SARET PHYS 120 Discussion 3 ① 10/4/1

## Resonance & Fourier components (3-1)

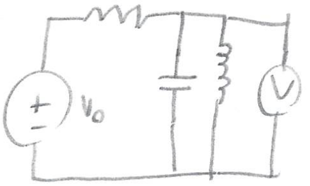
$f_0 \simeq 15\,\text{kHz}$ for easy numbers.

put in $f_{sq} = 5\,\text{kHz}$, $\sim (1/3) f_0$ so that

we have $V_{sq} = A\left(\sin(2\pi f_{sq} t) + \frac{1}{3}\sin(6\pi f_{sq} t) + \frac{1}{5}\sin(10\pi f_{sq} t)\right)$

$$|\Phi(\omega)| = \begin{bmatrix} 3.5\cdot 10^{-9} & f_{sq} & \rightarrow & \text{blocked} \\ 8.5\cdot 10^{-8} & 3f_{sq} & \rightarrow & \text{passed} \\ 1.1\cdot 10^{-8} & 5f_{sq} & \rightarrow & \text{blocked} \end{bmatrix}$$

## Decay.




$V_0 = V_R + V_C = V_R + V_L$

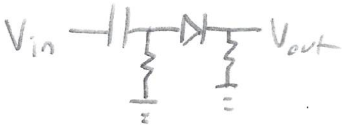
$= RI + \frac{1}{C}\int I\,dt = RI + L\dot{I}$

take time derivative

$0 = R\dot{I} + \frac{1}{C}I_C = R\dot{I} + L\ddot{I}_L$ also,

$I = I_C + I_L \Rightarrow \dot{I} = \dot{I}_C + \dot{I}_L$

## Diodes. (3-6)




Idealized diode: $V_D < 0.6$, $I = 0$

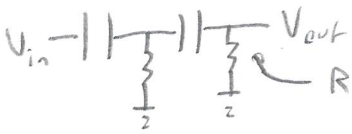
$V_D \geq 0.6$, $I =$ anything

Real diode: has some physical parameters

- IV curve
- capacitance!

→ see datasheet

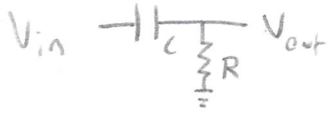
Question for class: what happens if you remove $2.2\,\text{k}\Omega$?




R=? (≠0!)

Setting up HW3 problems.

1. Transfer function. Ex. high pass from my lecture on 19/4/11.




$$\tilde{\Phi}(\omega) = R \Big/ \left(R + 1/i\omega C\right) \cdot \left(\frac{i\omega C}{i\omega C}\right) = \frac{i\omega\tau}{1+i\omega\tau}, \quad \tau = RC$$

Then, in frequency/spectral domain, $\tilde{V}_{out}(\omega) = \tilde{V}_{in}(\omega)\,\tilde{\Phi}(\omega)$

Fourier transform. (Hint is very useful!)

$$V_{in}(t) = A\sin\omega_0 t \Rightarrow \tilde{V}_{in}(\omega) = A\int_{-\infty}^{\infty} dt\, e^{-i\omega t}\sin(\omega_0 t)$$

$$= A\int_{-\infty}^{\infty} dt\, \frac{e^{i\omega_0 t} - e^{-i\omega_0 t}}{2i}\, e^{-i\omega t}$$

$$= A\frac{\pi}{i}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-i(\omega-\omega_0)t} - \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-i(\omega+\omega_0)t}\right]$$

$$= A\frac{\pi}{i}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right].$$

2. Circuit response. Table at end of Laplace 1.2 (also 19/4/11).

See Laplace transforms notes (2) attached!

Stepped input example. " 1.2 "

"

3. Fourier series of a square wave. Fourier notes Eq. 1.10+

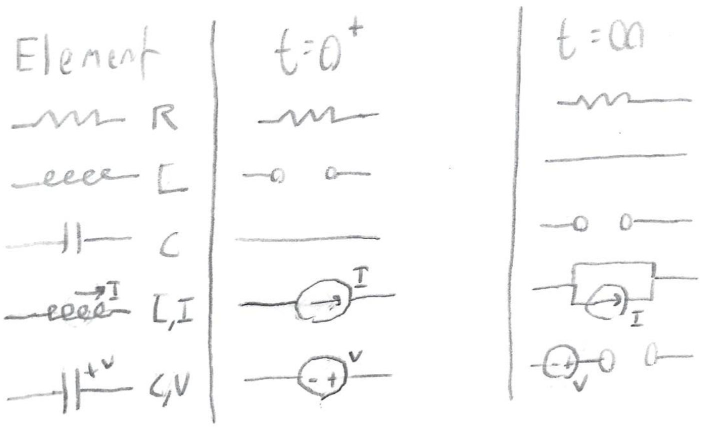
$$\tilde{C}_n = \frac{1}{T}\left[\int_{-T/2}^{0} dt\,(-1)\,e^{-in\omega_0 t} + \int_{0}^{T/2} dt\,(+1)\,e^{-in\omega_0 t}\right]$$

$$= 1/{-in\omega_0 T}\left[-e^{-in\omega_0 t}\Big|_{-T/2}^{0} + e^{-in\omega_0 t}\Big|_{0}^{T/2}\right] \qquad \omega_0 T = 2\pi, \quad \cos(n\pi) = \begin{cases} 1 & \text{odd} \\ 0 & \text{even}\end{cases}$$

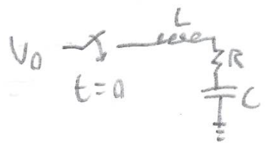
$$= \text{"}\left[-1 + e^{+in\omega_0 T/2} + e^{-in\omega_0 T/2} - 1\right] = 1/{in\omega_0 T}\left[2 - 2\cos(n\omega_0 T/2)\right] \Rightarrow \frac{4}{\pi}\cdot\left(1, \tfrac{1}{3}, \tfrac{1}{5}, \cdots\right)$$

phys 120 "guest" lecture: Laplace transforms (2) 19/4/11

Time evolution of circuit elements:




Stepped input.




$$-V_0 + L\dot{I} + RI + \frac{1}{C}\int_0^t dt' + V_C(0^-) \qquad (t)$$

Transform,

$$-V_0/s + sLI + RI + I/Cs + \frac{V_c(0^-)}{s} = 0 \qquad (s)$$

Solve,

$$I(s) = \frac{V_0}{L} \, 1/s^2+2\kappa s+\omega_0^2 \qquad \kappa = R/2L$$

$$\omega_0 = 1/\sqrt{LC}$$

with roots $a$ & $a^*$. $a = -\kappa + i\sqrt{\omega_0^2 - \kappa^2}$

$$I(s) = V_0/L \; 1/(s-a)(s-a^*).$$

$$I(t) = \frac{V_0}{L} \, 1/2\pi i \int_C ds \, \frac{e^{st}}{(s-a)(s-a^*)}$$