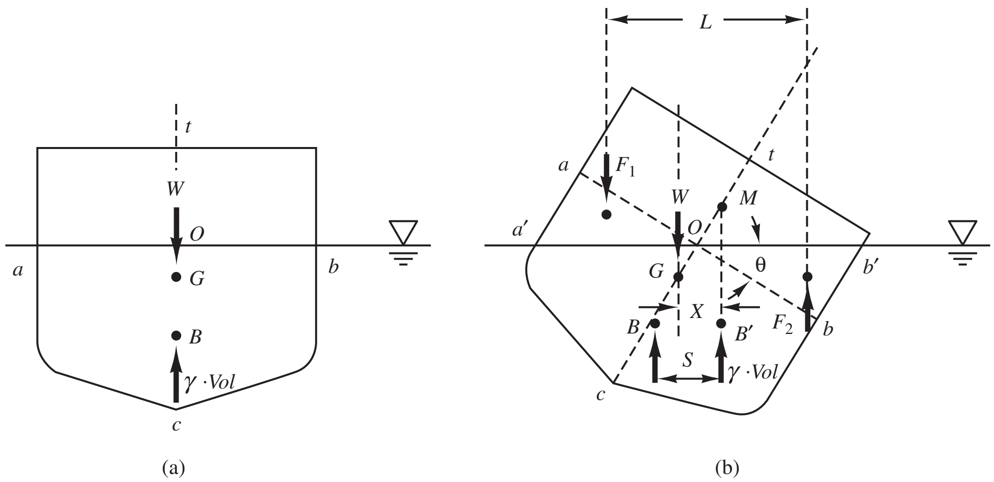


Figure 2.16 Center of buoyancy and metacenter of a floating body

change materially with the tilting position. The metacentric height and the righting moment can be determined in the following way.

Because tilting a floating body does not change the total body weight, the total displacement volume is not changed. The roll through an angle θ only changes the shape of the displaced volume by adding the immersion wedge *bob′* and subtracting the emersion wedge *aoa′*. In this new position, the total buoyancy force $(\gamma \cdot Vol)$ is shifted through a horizontal distance S to B'. This shift creates a couple F_1 and F_2 because of the new immersion and emersion wedges. The moment of the resultant force $(\gamma \cdot Vol)_{B'}$ about point B must equal the sum of the moments of the component forces:

$$\begin{aligned}(\gamma Vol)_{B'}(S) &= (\gamma Vol)_B(\text{zero}) + \text{moment of the force couple}\\ &= 0 + \gamma Vol_{\text{wedge}} L\end{aligned}$$

or

$$(\gamma Vol)_{B'} S = \gamma Vol_{\text{wedge}}\, L$$

$$S = \frac{Vol_{\text{wedge}}}{Vol} L \qquad \text{(a)}$$

where Vol is the total volume submerged, Vol_{wedge} is the volume of wedge *bob′* (or *aoa′*), and L is the horizontal distance between the centers of gravity of the two wedges.

But, according to the geometric relation, we have

$$S = \overline{MB}\sin\theta \qquad \textit{or} \qquad \overline{MB} = \frac{S}{\sin\theta} \qquad \text{(b)}$$

Combining Equations (a) and (b), we get

$$\overline{MB} = \frac{Vol_{\text{wedge}} L}{Vol \sin\theta}$$

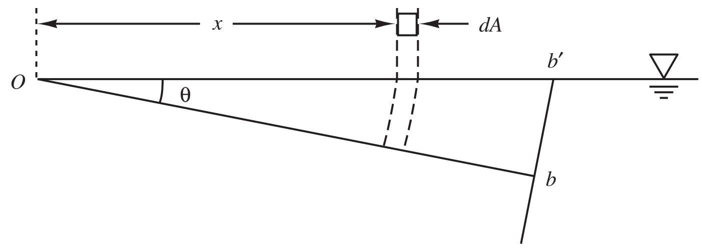


Figure 2.17

For a small angle, $\sin\theta \approx \theta$, the previous relationship may be simplified to

$$\overline{MB} = \frac{Vol_{\text{wedge}}L}{Vol\,\theta}$$

The buoyancy force produced by wedge *bob'*, as depicted in Figure 2.17, can be estimated by considering a small prism of the wedge. Assume that the prism has a horizontal area, *dA*, and is located at a distance *x* from axis of rotation *O*. The height of the prism is $x(\tan\theta)$. For a small angle θ, it may be approximated by $x\theta$. Thus, the buoyancy force produced by this small prism is $\gamma x\theta dA$. The moment of this force about the axis of rotation *O* is $\gamma x^2\theta dA$. The sum of the moments produced by each of the prisms in the wedge gives the moment of the immersed wedge. The moment produced by the force couple is, therefore,

$$\gamma Vol_{\text{wedge}}L = FL = \int_A \gamma x^2\,\theta dA = \gamma\theta\int_A x^2\,dA$$

But $\int_A x^2\,dA$ is the moment of inertia of the waterline cross-sectional area of the floating body about the axis of rotation *O*.

$$I_0 = \int_A x^2\,dA$$

Hence, we have

$$Vol_{\text{wedge}}L = I_0\theta$$

For small angles of tilt, the moment of inertia for upright cross section *aob*, and the inclined cross section *a'ob'* may be approximated by a constant value. Therefore,

$$\overline{MB} = \frac{I_0}{Vol} \qquad (2.17)$$

The metacentric height, defined as the distance between the metacenter *M* and the center of gravity *G,* can be estimated:

$$\overline{GM} = \overline{MB} \pm \overline{GB} = \frac{I_0}{Vol} \pm \overline{GB} \qquad (2.18)$$

The distance between the center of gravity and the center of buoyancy $\overline{GB}$ in the upright position, shown in Figure 2.16, can be determined by the sectional geometry or the design data of the vessel.

The ± sign indicates the relative position of the center of gravity with respect to the center of buoyancy. For greater flotation stability, it is advantageous to make the center of gravity as low as possible. If G is lower than B, then $\overline{GB}$ would be added to the distance to $\overline{MB}$ and produce a larger value of $\overline{GM}$.

The *righting moment*, when tilted as depicted in Figure 2.16 (b), is

$$M = W\overline{GM}\sin\theta \tag{2.19}$$

The stability of buoyant bodies under various conditions may be summarized as follows.

1. A floating body is stable if the center of gravity is below the metacenter. Otherwise, it is unstable.
2. A submerged body is stable if the center of gravity is below the center of buoyancy.

Example 2.7.
A 3 m × 4 m rectangular box caisson is 2 m deep (Figure 2.18). It has a draft of 1.2 m when it floats in an upright position. Compute (a) the metacentric height and (b) the righting moment in seawater (sp. gr. = 1.03) when the angle of heel (list) is 8°.

Solution
From Equation 2.18

$$\overline{GM} = \overline{MB} - \overline{GB}$$

where

$$\overline{MB} = \frac{I_0}{Vol}$$

and I_0 is the waterline area moment of inertia of the box about its longitudinal axis through O. Therefore,

$$\overline{GM} = \frac{\frac{1}{12}Lw^3}{Lw(1.2)} - \left(\frac{h}{2} - \frac{1.2}{2}\right)$$
$$= 0.225 \text{ m}$$

(*Note:* $L = 4$ m, $w = 3$ m, $h = 2$ m.)

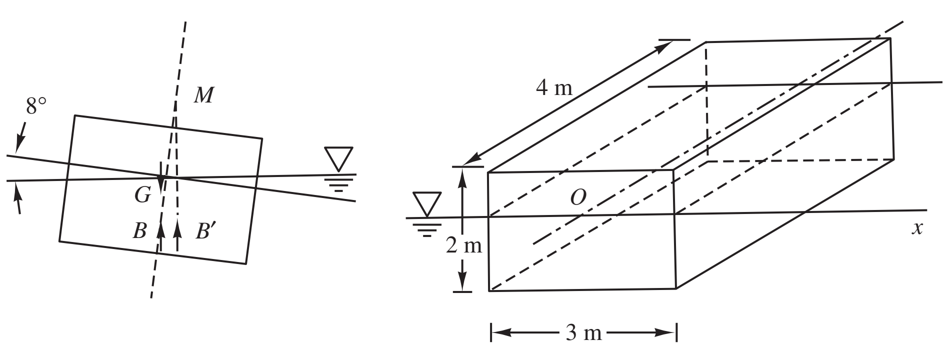


Figure 2.18

The specific gravity of seawater is 1.03; from Equation 2.19, the righting moment is

$$\begin{aligned} M &= W\overline{GM}\sin\theta \\ &= [(9{,}790\ \text{N/m}^3)(1.03)\{(4\ \text{m})(3\ \text{m})(1.2\ \text{m})\}](0.225\ \text{m})(\sin 8^\circ) \\ &= 4{,}550\ \text{N}\cdot\text{m} \end{aligned}$$

PROBLEMS
(SECTION 2.2)

2.2.1. Collapse depth (or crush depth) is the submerged depth that a submarine cannot exceed without collapsing because of the surrounding water pressure. The collapse depth of modern submarines is not quite a kilometer (730 m). Assuming seawater to be incompressible (sp. gr. = 1.03), what is the crush depth pressure in N/m^2 and psi (lb/in.^2)? Is the pressure you computed absolute or gauge pressure?

2.2.2. A cylindrical water tank (Figure P2.2.2) is suspended vertically by its sides. The tank has a 10-ft diameter and is filled with 20°C water to a 3-ft depth. Determine the force exerted on the tank bottom using two separate calculations (a) based on the weight of the water, and (b) based on the hydrostatic pressure on the bottom of the tank.

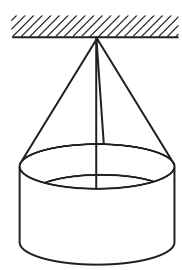


Figure P2.2.2

2.2.3. The simple barometer in Figure P2.2.3 uses water at 30°C as the liquid indicator. The liquid column rises to a height of 9.8 m from an original height of 8.7 m in the vertical tube. Compute the new atmospheric pressure, neglecting surface tension effects. What is the percentage error if the direct reading is used and vapor pressure is ignored?

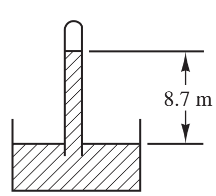


Figure P2.2.3

2.2.4. Mercury is often used in barometers as depicted in Figure P2.2.3. This is because the vapor pressure of mercury is low enough to be ignored, and because it is so dense (sp. gr. = 13.6) the tube can be shortened considerably. With the atmospheric pressure found in Problem 2.2.3 $(99.9\ \text{kN/m}^2$ at 30°C), determine the column height in meters (and feet) if mercury is used.

2.2.5. A storage tank $(6\ \text{m} \times 6\ \text{m} \times 6\ \text{m})$ is filled with water. Determine the force on the bottom and on each side.

2.2.6. A 30-ft-high, 1-ft-diameter pipe is welded to the top of a cubic container $(3 \text{ ft} \times 3 \text{ ft} \times 3 \text{ ft})$. The container and pipe are filled with water at 20°C. Determine the weight of the water and the pressure forces on the bottom and sides of the container.

2.2.7. A closed tank contains a liquid (sp. gr. = 0.80) under pressure. The pressure gauge depicted in Figure P2.2.7 registers a pressure of 4.50×10^4 N/m^2 (pascals). Determine the pressure at the bottom of the tank and the height of the liquid column that will rise in the vertical tube.

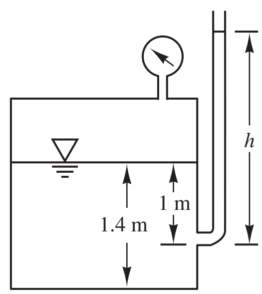


Figure P2.2.7

2.2.8. An underwater storage tank was constructed to store natural gas offshore. Determine the gas pressure in the tank (in pascals and psi; lb/in^2) when the water elevation in the tank is 6 m below sea level (Figure P2.2.8). The specific gravity of seawater is 1.03.

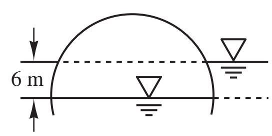


Figure P2.2.8

2.2.9. A closed tank contains oil with a specific gravity 0.85. If the gauge pressure at a point 10 feet below the oil surface is 23.7 psi (lb/in.^{2}), determine the absolute pressure and gauge pressure (in psi) in the air space at the top of the oil surface.

2.2.10. A multiple-piston hydraulic jack has two output pistons, each with an area of 250 cm^2. The input piston, whose area is 25 cm^2, is connected to a lever that has a mechanical advantage of 9:1. If a 50-N force is exerted on the lever, how much pressure (kN/m^2) is developed in the system? How much force (kN) will be exerted by each output piston?

(SECTION 2.4)

2.4.1. Referring to Figure 2.4 (c), if the height of water (at 4°C) above point 7 is 52.3 cm, what is the height of the oil (sp. gr. = 0.85) above point 8? (Note: Point 9 is 42.5 cm above point 7.)

2.4.2. A significant amount of mercury is poured into a U-tube with both ends open to the atmosphere. If water is poured into one leg of the U-tube until the water column is 3 feet above the mercury–water meniscus, what is the elevation difference between the mercury surfaces in the two legs?

2.4.3. An open tank in a petroleum company lab contains a layer of oil on top of a layer of water. The water height is 4 times the oil height h. The oil has a specific gravity of 0.82. If the gauge pressure at the bottom of the tank indicates 26.3 cm of mercury, what is the oil height h?

2.4.4. A mercury (sp. gr. = 13.6) manometer is used to measure water pressure in a pipe. Referring to Figure 2.5 (a), the value of y is 3.40 cm and the value of h is 2.60 cm. Determine the pressure in the pipe.

2.4.5. A manometer is mounted on a city water-supply pipe to monitor water pressure, as shown in Figure P2.4.5. However, the manometer reading of 3 ft (Hg) may be incorrect. If the pressure in the pipe is measured independently and found to be 16.8 lb/in.^{2} (psi), determine the correct value of the reading h.

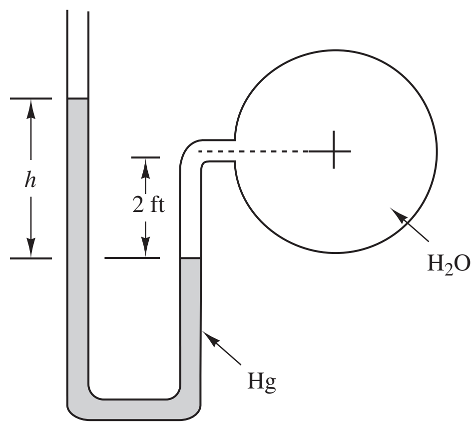


Figure P2.4.5

2.4.6. An open manometer, shown in Figure P2.4.6, is installed to measure pressure in a pipe carrying an oil (sp. gr. = 0.82). If the monometer liquid is carbon tetrachloride (sp. gr. = 1.60), determine the pipe pressure (in meters of water-column height).

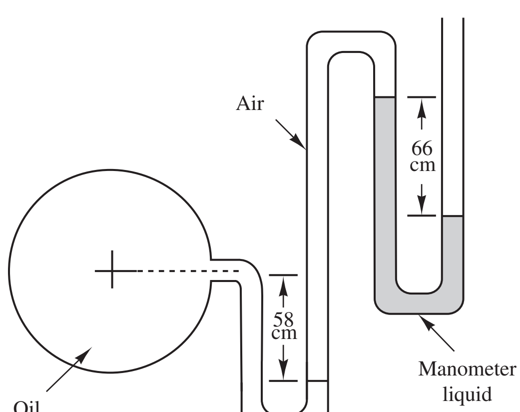


Figure P2.4.6

2.4.7. In Figure P2.4.7, a single-reading mercury manometer is used to measure water pressure in the pipe. What is the pressure (in psi) if $h_1 = 6.9$ in. and $h_2 = 24.0$ in.?

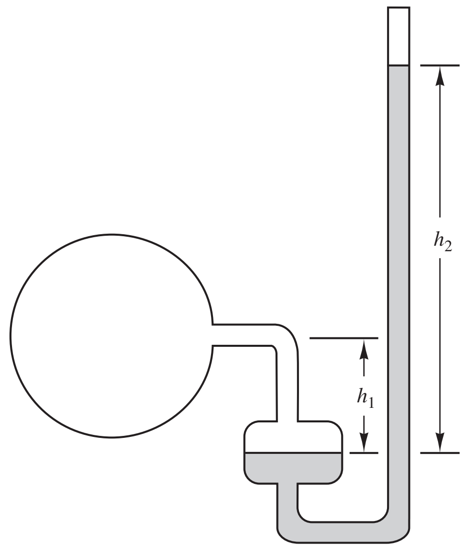


Figure P2.4.7

2.4.8. In Figure P2.4.7, determine the water pressure [in kilo-pascals (kPa)] in the pipe if $h_1 = 20.0$ cm and $h_2 = 67.0$ cm. Also determine the change in liquid height h_1 for a 10 cm change in h_2 if the diameter of the manometer tube is 0.5 cm and the diameter of the manometer fluid reservoir is 5 cm.

2.4.9. In Figure P2.4.9, water is flowing in pipe *A* and oil (sp. gr. = 0.82) is flowing in pipe *B*. If mercury is used as the manometer liquid, determine the pressure difference between *A* and *B* in psi.

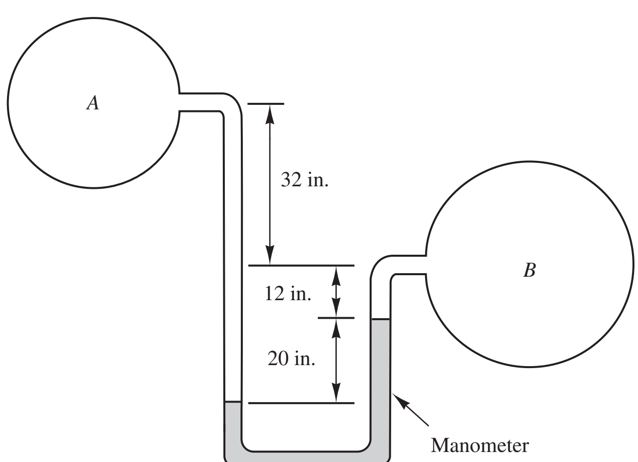


Figure P2.4.9

2.4.10. A micromanometer consists of two reservoirs and a U-tube as shown in Figure P2.4.10. Given that the densities of the two liquids are ρ_1 and ρ_2, determine an expression for the pressure difference $(P_1 - P_2)$ in terms of ρ_1, ρ_2, h, d_1, and d_2.

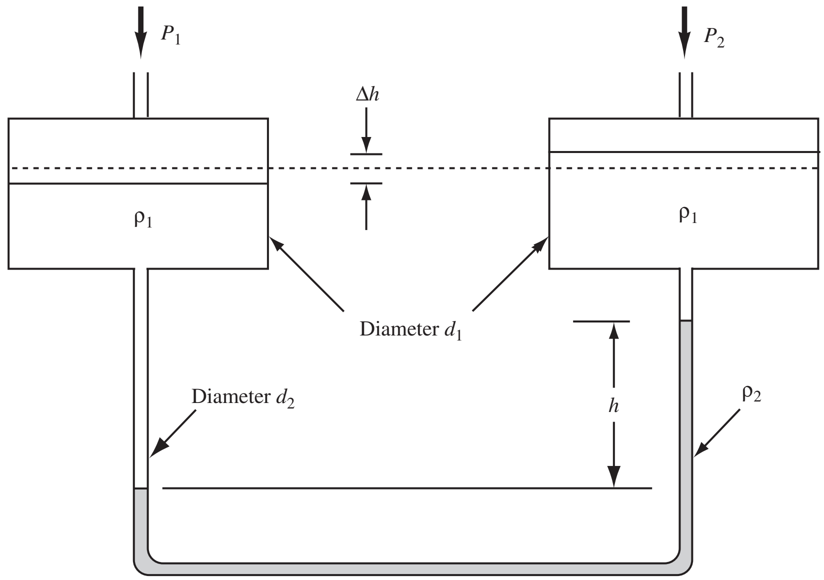


Figure P2.4.10

2.4.11. For the system of manometers shown in Figure P2.4.11, determine the differential reading h. Two different manometry fluids are being used with different specific gravities.

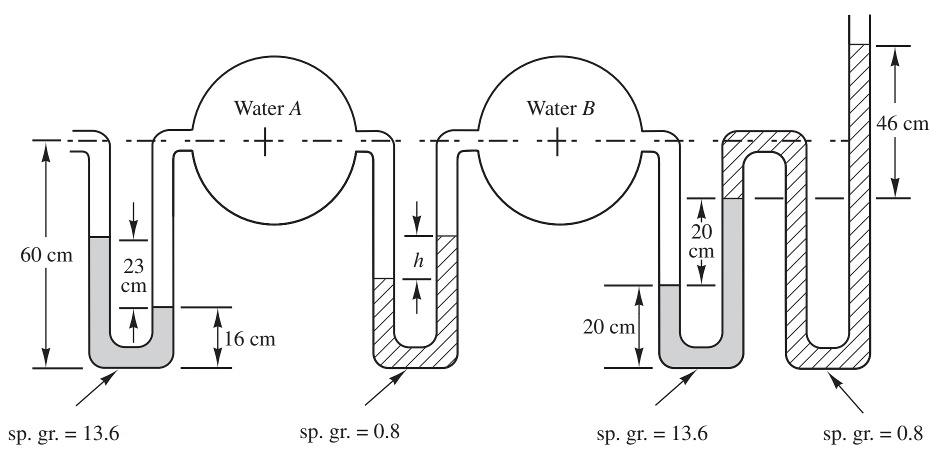


Figure P2.4.11

2.4.12. Determine the air pressure (kPa and cm of Hg) in the sealed left tank of Figure P2.4.12 if $E_A = 32.5$ m.

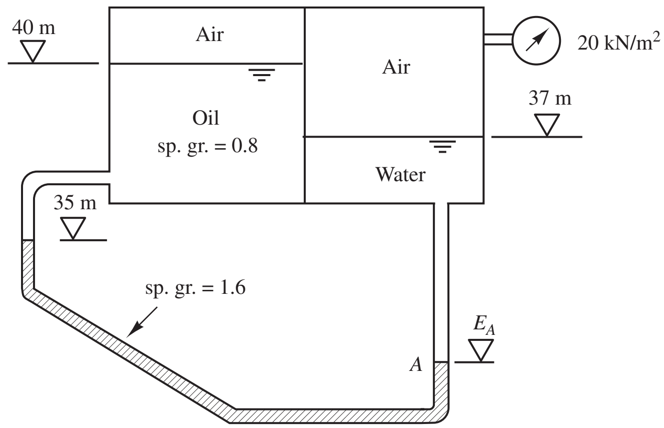


Figure P2.4.12

(SECTION 2.5)

2.5.1. A vertical gate keeps water from flowing in a triangular irrigation channel. The channel has a 4-m top width and a 3-m depth. If the channel is full, what is the magnitude of the hydrostatic force on the triangular gate and its location?

2.5.2. A concrete dam with a triangular cross section (Figure P2.5.2) is built to hold 30 ft of water. Determine the hydrostatic force on a unit length of the dam and its location. Also, if the specific gravity of concrete is 2.67, what is the moment generated with respect to the toe of the dam, *A*? Is the dam safe?

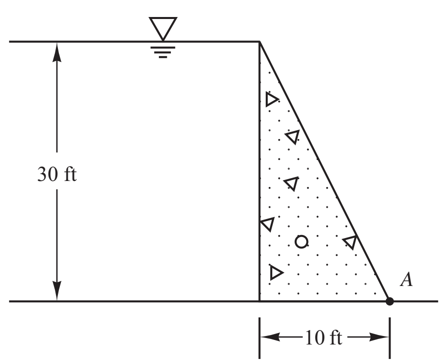


Figure P2.5.2

2.5.3. A 1-m-diameter circular (plane) gate is mounted into an inclined wall (45°). The center of the gate is located 1 m (vertically) below the water surface. Determine the magnitude of the hydrostatic force and its location with respect to the surface of the water along the incline.

2.5.4. A vertical plate, composed of a square and a triangle, is submerged so that its upper edge coincides with the water surface (Figure P2.5.4). What is the height to length ratio such that the pressure force on the square is equal to the pressure force on the triangle?

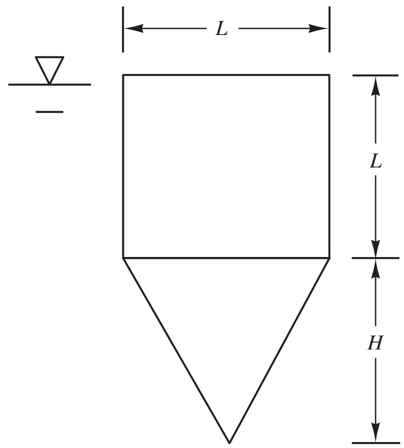


Figure P2.5.4 Front view of vertically submerged plate.

2.5.5. The rectangular gate in Figure P2.5.5 is hinged at *A* and separates water in the reservoir from the tail water tunnel. If the uniformly thick gate has a dimension of 2 m × 3 m and weighs 20 kN, what is the maximum height *h* for which the gate will stay closed? (Hint: Assume the water level *h* does not rise above the hinge.)

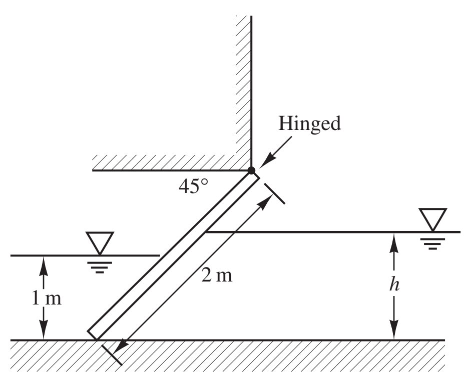


Figure P2.5.5

2.5.6. A circular gate is installed on a vertical wall as shown in Figure P2.5.6. Determine the horizontal force *P* necessary to hold the gate closed if the gate diameter is 6 ft and $h = 7$ ft. Neglect friction at the pivot.

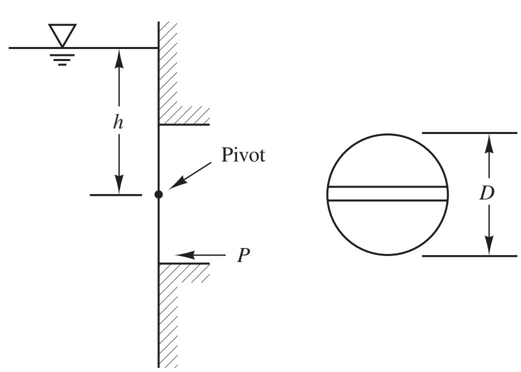


Figure P2.5.6

2.5.7. Figure P2.5.7 shows a 10-ft high (H), vertical, rectangular gate. The gate opens automatically when h increases to 4 ft. Determine the location of the horizontal axis of rotation 0–0'.

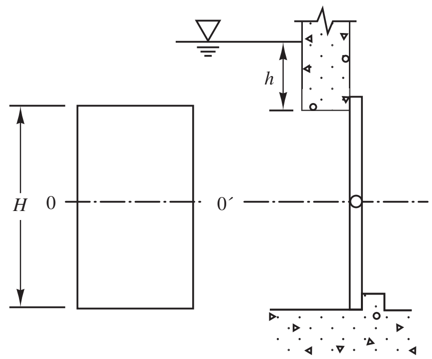


Figure P2.5.7

2.5.8. Calculate the magnitude and the location of the resultant pressure force on the annular gate shown in Figure P2.5.8.

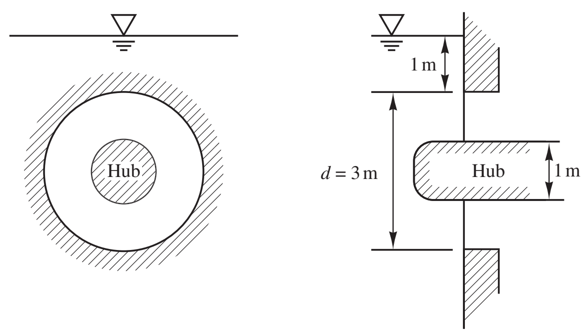


Figure P2.5.8

2.5.9. Calculate the magnitude and the location of the resultant pressure force on the annular gate shown in Figure P2.5.8 if the round central hub is replaced by a square hub (1 m by 1 m).

2.5.10. In Figure P2.5.10, the wicket dam is 5 m high and 3 m wide and is pivoted at its center. Determine the reaction force in the supporting member AB.

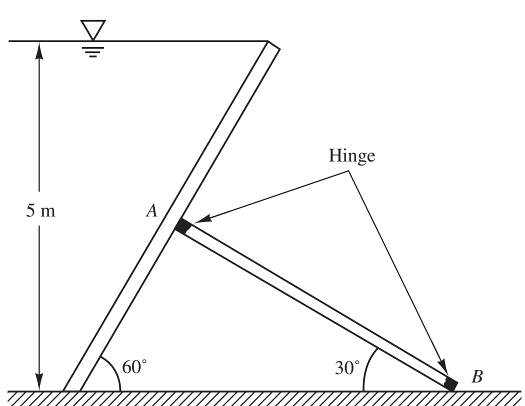


Figure P2.5.10

2.5.11. Determine the depth of the water (d) in Figure P2.5.11 that will cause the gate to open (lay down). The gate is rectangular and is 8 ft wide. Neglect the weight of the gate in your computations. At what depth will it close?

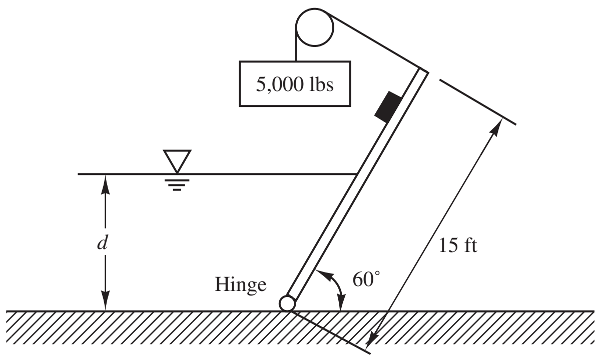


Figure P2.5.11

2.5.12. Neglecting the weight of the hinged gate, determine the depth h at which the gate will open in Figure P2.5.12.

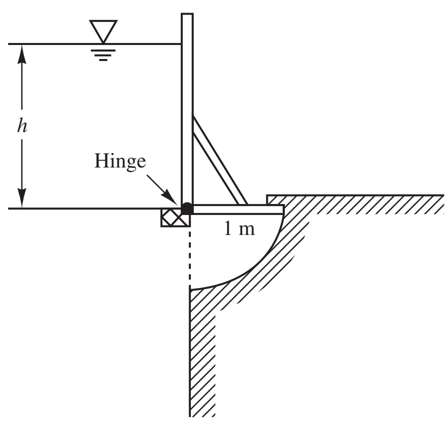


Figure P2.5.12

2.5.13. The circular gate shown in Figure P2.5.13 is hinged at the horizontal diameter. If it is in equilibrium, what is the relationship between h_A and h_B as a function of γ_A, γ_B, and d?

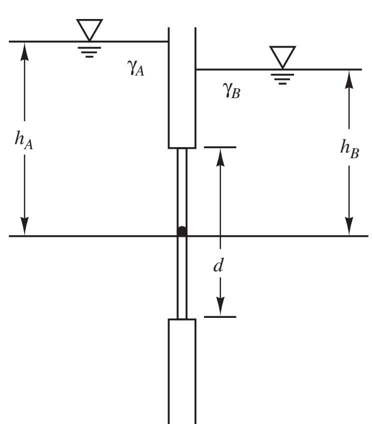


Figure P2.5.13

2.5.14. A sliding gate 10 ft wide and 6 ft high is installed in a vertical plane and has a coefficient of friction against the guides of 0.2. The gate weighs 3 tons, and its upper edge is 27 ft below the water surface. Calculate the vertical force required to lift the gate.

(SECTION 2.6)

2.6.1. A 10-m-long curved gate depicted in Figure P2.6.1 is retaining a 6-m depth of water in a storage tank. Determine the magnitude and direction of the total hydrostatic force on the gate. Does the force pass through point A? Explain.

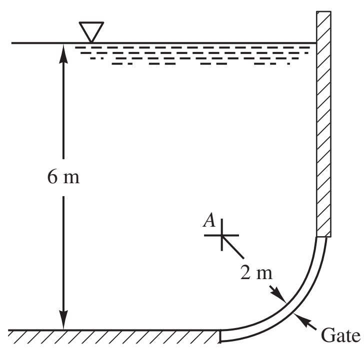


Figure P2.6.1

2.6.2. A hemispherical viewing port in a marine museum (Figure P2.6.2) has a 1-m radius, and the top of the port is 3 meters below the surface of the water (h). Determine the magnitude, direction, and location of the total hydrostatic force on the viewing port. (Assume the saltwater has a sp. gr. = 1.03.)

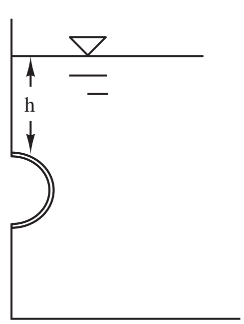


Figure P2.6.2

2.6.3. An inverted hemispherical shell of diameter d as shown in Figure P2.6.3 is used to cover a tank filled with water at 20℃. Determine the minimum weight the shell needs to be to hold itself in place if the diameter is 6 feet.

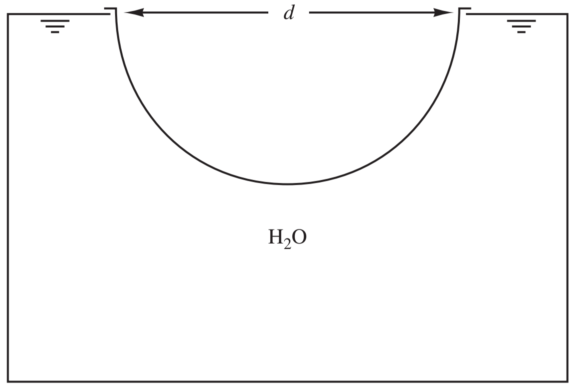


Figure P2.6.3

2.6.4. The corner plate of a barge hull is curved, with a radius of 1.75 m. The barge is leaking, and the depth of submergence (draft) is 4.75 m as depicted in Figure P2.6.4. The water on the inside is up to level *A*, producing hydrostatic pressure on the inside as well as the outside. Determine the resultant horizontal hydrostatic pressure force and the resultant vertical hydrostatic force on plate *AB* per unit length of hull.

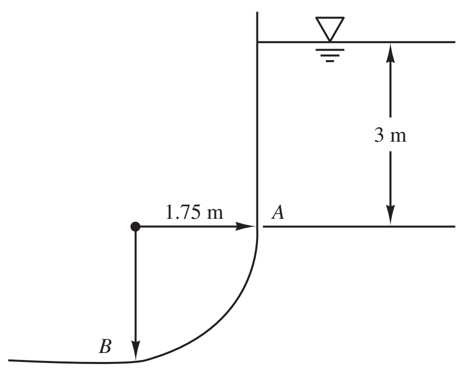


Figure P2.6.4

2.6.5. The tainter gate section shown in Figure P2.6.5 has a cylindrical surface with a 12-m radius; it is supported by a structural frame hinged at *O*. The gate is 10 m long (in the direction perpendicular to the page). Determine the magnitude, direction, and location of the total hydrostatic force on the gate (Hint: Determine the horizontal and vertical force components.)

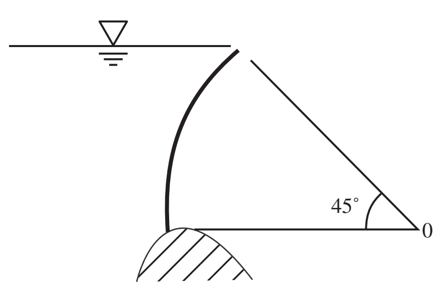


Figure P2.6.5

2.6.6. Calculate the magnitude, direction, and location of the total hydrostatic pressure force (per unit length) on the gate shown in Figure P2.6.6.

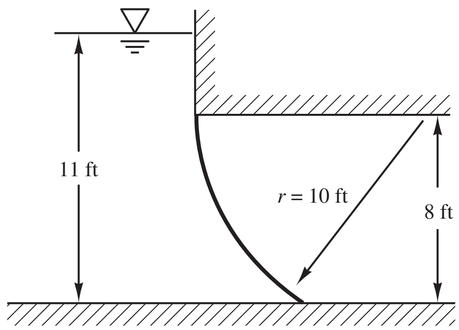


Figure P2.6.6

2.6.7. A 4-ft-diameter cylindrical tank lays on its side with its central axis horizontal. A 1.5-ft diameter pipe extends vertically upward from the middle of the tank. Oil (sp. gr. = 0.9) fills the tank and pipe to a level of 8 ft above the top of the tank. What is the hydrostatic force on one end of the tank? What is the total hydrostatic pressure force on one side (semicircle) of the tank if it is 10 ft long?

2.6.8. Calculate the horizontal and vertical forces acting on the curved surface *ABC* in Figure P2.6.8.

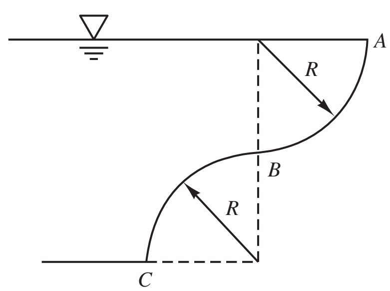


Figure P2.6.8

2.6.9. Calculate the magnitude and location of the vertical and horizontal components of the hydrostatic force on the surface shown in Figure P2.6.9 (quadrant on top of the triangle, both with a unit width). The liquid is water, and the radius $R = 4$ ft.

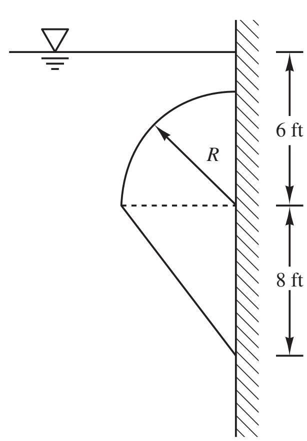


Figure P2.6.9

2.6.10. In Figure P2.6.10 a homogeneous cone plugs the 0.1-m-diameter orifice between reservoir *A* that contains water and reservoir *B* that contains oil (sp. gr. = 0.8). Determine the specific weight of the cone if it unplugs when h_0 reaches 1.5 m.

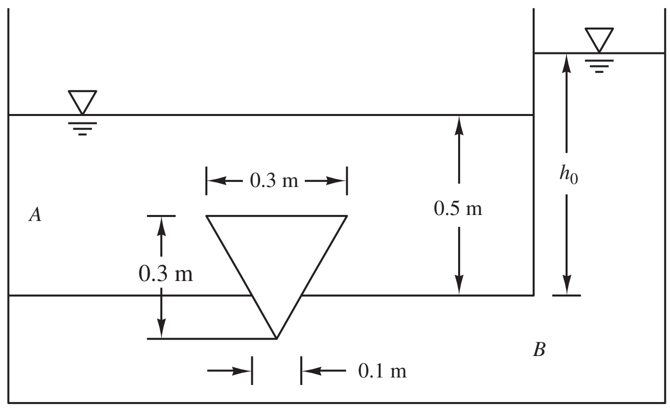


Figure P2.6.10

2.6.11. What would be the specific weight of the cone if reservoir *B* in Figure P2.6.10 contains air at a pressure of 8,500 N/m^2 instead of oil?

2.6.12. The homogeneous cylinder (sp. gr. = 2.0) in Figure P2.6.12 is 1 m long and $\sqrt{2}$ m in diameter and blocks a 1-m^2 opening between reservoirs *A* and *B* (sp. gr.$_A$ = 0.8, sp. gr.$_B$ = 1.5). Determine the magnitude of the horizontal and vertical components of the hydrostatic force on the cylinder.

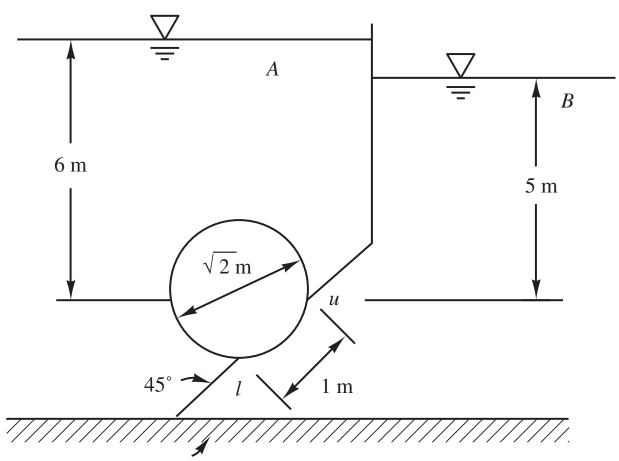


Figure P2.6.12

(SECTION 2.8)

2.8.1. A piece of irregularly shaped metal weighs 301 N. When the metal is completely submerged in water, it weighs 253 N. Determine the specific weight and the specific gravity of the metal.

2.8.2. The solid floating prism show in Figure P2.8.2 has two components. Determine γ_A and γ_B in terms of γ if $\gamma_B = 1.5 \cdot \gamma_A$.

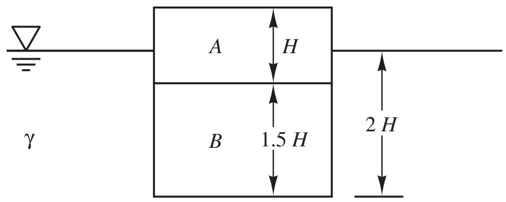


Figure P2.8.2

2.8.3. A solid brass sphere of 30-cm diameter is used to hold a cylindrical buoy in place (Figure P2.8.3) in seawater (sp. gr. $= 1.03$). The buoy (sp. gr. $= 0.45$) has a height of 2 m and is tied to the sphere at one end. What rise in tide, h, will be required to lift the sphere off the bottom?

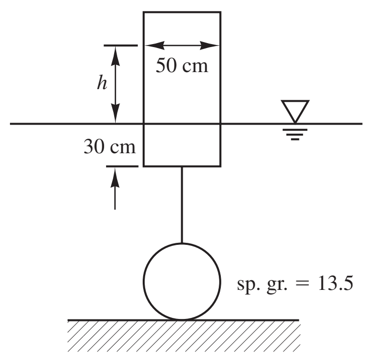


Figure P2.8.3

2.8.4. Three people are in a boat with an anchor. If the anchor is thrown overboard, will the lake level rise, fall, or stay the same theoretically? Explain.

2.8.5. A freshwater cylindrical anchor ($h = 1.2$ ft and $D = 1.5$ ft) is made of concrete (sp. gr. $= 2.7$). What is the maximum tension in the anchor line before the anchor is lifted from the lake bottom if the anchor line is at an angle of 60° with respect to the bottom?

2.8.6. In Figure P2.8.6, the spherical buoy of radius R opens the square gate AB when water rises to the half-buoy height. Determine R if the weight of the buoy and the gate are negligible.

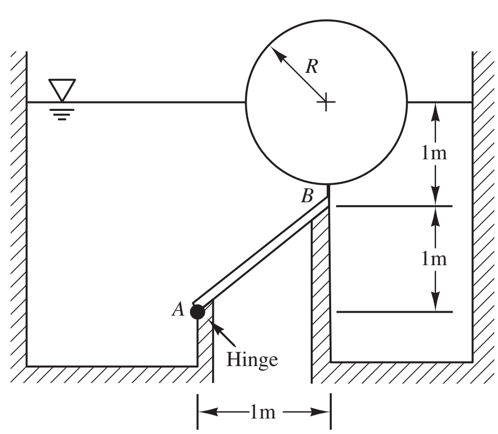


Figure P2.8.6

2.8.7. The floating rod shown in Figure P2.8.7 weighs 150 lbs, and the water's surface is 7 ft above the hinge. Calculate the angle θ assuming a uniform weight and buoyancy distribution.

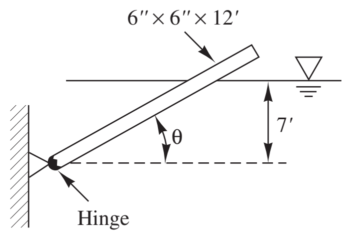


Figure P2.8.7

2.8.8. A rectangular barge is 14 m long, 6 m wide, and 2 m deep. The center of gravity is 1.0 m from the bottom, and the barge draws 1.5 m of seawater (sp. gr. = 1.03). Find the metacentric height and the righting moments for the following angles of heel (or list): 4°, 8°, and 12°.

2.8.9. Figure P2.8.9 shows a buoy that consists of a wooden pole 25 cm in diameter and 2 m long with a spherical weight at the bottom. The specific gravity of the wood is 0.62, and the specific gravity of the bottom weight is 1.40. Determine (a) how much of the wooden pole is submerged in the water, (b) the distance to the center of buoyancy from the water level, (c) the distance to the center of gravity from the water level, and (d) the metacentric height.

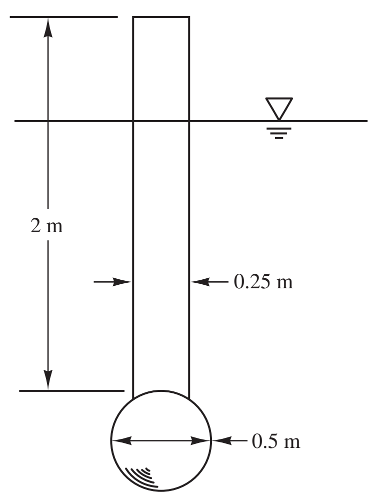


Figure P2.8.9

2.8.10. A wooden block is 2 m long, 1 m wide, and 1 m deep. Is the floating block stable if the metacenter is at the same point as the center of gravity? Explain.

2.8.11. A subway tunnel is being constructed across the bottom of a harbor. The process involves tugboats that pull floating cylindrical sections (or *tubes* as they are often called) across the harbor and sink them in place, where they are welded to the adjacent section already on the harbor bottom. The cylindrical tubes are 50 feet long with a diameter of 36 feet. When in place for the tugboats, the tubes are submerged vertically to a depth of 42 feet, and 8 feet of the tube is above the water (sp. gr. = 1.02). To accomplish this, the tubes are flooded with 34 feet of water on the inside. Determine the metacentric height and estimate the righting moment when the tubes are tipped through a heel (list) angle of 4° by the tugboats. (Hint: Assume the location of the center of gravity can be determined based on the water contained inside the tubes and the container weight is not that significant.)

2.8.12. A 12-m-long, 4.8-m-wide, and 4.2-m-deep rectangular pontoon has a draft of 2.8 m in seawater (sp. gr. = 1.03). Assuming the load is uniformly distributed on the bottom of the pontoon to a depth of 3.4 m, and the maximum design angle of list is 15°, determine the distance that the center of gravity can be moved from the center line toward the edge of the pontoon.