Lecture 3: Aerodynamic forces/moments

Saturday, September 5, 2020 9:26 PM

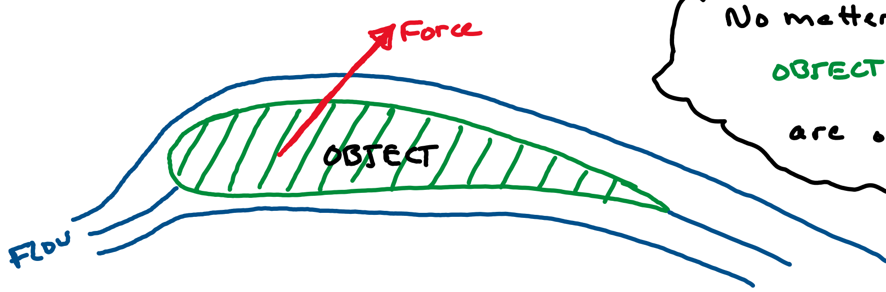
LAST TIME: Physical interpretation of AERO VARIABLES

$$\boxed{u,\ P,\ \rho,\ T,\ \mu}$$

TODAY: How these flow properties generate BODY FORCES/MOMENTS

C. AERODYNAMIC $\vec{\text{FORCES}}$/MOMENTS

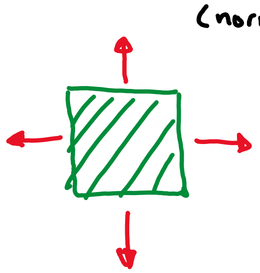
↳ An OBJECT in a FLOW feels FORCE



No matter how complicated our OBJECT/FLOW are, there are only 2 basic FORCES

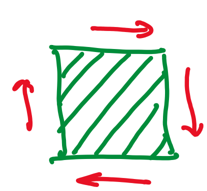
$\boxed{\text{PRESSURE}}$ P

(normal)

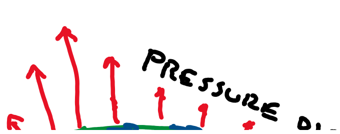


$\boxed{\text{SHEAR STRESS}}$, $\tau = \mu \frac{\partial u}{\partial y}$

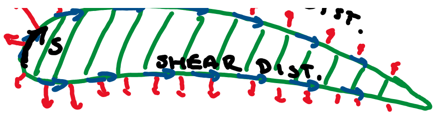
(tangential)



Complex surfaces make complex distributions



PRESSURE: $P(s)$ ← function of distance along s



SHEAR: $\tau(s)$

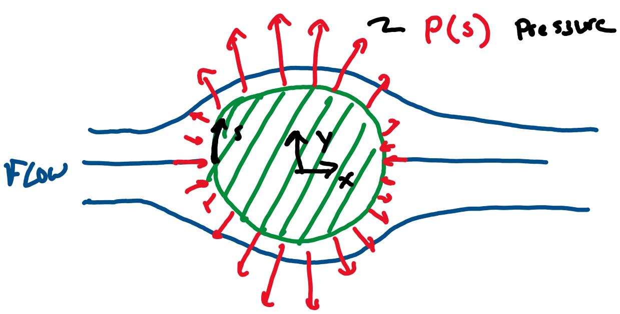
ASIDE: s vs. x & y

- In AERO there are a lot of complex OBJECT SURFACES
- We tend to care about FORCES on a surface
- $s = f(x, y)$ is a simpler way to follow a surface

PRESSURE + SHEAR $\Rightarrow$ NET FORCE

(1) Simple examples: the CYLINDER

CASE 1: STATIONARY + INVISCID

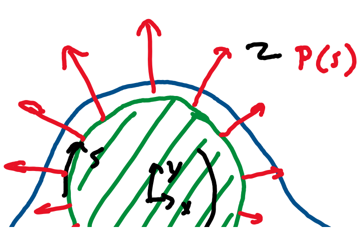


SAME PRESSURE DISTRIBUTION Top/Bottom Left/right

$\Rightarrow$ $F_x = 0$, $F_y = 0$ } no net force due to pressure

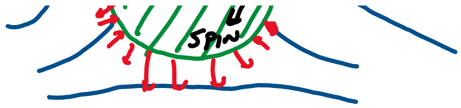
HISTORICAL NOTE: this caused D'Alembert's Paradox

CASE 2: Spinning + Inviscid



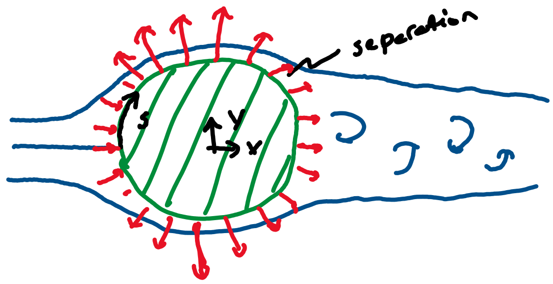
SAME P Front/Back

Different P Top/bottom



$\Rightarrow$ $\left.\begin{array}{l} F_x = 0 \\ F_y > 0 \end{array}\right\}$ LIFT force due to $P(s)$

CASE 3: Stationary + viscous



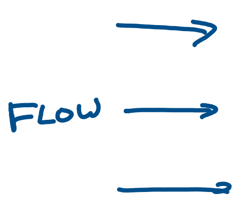
Different P front/back
Same P Top/bottom

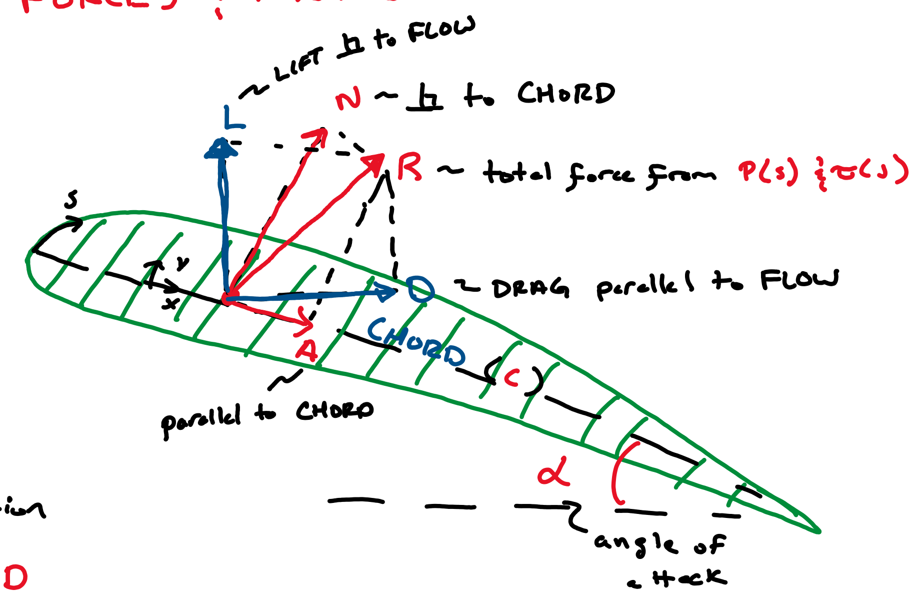
$\Rightarrow$ $\left.\begin{array}{l} F_x > 0 \\ F_y = 0 \end{array}\right\}$ DRAG due to $P(s)$

Depending on the PRESSURE & SHEAR $P(s) + \tau(s)$, we can get LIFT & DRAG

... back to airfoils!

(2.) AIRFOILS FORCES & MOMENTS





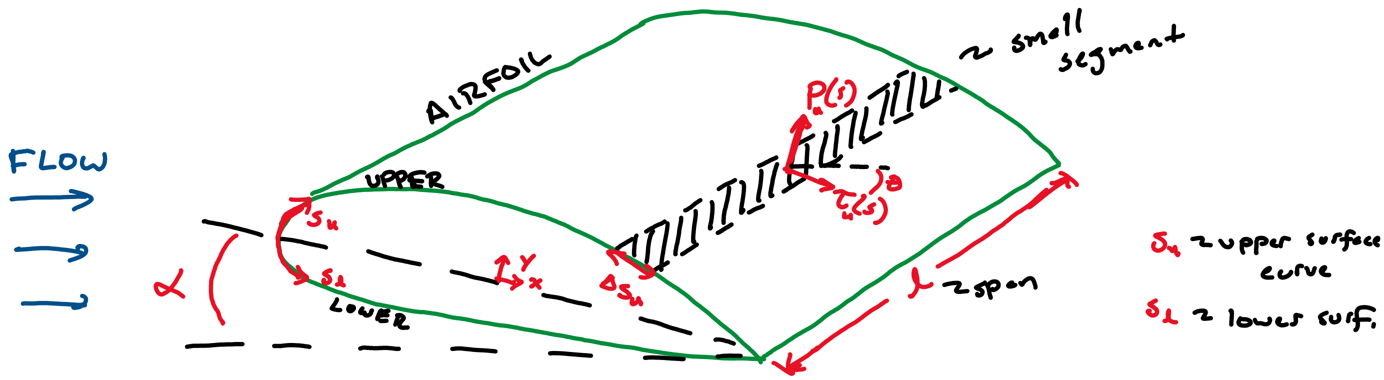
Depending on the situation

$N \,\&\, A \longleftrightarrow L \,\&\, D$

$$\left.\begin{array}{l} L = N\cos\alpha - A\sin\alpha \\ D = N\sin\alpha + A\cos\alpha \end{array}\right\} \text{by definition}$$

Say we're given $P(s)$ & $\tau(s)$, lets calculate N & A $\Rightarrow (L \,\&\, D)$

A derivation ...



For the SHADED segment, the FORCES are

$$(y) \quad \underbrace{\Delta N_u}_{\text{~force on segment}} = -P_u \,\Delta s_u\, l \cos\theta - \underbrace{\tau_u\, \overbrace{\Delta s_u\, l}^{\text{segment Area}}}_{\text{stress} \times \text{Area} = \text{force}} \sin\theta$$

$$\Delta A_u = -P_u \Delta s_u\, l \sin\theta + \tau_u \Delta s_u\, l \cos\theta$$

It's convenient to remove SPAN l and consider "per unit span" $\frac{(\cdot)}{l}$

$$\frac{A}{l} = A' \quad , \quad \frac{N}{l} = N' \quad \text{~ prime means } (\cdot)' = \frac{(\cdot)}{l}$$

$$\Rightarrow \quad \left.\begin{aligned} \Delta N'_u &= (-P_u \cos\theta - \tau_u \sin\theta)\,\Delta s_u \\ \Delta A'_u &= (-P_u \sin\theta + \tau_u \cos\theta)\,\Delta s_u \end{aligned}\right\} \text{Force per unit span of segment}$$

The BODY FORCE $= \sum_{\text{sum}}$ (segment forces)

↳ upper & lower surface

Integrate along $\vec{s}$

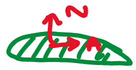
$$(1) \quad N' = \int^{TE} -(P_u \cos\theta + \tau_u \sin\theta)\, ds_u + \int^{TE} (P_l \cos\theta - \tau_l \sin\theta)\, ds_l$$

(TE: trailing edge; $\Delta \to d$)

$\int_{LE}$ $\underbrace{\qquad\qquad}_{\text{upper surface}}$ $\int_{LE}$ $\underbrace{\qquad\qquad}_{\text{lower surface}}$

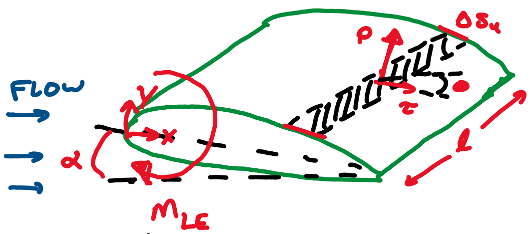
leading edge

$$(2) \quad A' = \int_{LE}^{TE} (-P_u \sin\theta + \tau_u \cos\theta)\, ds_u + \int_{LE}^{TE} (P_\ell \sin\theta + \tau_\ell \cos\theta)\, ds_\ell$$

Along with FORCES  we need the Moment 

x2 x1

(recall: $M = Fd$)



leading edge moment

per unit span

Stress x Area / span

distance to LE

upper

$$\Delta M'_{LE_u} = \underbrace{(P_u \cos\theta + \tau_u \sin\theta)\Delta S_u}_{\text{Stress x Area / span}} \times$$

$$+ (-P_u \sin\theta + \tau_u \cos\theta)\Delta S_u\, y$$

As we did before BODY MOMENT = $\sum_{\text{sum}}$ (segment moments) upper/lower

$$M'_{LE} = \int_{LE}^{TE} \left[(P_u \cos\theta + \tau_u \sin\theta)x - (P_u \sin\theta - \tau_u \cos\theta)y \right] ds_u$$

upper surface

(3)

$$+ \int_{LE}^{TE} \left[(-P_\ell \cos\theta + \tau_\ell \sin\theta)x + (P_\ell \sin\theta + \tau_\ell \cos\theta)y \right] ds_\ell$$

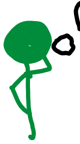
lower surface

So $\overset{(1)}{N'}$ & $\overset{(2)}{A'}$ & $\overset{(3)}{M'}$ $\longleftrightarrow$ $\int P(s) A + \int \tau(s) A$

pressure dist.

shear dist

(or L' & D')



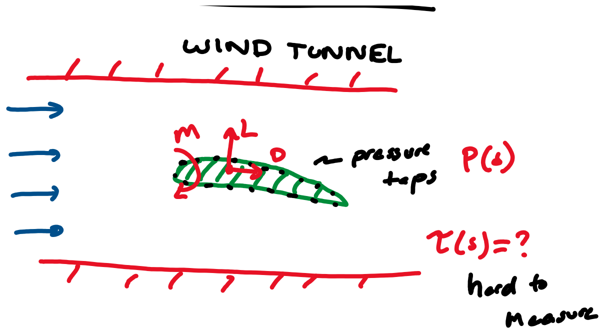
will I ever use this?

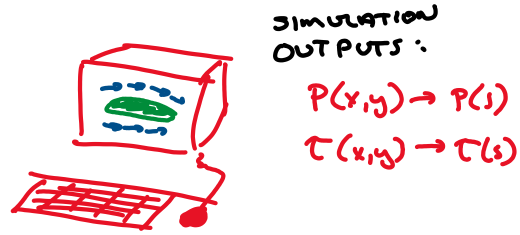
... maybe?

MEASUREMENTS

SIMULATIONS

* industry





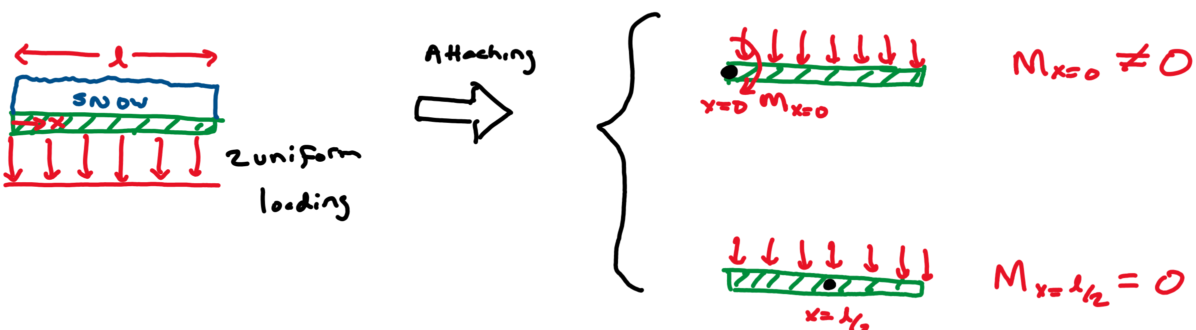
L, D, M – easy to measure directly!

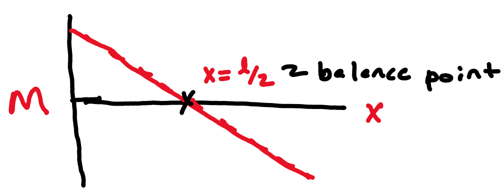
Need (1-3) to find L, D, M

Now you know N & A -or- L & D ... where do they go?

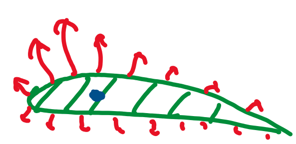
(3) CENTER OF PRESSURE (x_{cp})

Consider a PLATE covered in SNOW



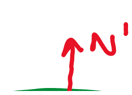


AIRFOILS have complicated P(s) & τ(s)

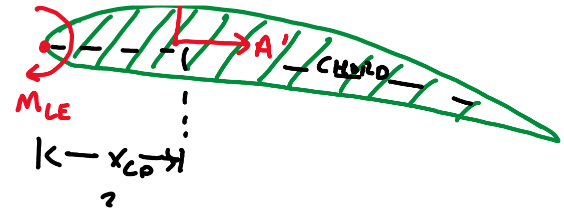


but they still have a BALANCE POINT (M = 0)

X_{cp} – CENTER OF PRESSURE



- Put A' (known from (2)) on the CHORD ($y=0$)



- N' (1) must cause M'_{LE} (3)

if Moment = force x distance then

negative because $+N' \to -M'_{LE}$

$$M'_{LE} = -x_{cp} N'$$

$$\boxed{x_{cp} = -\frac{M'_{LE}}{N'}} \approx -\frac{M'_{LE}}{L'} \quad (\text{small } \alpha \to N' \approx L')$$

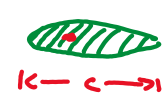
SPOILER:

In reality x_{cp} isn't too useful because it MOVES

$$x_{cp} = f(u, p, \tau, \dots)$$ ~ yuck.

In practice AIRFOIL performance is documented by $L', D', M'_{c/4}$

$\frac{1}{4}$ chord



Ex. NACA 0012 (look-up table)