Let's Revisit the Mortgage-Equity Analysis

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Introduction

The Appraisal of Real Estate, 14th edition, states, "Because most properties are purchased with debt and equity capital, the overall capitalization rate must satisfy the market return requirements of both investment positions. Lenders must anticipate receiving a competitive interest rate ... equity investors must anticipate receiving a competitive equity return."[1] This capitalization method described above is known as mortgage-equity analysis. The creator of the modern mortgage-equity analysis was Leon Ellwood, MAI. Morgan B. Gilreath, Jr. in his article "Perspectives on the Income Approach," claimed, "Perhaps the largest bombshell in recent history was dropped in 1958 with the publication of the *Ellwood Tables for Real Estate Appraising and Financing*, which enabled appraisers to perform a 10-year discounted cash flow valuation using the mortgage-equity analysis method."[2]

Ellwood's mortgage-equity analysis utilized a 10-year discounted cash flow procedure where the equity and debt components are valued separately and then added together to estimate the property's total value. Specifically, the property's net income is divided between the mortgage and equity components. The income attributed to the mortgage component is the interest on the debt during the 10-year holding period plus the repayment of the loan upon an assumed sale at the end of the 10th year. The income attributed to the equity component is the net income after payment of debt service during the 10-year holding period plus the proceeds

[1] Appraisal Institute, *The Appraisal of Real Estate*, 14th ed. (Chicago: Appraisal Institute, 2013), 495.
[2] Morgan B. Gilreath, Jr., "Perspectives on the Income Approach," *Volusia County Appraisal Manual*, 38, http://vcpa.vcgov.org/persp0379 (downloadable PDF).

from the assumed sale after deducting the remaining mortgage balance and any broker and legal costs associated with the sale. These streams of income are then discounted to the present value using the mortgage interest rate for the mortgage component and an equity yield for the equity component. The two components are then added together to form an estimate of the property's total value.

Appraisers have not embraced Ellwood's mortgage-equity analysis most likely because Ellwood was way ahead of his time—specifically, ahead of the financial calculator and personal computer. While his mathematic formulas were accurate, to utilize his mortgage-equity analysis the appraisers had to refer to Ellwood's book containing pages of confusing financial tables and calculations. The Appraisal Institute even had a two-day seminar that taught the Ellwood Method. William Kinnard in his article, "The Financial Logic of Investment Property Appraising," agrees, "The Ellwood analysis is unnecessarily complex. The basic ideas are quite simple and direct, but they are couched in unnecessarily complex mathematical forms."[3]

The greatest drawback to Ellwood's mortgage-equity analysis was the required assumption that the projected 10-Year Net Income remained level, escalated on a straight-line basis, or escalated exponentially. Ellwood had no way to handle an uneven Net Income stream.

After more than 50 years since Ellwood published his tables, *The Appraisal of Real Estate*, 14th edition, still describes the mortgage-equity analysis by showing the calculations for a level stream of net income, a straight-line escalation (the J factor), and an exponential growth (the K factor) of net income. Obviously, these constraints put on the projection of net income make

[3] Dr. William N. Kinnard, Jr., "The Financial Logic of Investment Property Appraising," *The Real Estate Appraiser* 35, no. 4 (May-June 1969): 20.

the mortgage-equity analysis completely unusable in today's dynamic economic environment in which appraisers need to deal with variable, random flows of net income. As a result, most appraisers employ a 10-year discounted cash flow utilizing a property yield (which is also referred to as an overall yield) where any variance in projected net income can be easily discounted to the present value.

According to *The Appraisal of Real Estate*, 14th edition, the property yield rate is "a rate of return on and of the total capital invested. … It does not, however, consider the effect of debt financing. Rather, it is calculated as if the property were purchased with no debt capital and thus is sometimes called an unleveraged rate."[4] Appraisers are taught to estimate the yield rate by considering a number of factors such as the perceived risk, expectations of future inflation, rates of return for alternative investments, historical rates of returns for comparable properties, availability of debt financing and prevailing tax law.[5] Essentially the impact of financing in the development of the yield rate is handled implicitly rather than explicitly.

Ellwood found the development of the property yield too subjective and believed that combining the knowledge and expertise of mortgage lenders together with equity investors results in a mortgage-equity analysis that produces a more supportable valuation.

Further support for using a mortgage-equity analysis comes from a number of the great appraisal educators, scholars, and practitioners.

Charles Akerson in his famous article, "Ellwood Without Algebra," observes, "The mortgage-equity appraiser views mortgage financing as the lifeblood of real estate investment,

[4] Appraisal Institute, *The Appraisal of Real Estate*, 14th ed. (Chicago: Appraisal Institute, 2013), 457.
[5] Appraisal Institute, The Appraisal of Real Estate, 14th ed., 457.

recognizing that the typical investor seeks the best mortgage financing available in order to obtain a maximum yield on a minimum down payment."[6] Akerson goes on to say mortgage-equity analysis is "a realistic system of analyzing and judging the relative attractiveness of real estate investments based on the economic facts of life."[7]

William Kinnard espoused that "the most appropriate route for appraisal research and appraisal practice to take is to embrace and to advance mortgage-equity analysis when income-producing properties are to be appraised."[8]

James Gibbons in his *Appraisal Journal* article, "Capitalization Rates," finds that the band of investment (or mortgage-equity) is based on "the determination of the customary structure of financing underlying the particular variety of property under examination."[9]

Today, with the calculating power of the personal computer, Ellwood's mortgage-equity method can handle any income stream and precisely calculate a total property value that will provide the mortgage lender with their required rate of return (interest rate) on their monies loaned and the equity investors with their desired rate of return (equity yield) on their invested capital.

This article will focus on the reasons why appraisers should be employing the mortgage-equity discount approach rather than discounting the income stream using a property yield rate. It will also demonstrate several mathematical formulas for performing a mortgage-equity analysis utilizing a loan-to-value ratio, a debt coverage ratio, and a debt yield.

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[6] Charles B. Akerson, MAI, "Ellwood Without Algebra," *The Appraisal Journal* 38, no. 3 (July 1970): 327.
[7] Charles B. Akerson, MAI, "Ellwood Without Algebra," *The Appraisal Journal* 38, no. 3 (July 1970): 327.
[8] Dr. William N. Kinnard, Jr., "The Financial Logic of Investment Property Appraising," *The Real Estate Appraiser* 35, no. 4 (May-June 1969): 20.
[9] James E. Gibbons, MAI, "Capitalization Rates," *The Appraisal Journal* 27, no. 2 (April 1959): 178.

How a Typical Real Estate Investor Buys Property

The mortgage-equity method is superior to an unlevered discounted cash flow analysis because it reflects the investment rational of both the lender and equity investor, and its accuracy is significantly enhanced through the use of easy-to-obtain and up-to-the-minute market and investment data.

As real estate appraisers, we need to put our feet into the shoes of typical buyers and employ the same procedures they use to establish their purchase price. Most buyers we have observed start with a projection of Net Income Before Debt Service (5 or 10 years). They then determine how much they can borrow and the terms of the financing through the utilization of a loan-to-value ratio, a debt coverage ratio, or debt yield. This gives them the amount of annual debt service and yearly equity returns (equity dividends). At the end of their holding period they estimate the proceeds from a sale and deduct the outstanding mortgage balance and selling expenses (equity residual). They then discount the annual equity dividends along with the equity residual from the sale to the present value to determine how much equity they can invest. The total of the mortgage component and equity component establishes what they can pay for the property. If their valuation results in the highest bid, they usually become the buyer rather than just a looker. Other than some all-cash institutional buyers who purchase properties with no debt, most real estate investors specifically consider the cost of financing in their purchase decisions.

In order to perform Ellwood's mortgage-equity analysis appraisers need to make a number of very specific (but usually well-supported) estimates and assumptions, which include:

Overall Assumptions
Net Income (Before Debt Service): Projected for 11 years

Mortgage Assumptions
Cost of the mortgage: Mortgage Interest Rate
Amount of the mortgage: Loan-to-Value Ratio or Debt Coverage Ratio or Debt Yield
Repayment of the mortgage: Amortization
Amount of debt service: Mortgage Constant times initial mortgage balance

Equity Assumptions
Desired return on equity: Equity Yield
Yearly Net Income less Debt Service: Equity Dividend

Assumed Sale at the End of the Holding Period
Terminal capitalization rate
Projected 11th Year's Net Income divided by the Terminal Capitalization Rate – Reversionary Value

Remaining mortgage balance
Selling expenses: Broker and legal costs
Reversionary Value less Remaining Mortgage Balance and Selling Expenses – Equity Residual

The Accuracy of the Mortgage-Equity Analysis is Enhanced through the Use of Up-to-the-Minute Market and Investment Data Representing a Significant Component of the Property's Value

L. W. Ellwood, MAI, in his article, "Selection of Capitalization Rates in Today's Market," starts by saying, "I believe there would be much less divergence among appraisers if they were required to employ more fact and less theory."[10]

Every real estate appraisal requires a number of estimates and assumptions. Some of these estimates and assumptions represent subjective decisions, while others are based on objective information. The accuracy of an appraisal is enhanced by minimizing the number of subjective decisions and utilizing more-supportable, current market and investment data.

[10] L. W. Ellwood, MAI, "Selection of Capitalization Rates in Today's Market," *The Appraisal Journal* 45, no. 3 (July 1977): 325.

The main difference between a mortgage-equity analysis and the use of an unlevered discounted cash flow analysis is that the mortgage-equity analysis utilizes specific mortgage and equity data such as loan-to-value ratio, debt coverage ratio, debt yield, interest rate, amortization rate, and equity yield. The mortgage information is usually highly accurate data that can be obtained easily. Gibbons agrees that this mortgage information "is, therefore, used with a sense of assurance and confidence that the results obtained can be substantiated and justified."[11] The equity yield, on the other hand, can be somewhat subjective since it requires data not as readily available. Good appraisers should be having ongoing conversations with investors about their return requirements. In addition, appraisers can derive equity yields from sales transactions of properties appraised at the time of sale by hypothesizing market-rate debt and solving for the equity IRR.

The property yield is a highly subjective blend of perceived risk, expectations of future inflation, rates of return for alternative investments, historical rates of returns for comparable properties, availability of debt financing, and prevailing tax law. Property yield rates are often obtained from investor surveys, transactions, and the appraiser's "experience and local knowledge," which represent a subjective historical source that probably is not "up-to-the-minute" information. An additional reason to utilize a mortgage-equity analysis over an unlevered discounted cash flow analysis is that the timing of the cash flows in a leveraged real estate investment can greatly impact the actual yield earned by the equity investor.[12] An unlevered discounted cash flow analysis where debt service and return on equity are not

[11] James E. Gibbons, MAI, "Capitalization Rates," *The Appraisal Journal* 27, no. 2 (April 1959): 178.
[12] Suzanne R. Mellen, MAI, "Does Your Underwriting Adequately Compensate for the Timing of Future Cash Flow?" *HVS Journal* (January 2007).

explicitly considered can result in overvaluing an asset when net income is projected to ramp-up over time.

If one assumes a loan-to-value ratio of 75%, then 75% of the property's value is the mortgage and 25% is the equity. Thus, the mortgage component represents 75% of the overall investment. Since the mortgage data and information are highly objective and easily obtained, then 75% of the appraised value should have a high level of accuracy. The value of the 25% equity component is subjective because of its reliance on equity yield data, which can result in a lower level of accuracy.

On the other hand, the property yield used to calculate 100% of the property's value is highly subjective, which can have a negative impact on the appraisal's accuracy. So, which is preferable—a valuation where 75% of the value is highly accurate and 25% is subject to a high level of subjectivity, or a valuation where the entire value is subject to a high level of subjectivity?

Why is Mortgage Data Highly Objective, Up to Date, and Easy to Obtain?

L.W. Ellwood cites in his article that the following mortgage factors "are reasonably factual" and represent "market data concerning which the professional appraiser should be informed at all times."[13]

1. Ratio of mortgage money to fair market value.
2. Interest rate that will attract mortgage money at time of appraisal.
3. Maximum full mortgage amortization term available at time of appraisal.

[13] L. W. Ellwood, MAI, "Selection of Capitalization Rates in Today's Market," *The Appraisal Journal* 45, no. 3 (July 1977): 328.

Accurate and up-to-date mortgage data is easy to obtain from many sources. If the appraisal is for new financing then the mortgage commitment information can provide a benchmark for the interest rate, amortization, and loan-to-value and debt coverage ratios. A number of real estate research and consulting firms provide information on current financing terms. Additionally, most mortgage lenders will assist appraisers in obtaining financing information.

Appraisers can also develop their own highly accurate and up-to-the minute interest rate estimates by using sources such as the American Council of Life Insurance (ACLI). The ACLI is a group of life insurance companies that are active in real estate lending. Each quarter they pool their property-specific mortgage origination data such as interest rates, loan-to-value ratios, and amortization terms, and publish the results. This highly accurate lender information is perfect for developing the financing terms for the mortgage component. The challenge is that the ACLI-published data is at least one quarter and sometimes six months old and, in a changing interest rate environment, the results can be skewed.

To correct for this timing issue appraisers need to find a proxy for the interest rate information through the use of a regression analysis. We found that, depending on the type of property being appraised, there is readily available financial data (such as Moody's corporate bond yields) that when regressed against historical ACLI interest rate information produces a regression formula with a high level of correlation. All appraisers then need to do is obtain the current financial data and plug it into the regression formula, which produces a current mortgage interest rate estimate.

The following is an example of how to develop regression formulas from ACLI mortgage interest rate information.

The ACLI provides quarterly mortgage information for five property types:

- Apartment
- Office
- Retail
- Industrial
- Hotel

For each of these property types quarterly rate data dating back to the fourth quarter of 1999 to the first quarter of 2019 was obtained from the ACLI, plus data from the following financial sources:

- Moody's Aaa Corporate Bond Yield
- Moody's Avg A Corporate Bond Yield
- Moody's Baa Corporate Bond Yield
- 30-Year Mortgage Rate (Federal Reserve Economic Data)
- 15-Year Mortgage Rate (Federal Reserve Economic Data)
- 20-Year Treasury Bond Yield

A series of linear regressions were run using the ACLI data as the dependent variable (y) and each of the financial sources as the independent variable (x). The results were then compared using the R squared results from each of the regressions.

R Square is the coefficient of determination, which is used as an indicator of the goodness of fit for linear regression models. R Square ranges from 0% to 100%. Any result over 90% is considered a good fit.

Exhibit 1 shows the R Square results from the various regressions. The percentages highlighted in yellow shows which financial source scored the highest R Square for each property type.

EXHIBIT 1

R Square

R Square	Moody's Aaa	Moody's Avg A	Moody's Baa	30-Yr Mort	15-Yr Mort	20-Yr Treas
Apartment	0.918	0.913	0.900	0.931	0.929	0.855
Office	0.917	0.917	0.892	0.923	0.923	0.867
Retail	0.914	0.912	0.895	0.901	0.900	0.843
Industrial	0.910	0.904	0.891	0.890	0.888	0.840
Hotel	0.857	0.948	0.828	0.815	0.812	0.776

The data shows that for the Retail and Industrial categories, Moody's Aaa Bond Yields result in the highest R Squared. For Apartment and Office, the 30 Year Mortgage Rate is most appropriate; office mortgage interest rates can also come from the 15 Year Mortgage Rate. Moody's Average A Corporate Bond Yield is the best for Hotels.

Exhibit 2 shows the regression formulas that were developed from these financial sources.

EXHIBIT 2

y	dependent variable				
a	Y-intercept				
b	slope of line				
x	independent variable				
y = a +bx	a	b			Regression Formulas
Apartment	0.13271	0.96338		Apartment	y = .13271 + .96338x
Office	0.27441	0.95497		Office	y = .27441 + .95497x
Retail	0.23184	1.02004		Retail	y = .23184 + 1.02004x
Industrial	-0.01581	1.04364		Industrial	y = -.01581 + 1.04364x
Hotel	0.76607	0.95655		Hotel	y = .76607 + .95655x

Example:

The yield on a Moody's AAA Corporate Bond as of 8/12/2019 was 3.09%. The projected mortgage interest rate as of this date for a Retail would be calculated as follows:

y = .23184 + (1.02004 * 3.09)
y = 3.38%

These regression formulas were based on six different financial sources. The key is to use a financial source where data is published either daily or weekly so the calculated mortgage interest rate is completely up to date. In addition, the regression analysis should be updated every quarter based on the data from the ACLI.

Mortgage-Equity Analysis for an Uneven Stream of Net Income

Now that we have financial calculators and personal computers, we can bring Ellwood's mortgage-equity analysis up to date to handle uneven streams of net income. Suzanne Mellen, MAI, developed the algebraic equation that performs this analysis in her April 1983 *Appraisal Journal* article, "Simultaneous Valuation: A New Capitalization Technique for Hotel and Other Income Properties."[14] Unfortunately, her article was oriented specifically toward the valuation of hotels and did not receive much traction for other types of commercial properties. In addition, the use of the term "simultaneous valuation formula equation" in the title was a misnomer because the algebraic equation is actually linear, and not a simultaneous equation. The authors are thus resurrecting this formula (and adding several important enhancements) to provide the appraisal community with a more accurate method of performing a discounted cash flow by using the mortgage-equity analysis. Lastly, this mortgage-equity analysis is appropriate for all forms of commercial real estate.

Mellen's mortgage-equity algebraic formula utilized a loan-to-value ratio to size the mortgage component. This article will also demonstrate how mortgage-equity analysis can be performed to size the mortgage based on achieving a defined debt-coverage ratio or a debt yield as of a

[14] Suzanne R. Mellen, MAI, "Simultaneous Valuation: A New Capitalization Technique for Hotel and Other Income Properties," *The Appraisal Journal* 51, no. 2 (April 1983): 165-189.

specific year's Net Income. Thus, if one wanted to achieve a debt-coverage ratio of 1.5 as of the third year, the mortgage-equity formula calculates the overall value that produces this desired result. Each of these three formulas to size the mortgage component will now be demonstrated.

Formula 1: Mortgage-Equity Analysis Using a Loan-to-Value Ratio

The mortgage-equity analysis using a loan-to-value ratio calculates the exact amount of debt and equity that the property will be able to support based on the anticipated cash flow derived from the forecast of net income and the return requirements of the mortgage lender (interest rate) and the equity investor (equity yield) to produce a specified loan-to-value ratio.

Overall the valuation process is as follows. To solve for the value of the mortgage and equity components, the yearly debt service is deducted from the forecast of net income before debt service, leaving the net income to equity for each year in the forecast. The net income as of Year 11 is capitalized into a reversionary value using the terminal capitalization rate. The equity residual, which is the total reversionary value minus the mortgage balance at that point in time and any broker and legal cost associated with the sale, is discounted to the date of value at the equity yield rate. The net income to equity for each of the forecast years is also discounted. The sum of these discounted values equates to the value of the equity component. Since the equity component represents a specific percentage of the total value (loan-to-value ratio), the value of the mortgage and the total property value can be easily computed.

The process described above can be expressed in an algebraic equation, which sets forth the mathematical relationships between known and unknown variables. The symbols used to represent these variables are listed below.

NI	Net income available for debt service
V	Value
M	Loan-to-value ratio
I	Mortgage interest rate
f	Annual debt service constant
n	Number of years in projection period
d_e	Annual cash available to equity
d_r	Residual equity value
b	Brokerage and legal cost percentage
P*	Fraction of loan paid off in projection period
fp	Annual constant required to amortize the entire loan during the projection period
R_r	Overall terminal capitalization rate applied to net income to calculate total property reversion (sale price at end of the projection period)
1/Sn	Current worth of 1$ (discount factor) at the equity yield rate

*P = (f-i) ÷ (fp - i) where I = the interest rate of the mortgage

Using these symbols, a series of formulas can be derived to express the components making up this mortgage-equity valuation process.

Debt service. To calculate a property's debt service, the appraiser first determines the amount of the mortgage, which is the total property value (V) multiplied by the loan- to-value ratio (M). Then the amount of the mortgage is multiplied by the annual debt service constant (f) using the following formula:

$$f \times M \times V = \text{debt service}$$

Net income to equity (equity dividend). The net income to equity (de) is the property's net income before debt service (NI) minus the debt service. The following formula represents net income to equity:

$$NI - (f \times M \times V) = d_e$$

Reversionary value. The value of the property at the end of Year 10 is calculated by dividing the net income in Year 11 before debt service (NI") by the terminal capitalization rate (Rr). The following formula calculates the property's reversionary value in Year 10:

NI''/R_r = reversionary value

Broker and legal costs. When a property is sold, costs associated with the transaction normally include a broker's commission and attorneys' fees. For a typical property transaction broker and legal costs typically range from 1% to 4% of the sale price. Because these expenses reduce the proceeds to the seller, they are usually deducted from the reversionary value in mortgage-equity analysis. Broker and legal costs (b) expressed as a percentage of the reversionary value (NI"/Rr) can be calculated with the following formula:

$(b\ (NI''/\ R_r))$ = broker and legal costs

Ending mortgage balance. The balance of the mortgage at the end of Year 10 must be deducted from the total reversionary value (debt and equity) to isolate the equity residual. A financial formula is used to calculate the fraction of the loan paid off, which is expressed as a percentage of the original loan balance at a particular point in time. The mortgage interest rate (i) is deducted from the annual debt service constant of the loan over the entire amortization period (f) and the result is divided by the annual constant required to amortize the entire loan over the projection period (sub p) minus the mortgage interest rate. The formula is

$(f-i)\ /(fp-i) = P$

If the fraction of the loan paid off expressed as a percentage of the initial loan balance is P, then the percentage of the loan remaining can be expressed as 1 - P. Thus, the ending mortgage balance is the fraction of the loan remaining (1-P) multiplied by the amount of the initial loan (M x V). The formula is

$(I\ -P) \times M \times V$ = ending mortgage balance

Equity residual value. The value of the equity when the property is sold at the end of the projection period (d) is the reversionary value minus broker and legal costs and the ending mortgage balance. The following formula represents the equity residual value:

$$(NI''/ R_r) - (b(NI''/ R_r)) - ((1-P) \times M \times V)) = d_r$$

Annual cash flow to equity. The annual cash flow to equity consists of the equity dividend for each of the 10 projection years plus the equity residual at the end of Year 10. The following formulas represent the annual cash flow to equity:

$$NI' - (f \times M \times V) = d_e'$$
$$NI^2 - (f \times M \times V) = d_e^2 \ldots$$
$$NI^{10} - (f \times M \times V) = d_e^{10}$$

Value of the equity. If the initial amount of the mortgage is calculated by multiplying the loan-to-value ratio (M) by the value of the property (V), then the equity value will be 1 minus the loan-to-value ratio times the property value. The formula is

$$(1 - M) V$$

Discounting the cash flow to equity to present value. The cash flow to equity for each of the projection years is discounted to present value at the equity yield rate ($1/S^n$). The sum of all these cash flows is the value of the equity (I-M)V. The following formula calculates equity as the sum of the discounted cash flows:

$$(d_e' \times 1/S^1) + (d_e^2 \times 1/S^2) + \ldots + (d_e^{10} \times 1/S^{10})$$
$$+(d_r \times 1/S^{10}) = (I - M)V$$

Combining equations: The annual cash flow to equity and cash flow to equity are discounted to present value. The final step in the process is to make one, overall equation that shows that the

annual cash flow to equity plus the yearly cash flows discounted to present value equal the value of the equity.

$$((NI' - (f \times M \times V))\ 1/S^1) + ((NI^2 - (f \times M \times V))\ 1/S^2) + \ldots$$
$$\ldots + ((NI^{10} - (f \times M \times V))\ 1/S^{10}) +$$
$$\ldots + (NI''/ R_r) - (b(NI''/ R_r)) - ((1 - P) \times M \times V)\ 1/S^{10}) = (1-M)V$$

Since the only unknown is the property value (V), this equation is easy to solve.

Example:

Estimate the market value of a commercial property using the Mortgage-Equity Analysis Using a Loan-to-Value Ratio.

A four-year projection of income and expense was made up to the point where the property is expected to stabilize. After that point the Net Income Before Debt Service is projected to grow at 3% per year.

EXHIBIT 3

Year	Net Income Available For Debt Service ($000)
1	$1,000
2	$1,100
3	$1,300
4	$1,500
5	$1,545
6	$1,591
7	$1,639
8	$1,688
9	$1,739
10	$1,791

Mortgage

Based on discussions with lenders and mortgage data from the ACLI the following are the mortgage financing assumptions:

Mortgage Interest Rate (i)	7.0%
Loan-to-Value Ratio (M)	75%
Amortization	25 Years
Constant (f)	.084814
Term	10 Years

Terminal Value

Holding Period	10 Years
Terminal Capitalization Rate (R_r)	10%
Broker and Legal Cost (b)	3%

Equity Requirements

Equity Yield	18%

The Following Calculations Can be Made with the Available Data

Annual Debt Service

f x M x V = Debt Service

.084181 x .75 x V = .06361V

Percentage of the Loan Paid Off During Holding Period

P = (f-i) ÷ (fp - i)

Mortgage Constant 25 Year Term: .08481
Mortgage Constant 10 Year Term: .13933

P = (.08481 - .07000) / (.13933 - 0.07000) = .21367

Reversionary Value

NI"/R_r = reversionary value

$1,845/.10 = $18,448

Broker and Legal Costs

(b (NI"/ R_r)) = broker and legal costs

(.03($1,845/.10)) = $553

The Following Calculations Require an Algebraic Equation to Solve for Value (V)

Ending Mortgage Balance

(I -P) x M x V = ending mortgage balance

(1 - .21367) x .75 x V = .83975V

Equity Residual Value (d_r)

(NI"/ R_r) - (b(NI"/ R_r)) - ((1-P) x M x V)) = d_r

$18,448 - $553 - ((1-.21367) x .75 x V)) = d_r

$17,894 - (.78633 x .75} = d_r

$17,894 - 0.5897V = d_r

Annual Cash Flow to Equity

NI' - (f x M x V) = d_e'
NI^2 - (f x M x V) = d_e^2 . . .
NI^{10} - (f x M x V) = d_e^{10}

EXHIBIT 4

Year					
1	(	$1,000	-	0.0636 V)	$= d_e^1$
2	(	$1,100	-	0.0636 V)	$= d_e^2$
3	(	$1,300	-	0.0636 V)	$= d_e^3$
4	(	$1,500	-	0.0636 V)	$= d_e^4$
5	(	$1,545	-	0.0636 V)	$= d_e^5$
6	(	$1,591	-	0.0636 V)	$= d_e^6$
7	(	$1,639	-	0.0636 V)	$= d_e^7$
8	(	$1,688	-	0.0636 V)	$= d_e^8$
9	(	$1,739	-	0.0636 V)	$= d_e^9$
10	(	$1,791	-	0.0636 V)	$= d_e^{10}$

Discounting the Cash Flow and Equity Residual to Present Value

EXHIBIT 5

Year	Discount Factors At 18% Equity Yield
1	0.84746
2	0.71818
3	0.60863
4	0.51579
5	0.43711
6	0.37043
7	0.31393
8	0.26604
9	0.22546
10	0.19106

$$(d_e' \times 1/S^1) + (d_e^2 \times 1/S^2) + \ldots + (d_e^{10} \times 1/S^{10})$$

$$+(d_r \times 1/S^{10}) = (I - M)V$$

EXHIBIT 6

Year								
1	(	$1,000	-	0.06361 V)	x	0.84746	+	
2	(	$1,100	-	0.06361 V)	x	0.71818	+	
3	(	$1,300	-	0.06361 V)	x	0.60863	+	
4	(	$1,500	-	0.06361 V)	x	0.51579	+	
5	(	$1,545	-	0.06361 V)	x	0.43711	+	
6	(	$1,591	-	0.06361 V)	x	0.37043	+	
7	(	$1,639	-	0.06361 V)	x	0.31393	+	
8	(	$1,688	-	0.06361 V)	x	0.26604	+	
9	(	$1,739	-	0.06361 V)	x	0.22546	+	
10	(	$1,791	-	0.06361 V)	x	0.19106	+	
Residual	(	$17,894	-	0.5897 V)	x	0.19106	=	(1-M)V

Combine Terms

EXHIBIT 7

Year	Total		Total
1	$847	-	0.05391 V
2	$790	-	0.04568 V
3	$791	-	0.03872 V
4	$774	-	0.03281 V
5	$675	-	0.02780 V
6	$589	-	0.02356 V
7	$515	-	0.01997 V
8	$449	-	0.01692 V
9	$392	-	0.01434 V
10	$342	-	0.01215 V
Residual	$3,419	-	0.11267 V
	$9,584	-	0.39854 V = (1-M)V

Solve for Value (V)

EXHIBIT 8

$$\$9{,}584 - 0.39854\,V = 0.25\,V$$
$$\$9{,}584 = 0.64854$$
$$V = \$14{,}778$$

Proof

To determine whether all assumptions have been met, a proof needs to be performed to determine if the value is actually $14,778. The following is the proof:

As shown in Figure 9, based on a 75% loan-to-value ratio, the amount of the mortgage component is $11,083 and the 25% equity component is $3,694, which together equates to $14,778.

EXHIBIT 9

10-Yr. LTV Model:	Value $(000)	% of Total Value	IRR
Value of the Property	$14,778	100.0%	10.85%
Value of the Mortgage Component	$11,083	75.0%	7.00%
Value of the Equity Component	$3,694	25.0%	18.00%

Using the debt service constant for a 7%, 25-year mortgage of .08481 results in annual debt service of:

$11,083 x .08481 = $940 (Annual Debt Service)

Exhibit 10 indicates the annual Cash Flow to the Mortgage and Equity.

EXHIBIT 10

Year	1	2	3	4	5	6	7	8	9	10
Total Property	$1,000	$1,100	$1,300	$1,500	$1,545	$1,591	$1,639	$1,688	$1,739	$1,791
Mortgage	$940	$940	$940	$940	$940	$940	$940	$940	$940	$940
Equity	$60	$160	$360	$560	$605	$651	$699	$748	$799	$851

The Equity Residual is calculated as follows in Exhibit 11:

EXHIBIT 11

Year 11 Cash Flow of $1845 capitalized at 10% = $18,448

Less: Selling Expenses	$553
Equals: Net sales proceeds	$17,895

Exhibit 12 shows the Remaining Mortgage Balance.

EXHIBIT 12

Amount Paid Off (1 - .21367) = 0.78633
Original Mortgage $11,083 x 0.78633 = $8,715

Net Sales Proceeds	$17,895
Less: Remaining Mortgage Balance	$8,715
Equity Residual	$9,180

Figure 13 proves that when the Net Income to Equity plus the Equity Residual are discounted to the present value at the 18% Equity Yield Rate, the resulting value of the Equity Component is $3,694.

EXHIBIT 13

	Equity Component Present Value		
Year	Net Income to Equity	PV Factor @ 18.0%	Discounted Cash Flow
2020	$60	0.8475	$51
2021	$160	0.7182	$115
2022	$360	0.6086	$219
2023	$560	0.5158	$289
2024	$605	0.4371	$264
2025	$651	0.3704	$241
2026	$699	0.3139	$219
2027	$748	0.2660	$199
2028	$799	0.2255	$180
2029	$10,030	0.1911	$1,916
	Equity Component Value		$3,694

Year 10 net inc. to equity of	$851
plus the equity residual of	$9,179
	$10,030

Net Sales Price	$17,895
Less: RMB	$8,715
Equals: Equity Residual	$9,179

Figure 14 proves that when the Annual Debt Service plus the Remaining Mortgage Balance are discounted to the present value at the 7% Mortgage Interest Rate, the resulting value of the Mortgage Component is $11,083.

EXHIBIT 14

	Mortgage Component Present Value		
Year	Mortgage Payment	PV Factor @ 7.00%	Discounted Cash Flow
2020	$940	0.9352	$879
2021	$940	0.8746	$822
2022	$940	0.8179	$769
2023	$940	0.7649	$719
2024	$940	0.7153	$672
2025	$940	0.6690	$629
2026	$940	0.6256	$588
2027	$940	0.5851	$550
2028	$940	0.5472	$514
2029	$9,655	0.5117	$4,941
	Mortgage Component Value		$11,083
	Year 10 mort. payment of		$940
	plus the RMB of		$8,715
			$9,655

The reversion is the remaining mortgage balance (RMB) of the loan at the end of year 10.

Combining the Value of the Equity Component of $3,694 with the Value of the Mortgage Component of $11,083 equates to the Total Value of the Property of $14,778. Furthermore, the Equity Investor and Mortgage Lender both received their desired rate of return, confirming the valuation is correct by this Mortgage-Equity Analysis.

If the property were appraised using an unlevered discounted cash flow analysis, the Property Yield would have to be 10.9% to result in the same $14,778 Total Property Value, as shown in Exhibit 15.

EXHIBIT 15

Year	Total Property Present Value Net Income	PV Factor @ 10.9%	Discounted Cash Flow
2020	$1,000	0.9021	$902
2021	$1,100	0.8138	$895
2022	$1,300	0.7341	$954
2023	$1,500	0.6623	$993
2024	$1,545	0.5974	$923
2025	$1,591	0.5390	$858
2026	$1,639	0.4862	$797
2027	$1,688	0.4386	$740
2028	$1,739	0.3957	$688
2029	$19,686	0.3569	$7,027
	Total Property Value		$14,778
	Year 10 Cash Flow of		$1,791
	plus reversion of		$17,895
			$19,686
	Year 11 Cash Flow of $1845		
	capitalized at 10% equals		$18,448
	Less: Selling Expenses		$553
	Equals: Net sales price		$17,895

The question is: Doesn't utilizing the Mortgage-Equity Analysis, where 75% of the discount rate is derived from mortgage interest rate data, have more support than an unlevered discount cash flow analysis with an applied 10.9% Property Yield?

Formula 2: Mortgage-Equity Analysis Using a Debt Coverage Ratio

The mortgage-equity analysis using a debt coverage ratio calculates the exact amount of debt and equity that the property will be able to support based on the anticipated cash flow derived from the forecast of net income and the return requirements of the mortgage lender (interest) and the equity investor (equity yield), to produce a specified Debt Coverage Ratio as of a specific projection year.

The algebra for the mortgage-equity analysis using a debt coverage ratio is simpler than the loan-to-value ratio because the size of the mortgage can immediately be determined. Using the

data from the previous example, the following shows how the amount of the mortgage and debt service is calculated.

Assume the mortgage lender wants a Debt Coverage Ratio of 1.3 based on Year 3 projected Net Income, which is $1,300.

The mortgage constant of .08481 is multiplied by the debt coverage ratio of 1.3, producing a mortgage cap rate:

.08481 x 1.3 = .11026

The mortgage amount is calculated by dividing the Year 3 projected Net Income by the mortgage cap rate:

$1,300/.11026 = $11,791

The debt service can then be calculated:

$11,791 x .08481 = $1,000

Once the mortgage amount and debt service are determined, the net income to equity plus the equity residual can be calculated and discounted to the present value at the equity yield rate and added to the mortgage amount to produce the total property value.

The overall equation for the Mortgage-Equity Analysis Using a Debt Coverage Ratio is as follows:

Terms:

Debt Coverage Ratio:	DCR
Year (N) to Achieve the DCR:	DCR^n
Net Income for DCR^n:	NI^{DCR}
Initial Mortgage Balance:	B^0

Initial Mortgage Balance (Value of the Mortgage Component):

$$NI^{DCR}/(f \times DCR) = B^0$$

Mortgage-Equity Analysis Using a Debt Coverage Ratio Equation:

$$B^0 + ((NI' - (f \times B^0))\ 1/S^1) + ((NI^2 - (f \times B^0))\ 1/S^2) + \ldots$$
$$\ldots + ((NI^{10} - (f \times B^0))\ 1/S^{10}) +$$
$$\ldots + (NI''/ R_r) - (b(NI''/ R_r)) - ((1 - P) \times B^0)\ 1/S^{10}) = V$$

Example

Using all the Projected Net Income Data and the Mortgage and Equity Data from the previous example the property will be valued using the Mortgage-Equity Analysis using a Debt Coverage Ratio rather than the Loan-to-Value Ratio.

Assume the Mortgage Lender is sizing the loan so a debt coverage ratio of 1.3 is achieved in the third year. The projected third-year Net Income is $1,300.

The mortgage constant of .08481 is multiplied by the debt coverage ratio of 1.3 producing a mortgage cap rate:

.08481 x 1.3 = .11026

The initial mortgage balance (or the value of the mortgage component) is calculated by dividing the Year 3 projected Net Income by the mortgage cap rate:

$1,300/.11026 = $11,791

The debt service can then be calculated by multiplying the initial mortgage balance by the mortgage constant:

$11,791 x .08481 = $1,000

The net income to equity after the payment of debt service is discounted at the desired equity yield rate of 18%.

Discounting the Cash Flow and Equity Residual to Present Value

EXHIBIT 16

	Net Income		Debt Service		NI to Equity		Equity Yield		
1	$1,000	-	$1,000	=	$0	x	0.84746	=	$0
2	$1,100	-	$1,000	=	$100	x	0.71818	=	$72
3	$1,300	-	$1,000	=	$300	x	0.60863	=	$183
4	$1,500	-	$1,000	=	$500	x	0.51579	=	$258
5	$1,545	-	$1,000	=	$545	x	0.43711	=	$238
6	$1,591	-	$1,000	=	$591	x	0.37043	=	$219
7	$1,639	-	$1,000	=	$639	x	0.31393	=	$201
8	$1,688	-	$1,000	=	$688	x	0.26604	=	$183
9	$1,739	-	$1,000	=	$739	x	0.22546	=	$167
10	$1,791	-	$1,000	=	$791	x	0.19106	=	$151
					Discounted Net Income to Equity				$1,671

Value of the Reversion $1,845/.10 =	$18,448
Less Broker and Legal: $18,448 x .03	$553
	$17,895
Ending Mortgage Balance (1 - .21367) = .78633 x $11,791 =	$9,271
Equity Residual	$8,623
Discounted Value of Equity Residual ($8,623 x .19106) =	$1,648
Value of Equity Component ($1,671 + $1,648) =	$3,319
Original Mortgage Balance	$11,791
Total Property Value	$15,109

Proof:

Exhibit 17 shows the Net Income, Mortgage Debt Service, and Equity Dividend. It also verifies the 1.3 Debt Coverage Ratio in Year 3.

EXHIBIT 17

Year	1	2	3	4	5	6	7	8	9	10
Total Property	$1,000	$1,100	$1,300	$1,500	$1,545	$1,591	$1,639	$1,688	$1,739	$1,791
Mortgage	$1,000	$1,000	$1,000	$1,000	$1,000	$1,000	$1,000	$1,000	$1,000	$1,000
Equity	$0	$100	$300	$500	$545	$591	$639	$688	$739	$791
Debt Coverage Ratio	1.00	1.10	1.30	1.50	1.55	1.59	1.64	1.69	1.74	1.79

Exhibit 18 shows the Total Property Value, and the value of the Mortgage and Equity Components. Because the size of the mortgage was determined by the debt coverage ratio rather than the loan-to-value ratio the mortgage equates to 78% of the total property value. The last column shows the mortgage component having a yield of 7% (which is the assumed interest rate) and the equity receiving an 18% equity yield.

EXHIBIT 18

10-Yr DCR Model:	Value $(000)	% of Total Value	IRR
Value of the Property	$15,109	100.0%	10.51%
Value of the Mortgage Component	$11,791	78.0%	7.00%
Value of the Equity Component	$3,319	22.0%	18.00%

Exhibit 19 shows each component receiving the desired rate of return.

EXHIBIT 19

	Total Property Present Value				Mortgage Component Present Value				Equity Component Present Value			
Year	Net Income	PV Factor @ 10.51%	Discounted Cash Flow	Year	Mortgage Payment	PV Factor @ 7.00%	Discounted Cash Flow	Year	Net Income to Equity	PV Factor @ 18.00%	Discounted Cash Flow
2020	$1,000	0.9049	$905	2020	$1,000	0.9352	$935	2020	$0	0.8475	$0
2021	$1,100	0.8188	$901	2021	$1,000	0.8746	$875	2021	$100	0.7182	$72
2022	$1,300	0.7409	$963	2022	$1,000	0.8179	$818	2022	$300	0.6086	$183
2023	$1,500	0.6704	$1,006	2023	$1,000	0.7649	$765	2023	$500	0.5158	$258
2024	$1,545	0.6066	$937	2024	$1,000	0.7153	$715	2024	$545	0.4371	$238
2025	$1,591	0.5489	$874	2025	$1,000	0.6690	$669	2025	$591	0.3704	$219
2026	$1,639	0.4967	$814	2026	$1,000	0.6256	$626	2026	$639	0.3139	$201
2027	$1,688	0.4494	$759	2027	$1,000	0.5851	$585	2027	$688	0.2660	$183
2028	$1,739	0.4067	$707	2028	$1,000	0.5472	$547	2028	$739	0.2255	$167
2029	$19,686	0.3680	$7,244	2029	$10,271	0.5117	$5,256	2029	$9,414	0.1911	$1,799
	Total Property Value		$15,109		Mortgage Component Value		$11,791		Equity Component Value		$3,319
	Year 10 Cash Flow of		$1,791		Year 10 mort. payment of		$1,000		Year 10 net inc. to equity o		$791
	plus reversion of		$17,895		plus the RMB of		$9,271		plus the equity residual of		$8,623
			$19,686				$10,271				$9,414
	Year 11 Cash Flow of $1845										
	capitalized at 10% equals		$18,448		The reversion is the remaining mortgage				Net Sales Price		$17,895
	Less: Selling Expenses		$553		balance (RMB) of the loan at the end				Less: RMB		$9,271
	Equals: Net sales price		$17,895		of year 10.				Equals: Equity Residual		$8,623

When a buyer looks at a potential acquisition of a commercial property the size of the mortgage becomes the critical factor. A debt coverage rate is one way a lender can size a loan. Once a lender provides guidance as to the debt coverage ratio, the buyer can determine how much can be borrowed and apply the mortgage-equity analysis using a debt coverage ratio to

determine the price that can be paid. The precision of this analysis cannot be matched by utilizing an unlevered discounted cash flow analysis with a property yield rate.

Formula 3: Mortgage-Equity Analysis Using a Debt Yield

The mortgage-equity analysis using a debt yield is the method that currently probably best reflects the actions of typical real estate buyers. The mortgage-equity analysis using a debt yield calculates the exact amount of debt and equity that the property will be able to support based on the anticipated cash flow derived from the forecast of net income and the return requirements of the mortgage lender (interest) and the equity investor (equity yield) to produce a specified Debt Yield as of a specific projection year.

The algebra for the mortgage-equity analysis using a debt yield is similar to the mortgage-equity analysis using a debt coverage ratio because the size of the mortgage can immediately be determined. Using the data from the previous example, the following shows how the amount of the mortgage and debt service is calculated.

Assume the Mortgage Lender is sizing the loan so a debt yield of 11% is achieved in the third year. The projected third-year Net Income is $1,300.

The initial mortgage balance is:

The third-year Net Income of $1,300 divided by the debt yield of 11%:

$1,300/.11 = $11,818

The debt service can then be calculated by multiplying the initial mortgage balance by the mortgage constant:

$11,818 x .08481 = $1,002

The remaining mortgage-equity analysis calculations are the same as those used with the debt coverage ratio.

Proof

Exhibit 20 shows the Net Income, Mortgage Debt Service, and Equity Dividend. It also verifies the 11% Debt Yield in Year 3.

EXHIBIT 20

Year	1	2	3	4	5	6	7	8	9	10
Total Property	$1,000	$1,100	$1,300	$1,500	$1,545	$1,591	$1,639	$1,688	$1,739	$1,791
Mortgage	$1,002	$1,002	$1,002	$1,002	$1,002	$1,002	$1,002	$1,002	$1,002	$1,002
Equity	($2)	$98	$298	$498	$543	$589	$637	$686	$737	$789
Debt Coverage Ratio	1.00	1.10	1.30	1.50	1.54	1.59	1.64	1.68	1.73	1.79
Debt Yield	8.46%	9.31%	11.00%	12.69%	13.07%	13.47%	13.87%	14.29%	14.71%	15.16%

Exhibit 21 shows the Total Property Value, and the value of the Mortgage and Equity Components. Because the size of the mortgage was determined by the debt yield rather than the loan-to-value ratio the mortgage equates to 78.2% of the total property value. The last column shows the mortgage component yielding 7% (which is the assumed interest rate) and the equity yielding 18%.

EXHIBIT 21

10-Yr Debt Yield Model:	Value $(000)	% of Total Value	IRR
Value of the Property	$15,122	100.0%	10.50%
Value of the Mortgage Component	$11,818	78.2%	7.00%
Value of the Equity Component	$3,304	21.8%	18.00%

Exhibit 22 shows each component receiving the desired rate of return.

EXHIBIT 22

	Total Property Present Value				Mortgage Component Present Value				Equity Component Present Value			
Year	Net Income	PV Factor @ 10.50%	Discounted Cash Flow	Year	Mortgage Payment	PV Factor @ 7.00%	Discounted Cash Flow	Year	Net Income to Equity	PV Factor @ 18.00%	Discounted Cash Flow
2020	$1,000	0.9050	$905	2020	$1,002	0.9352	$937	2020	($2)	0.8475	($2)
2021	$1,100	0.8190	$901	2021	$1,002	0.8746	$877	2021	$98	0.7182	$70
2022	$1,300	0.7411	$963	2022	$1,002	0.8179	$820	2022	$298	0.6086	$181
2023	$1,500	0.6707	$1,006	2023	$1,002	0.7649	$767	2023	$498	0.5158	$257
2024	$1,545	0.6070	$938	2024	$1,002	0.7153	$717	2024	$543	0.4371	$237
2025	$1,591	0.5493	$874	2025	$1,002	0.6690	$671	2025	$589	0.3704	$218
2026	$1,639	0.4971	$815	2026	$1,002	0.6256	$627	2026	$637	0.3139	$200
2027	$1,688	0.4499	$759	2027	$1,002	0.5851	$586	2027	$686	0.2660	$182
2028	$1,739	0.4071	$708	2028	$1,002	0.5472	$548	2028	$737	0.2255	$166
2029	$19,686	0.3684	$7,253	2029	$10,295	0.5117	$5,268	2029	$9,390	0.1911	$1,794
	Total Property Value		$15,122		Mortgage Component Value		$11,818		Equity Component Value		$3,304
	Year 10 Cash Flow of		$1,791		Year 10 mort. payment of		$1,002		Year 10 net inc. to equity of		$789
	plus reversion of		$17,895		plus the RMB of		$9,293		plus the equity residual of		$8,602
			$19,686				$10,295				$9,390
	Year 11 Cash Flow of $1845										
	capitalized at 10% equals		$18,448		The reversion is the remaining mortgage				Net Sales Price		$17,895
	Less: Selling Expenses		$553		balance (RMB) of the loan at the end				Less: RMB		$9,293
	Equals: Net sales price		$17,895		of year 10.				Equals: Equity Residual		$8,602

Conclusion

Ellwood, Akerson, Kinnard, and Gibbons, several of the great minds of the real estate appraisal profession, all saw mortgage-equity analysis as the preferred method for valuing real property typically acquired with both mortgage and equity financing. Because they were constrained to models that assumed specific flows of net income (flat, straight-line, and exponential growth), mortgage-equity analysis lost out to the more-simple property yield discounting procedure, which inherently assumed an all-cash buyer. Now with better computing power and Suzanne Mellen's algebraic equation, the mortgage-equity analysis can handle any type of income flow for all forms of commercial properties; the results are highly supported with market data and truly reflect the actions of typical buyers.