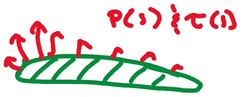
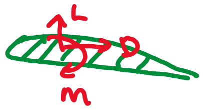
Lecture 4: Non-dimensional #'s & Similarity

Sunday, September 13, 2020 12:37 AM

LAST TIME : Go from DISTRIBUTED FORCES to BODY FORCES





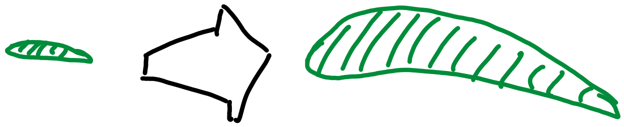
TODAY : - Non-dimensionalization

$L \rightarrow C_L$

$D \rightarrow C_D$

$\vdots$

- Similarity



(D) REMOVING DIMENSIONAL DEPENDENCE

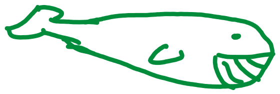
Example : travel 5 km

→ you assume based on past experience

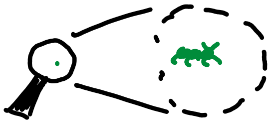
5 km :

"not bad"

"easy"



"I'll die"



When considering the distance, we naturally NON-DIMENSIONALIZE (i.e., think RELATIVELY)

$$D_i^* = \frac{D \;\sim \text{distance}}{L_B \;\approx \text{body-length}}$$

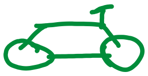
 $D_i^* = 3{,}100$

 $D_i^* = 167$

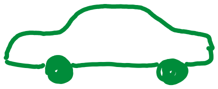
 $D_i^* = 6{,}700{,}000$!

There are other ways to NON-DIMENSIONALIZE based on problem...

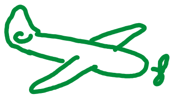
5 km:



vs.



vs.



D_1^* → not too different for each ... BODY LENGTH not a good length scale

Try something new: velocity × time = length ~ new length scale

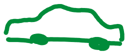
V vehicle speed

T relevant time

1 hr?, 1 day?

$$D_2^* = \frac{D}{VT}$$

$D_2^* = 0.21$



$D_2^* = 0.07$ much better!

$D_2^* = 0.017$

In AERODYNAMICS, we NON-DIMENSIONALIZE frequently

DIMENSIONAL QUANTITY →	RELEVANT SCALING →	NON-DIM #
L (lift)	$\frac{1}{2}\rho u_\infty^2 A$	C_L (lift coefficient)
μ (viscosity)	$\rho L u_\infty$	Re (Reynolds #)

(1) LIFT/DRAG/MOMENT COEFFICIENTS

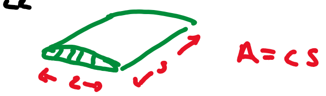
FORCE ⇒ need relevant reference force

Typically: **INERTIA FORCE**

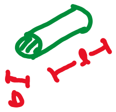
$KE = F \cdot L$ (kinetic energy) $(F = KE/L)$; $KE = \frac{1}{2} \overset{=\rho L^3}{m} V^2 = \frac{1}{2} \rho L^3 V^2$

$$F_{inertia} = \frac{1}{2} \rho \underbrace{L^2}_{\text{reference area}} \overset{\text{reference velocity (Freestream } U_\infty)}{V^2}$$

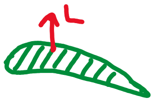
* ← our reference force



$A = cs$



$A = D\ell$



LIFT COEFFICIENT:

$$C_L = \frac{L}{F_{inertia}} = \frac{L}{\frac{1}{2}\rho c s U_\infty} \Rightarrow$$

LIFT EQN

$$\boxed{L = \frac{1}{2} \rho U_\infty^2 s c C_L}$$

"per unit span" $C_\ell = \frac{L/s}{F/s} = \frac{L'}{\frac{1}{2}\rho c U_\infty^2}$

DRAG COEFFICIENT:

$$C_D = \frac{D}{\frac{1}{2}\rho c s U_\infty^2} \Rightarrow$$

DRAG EQN.

$$\boxed{D = \frac{1}{2} \rho U_\infty^2 s c C_D}$$

"per unit span": $C_d = \frac{D'}{\frac{1}{2}\rho c U_\infty^2}$

MOMENT COEFFICIENT:

M . $\underset{F_{inertia}}{\text{force}} \times \underset{c \text{ (chord)}}{\text{distance}}$

$$C_M = \frac{M}{F_{inertia} c} = \frac{M}{\frac{1}{2}\rho s c^2 U_\infty^2}$$

MOMENT EQN.

$$\boxed{M = \frac{1}{2} \rho U_\infty^2 s c^2 C_m}$$

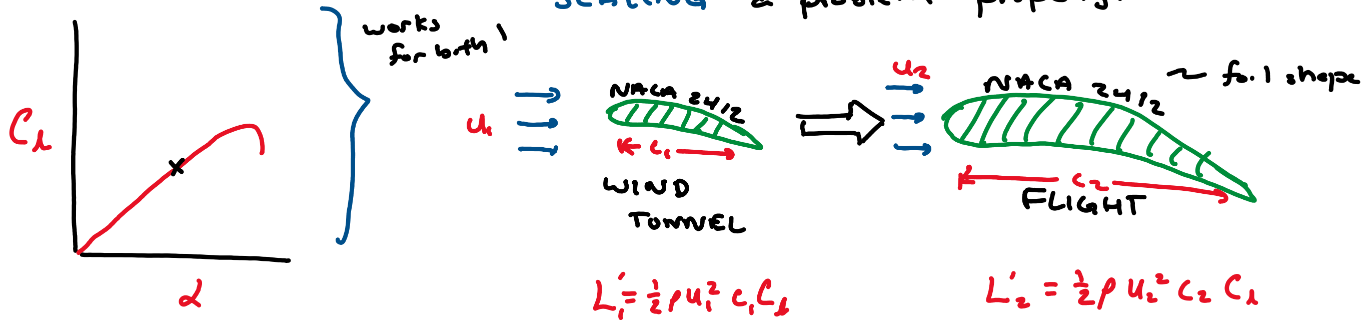
"per unit span": $C_m = \frac{M'}{\frac{1}{2}\rho U_\infty^2 c^2}$

HISTORICAL NOTE: – these **COEFFICIENTS** come from **WRIGHT BROS** for **AERODYNAMICS**

– In **FLUID MECHANICS**, these are **EULER #s** (Eu)

(2) SIMILARITY VARIABLES

↳ Non-dim. numbers held CONSTANT when SCALING a problem properly.



$L'_1 = \frac{1}{2}\rho u_1^2 c_1 C_l$ $\quad$ $L'_2 = \frac{1}{2}\rho u_2^2 c_2 C_l$

Other than the COEFFICIENTS (C_L, C_D, C_m..), we typically match

- REYNOLDS NUMBER (Re)
- Mach Number (M)

REYNOLDS NUMBER : $\frac{\text{INERTIA FORCE}}{\text{VISCOUS FORCE}}$

~ the fluid's forward momentum

the flow's main resisting force

$$Re = \frac{F_{inertia}}{F_{viscous}}$$

$F_{inertia}$ ~ from above: $\rho L^2 V^2$

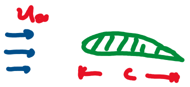
$F_{viscous}$ ~ $\tau A = \mu \frac{\Delta V}{\Delta y} L^2 = \mu \frac{V}{L} L^2 = \mu V L$

velo scale (V)

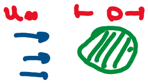
length scale (L)

$$\boxed{Re = \frac{\rho L V}{\mu}}$$

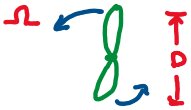
NOTE: choice of L & V depend on situation.



: $L = c$, $V = u_\infty$

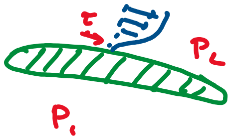


: $L = D$, $V = u_\infty$

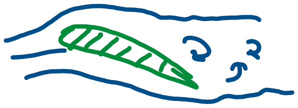


: $L = D$, $V = \overbrace{\Omega \frac{D}{2}}^{\text{tip velo}}$

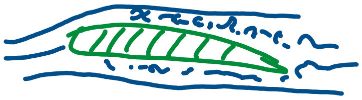
Preserving Re ensures: (1) similar PRESSURE vs. FRICTION DRAG



(2) similar FLOW SEPARATION



(3) Similar LAMINAR / TURBULENCE



MACH NUMBER:

$\frac{\text{INERTIA FORCE}}{\text{COMPRESSION}}$ ~ flow's forward momentum

~ flow's compression force

$$M = \left(\frac{F_{inertia}}{F_{compress}}\right)^{1/2}$$

from above $\rho L^2 V^2$

$= E_\forall A$ where $E_\forall = \frac{\Delta P}{\Delta\forall/\forall} = \frac{\Delta P}{\Delta\rho/\rho}$ ~ $\left(\text{speed of sound}\right)^2$ a (or c)

~ bulk modulus

~ area

~ pressure

~ % volume

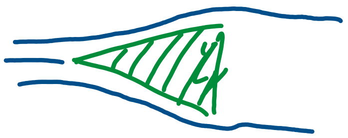
$= a^2 \rho L^2$

$$M = \left(\frac{\rho L^2 V^2}{\rho L^2 a^2}\right)^{1/2} \Rightarrow \boxed{M = \frac{V}{a}}$$

Perserving M ensures: (1) same COMPRESSION characteristics

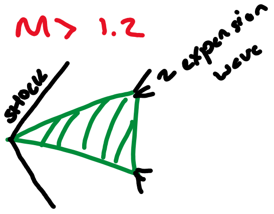
(2) same FLOW REGIME

M < 0.8



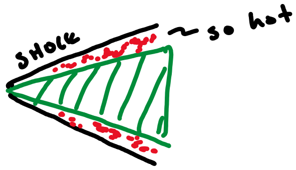
SUBSONIC

M > 1.2



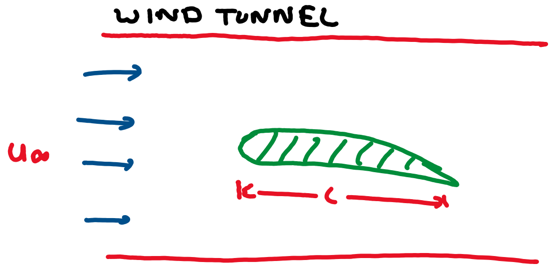
SUPERSONIC

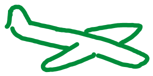
M > 5



HYPERSONIC

PRACTICAL NOTE: matching Re & M is tough for SCALED-DOWN EXPERIMENTS



Consider trying to match Re ~ 

& keeping M<0.3 (incompressible)

PLANE: $Re = \frac{\rho c u_\infty}{\mu} \sim 10^7$

EXPERIMENT: $c = 0.75$ m

$\frac{\mu}{\rho} = 1.5\times10^{-5}\ \frac{m^2}{s}$

$\Rightarrow u_\infty = 200$ m/s

but $M = \frac{200}{343} = 0.6$!!

Only options: (1) go BIGGER $

(2) change fluid prop. H_2O

(3) don't bother matching Re and hope it's okay.