Amplifier refresher.
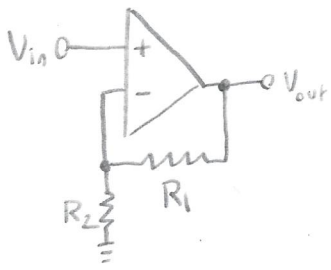

Voltage divider: $V_- = \dfrac{R_2}{R_1 + R_2} V_{out}$

Op-Amp equation: $V_{out} = A(V_+ - V_-) = A\left(V_{in} - \dfrac{R_2}{R_1 + R_2} V_{out}\right)$

Solve: $A V_{in} = \left(1 + \dfrac{A R_2}{R_1 + R_2}\right) V_{out}$

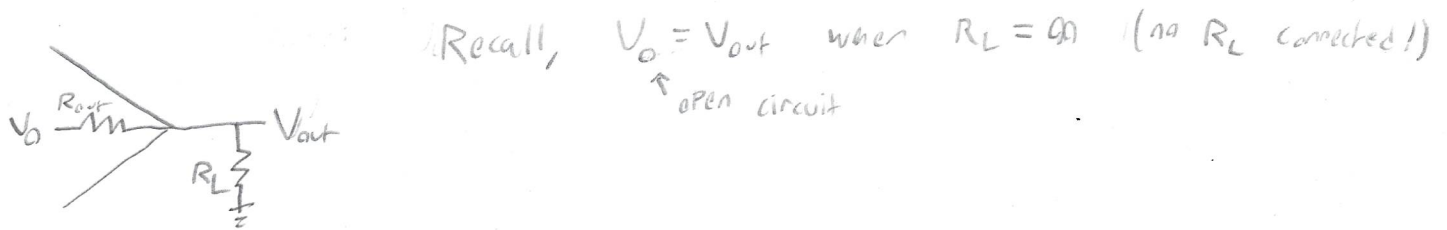$\lim A \to \infty$: $V_{in} = \dfrac{R_2 V_{out}}{R_1 + R_2} \Rightarrow V_{out} = \dfrac{R_1 + R_2}{R_2} V_{in}$

$= \left(1 + \dfrac{R_1}{R_2}\right) V_{in} = G V_{in}$

$\boxed{\text{Limit check: buffer; } R_1 = 0, R_2 = \infty \qquad G = 1 + \dfrac{0}{\infty} = 1 \quad \checkmark}$

$\Rightarrow G = 1 + \dfrac{R_1}{R_L}$

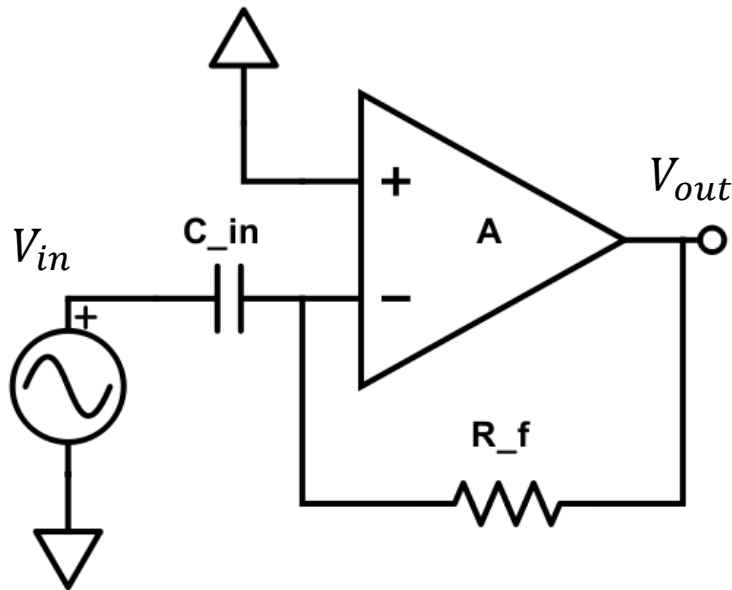## Output impedance of this amplifier.

As we've said plenty in lab, (and in the handout!) reporting a bound is sufficient. Using the resistor box of observing the (lack of) effect at small R_L should be enough or you to argue (w/ justification if you want credit!) that Z_{out} is small.

An experimental method that works quite well is to collect several non-clipped data points for various R_L, and treat it as a voltage divider:

Recall, $V_0 = V_{out}$ when $R_L = \infty$ (no R_L connected!)
          ↑
       open circuit

Op-Amp Differentiator: frequency domain analysis

V_{in} C_in V_{out}

A

$+$

$-$

R_f

$$\tilde{V}_{out} - \tilde{V}_- = I Z_f$$
$$\tilde{V}_- - \tilde{V}_{in} = I Z_{in}$$
$$V_+ = 0$$
$$\tilde{V}_{out} = \tilde{A}(\omega)\left(\tilde{V}_+(\omega) - \tilde{V}_-(\omega)\right) = -\tilde{A}(\omega)\tilde{V}_-$$
$$Z_f = R_f, Z_{in} = (i\omega C_{in})^{-1}$$

$$\left(\tilde{V}_{out} - \tilde{V}_-\right) = \frac{(\tilde{V}_- - \tilde{V}_{in})Z_f}{Z_{in}}$$
$$\Rightarrow \tilde{V}_-\left(\frac{Z_f}{Z_{in}} + 1\right) = \tilde{V}_{out} + \frac{Z_f}{Z_{in}}\tilde{V}_{in}$$
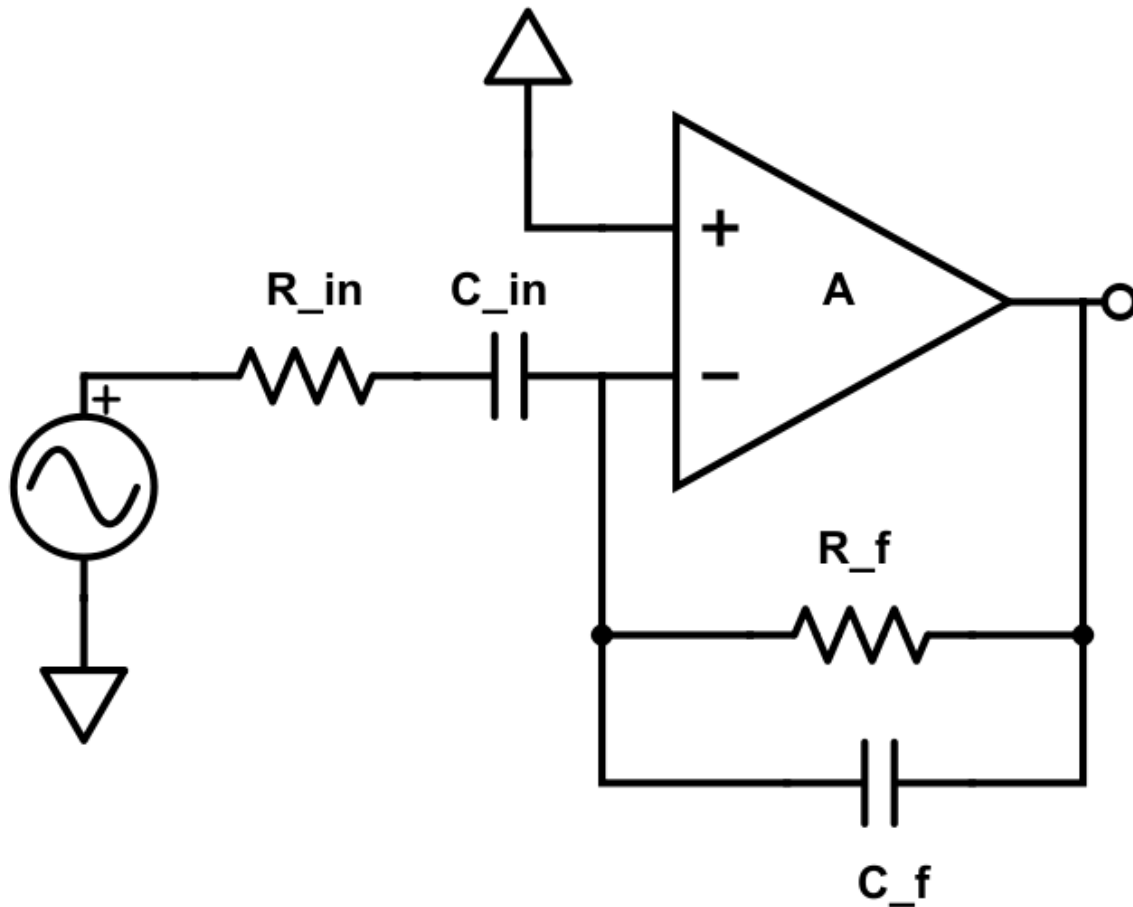
$$\tilde{V}_{out} = -\frac{\tilde{A}(\omega)\left(\tilde{V}_{out} + \frac{Z_f}{Z_{in}}\tilde{V}_{in}\right)}{\left(\frac{Z_f}{Z_{in}} + 1\right)}$$

$$\tilde{V}_{out} = \frac{-\dfrac{\tilde{A}(\omega)Z_f}{Z_f + Z_{in}}}{1 + \dfrac{\tilde{A}(\omega)Z_{in}}{Z_f + Z_{in}}}\tilde{V}_{in} = \frac{(-\tilde{A}(\omega)Z_f)}{Z_f + (1 + \tilde{A}(\omega))Z_{in}}\tilde{V}_{in}$$

Physics 120, Winter 2019, Homework 7

Due: 2018-02-28, at beginning of lab

The following problems refer to this differentiator circuit, which I introduced in lecture.

1. Derive the complex expression for the closed loop gain $\tilde{G}(\omega)$, assuming $\tilde{A}(\omega) = \tilde{A}$ is constant and large for all frequencies of interest.

$$Z_f = \left[\frac{1}{R_f} + i\omega C_f\right]^{-1} = \frac{R_f}{1 + i\omega C_f R_f}$$

$$Z_{in} = R_{in} + \frac{1}{i\omega C_{in}} = \frac{1 + i\omega R_{in} C_{in}}{i\omega C_{in}}$$

From lecture:

$$\tilde{G}(\omega) = \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = \frac{(-\tilde{A}(\omega)Z_f)}{Z_f + \left(1 + \tilde{A}(\omega)\right)Z_{in}} = \frac{-\tilde{A}\dfrac{R_f}{1 + i\omega C_f R_f}}{\dfrac{R_f}{1 + i\omega C_f R_f} + (1 + \tilde{A})\dfrac{1 + i\omega R_{in}C_{in}}{i\omega C_{in}}}$$

$$\approx \frac{R_f i\omega C_{in}}{(1 + i\omega C_f R_f)(1 + i\omega R_{in}C_{in})}$$

2. Derive the magnitude of the gain $|\tilde{G}(\omega)|$ and the phase change of the output vs input wave $\Delta\phi$. What is the behavior of each as a function of frequency in the high- and low-frequency limit?

Magnitude:

$$|\tilde{G}(\omega)| = \sqrt{\frac{R_f i\omega C_{in}}{(1 + i\omega C_f R_f)(1 + i\omega R_{in}C_{in})}\frac{-R_f i\omega C_{in}}{(1 - i\omega C_f R_f)(1 - i\omega R_{in}C_{in})}}$$

$$= \sqrt{\frac{\omega^2 R_f^2 C_{in}^2}{(1 + \omega^2 \tau_f^2)(1 + \omega^2 \tau_{in}^2)}}$$

Where I've defined $\tau_{in} = C_{in}R_{in}$ and $\tau_f = C_f R_f$.

Low- and high-frequency limits:

$$\lim_{\omega \to 0}|\tilde{G}(\omega)| = \omega R_f C_{in} \propto \omega$$

$$\lim_{\omega \to \infty}|\tilde{G}(\omega)| = \frac{\omega R_f C_{in}}{\omega R_f C_f \omega R_{in}C_{in}} = \frac{1}{\omega C_f R_{in}} \propto \frac{1}{\omega}$$

Phase:

$$\delta\phi = \mathrm{atan}\left(\frac{Imag\left(\tilde{G}(\omega)\right)}{Real\left(\tilde{G}(\omega)\right)}\right)$$

$$\tilde{G}(\omega) = \frac{R_f i\omega C_{in}}{(1 + i\omega C_f R_f)(1 + i\omega R_{in}C_{in})}\frac{(1 - i\omega C_f R_f)(1 - i\omega R_{in}C_{in})}{(1 - i\omega C_f R_f)(1 - i\omega R_{in}C_{in})}$$

$$= \frac{i\omega R_f C_{in}(1 - i\omega \tau_f - i\omega \tau_{in} - \omega^2 \tau_{in}\tau_f)}{(1 + \omega^2 \tau_f^2)(1 + \omega^2 \tau_{in}^2)} = \frac{\omega^2 R_f C_{in}(\tau_f + \tau_{in}) + i\omega R_f C_{in}(1 - \omega^2 \tau_{in}\tau_f)}{(1 + \omega^2 \tau_f^2)(1 + \omega^2 \tau_{in}^2)}$$

$$\delta\phi = \mathrm{atan}\left(\frac{\omega R_f C_{in}(1 - \omega^2 \tau_{in}\tau_f)}{\omega^2 R_f C_{in}(\tau_f + \tau_{in})}\right) = \mathrm{atan}\left(\frac{(1 - \omega^2 \tau_{in}\tau_f)}{\omega(\tau_f + \tau_{in})}\right)$$

Low- and high-frequency limits:

$$\lim_{\omega \to 0}\Delta\phi = \mathrm{atan}\left(\frac{1}{\omega(\tau_f + \tau_{in})}\right) \to 0$$

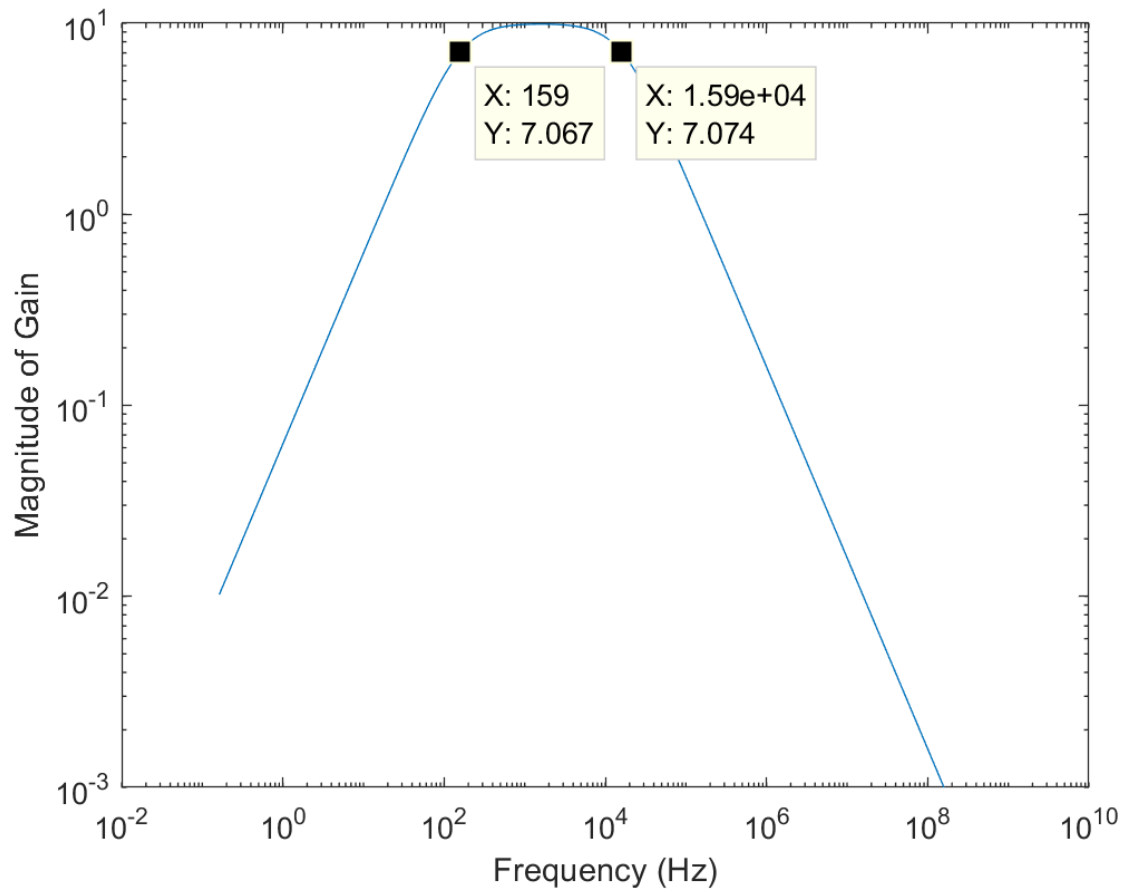$$\lim_{\omega \to \infty}\Delta\phi = \mathrm{atan}\left(\frac{\omega \tau_{in}\tau_f}{\tau_{in} + \tau_f}\right) \to \frac{\pi}{2}$$

3. Make two plots with frequency on the horizontal axis, one for magnitude (log-log plot) and one for phase (linear-log plot). Make the plots for $R_{in} = 10k\Omega$, $C_{in} = 0.1\ \mu F$, $R_f = 100\ k\Omega$, $C_f =$

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$100 \ pF$. Identify the frequencies such that $2\pi f R_{in} C_{in} = 1$, and $2\pi f R_f C_f = 1$, and show them on the plot.

$$2\pi f R_{in} C_{in} = 1 \Rightarrow f = 159 \ Hz$$
$$2\pi f R_f C_f = 1 \Rightarrow f = 1.59 \cdot 10^4 \ Hz$$

X: 159
Y: 7.067

X: 1.59e+04
Y: 7.074

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