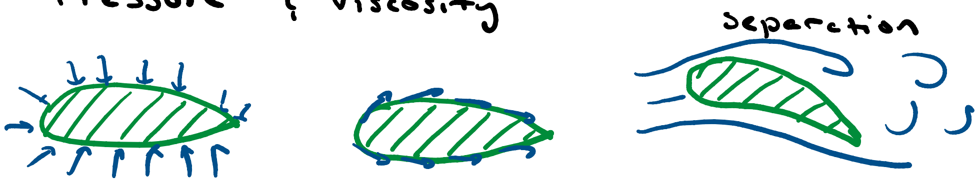
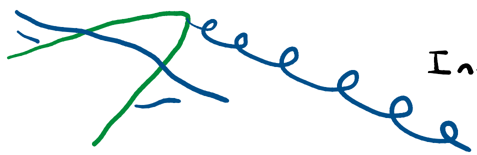
# Lecture 15: Finite wing effects

Friday, October 30, 2020 10:50 PM

**Last Time:** Drag: Pressure & Viscosity



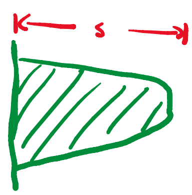

**Today:** Finite wing effects



Induced drag / downwash / effective a.o.a.

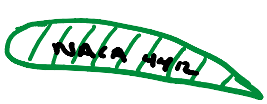
---

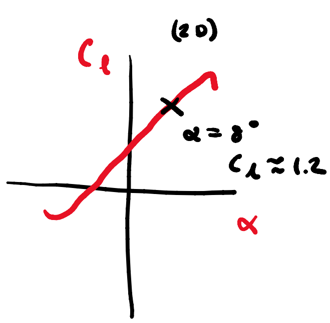
## (N) Finite Wings
(physical effects)

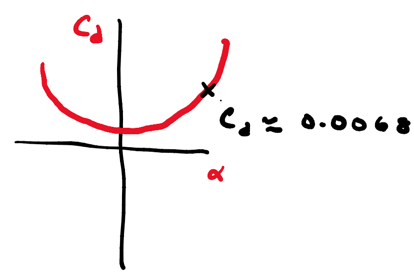



Beginning to think about
REAL (3D) not IDEAL (2D)

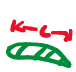
Consider a NACA 4412 foil



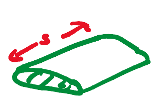



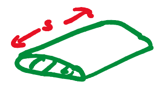




Say we go from 2D




3D



Say we go from 2D  to 3D 




**2D** $\alpha = 8^\circ$

$C_l = 1.2$
$C_d = 0.0068$

} these are "per-unit-span"

> Recall,
>
> $C_l = \dfrac{L'}{\frac{1}{2}\rho U_\infty^2 c}$ (Lift per unit span)
>
> $C_L = \dfrac{L}{\frac{1}{2}\rho U_\infty^2 s c}$

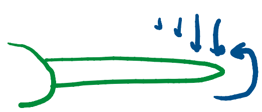
**3D** $C_l = C_L$ ? ⇒ **NO** !

$C_d = C_D$

"Per unit span" are IDEAL & assume $s = \infty$

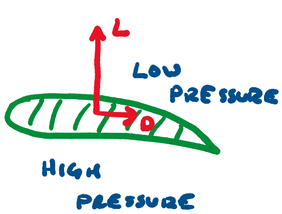
⇒ LIFT & DRAG come from 2D stuff

In REAL cases, $s \neq \infty$, leads to finite-span effects:



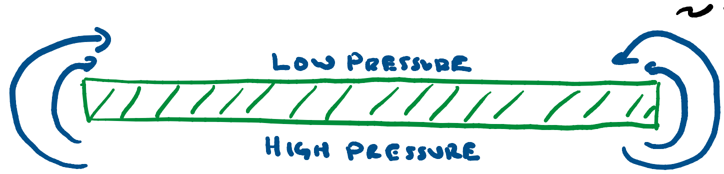
DOWNWASH / EFFECTIVE A.O.A / INDUCED DRAG

Consider a foil



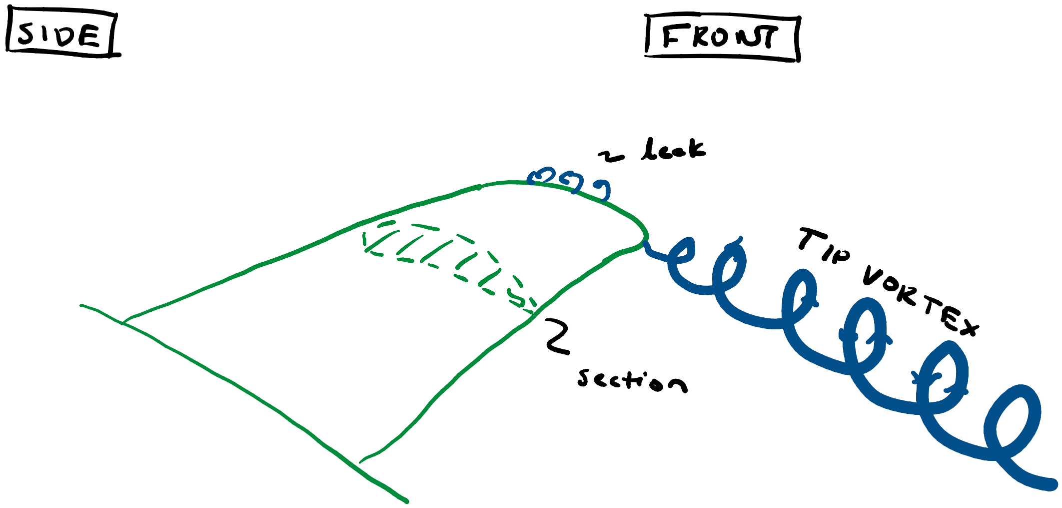


SIDE

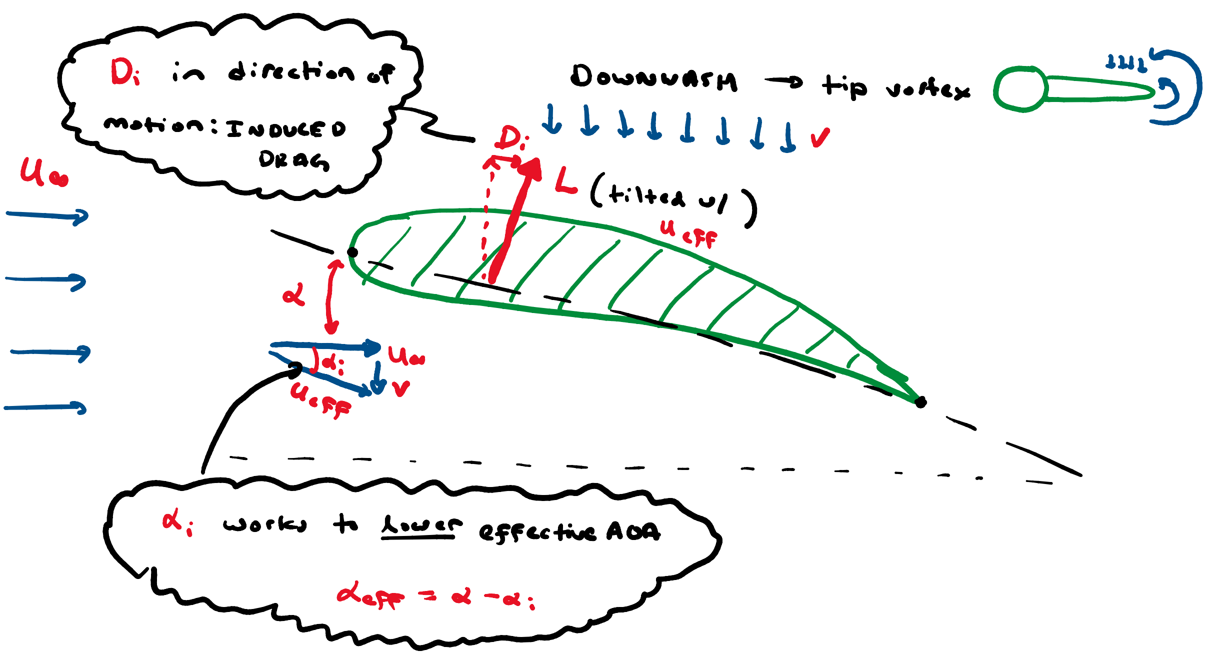




FRONT

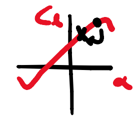



Consider a SECTION near the tip



**DOWNWASH** leads to:

(1) Change in $\alpha$ locally

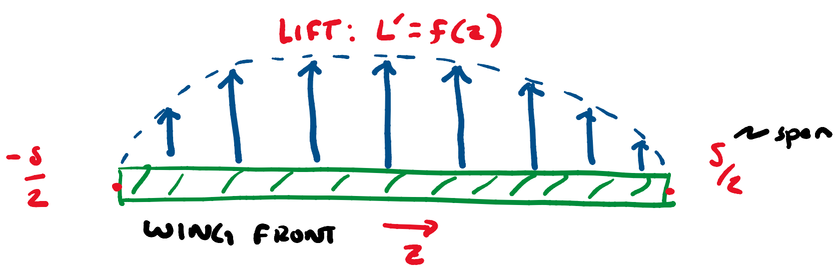



(2) Portion of LIFT in direction of motion becomes INDUCED DRAG $C_D$:

For a finite wing, drag comes from 3 PLACES

- SKIN FRICTION (viscous)
- SEPARATION (pressure)
- INDUCED (pressure)

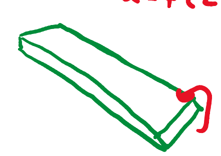
**DOWNWASH** also leads to **LIFT DISTRIBUTION**



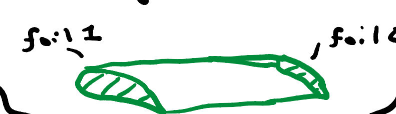

NOTE: other things cause $L' = f(z)$

- chord variation


- geometric twist $\alpha = f(z)$



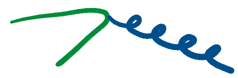
- aerodynamic twist




KUTTA - JOUKOWSKI: if $L' = f(z)$
($L' = \rho U_\infty \Gamma$)

$\Rightarrow \Gamma = g(z)$

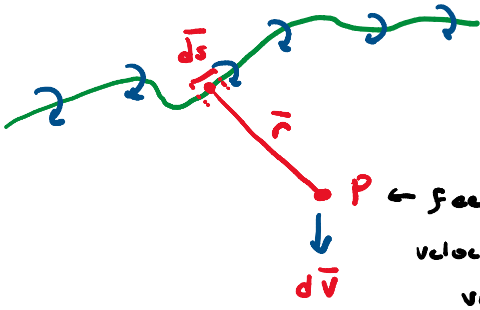
Since this is all from VORTEX

Since this is all from VORTEX 

let's explore them

## (1) BIOT - SAVART LAW : understanding semi-infinite vortices (i.e. tip vortex)

Consider a VORTEX FILAMENT




circulation of entire filament (const.)

$$d\bar{V} = \frac{\Gamma}{4\pi} \frac{\bar{ds} \times \bar{r}}{|\bar{r}^3|}$$

General : also found in electromagnetism

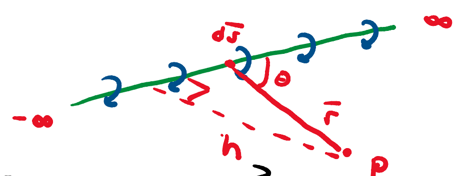
What if VORTEX is straight from $-\infty \rightarrow +\infty$

"INFINITE VORTEX"

add up BIOT - SAVART along path

$$\bar{V} = \int_{-\infty}^{\infty} \frac{\Gamma}{4\pi} \frac{\bar{ds} \times \bar{r}}{|\bar{r}^3|}$$

Cross product
$\bar{A} \times \bar{B} = |A||B| \sin\theta$




h ≥ perpendicular to VORTEX ↔ P

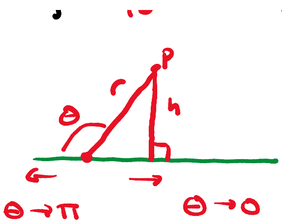
we just want magnitude $V = |\bar{V}|$

$$V = \frac{\Gamma}{4\pi} \int_{-\infty}^{\infty} \frac{\sin\theta}{r^2} ds$$

Change $r, s$ to $h, \theta$


$r = \frac{h}{\sin\theta}$

$$V = \frac{\Gamma}{4\pi}\int_{-\infty} \frac{\sin\theta}{r^2}\,ds$$




$r = \frac{h}{\sin\theta}$

$s = \frac{h}{\tan\theta}$

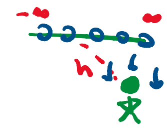
$\Rightarrow ds = -\frac{h}{\sin^2\theta}\,d\theta$

$$V = -\frac{\Gamma}{4\pi h}\int_{\pi}^{0} \sin\theta\, d\theta$$

$$\boxed{V = \frac{\Gamma}{2\pi h}}$$

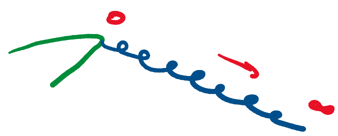
infinite line-vortex

← how you feel standing distance $h$ from




what if the VORTEX goes from $0 \to \infty$

"semi-infinite vortex"

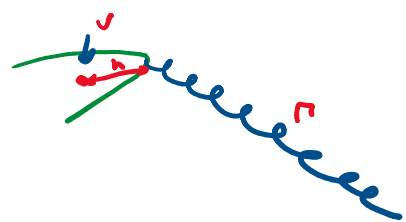



$$V = -\frac{\Gamma}{4\pi}\int_{\pi}^{\pi/2} \sin\theta\, d\theta$$

$$= \frac{\Gamma}{4\pi}\cos\theta\Big|_{\pi}^{\pi/2}$$

$$\boxed{V = \frac{\Gamma}{4\pi h}}$$

semi-infinite vortex




There are RULES to VORTEX FILAMENTS

## HELMHOLTZ VORTEX THEOREM

(1) The STRENGTH of filament CONSTANT along length.

(2) VORTICES cannot just STOP in fluid

- must terminate @ boundary 

- must terminate @ boundary 

-or- loop  (vortex ring)

(3) If nothing causes ROTATION externally, IRROTATIONAL stays that way

Goal moving forward: PREDICT / CALCULATE

INDUCED DRAG $C_{D_i}$

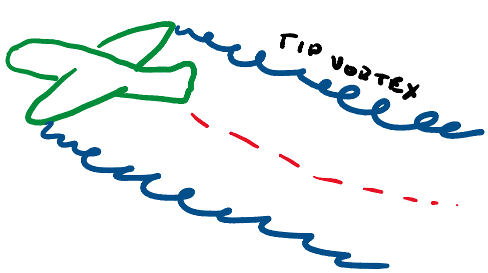
LIFT $L$

LIFT DISTRIBUTIONS $L'(z)$

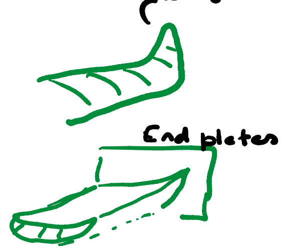
... for finite wings.

---

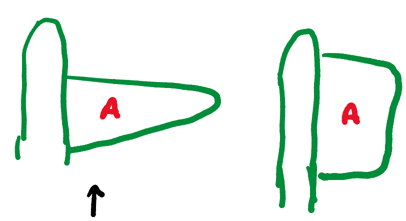
In PRACTICE, tip vortices & induced drag are important




• Dictate TAKEOFF/LANDING schedules at airports




• Avoidance is used (flow control)




• leads to higher ASPECT RATIOS