

# Measurements

## 1. Key points to know

You must know:

- what is meant by random errors and systematic errors
- what is meant by absolute, fractional and percentage uncertainties
- that error bars are used on graphs to indicate uncertainties in data
- that gradients and intercepts on graphs have uncertainties.
- explain how random and systematic errors can be identified and reduced
- collect data that include absolute and/or fractional uncertainties and go on to state these as an uncertainty range
- determine the overall uncertainty when data with uncertainties are combined in calculations involving addition, subtraction, multiplication, division and raising to a power
- determine the uncertainty in gradients and intercepts of graphs.

## 1.2 Uncertainties

Uncertainty in measurement is expressed in three ways.

-Absolute uncertainty: the numerical uncertainty associated with a quantity. For example, when a length of quoted value 5.00 m has an actual value somewhere between 4.95 m and 5.05 m, the absolute uncertainty is  $\pm 0.05$  m. The length will be expressed as  $[5.00 \pm 0.05]$ m.

-Fractional uncertainty  
=  $\frac{\text{absolute uncertainty in quantity}}{\text{numerical value of quantity}}$ . A fractional uncertainty has no unit.

-Percentage uncertainty = fractional uncertainty  $\times 100$  expressed as a percentage. There is no unit.

## 1.2 Errors

-Random errors are unpredictable changes in data collected in an experiment. Examples include fluctuations in a measuring instrument or changes in the environmental conditions where the experiment is being carried out.

-Systematic errors are often produced within measuring instruments. Suppose that an ammeter gives a reading of +0.1 A when there is no current between the meter terminals. This means that every reading made using the meter will read 0.1 A too high. The effect of a systematic error can produce a non-zero intercept on a graph where a line through the origin is expected.

## 1.3 Combining uncertainties

- The absolute uncertainties are added when quantities are added and subtracted.

When  $y = a \pm b$  then  $\Delta y = \Delta a + \Delta b$

- The fractional uncertainties are added when quantities are multiplied or divided.

When  $y = \frac{ab}{c}$  then  $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$

- When a quantity is raised to a power  $n$ , the fractional uncertainty is multiplied by  $n$ .

When  $y = a^2$ , this is the same as  $a \times a$  so using the algebraic rule above:

$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta a}{a} = \frac{2\Delta a}{a}$ . In the general case, when

$y = a^n$ ,  $\frac{\Delta y}{y} = \left| n \frac{\Delta a}{a} \right|$ , where  $\|$  means the absolute value or magnitude of the expression.

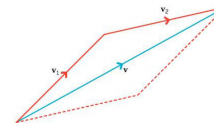
## 1.4 Scalars and Vectors

-Scalars are quantities that have magnitude (size) but no direction. They generally have a unit associated with them.

-Vectors are quantities that have both magnitude and a physical direction. A unit is associated with the number part of the vector.

## 1.5 Vector operations

Figure below shows the addition of two vectors. The vectors must be drawn to the same scale and the direction angles drawn accurately too. A further construction produces the parallelogram with the red solid and dashed lines. Then the magnitude of the new vector  $\mathbf{v}_1 + \mathbf{v}_2$  is given by the length of the blue vector with the direction as shown.



-Vector Resolution:

