A new measure to inform policy on access to surgery

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This technical report presents a new measure to inform policy on access to surgery for hip fracture. While having surgery on admission or the following day has been shown to reduce postoperative mortality, this average effect does not account for patients who do not benefit or are not affected by timing. The proposed measure indicates, instead, how often a patient would benefit from early surgery and be harmed by delayed surgery. This makes it a valuable tool for policy makers, allowing for policy evaluation within patient subgroups.

1 Introduction

After breaking a hip, older adults face a high risk of death: 30% die within a year, with 7% dying during hospitalization. Delays in repairing a hip fracture can increase mortality rates by prolonging exposure to inflammation and immobilization. A 2018 study examined the medical records of 140,000 patients 65 years or older who underwent surgery for hip fractures between 2004 and 2012 in Canada.[1] The study found that having surgery on admission day or the following day reduced postoperative mortality in this patient population. If all surgeries were performed within two days, there would be a projected reduction of eight deaths for every 1,000 surgeries.

This estimate provides information on the average effect of timing for hip fracture surgery on the patient population. Lizaur-Utrilla et al argue that there is no single optimal timing that is appropriate for all patients because of variations in injury, care requirements, and individual characteristics.[2] Our study supports this claim, as we found significant differences in the magnitude of mortality reduction in the early surgery group across strata of the study population. However, the numbers indicating reduction in mortality still represent the average effect within the strata. They are not representative of patients who who did not benefit from early surgery and those who were not affected by the timing of the surgery.

This technical report explores the methods for calculating how often the same patient would have benefited if the surgery was performed early and would not have benefited if the surgery was delayed.

2 Probability of individual effects

In health research, randomized experiments and observational studies aim to recover the average treatment effect (ATE) . This target quantity describes changes in the summary outcome measure as if the same population of patients would have received different treatments.

Mueller and co-authors addressed a different question: how likely is it that the same patient would have a favorable outcome with treatment and an unfavorable outcome without treatment.[3] The authors related it to the probability that treatment is a necessary and sufficient cause of the outcome (PNS) presented earlier.[4]

To better understand the difference between the two target quantities, let us imagine the possible value that the outcome variable Y would have if a specific unit u were assigned to receive treatment $X=x$ in isolation from any other factors that might influence treatment choice. We will refer to this value as the possible outcome of treatment x for unit u and denote it as $Y^x(u)$. Considering the event $\{Y^x=y\}:=\{u\colon Y^x(u)=y\}$, the probability of outcome y in the population of units U that were to receive the same treatment x is found as

$$\begin{aligned} P(Y^x=y) &= E(\mathbb{1}(Y^x=y)) \\ &= \sum_{u\in U} \mathbb{1}(Y^x(u)=y)\,P(u). \end{aligned}$$

In other words, it is calculated by aggregating the probability mass $P(u)$ across all units u that are consistent with the realization $Y^x(u)=y$.

We say the variable X has a causal effect on the variable Y in *the population* if there exists two distinct values x_0 and x_1 such that the distribution $P(Y^{x_0}=y)$ differs from the distribution $P(Y^{x_1}=y)$. The difference between their means is called the average treatment effect (ATE). When the outcome variable takes only two values the average treatment effect takes a simple form

$$\mathrm{ATE} = P(Y^1=1) - P(Y^0=1).$$

This target quantity is the treatment effect on the population.

In contrast, the individual treatment effect is unit-specific. Mill reasons that the effect of a treatment is the difference in the state of two identical units after treating only one.[5] A century later, Neyman considers different treatments on the same unit and then defines the causal effect as the difference between their possible outcomes[6]

$$\mathrm{ITE}(u) = Y^1(u) - Y^0(u).$$

When the potential outcome is 1 for a specific unit u receiving treatment and 0 without treatment, we say the unit is causally affected by the treatment. We can express this dichotomy as the conjunction of two events, $\{Y^1(u)=1\}$ and $\{Y^0(u)=0\}$. They are not observed at the same time in a specific unit. However, the probability of event

$$\{Y^1=1, Y^0=0\}:=\{u\colon Y^1(u)=1 \cap Y^0(u)=0\}$$

is well defined as

$$P(Y^1=1, Y^0=0) = \sum_{u\in U} \mathbb{1}(Y^1(u)=1 \cap Y^0(u)=0)\,P(u).$$

This probability tells us how likely it is that a treatment is necessary and sufficient cause of a particular outcome (PNS). It describes the proportion of units for which the outcome occurs only if the unit receives the treatment, and does not occur if it does not. Considering the dichotomy of the possible outcomes $Y^1(u)$ and $Y^0(u)$ in a unit u the probability of a necessary and sufficient cause could be presented as

$$\mathrm{PNS} := P(Y^1 - Y^0 = 1).$$

We cannot determine this probability directly from experimental data. However, Tian and Pearl found its bounds using observational and experimental data.[7] Mueller et al. refined these bounds by considering factors that collectively block all non-causal paths between treatment and outcome.[3]

3 PNS bounds after stratification

In the population stratified by values of a covariate Z

$$\begin{aligned} \mathrm{PNS} &= \sum_z P(Y^1=1, Y^0=0 \mid Z=z)\,P(z) \\ &= \sum_z \mathrm{PNS}(z)\,P(z). \end{aligned}$$

Here, z-specific PNS refers to all units $u\in U$ that have the same value z of Z.

Let us consider covariates Z that have the power, directly or through a chain of dependencies, to influence what treatment units receive and what outcomes they produce. Pearl established that attribution of variation in outcome to changes in exposure could be achieved by conditioning on a set of factors sufficient for blocking all biasing influences between exposure and outcome.[8] In particular, if Z is a set of covariates satisfying the backdoor criterion, then in each stratum $Z=z$ we have

$$P(Y^x=y \mid z) = P(Y=y \mid X=x, z).$$

For z-specific PNS, Mueller et al. identified the bounds

$\text{LB} \le \text{PNS}(z) \le \text{UB}$ as follows:[3]

$$\text{LB} = \max\{0, P(Y=1 \mid X=1,z) - P(Y=1 \mid X=0,z)\},$$
$$\text{UB} = \min\{P(Y=1 \mid X=1,z), P(Y=0 \mid X=0,z)\}.$$

We are ready now to apply the methods described above to estimate the proportion of patients each of whom would benefit if treated early but would not benefit if the surgery was delayed.

4 Strata of hip fracture patients

We use a binary variable, X, to represent the occurrence of delayed surgery ($X=1$) or early surgery ($X=0$). Another binary variable, Y, is used to denote in-hospital death ($Y=1$). A categorical variable Z represents the 64 multifactorial strata constructed by combining five factors: treatment era (2004–2007, 2008–2012), hospital type (teaching, community), procedure type (bone fixation, arthroplasty), age at admission (65–84 years, 85 years and older), and prefracture health status (admitted from home without comorbidity, admitted from home with comorbidity or with home care services, admitted from a long-term care facility, or admitted from other care facility). We chose these factors from a causal diagram encoding all known dependencies among factors that directly or indirectly influence surgical timing and mortality.[1] Using the backdoor criterion, we concluded that stratifying on Z would be sufficient to render timing and mortality independent in the absence of causal connection between them.[4]

Figure 1 shows the PNS bounds in 32 strata relative to the overall ATE of eight deaths per 1000 surgeries. The upper bound coincides with the mortality rate in the delayed surgery group, $P(Y=1 \mid X=1,z)$. The lower bound coincides, when it is not zero, with the stratum-specific ATE, $P(Y=1 \mid X=1,z) - P(Y=1 \mid X=0,z)$. The dots show the mortality rate in the delayed and early surgery groups.

Our study reveals that the mortality effect of surgical timing varies across the strata. As depicted in the figure, three principal patterns emerged across the strata: the probability of in-hospital death in the early surgery group was either between the bounds, below the lower bound, or above the upper bound. Patients aged 85 years and older benefited more from early surgery than their younger counterparts. Patients admitted from home care or from home with major comorbidity experienced the most pronounced benefit.

5 Main message

Our study identifies the probability bounds for the *causal* reduction in mortality resulting from early surgery. We emphasize the causal interpretation of the estimate because of its connection to the individual treatment effect, which refers to a favorable outcome when treated and an unfavorable outcome when untreated for the same individual.

Previous literature has reported on the average effect of timing for *all* patients, but this measure conceals the fact that early surgery benefits some patients and has no effect on others. We have estimated the bounds for the frequency with which patients would fare better with early surgery and worse without it in strata created by combining factors that prevent other known factors from producing covariation between the timing and outcome of surgical treatment for broken hips. We have found that the proportion of patients with an individual treatment effect varies across strata, and in some cases, the benefit of early surgery substantially exceeds the overall ATE.

6 Conclusion

In conclusion, this study builds on the theoretical foundation laid by Pearl in developing the probability of causation.[9] We propose a new healthcare quality indicator, the proportion of patients who would survive with early surgery but would die if surgery is delayed. By comparing this indicator with the commonly used indicator, ATE, we have demonstrated that they measure different aspects of treatment effects. While ATE captures the projected difference in outcome probabilities between treatment groups with the same patient population,

$$\text{ATE} = P(Y^1=1) - P(Y^0=1).$$

PNS measures the probability of the difference in possible outcomes of two treatments in the same patient

$$\text{PNS} = P(Y^1 - Y^0 = 1).$$

We believe that the PNS provides a more nuanced and patient-centered approach to evaluating healthcare quality and treatment effectiveness, and we encourage further research and evaluation of this indicator in practice.

References

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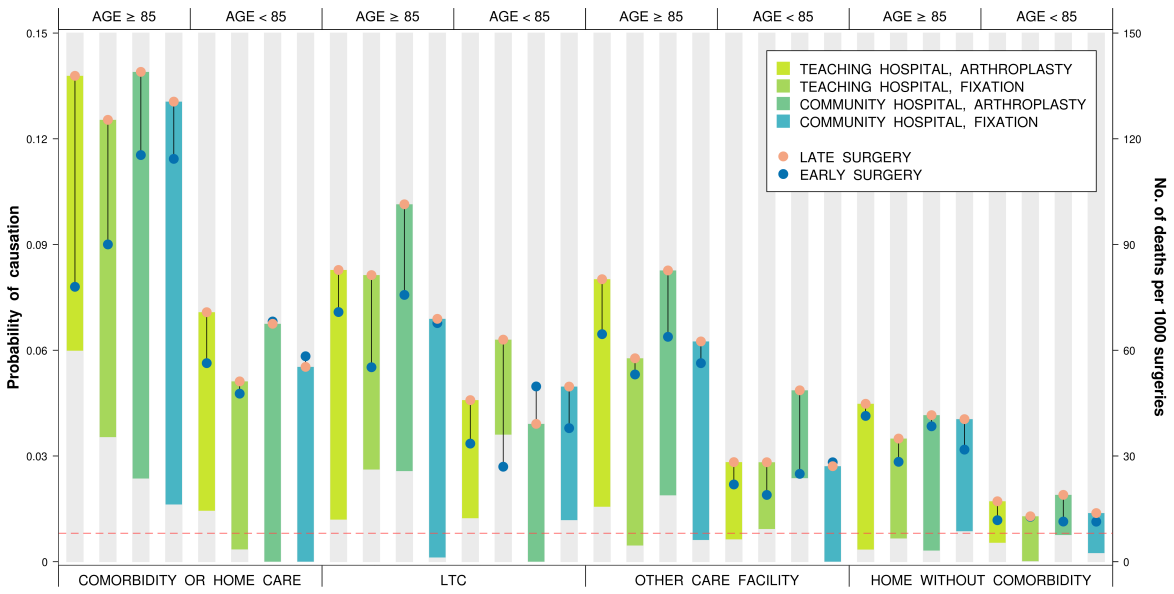

Figure 1: *The bounds of probability of causal mortality reduction due to early surgery in patients treated for hip fracture. The bars represent the range between the lower and upper bounds for the probability that the early surgery would be beneficial and the late surgery would be detrimental for the same individuals 32 strata of the recent treatment era. The dashed red lines shows the average treatment effect of early surgery in all patients.*