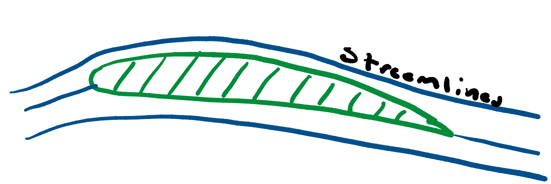
Lecture 6: Streamlines and Stream Function

Sunday, September 20, 2020 11:48 PM

Last Time: Conservation of
- Mass $\dot{m}_{in} - \dot{m}_{out} = \dot{m}_{cube}$
- Momentum $F = ma$
- Energy $\delta w - \delta q = de$

} Rules of a flow

Today: Flow Visualization



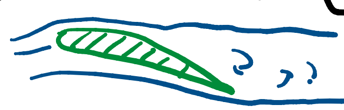
Stream Function: $\psi(x,y) = \text{const.}$

(F) Visualizing a Flow

↳ In Aero, we like to "see" or picture the flow

This tells us...

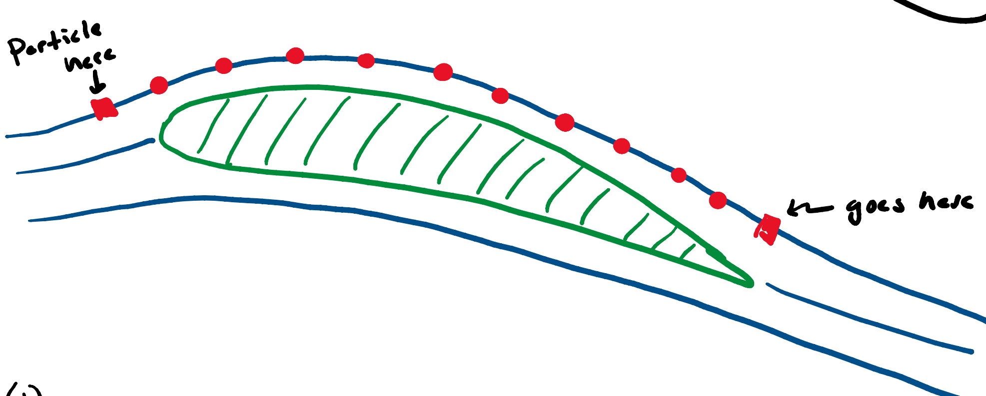
SEPARATION



TURBULENT

Recirculating

So, we draw:
- path lines
- Streak lines
- Stream lines



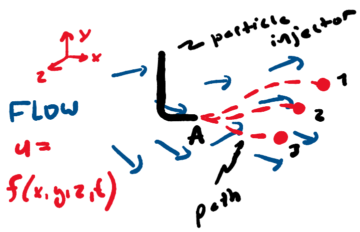
(1)

PATHLINES:	STREAK LINES:	STREAM LINES:

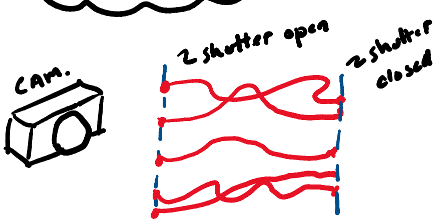
(1)

PATH LINES | STREAKLINES | STREAM LINES

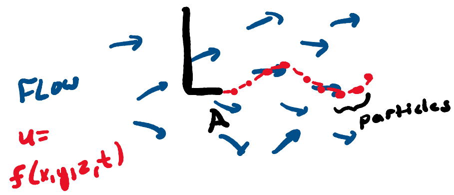
PATH LINES



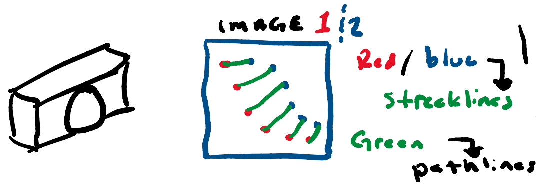
Traces the curve of a single particle path passing through A.



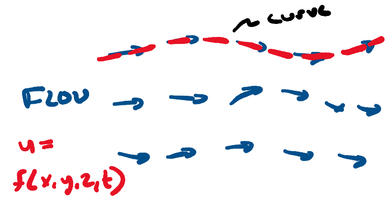
STREAKLINES



Traces the curve created by a series of particles passing through A.

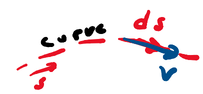


STREAM LINES



Traces a curve whose tangent at any point is aligned with velocity

Less physical → more mathy

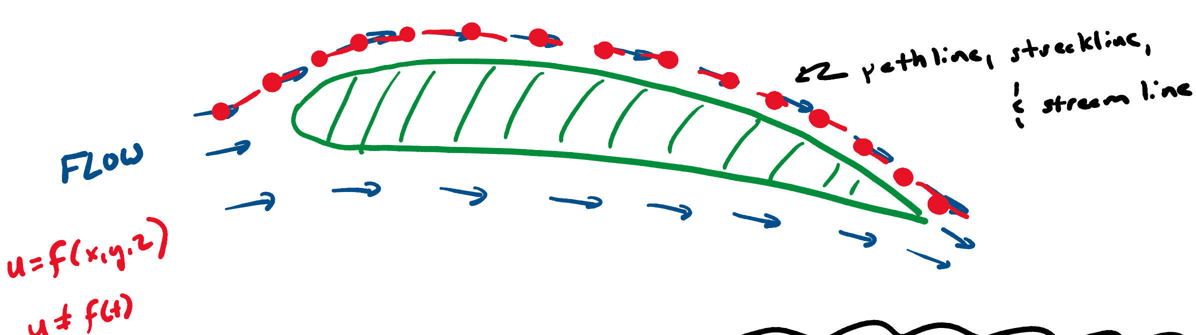


$d\mathbf{s} \times \mathbf{V} = 0$

For AERO → flow is ASSUMED STEADY (mostly)

$$\frac{\partial(\cdot)}{\partial t} = 0$$

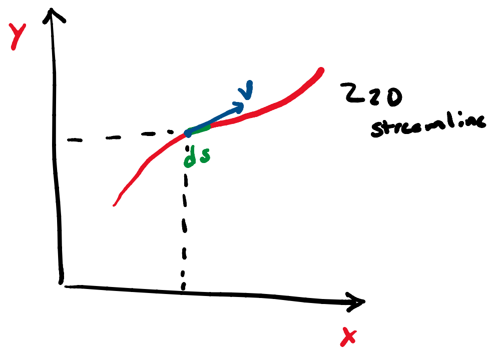
Therefore PATHLINES = STREAKLINES = STREAMLINES !!



So, good to know the difference but you'll rarely use it.

but you'll rarely use it.

Let's MATHEMATICALLY define the STREAMLINE



↳ $\vec{ds} \times \vec{V} = 0$ ⇐ means ds & V are parallel

Let's equate the slopes

Slope of $\vec{ds} = \frac{dy}{dx}$

Slope of $\vec{V} = \frac{v}{u}$

Therefore, $\frac{dy}{dx} = \frac{v}{u}$ ... $\underline{v\,dx - u\,dy = 0}$

2D streamline

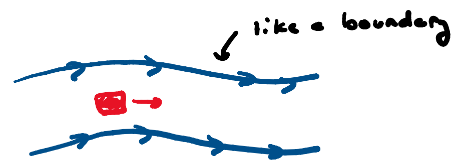
STREAMLINE EQNS (3D)

$$\boxed{\begin{aligned} w\,dy - v\,dz &= 0 \\ u\,dz - w\,dx &= 0 \\ v\,dx - u\,dy &= 0 \end{aligned}}$$

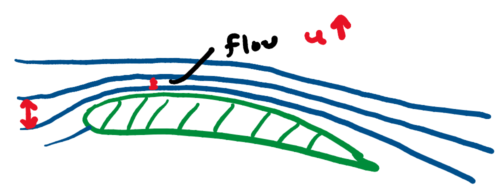
simply mathematically equating slopes of $\vec{ds}$ and $\vec{V}$

Useful STREAMLINE properties:

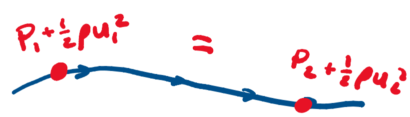
(1) FLOW does not cross streamline



(2) If streamlines converge or expand flow changes velocity.



(3) BERNOULLI is typically along streamline



(2) STREAM FUNCTION

A scalar function of 2D incompressible flow made from u, v to represent streamlines

Note: we build it, not derive it.

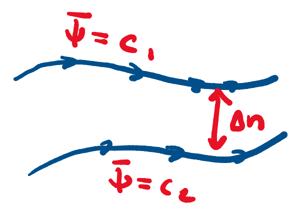
From above:

$$\frac{dy}{dx} = \frac{v}{u} \Rightarrow \int \frac{1}{v}\, dy = \int \frac{1}{u}\, dx \Rightarrow g(x,y) + C_1 = h(x,y) + C_2$$

~some function ~some constant

if: $\dfrac{g(x,y)}{h(x,y)} = \Psi(x,y)$ ~new function & $C_2 - C_1 = C$ ~new const.

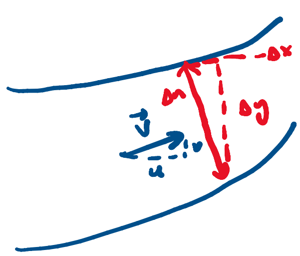
then: $\Psi(x,y) = C$



The constant C is arbitrary ... let's give it meaning!

$\Delta\Psi = C_2 - C_1$ ~ let's define this to be $\dot{m}$ ≡ mass flow rate per length in/out of page

$= \rho V \Delta n$



$$\Delta\Psi = \rho \underbrace{V \Delta n}_{\text{break this into } (x) \,\&\, (y) \text{ components}}$$

$\Delta\Psi = \rho u \Delta y - \rho v \Delta x$ ... assume small changes $(\Delta \to d)$

~mass flow

Chain rule: $d\Psi = \dfrac{\partial \Psi}{\partial x} dx + \dfrac{\partial \Psi}{\partial y} dy$

Stream function

CARTESIAN

$$\rho u = \frac{\partial \Psi}{\partial y}$$

CYLINDRICAL

$$\rho u_r = \frac{1}{r} \frac{\partial \Psi}{\partial \theta}$$

= Constructed Fcn:

Stream function (general)

$$\boxed{\begin{aligned} \rho u &= \frac{\partial \Psi}{\partial y} \\ \rho v &= -\frac{\partial \Psi}{\partial x} \end{aligned}}$$

$$\boxed{\begin{aligned} \rho u_r &= \frac{1}{r}\frac{\partial \Psi}{\partial \theta} \\ \rho u_\theta &= -\frac{\partial \Psi}{\partial r} \end{aligned}}$$

- Constructed Eqns
- 2D, steady
- $u, v \rightarrow \psi$ helpful for math
- Gives STREAMLINES $\psi = c$

if incompressible $\psi \equiv \Psi / \rho$

Stream function (incomp)

CARTESIAN

$$\boxed{\begin{aligned} u &= \frac{\partial \psi}{\partial y} \\ v &= -\frac{\partial \psi}{\partial x} \end{aligned}}$$

CYLINDRICAL

$$\boxed{\begin{aligned} u_r &= \frac{1}{r}\frac{\partial \Psi}{\partial \theta} \\ u_\theta &= -\frac{\partial \Psi}{\partial r} \end{aligned}}$$

In practice, stream functions are rarely used, but

Flow Viz is used a lot.