SL/HL Probability

Worksheet # 5.2

Topics Covered

1. Probability concepts involving Venn Diagrams, Tree Diagrams, Conditional, Independent and Mutually exclusive events

2. Discreet Probability distribution and its Application

3. Miscellaneous Problems

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1 Probability concepts and application

1. An annual squash tournament groups players into 5 divisions according to their skill level.

The table shows the number of players at the tournament over 3 years.

Division	2017	2018	2019
1	4	5	5
2	6	7	8
3	13	12	14
4	18	10	14
5	20	17	16
Total	61	51	57

Find the probability that a player:

a) in the 2017 tournament played in division 1

b) in any of the past tournaments played in division 3

c) in the 2019 tournament did not play in division 2 or 4 .

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2. A hospital recorded the age and gender of its 1020 melanoma patients over one year. The data is shown below.

	< 40	$40 - 59$	60	Total
Male	56	127		
Female	75	113	230	
Total				1020

a) Complete the table.

b) Find the probability that a randomly selected melanoma patient was:

i) male

ii) female and younger than 40

iii) 60 or older, given they were female

iv) male, given they were 40 or older.

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3. A die is rolled, and a square spinner with sectors $1,2,3$, and 4 is spun.

a) Draw a grid to illustrate the sample space of possible outcomes.

b) Use your grid to find the probability of getting:

i)two 1 s	ii) two 5 s	iii) a sum of 6
iv) a 2 and a 3	**v**) a 2 or a 3 (or both)	vi) exactly one 4.

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4. Suppose $\mathrm{P}(A)=0.37, \mathrm{P}(B)=0.41$, and $\mathrm{P}(A \cup B)=0.78$.

a) Find $\mathrm{P}(A \cap B)$.

b) What can you say about A and B ?

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5. Given that $\mathrm{P}(A)=\frac{23}{50}, \mathrm{P}(B)=\frac{5}{7}$, and $\mathrm{P}\left((A \cup B)^{\prime}\right)=\frac{1}{12}$, find $\mathrm{P}(A \cap B)$.

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6. The events A and B are such so that

$$\mathrm{P}(A)=0.7, \quad \mathrm{P}(B)=0.4 \quad \text{and} \quad \mathrm{P}(A \cup B)=0.8$$

a) Illustrate this information in a fully completed Venn diagram.

b) Determine the probability ...

i. … of either event A or event B occurring, but not both.

ii. … of neither event A nor event B occurring.

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7. The events A and B are such so that

$$\mathrm{P}(A)=0.48, \mathrm{P}(B)=0.38 \text{ and } \mathrm{P}(A \cap B)=0.28$$

Determine the value of …

a) $\ldots \mathrm{P}(A \cup B)$.

b) $\ldots \mathrm{P}\left(A^{\prime} \cap B^{\prime}\right)$.

c) $\ldots \mathrm{P}\left(A^{\prime} \cup B^{\prime}\right)$.

8. Of the families in a village, 70% have dogs and 45% have dogs and cats.

Of the families in this village, 18% have neither a dog, nor a cat.

a) Illustrate this information in a fully completed Venn diagram.

b) Find the percentage of families that do not have a cat.

c) Determine the probability that a family picked at random will own dogs or cats, but not both.

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9. The events C and D are such so that

$$\mathrm{P}(C) = 0.4, \quad \mathrm{P}(D) = 0.5 \quad \text{and} \quad \mathrm{P}\left(C' \cap D'\right) = 0.25$$

Determine the value of …

a) … $P(C \cap D)$.

b) … $\mathrm{P}(C' \cap D)$.

c) … $\mathrm{P}(C' \cup D')$.

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10. The events E and F satisfy

$$\mathrm{P}(E) = 0.35, \mathrm{P}(F) = 0.45 \text{ and } \mathrm{P}\left(E \cap F'\right) = 0.15.$$

a) By completing a suitably labelled Venn diagram, or otherwise, find $\mathrm{P}(E \cap F)$.

b) Determine

i. … $\mathrm{P}(E \mid F)$.

ii. ... $\mathrm{P}(F \mid E')$.

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11. The events C and D satisfy

$\mathrm{P}(C \cap D') = 0.1, \mathrm{P}(C' \cap D) = 0.15$ and $\mathrm{P}(C' \cap D') = 0.2$

a) Illustrate the above information in a fully completed Venn diagram.

b) Determine .

i. ... $\mathrm{P}(C \mid D')$.

ii. ... $\mathrm{P}(D' \mid C)$.

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12. The events A and B are independent with

$$\mathrm{P}(A) = 0.3 \text{ and } \mathrm{P}(B) = 0.5$$

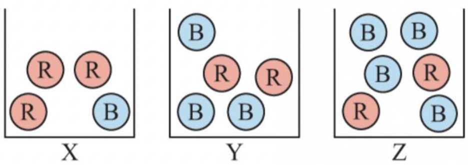
Determine

a) .... $\mathrm{P}(A \cup B)$.

b) ... $\mathrm{P}(A' \cap B)$.

c) ... $\mathrm{P}(B \mid A')$.

13.



One ball is drawn from each of the boxes shown.

a) Draw a tree diagram to illustrate the situation.

b) Find the probability that:

i) exactly two red balls are drawn X Y

ii) blue balls are drawn from boxes X and Z

iii) at most one blue ball is drawn.

c) Suppose an extra red ball is added to box Y. Which of the probabilities in **b** will be affected?

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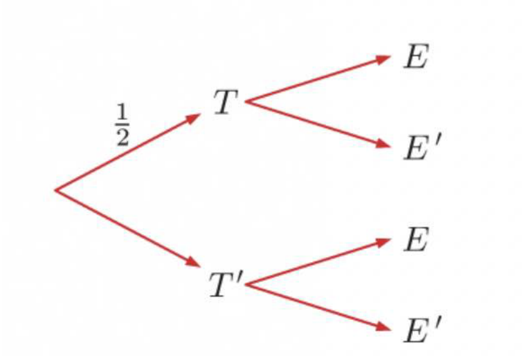
14. Suppose you toss a coin and roll a die simultaneously.

Let T represent a tail with the coin and E represent a 2 or a 5 with the die.

a) Complete the tree diagram showing the probabilities of the different outcomes.



b) Find: i) $\mathrm{P}(T \cap E')$ ii) $\mathrm{P}(T \cup E')$

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15. Events A and B are independent. Given that $\mathrm{P}(A \cup B) = 0.63$ and $\mathrm{P}(B) = 0.36$, find $\mathrm{P}(A)$.

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16. A box of chocolates contains 6 dark brown, 4 light brown, and 2 white truffles. Two truffles are selected from the box without replacement.

Find the probability of selecting:

a) 2 white truffles

b) different coloured truffles.

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17. A box contains 4 blue balls and n red balls. When two balls are drawn from the box without replacement, the probability that both are red is $\frac{1}{3}$. Find n.

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18. 40% of students in a class own an orange highlighter, 20% own a blue highlighter, and 50% do not own either coloured highlighter.

a) Draw a Venn diagram to describe the situation.

b) Find the probability that a randomly selected student:

i) owns a blue highlighter, given they own an orange highlighter

ii) owns an orange highlighter, given they do not own a blue highlighter.

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19. Suppose $\mathrm{P}(X)=\frac{3}{7}, \mathrm{P}(Y)=\frac{2}{9}$, and $\mathrm{P}(X \cup Y)=\frac{3}{5}$.

a) Find:

i) $\mathrm{P}(X \cap Y)$

ii) $\mathrm{P}(X \mid Y)$

iii) $\mathrm{P}(Y \mid X)$

b) Are X and Y independent events? Explain your answer.

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20. Suppose $\mathrm{P}(A \cap B)=0.2$ and $\mathrm{P}\left(A^{\prime} \cap B\right)=0.3$.

Given that $A^{\prime}$ and B are independent, find $\mathrm{P}\left(A^{\prime} \cup B\right)$.

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21. a) If 3 coins are tossed, find the probability that two fall heads and the other falls tails.

b) Suppose 3 coins are tossed 400 times. On how many occasions would you expect to see exactly one tail?

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22. a) Tickets in a raffle are numbered 1 to 100 . A ticket is drawn at random. A is the event that a ticket with a number less than 45 is drawn, and B is the event that a ticket with a number between 40 and 55 is drawn.

i) Are A and B mutually exclusive events? Explain your answer.

ii) Find $\mathrm{P}(A \cup B)$.

b) Suppose $\mathrm{P}(A)$ and $\mathrm{P}(B)$ are both non-zero. Explain why events A and B cannot be both independent and mutually exclusive at the same time.

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23. A national cricket team plays 30% of their matches in their home country. When playing at home, the team wins 40%, draws 35%, and loses 25% of their matches. When playing away from home, the team wins 15% and loses 45% of their matches.

Given that the team did not win their last match, find the probability that the match was played in their home country.

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24. The quality controller in a factory examined 250 components for three types of minor faults, known as fault A, fault B and fault C.

His results are summarized as follows.

4 components with type C fault only.

- 8 components with type A and C but no B fault.

- 9 components with type A fault only.

- 7 components with type B and C but no A fault.

- 1 component with type B fault only.

- 5 components with type A and B but no C fault.

- 2 components with all three types of fault.

a) Draw a fully completed Venn diagram to represent this information.

A component is selected at random from the quality controller's sample.

b) Find the probability that the selected component has a type C fault.

c) Given that the selected component has a type B fault, find the probability that the component has all three types of fault.

d) Given instead that the selected component has a fault, find the probability it has two faults.

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25. The events A and B are such so that

$$\mathrm{P}(A) = 0.2, \mathrm{P}(B) = 0.5 \text{ and } \mathrm{P}(A \cup B) = 0.6$$

Determine whether A and B are independent events.

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26. The events A and B are statistically independent and further satisfy

$$\mathrm{P}(A) = 0.4 \text{ and } \mathrm{P}(A \cap B) = 0.12.$$

Determine ... a) $\ldots \mathrm{P}(B)$.

b) $\ldots \mathrm{P}(A \cup B)$.

c) $\ldots \mathrm{P}(A \cap B')$.

d) $\ldots \mathrm{P}(B' \mid A')$.

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27. The events A and B satisfy

$$\mathrm{P}(A) = 0.5, \mathrm{P}(B) = 0.2 \text{ and } \mathrm{P}(A \mid B) = 0.3.$$

Determine ...

a) $\ldots \mathrm{P}(A \cap B)$.

b) $\ldots \mathrm{P}(A \cup B)$.

c) $\ldots \mathrm{P}(B \mid A)$.

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28. The following sets are given.

$$U = \left\{x \mid x \in \mathbb{Z}^+, x \leq 3\right\}$$
$$A = \{x \in U : x \text{ is multiple of } 3\}$$
$$B = \{x \in U : x \text{ is multiple of } 5\}.$$

a) Use a Venn diagram to represent the information above.

b) Find the following probabilities.

i) $\mathrm{P}(A)$

ii) $\mathrm{P}(B)$

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29. The probability that a train is late is 0.3. If the train is late, the probability that Florian will make it to work on time is 0.6. If the train is not late, the probability that Florian will be on time for work is 0.9. Let A be the event "the train is late" and let B be the event "arrives at work on time".

a) Draw a tree diagram to represent these events.

b) Find the probability that the train is late and Florian is on time for work.

c) Find the probability that Florian will make it on time.

d) Show that the train being late and Florian being on time for work are not independent events.

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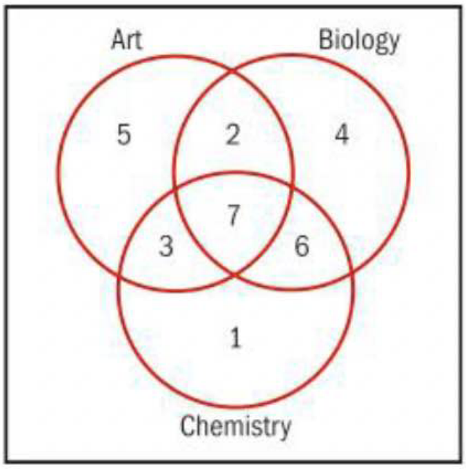
30. The Venn diagram shows the homework for 35 students



A student is chosen at random. Find the probability that the student:

a) had art homework

b) had biology homework

c) had chemistry homework

d) had art homework, given that they had homework in biology

e) had biology homework given that they chemistry homework

f) had chemistry homework given that they they had not been assigned any homework in art

g) had homework in all three subjects, given that they were assigned homework on that day.

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31. Given that

$\mathrm{P}(A)=\frac{1}{3}, \mathrm{P}(B \mid A)=\frac{3}{5}$ and $\mathrm{P}(B \mid A')=\frac{1}{2}$, find

a) $\mathrm{P}(B')$

b) $\mathrm{P}(A' \cup B')$.

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32. Owen shoots two free throws in a basketball game. The probability that he scores the first shot is 0.85. The probability that he misses the second shot given that he scored the first shot is 0.10. The probability that he is going to score the second shot given that he missed the first shot is 0.75.

Find the probability that Owen scores only one shot.

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33. The events A and B are such so that

$$\mathrm{P}(A)=0.2, \quad \mathrm{P}(B)=0.6 \quad \text{and} \quad \mathrm{P}\left(A' \cap B'\right)=0.25$$

Determine

a) $\ldots \mathrm{P}(A \cap B)$.

b) $\ldots \mathrm{P}(A \cap B') \cup \mathrm{P}(A' \cap B')$.

c) $\ldots \mathrm{P}(A' \cup B)$

34.The events A and B are such so that

$$\mathrm{P}(B)=0.35, \quad \mathrm{P}(A \mid B)=0.1, \quad \mathrm{P}\left(A \cap B'\right)=0.15$$

Determine ...

a) $\ldots \mathrm{P}(A \cap B)$.

b) $\ldots \mathrm{P}(A \cup B)$.

c) $\ldots \mathrm{P}(B \mid A')$.

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35. - The probability that one of the members of the W.M.T. Club reads the Economist is 0.65.

-The probability that one of the members of the W.M.T. Club reads the Financial Times is 0.45.

- The probability that one of the members of the W.M.T Club only reads the Economist is 0.4.

Member of the W.M.T. Club is selected at random.

a) Determine the probability that this member reads the Economist and the Financial Times.

b) If this member reads the Financial Times determine the probability that he also reads the Economist.

c) Given instead that this member does not read the Economist, determine the probability he reads the Financial Times.

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36. The events A and B are independent and further satisfy

$$\mathrm{P}(A) = 0.2 \quad \text{and} \quad \mathrm{P}(A \cup B) = 0.68.$$

a) Determine $\mathrm{P}(B)$.

b) Find the probability that exactly one of the two events occur.

c) Given that exactly one of the two events occur, find the probability that event A occurs.

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37. The events A and B satisfy

$$\mathrm{P}(A) = \mathrm{P}(B) = p \quad \text{and} \quad \mathrm{P}(A \cup B) = \frac{11}{36}$$

Given that A and B are statistically independent, determine the value of p.

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38. The events A and B are such so that

$$\mathrm{P}(A) = 0.3, \quad \mathrm{P}\left(A \cap B'\right) = 0.1, \quad \mathrm{P}\left(A \cup B'\right) = 0.55$$

a) Find $P(B)$.

b) Illustrate the above information in a fully completed Venn diagram.

c) Determine ...

i. ... $\mathrm{P}(A \mid B)$.

ii. ... $\mathrm{P}(B' \mid A')$.

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39. The events A and B satisfy

$$\mathrm{P}(A) = \frac{3}{5}, \quad \mathrm{P}(B) = \frac{5}{8}, \quad \mathrm{P}(A \cup B) = \frac{7}{10}$$

a) Illustrate this information in a fully completed Venn diagram.

b) Determine, showing all the relevant workings, whether A and B are statistically independent.

The events A and C satisfy

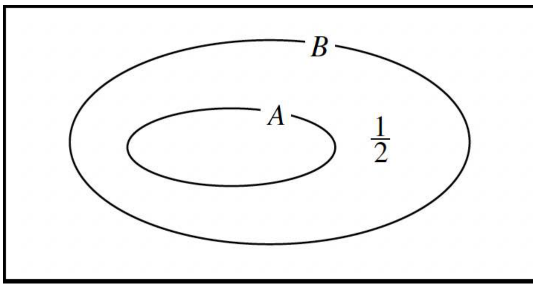
$$\mathrm{P}(A \cup C) = \frac{7}{10} \quad \text{and} \quad \mathrm{P}(C \mid A) = \frac{1}{3}$$

c) Determine ...

i. ... $\mathrm{P}(A \mid C)$.

ii. .. $\mathrm{P}[(A \cap C)']$.

40.



The Venn diagram above, shows the probability sample space for two events A and B, where

$$A \subseteq B, \quad \mathrm{P}(A) \times \mathrm{P}(B) = \frac{1}{9} \text{ and } \quad \mathrm{P}\left(B \cap A'\right) = \frac{1}{2}$$

a) Show clearly that $P(A) = \frac{1}{6}$.

b) Determine $\mathrm{P}(B \mid A')$.

41. The events A and B are such so that

$$\mathrm{P}(A) = \frac{1}{2}, \quad \mathrm{P}(A \cap B) = \frac{1}{4}, \quad \mathrm{P}\left[\left(A' \cap B\right)'\right] = \frac{2}{3}.$$

a) Find the value of $\mathrm{P}(B)$.

b) Illustrate the probability space of A and B in a suitably labelled, fully completed Venn diagram.

c) Given that at most one of the two events occurred, determine the probability that event A did not occur.

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42. The events A and B are such so that

$$\mathrm{P}(A) = \frac{1}{2}, \quad \mathrm{P}(B) = \frac{1}{4}, \quad \mathrm{P}\left(A \cap B'\right) + \mathrm{P}\left(A' \cap B\right) = \frac{1}{3}.$$

Illustrate the probability space of these two events in a fully completed Venn diagram.

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43. The events A and B satisfy

$$\mathrm{P}(A) = x, \quad \mathrm{P}(B) = y, \quad \mathrm{P}(A \cup B) = 0.6, \quad \mathrm{P}(B \mid A) = 0.2.$$

a) Show clearly that

$$4x + 5y = 3$$

The events B and C are mutually exclusive such that

$$\mathrm{P}(B \cup C) = 0.9, \quad \mathrm{P}(C) = x + y$$

b) Find the value of x and the value of y.

c) Determine, showing all the relevant workings, whether A and B are statistically independent.

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44. The events C and D are such so that

$$\mathrm{P}(C) = \frac{1}{3}, \quad \mathrm{P}(D) = \frac{7}{36}, \quad \mathrm{P}\left[\left(C \cap D'\right) \cup \left(C' \cap D\right)\right] = \frac{13}{36}$$

a) Find, showing a full clear method, the value of $\mathrm{P}(C \cap D)$.

If instead the events C and D satisfy

$$\mathrm{P}(C) = \frac{k}{k+2}, \quad \mathrm{P}(D) = \frac{2}{k}$$

where k is a positive constant such that $\mathrm{P}(C) < 1, \mathrm{P}(D) < 1$.

b) Show that C and D cannot be mutually exclusive.

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45. During the winter, Ned attends weekly business meetings in Newcastle and always travels to these meetings by car.

The probability of being dry, raining or snowing during his travel to these meetings is $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$, respectively.

The respective probabilities of Ned arriving on time when is dry, raining or snowing are $\frac{4}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$.

a) Determine the probability that Ned will arrive late for his next winter business meeting.

Ned arrived late for his meeting last week.

b) Find the probability that it was raining on that day.

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46. A bag contains blue, yellow and red discs and these discs show on their face a single digit whole number.

The probability of drawing a blue disc is $\frac{1}{2}$, the probability of drawing a yellow disc is $\frac{1}{3}$ and the probability of drawing a red disc is $\frac{1}{6}$.

$\frac{5}{12}$ of the blue discs show an even number, $\frac{5}{8}$ of the yellow discs show an even number and $\frac{1}{4}$ of the red discs show an even number.

A disc is drawn at random from the bag.

a) Determine the probability that the disc will show an even number.

b) Given that the disc that was drawn was showing an even number, find the probability that the disc was not red.

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47. A test is developed to determine whether someone has or has not got a disease, which is known to be present in 3% of the population.

Given a person has the disease the test is positive with probability of 98%.

Given a person does not have the disease the test is positive with probability of 5%.

a) Draw a tree diagram to represent this information.

A person is selected at random from the population and tested for the disease.

b) Find the probability that this person's test is positive.

A person who tested positive is selected.

c) Find the probability that the person does not have the disease.

d) Comment on the effectiveness of this test with reference to the answer given in part (c).

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48. Bag X contains 3 one pound coins and 2 two pound coins.

Bag Y contains 1 one pound coin and 3 two pound coins.

A statistical experiment consists of

- a coin being picked at random from bag X and placed into bag Y.

- then a coin being picked at random from bag Y and placed back into bag X.

Find the probability ...

a) ... that a one pound coin will be picked on both occasions in this experiment.

b) ... that at the end of the experiment both bags contain $7 pounds$.

A third coin is picked at random from bag X at the end of the experiment.

c) Determine the probability that it will be a two pound coin.

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49. Taxis in Pajan have to pass an additional safety test consisting of three parts, one for the brakes, one for the tyres and one for the lights. A taxi must pass all three parts.

The individual probabilities that a taxi will fail the "brake part", the "tyre part" and the "light part" are $\frac{1}{6}$, $\frac{1}{4}$ and $\frac{1}{5}$, respectively.

These probabilities are independent of one another.

A taxi from Pajan is tested at random.

a) Find the probability that it will fail exactly one of the three parts of the test.

Safety regulations change so that the test has to be performed in the order "brake part" first, "tyre part" next and "lights part" last.

If the taxi fails one of the three parts the test results in failure, without any of remaining parts of the test having to be carried out.

A taxi from Pajan is tested at random under these new regulations.

b) Find the probability that it will fail the test.

c) Given a taxi failed the test, determine the probability it failed the ”lights part”.

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50. Arnie and Ned play each other in a darts match, which consists of up to three games.

The winner is the first person to win two games.

The probability that Arnie wins the first game is 0.7.

Whenever Arnie wins a game the probability he wins the next is 0.6.

Whenever Ned wins a game the probability he wins the next is 0.8.

a) Find the probability that Arnie wins the match.

b) Given that Arnie won the match find the probability he won in two games.

c) Given Arnie won the first game find the probability he won the match.

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51. The scheduled flight DM104, of a certain airline, can be delayed due to the following three reasons.

- aircraft technical problems, denoted by the event T

- weather conditions, denoted by the event W

- air traffic congestion, denoted by the event C.

These events are assumed to be independent of one another and the flight will be delayed if one or more of these events occur.

It is given that $\mathrm{P}(T)=0.1, \mathrm{P}(W)=0.05$ and $\mathrm{P}(C)=0.2$.

a) Find the probability that the next DM104 flight ...

i. ... will be delayed due to exactly one reason.

ii. ... will be delayed.

b) Given that the next DM104 flight was delayed, find the probability that it was delayed due to one reason only.

c) Given that the next DM104 flight had no technical problems, find the probability that it was delayed.

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52. A medical test for a certain disease will produce a positive result for 2% of the population and a negative result for 95% of the population. The test is classed inconclusive for 3% of the population.

95% of the people who test positive have the disease, 1% of the people who test negative have the disease, while 20% of the people whose test is classed as inconclusive have the disease.

a) Draw a tree diagram to represent the above information.

A person is selected at random.

b) Find the probability that this person will ...

i. ... test negative and will have the disease.

ii. ... will have the disease.

c) Given a person has the disease, determine the probability he tested negative.

d) Comment on the effectiveness of the test with reference to part (c).

For the people whose test is classed as inconclusive a second more expensive test is carried out. This test will always identify if a person does not have the disease. If a person has the disease the test will correctly identify this in 95% of the cases.

Another person is selected from the population.

e) Determine the probability that this person either tested negative originally or his test was inconclusive but eventually was told that he does not have the disease when the second test was carried out.

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53. A box contains 3 black balls and 7 white balls, all identical in size. An experiment consists of drawing a ball out of the box and recording its colour.

- If the ball drawn is black, after its colour is recorded, the ball is replaced back into the box and an extra black ball is also placed in the box.

- If the ball drawn is white, after its colour is recorded, the ball is not replaced back into the box.

The experiment is attempted 3 consecutive times.

Given that in these 3 attempts both colours were recorded, determine the probability that a black colour was recorded twice.

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54. A market research company is conducting a survey on pets.

There were 144 pet owners in the survey of whom 66 were dog owners, and of these dog owners 18 were cat owners too.

There were 42 pet owners that owned no dog or cat.

As an incentive for pet owners to participate in the survey, a year's free supply of pet food will be given to one of the entrants as a prize.

An entrant is drawn at random to determine the winner of the prize.

Find the probability that the prize-winner ...

a) ... owns a cat but does not own a dog.

b) ... owns a dog.

c) ... does not own a cat given he does not own a dog.

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55. People are watching a film in a cinema.

The counterfoil of their tickets was retained as they went in.

The management is going to draw two ticket counterfoils at random and the recipients of these tickets will receive free cinema tickets.

Of the people watching the film, there are 10 men, 20 women, 25 boys and 45 girls.

Calculate the probability that the two free tickets will be won by two children of the same gender.

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56. Two bags contain numbered cards.

Bag A contains 5 cards, numbered $1, 2, 3, 4$ and 5 .

Bag B contains 5 cards, numbered $2, 3, 4, 5$ and 6 .

A card is selected at random from bag A and placed into bag B. A card is then selected at random from bag B.

Determine the probability that ...

a) ... both the cards selected will show the number 3 .

b) ... both the cards selected will show the same number.

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57. A school committee consists of 2 teachers, 3 boys and 4 girls. A subcommittee is to be formed by picking at random 3 members out of the original members of the school committee.

Calculate the probability that the subcommittee will contain ...

a) . . . no teachers.

b) ... one teacher, one boy and one girl.

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58. A box contains 6 jam doughnuts, 5 cream doughnuts and 4 plain doughnuts. The doughnuts all look the same on the outside. Three children pick a doughnut each.

a) Find the probability that the three doughnuts the children picked are all ...

i. ... jam doughnuts.

ii. . . . of the same type.

iii. ... of different type.

b) Given that the doughnuts the children picked are all of the same type, find the probability they contained jam.

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59. A bag contains 6 red pegs, 7 green pegs, 8 blue pegs and 9 yellow pegs. Andrea picks two pegs at random from the bag to hang a dress on a washing line.

Determine the probability that the pegs will be of the same colour.

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60. A school's staff consists of 90 teachers and 10 administrators.

There are 7 female administrators and 53 female teachers.

Two members of staff are selected at random to attend a meeting.

Find the probability of the two members selected, one will be a female teacher and the other a male.

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61. A box contains 5 red and 4 black balls.

Three balls are selected from the box and are not replaced.

Determine the probability that ...

a) . . . all three balls selected are red.

b) ... more black balls are selected that red balls.

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62. Katerina and Mathew are taking part in a quiz game where they will be asked two questions each.

The probability that Katerina answers any of her questions correctly is $\frac{2}{3}$.

The probability that Mathew answers any of his questions correctly is $\frac{3}{4}$.

Determine the probability that Katerina and Mathew will answer correctly exactly three out of the four questions.

---

63. A bag contains four coins, of which three are fair while the fourth coin is double headed so that when this coin is tossed a head is always obtained.

One of these four coins is selected at random and tossed three times.

a) Find the probability that a head will be obtained on all three tosses.

b) Given three heads were obtained, determine the probability that the double headed coin was picked from the bag.

The selected coin is tossed a fourth time.

c) Find the probability that a head is obtained.

---

64. Andrew and Bradley are tennis players playing each other regularly. Their matches can last up to 5 sets, the winner being the first to win 3 sets.

Andrew is assumed to have constant probability $\frac{3}{5}$ of winning any set.

a) Find the probability that Andrew will win the match in ...

i. …3 sets.

ii. …4 sets.

b) Hence determine the probability that Andrew will win the match.

c) If Andrew wins a match, find the probability that more than 3 sets were played.

---

65. Tupac and Lutac are two friends that go weigh training on Mondays and Thursdays.

The probability that Tupac trains on a Monday is 0.9 and on a Thursday 0.8.

- If Tupac trains on a Monday the probability that Lutac also trains is 0.8.

- If Tupac does not train on a Monday the probability that Lutac also does not train is 0.2.

- If Tupac trains on a Thursday the probability that Lutac also trains is 0.9.

- If Tupac does not train on a Thursday the probability that Lutac also does not train is 0.6.

Determine the probability that on a given week ...

a) … they both trained on both days.

b) ... one of them trained on both days and the other only once.

---

66. A market research company is conducting a survey on health, diet and exercise on the staff of a large factory.

They found that $\frac{1}{3}$ of the factory workers took regular exercise and $\frac{1}{2}$ of the factory workers ate breakfast. They further found that of those factory workers who took regular exercise $\frac{1}{8}$ also ate breakfast.

A factory worker is selected at random.

a) Find the probability that the worker does not eat breakfast and does not take regular exercise.

b) Determine, with a reason, whether the events ”take regular exercise” and ”eat breakfast” are statistically independent.

---

67. The probability of a biased coin showing heads is $\frac{1}{7}$.

Aaron, Benjamin, Caleb and David toss this coin repeatedly.

They start this experiment alphabetically and continue alphabetically until a head is obtained. The winner is the first person to obtain a head and once a winner is declared the experiment is over.

Determine the probability David ...

a) ... wins in his first toss.

b) ... wins in his second toss.

c) … gets a third toss.

d) … gets exactly three tosses.

---

68. The two way table below summarizes the sales of three different types of number of smart phones classified by their contract length in months.

	Length of Contract			
	12 month	24 month	36 month	TOTAL
Hamsung	17	16	17	50
Mokia	29	26	15	70
Ephone	30	27	23	80
TOTAL	76	69	55	200

Let A be the event that a customer chose a Hamsung smart phone. Let B be the event that a customer chose a 24 month contract. Let C be the event that a customer chose a Mokia smart phone.

A customer is selected at random.

a) Determine the values of ...

i. $\ldots \mathrm{P}(B)$.

ii. $\ldots .\mathrm{P}(A \cap B)$.

iii. $\ldots .\mathrm{P}(A \mid B)$.

iv. $\ldots \mathrm{P}(B' \cup C')$

Of the customers who bought a Hamsung phone 26% bought a styling skin. The respective percentages of hose customers who bought Mokia or EPhone smart phones are 60% and 25%.

b) Find the probability that a customer selected at random bought ...

i. ... a skin.

ii. … a Ephone given he did not buy a skin.

---

69. A regular daily bus service leaves the Voder bus Station and arrives in the Hostend bus station some time later on the same day.

From its scheduled departure time from Voder, the bus will either leave on time or late but never early.

From its scheduled arrival time to Hostend, the bus will either arrive early, on time or late.

If the bus leaves Voder on time, the probability it arrives in Hostend early is 4%, on time 52% and late 21%.

The probability that the bus arrives in Hostend early is 6% and on time 69%.

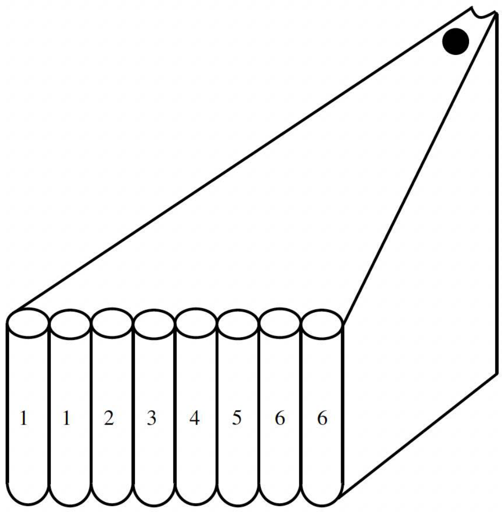
a) Given that the bus arrives early in Hostend, determine the probability that it left Voder on time.

b) Given that the bus arrives late in Hostend, find the probability that it left Voder late.

Three consecutive days for this bus service are monitored.

c) Find the probability that these three days, the bus arrives early once, late once and on time once.

70.



The figure below shows a contraption where balls are rolling down a slope and fall into one of 8 vertical tubes. There is equal chance of a ball falling into any of these 8 vertical tubes. The ball scores according to which tube it falls in, and these scores are marked clearly in the figure below.

Two balls are rolled in succession.

Determine the probability that ...

a) . . . their sum is even.

b) . . . the second ball will score higher than the first.

c) . . . their sum is even and the second ball will score higher than the first.

Three balls are rolled in succession.

Determine the probability that . . .

d) . . . they will fall into different tubes.

e) ... their scores will be different.

---

71. A box contains four yellow balls and one purple ball. Chester and David play a game by each taking it in turn to take a ball from the box, without replacement. The first player to take a purple ball is the winner. Chester plays first.

(a) Find the probability that he wins.

The game is now changed so that the ball chosen is replaced after each turn. Chester still plays first.

(b) Find the probability that he wins.

---

72. Peter and Quinn play a game with a bag containing one red marble and three blue marbles. Each player in turn randomly selects a marble from the bag, notes its colour and replaces it. The first player to take a red marble is the winner. Quinn starts the game.

(a) Find the probability that Quinn eventually wins.

(b) The probability that Peter wins on his nth turn is $\frac{243}{4096}$. Find the value of n.

---

73. In a school, students are required to learn at least one language, Japanese or Korean. It is known that 80% of the students learn Japanese, and 55% learn Korean.

(a) Find the percentage of students who learn both Japanese and Korean.

(b) Find the percentage of students who learn Japanese, but not Korean.

At this school, 58% of the students are female, and 75% of the female learn Japanese.

(c) A student is chosen at random. Let F be the event that the student is a female, and let J be the event that the student learns Japanese.

(i) Find $\mathrm{P}(F \cap J)$.

(ii) Show that F and J are not independent.

(d) A male is chosen at random. Find the probability that he learns Japanese.

74.Consider the events A and B. It is given that $\mathrm{P}(A) = 0.4, \mathrm{P}(B) = 0.65$ and $\mathrm{P}(A \cup B) = 1$.

(a) Find $\mathrm{P}(A \cap B)$.

(b) Find $\mathrm{P}(A' \cap B)$.

Consider another event C. It is also given that $\mathrm{P}(C) = 0.7$ and $\mathrm{P}(A \mid C) = 0.78$.

(c) (i) Find $\mathrm{P}(A \cap C)$.

(ii) Show that A and C are not mutually exclusive.

(iii) Show that A and C are not independent.

(d) Find $\mathrm{P}(C \mid A')$.

---

75. In a class of 100 boys, 85 boys play football and 45 boys play rugby. Each boy must play at least one sport from football and rugby.

(a) Find the percentage of boys who play both sports.

(b) Find the percentage of boys who play only one sport.

(c) One boy is selected at random.

(i) Given that the boy selected plays football, find the probability that he plays rugby also.

(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

It is also given that 60% of the boys are taller than 180 cm and 90% of these boys play football.

Let F be the event that a boy plays football and T be the event that a boy is taller than 180 cm.

(d) (i) Explain why F and T are not mutually exclusive.

(ii) Show that F and T are not independent.

(iii) Find the percentage of boys who is not taller than 180 cm and play football.

---

76. A survey of the reading habits of a group of students revealed that 35% read quality newspapers, 50% read tabloid newspapers and 25% do not read newspapers at all.

(a) Find the percentage of students who read both types of newspapers.

(b) Find the percentage of students who only read tabloid newspapers.

(c) One student is selected at random.

(i) Given that the student selected reads newspapers, find the probability that he reads tabloid newspapers only.

(ii) Given that the student selected reads tabloid newspapers, find the probability that he reads quality newspapers also.

It is also given that 40% of the students wear glasses and 95% of these students read tabloid newspapers.

Let G be the event that a student wear glasses and T be the event that a student read tabloid newspapers.

(d) (i) Explain why G and T are not mutually exclusive.

(ii) Show that G and T are not independent.

(iii) Find the percentage of students who do not wear glasses and read tabloid newspapers.

---

77. In a school, students are required to take at least one subject from art and technology. It is known that 55% of the students take art, and 70% take technology.

(a) Find the percentage of students who take both art and technology.

(b) Find the percentage of students who take one subject only.

At this school, 63% of the students are male, and 72% of the male take art.

(c) A student is chosen at random. Let M be the event that the student is a male, and let A be the event that the student takes art.

(i) Find $\mathrm{P}(M \cap A)$.

(ii) Show that M and A are not independent.

(d) A female is chosen at random. Find the probability that she takes art.

---

2 Discreet Distribution

78. The random variable X has the following probability distribution.

x	2	4	6
$P(X = x)$	a	b	0.25

Given that $E(X) = 3.4$, find a.

---

79. Two fair dice are rolled. Let X be the difference between the numbers rolled.

a) Explain why X is a discrete random variable.

b) State the possible values of X.

c) Find $P(X = 3)$.

---

80. The random variable X has the following probability distribution.

x	1	2	3	4
$P(X = x)$	0.2	0.3	a	b

Given that $E(X) = 2.62$, find a.

---

81. The random variable X has the following probability distribution.

x	20	30	40	50
$P(X = x)$	0.1	a	b	0.1

Given that $E(X) = 33$, find the values of a and b.

---

82. The random variable X has the following probability distribution.

x	0	10	20	30
$P(X = x)$	0.1	a	b	c

Given that $P(X < 15) = 0.5$ and $E(X) = 16$, find the values of b and c.

---

83. The random variable X has the following probability distribution.

x	0	3	6	9
$\mathrm{P}(X=x)$	0.4	a	b	c

Given that $\mathrm{P}(2<X<7)=0.3$ and $\mathrm{E}(X)=4.2$, find the values of a and b.

---

84. The following table shows the probability distribution of a discrete random variable X.

x	0	2	4	6
$\mathrm{P}(X=x)$	0.4	0.1	$2k$	$3k$

(a) Find the value of k.

(b) Find $\mathrm{E}(X)$.

---

85. The following table shows the probability distribution of a discrete random variable X.

x	0	1	2	3
$\mathrm{P}(X=x)$	$9k$	k	0.1	0.4

(a) Find the value of k.

(b) Find $\mathrm{E}(X)$.

---

86. The following table shows the probability distribution of a discrete random variable X.

x	k	$k+1$	$k+2$	8
$\mathrm{P}(X=x)$	$\frac{k}{2}$	$\frac{1}{8}$	$\frac{k}{4}$	$\frac{1}{8}$

(a) Find the value of k.

(b) Find $\mathrm{E}(X)$.

---

87. Two unbiased, standard, 6 sided dice are thrown and the scores are recorded. Let the random variable X be the minimum of these two scores. The probability distribution of X is given in the following table.

x	1	2	3	4	5	6
$\mathrm{P}(X=x)$	$\frac{11}{36}$	p	$\frac{7}{36}$	q	$\frac{3}{36}$	$\frac{1}{36}$

(a) Find the value of p and the value of q.

(b) Find the expected value of X.

---

88. The probability distribution function of a discrete random variable X is given by $\mathrm{P}(X = x_i) = \frac{1}{k}x^2, x = 0,1,2,3,4$

a) Complete a probability distribution table. Find the constant k.

b) Find the probability that X is less than or equal to 2 .

c) Find the mode of the distribution.

d) Represent the distribution with a vertical line graph.

---

89. By drawing the probability distribution table, determine whether the following functions describe a discrete random variable. If a discrete random variable is described find

a) the mode b $\mathrm{P}(X > 3)$

i) $\mathrm{P}(X = x_i) = \frac{1}{2}x^3, x = 0,1,2,3,4$

ii) $\mathrm{P}(X = x_i) = \frac{x!}{5x+2}, x = 0,1,2,3,4$

---

90. The discrete random variable has PDF given by:

$\mathrm{P}(X = kx), x = 10,11,12$

a Find the value of k.

b Find the mode.

c Find the probability that X takes an even number.

---

91. Petr travels to school on a train. On any day, the probability that Petr will miss the train is 0.2.

If he misses the train, the probability that he will be late for school is 0.9.

If he does not miss the train, the probability that he will be late is 0.25.

Let A be the event "he misses the train" and B the event "he is late for school".

(a) Draw the tree diagram of the situation

(b) Find (i) $\mathrm{P}(A \cap B)$;

(ii) $\mathrm{P}(B)$.

(c) Find the probability that

(i) Petr does not miss the train and is not late for school;

(ii) Petr does not miss the train, given that he is late for school.

The cost for each day that Petr catches the train is 5 dollars. Petr goes to school on Monday and Friday.

(d) Copy and complete the probability distribution table.

X	0	5	10
$\mathrm{P}(X=x)$			

(e) Find the expected cost for Petr for both days.

---

92. A five-sided fair die has three blue faces and two red faces. The die is rolled for two times.

Let F be the event a blue face lands down in the first roll, and S be the event a blue face lands down in the second roll.

(a) Draw the tree diagram

(b) Find (i) $\mathrm{P}(F \cap S)$;

(ii) $\mathrm{P}(S)$.

(c) Find the probability that

(i) a red face lands down for two times;

(ii) a blue face lands down in the first roll, given that a red face lands down in the second roll.

Rossi plays a game where he rolls the die for two times. If a blue face lands down, he scores 4. If the red face lands down, he scores 1. Let X be the total score obtained.

(d) Copy and complete the probability distribution table.

X	2	5	8
$\mathrm{P}(X=x)$			

(e) Calculate the expected value of X.

---

93. Sandy and Katie both wish to go to the shopping mall but one of them has to stay home to baby-sit.

The probability that Sandy goes to the shopping mall is 0.2. Only one of them will go.

If Sandy goes to the shopping mall the probability that she is late home is 0.7.

If Katie goes to the shopping mall the probability that she is late home is 0.4.

Let S be the event that Sandy goes to the shopping mall, and L be the event that the person who goes to the shopping mall arrives home late.

(a) Draw the Tree diagram.

(b) Find

(i) $\mathrm{P}(S \cap L')$;

(ii) $\mathrm{P}(L')$.

(c) Find the probability that

(i) Katie goes to the shopping mall and she is not late;

(ii) Sandy goes to the shopping mall, given that the person who goes to the shopping mall arrives home late.

The attendance of Sandy is recorded. If one lateness is marked, she will be penalized by a \$10 fee reduction. If two latenesses are marked, she will be penalized by a \$25 fee reduction. Let X be the total amount of fee reduction for Sandy.

(d) Copy and complete the probability distribution table.

X	0	10	25
$\mathrm{P}(X=x)$			

(e) Calculate the expected value of X.

---

94. Thierry and Jake share a flat. Thierry cooks dinner five nights out of ten.

If Thierry does not cook dinner then Jake does. If Thierry cooks dinner the probability that they have lasagna is $\frac{9}{10}$.

If Jake cooks dinner the probability that they have lasagna is $\frac{3}{10}$.

Let T be the event that Thierry cooks dinner, and L be the event that they have lasagna in their dinner.

(a) Draw the Tree Diagram.

(b) Find (i) $\mathrm{P}(T' \cap L')$;

(ii) $\mathrm{P}(L')$.

(c) Find the probability that

(i) Thierry cooks dinner and they do not have lasagna;

(ii) they do not have lasagna, given that Jake cooks dinner.

The cost for making lasagna for once is 125 dollars. Consider the expenditure on making lasagna on a particular three days. Let X be the total amount of expenditure.

(d) Copy and complete the probability distribution table.

X	0	125	250	375
$\mathrm{P}(X=x)$			$\frac{54}{125}$	

(e) Find the expected expenditure.

---

95. On a shelf there are two tins, one red and one blue. The red tin contains two orange candies and eight apple candies, and the blue tin contains six orange candies and four apple candies. Oliver randomly chooses either the red or the blue tin and randomly selects a candy.

Let R be the event that the red tin is chosen, and A be the event that an apple candy is selected.

(a) Draw the tree diagram.

(b) Find

(i) $\mathrm{P}(R' \cap A)$;

(ii) $\mathrm{P}(A)$.

(c) Find the probability that

(i) an orange candy is selected from the red tin;

(ii) he selected a candy from the red tin, given that the candy selected is an apple candy.

Oliver designed a game by using the above two tins. If an orange candy is chosen, then the participant will get 4 tokens. If an apple candy is chosen, no tokens would be awarded.

Let X be the total number of tokens of three different participants of the game.

(d) Copy and complete the probability distribution table.

X	0	4	8	12
$\mathrm{P}(X=x)$			$\frac{36}{125}$	

(e) Find the expected expenditure.

96. In a game, two standard six-sided dice are tossed. Let X be the sum of the scores on the two dice.

(a) Find (i) $\mathrm{P}(X=10)$;

(ii) $\mathrm{P}(X>10)$;

(iii) $\mathrm{P}(X=11 \mid X>9)$.

(b) Jenna plays a game where she tosses two dice.

If the sum is 10 , she wins 2 points.

If the sum is greater than 10 , she wins 1 point.

If the sum is less than 10 , she loses k points.

Find the value of k for which Jenna's expected number of points is zero.

---

97. In a game, two standard six-sided dice are tossed. Let X be the sum of the scores on the two dice.

(a) Find

(i) $\mathrm{P}(X=8)$;

(ii) $\mathrm{P}(X>8)$;

(iii) $\mathrm{P}(X>9 \mid X>8)$.

(b) Rick plays a game where he tosses two dice.

If the sum is 8 , he wins 5 points.

If the sum is greater than 8 , he wins k points.

If the sum is less than 8 , he loses 1 point.

Find the value of k for which Rick's expected number of points is one.

---

98. In a game, there are two different boxes containing different numbered cards. The first box contains cards numbered 1,2 and 3 . The second box contains cards numbered 10,20 and 30 . Two cards are drawn at random, one from each box. Let X be the sum of the numbers on the two cards.

(a) Find

(i) $\mathrm{P}(X=21)$;

(ii) $\mathrm{P}(X > 21)$;

(iii) $\mathrm{P}(30 < X < 33 \mid X > 21)$.

(b) Stephen plays a game where he draws two cards at random, one from each box.

If the sum is 21 , he wins $3k$ points.

If the sum is greater than 21 , he wins k points.

If the sum is less than 21 , no points would be awarded

Find the value of k for which Stephen's expected number of points is eight.

---

99. In a game, there are two different boxes containing different numbered cards. The first box contains cards numbered 2,3 and 5 . The second box contains cards numbered 2,7 and 11. Two cards are drawn at random, one from each box. Let X be the product of the numbers on the two cards.

(a) Find

(i) $\mathrm{P}(X = 33)$;

(ii) $\mathrm{P}(X \geq 35)$;

(iii) $\mathrm{P}(X < 22 \mid X < 33)$.

(b) Jessie plays a game where she draws two cards at random, one from each box.

If the product is 33 , she wins $4k$ points.

If the product is greater than 33 , she wins $3k$ points.

If the product is less than 33 , she loses $2k$ points.

Find the value of k for which Jessie's expected number of points is -16.

---

100. A discrete random variable X has the following probability distribution.

x	1	2	3	4
$\mathrm{P}(X = x)$	$5k^2$	$\frac{1}{2}k$	$\frac{3}{2}k$	$3k^2$

(a) Find the value of k.

(b) Find $\mathrm{P}(X = 4 \mid X > 2)$.

---

101. A discrete random variable X has the following probability distribution.

x	4	8	12
$P(X=x)$	$10k^2$	k	$20k^2$

(a) Find the value of k.

(b) Find $P(X = 12 \mid X > 6)$.

---

102. A discrete random variable X has the following probability distribution.

x	12	24	30	36
$P(X=x)$	k	$7k^2$	$8k^2$	k

(a) Find the value of k.

(b) Find $P(X = 24 \mid X > 20)$.

---

103. A discrete random variable X has the following probability distribution.

x	0	1	2	3	4	5
$P(X=x)$	k^2	k	$4k^2$	$8k^2$	$4k$	k^2

(a) Find the value of k.

(b) Find $P(2 < X \leq 4 \mid 1 < X \leq 4)$.

---

3 Miscellaneous Problems of probability

104. In a school there are 148 students in Years 12 and 13 studying Science, Humanities or Arts subjects. Of these students, 89 wear glasses and the others do not. There are 30 Science students of whom 18 wear glasses. The corresponding figures for the Humanities students are 68 and 44 respectively.

A student is chosen at random.

Find the probability that this student

(a) is studying Arts subjects,

(b) does not wear glasses, given that the student is studying Arts subjects.

Amongst the Science students, 80% are right-handed. Corresponding percentages for Humanities and Arts students are 75% and 70% respectively.

A student is again chosen at random.

(c) Find the probability that this student is right-handed.

(d) Given that this student is right-handed, find the probability that the student is studying Science subjects.

---

105. A group of 100 people produced the following information relating to three attributes. The attributes were wearing glasses, being left handed and having dark hair. Glasses were worn by 36 people, 28 were left handed and 36 had dark hair. There were 17 who wore glasses and were left handed, 19 who wore glasses and had dark hair and 15 who were left handed and had dark hair. Only 10 people wore glasses, were left handed and had dark hair.

(a) Represent these data on a Venn diagram.

A person was selected at random from this group.

Find the probability that this person

(b) wore glasses but was not left handed and did not have dark hair,

(c) did not wear glasses, was not left handed and did not have dark hair,

(d) had only two of the attributes,

(e) wore glasses given that they were left handed and had dark hair.

---

106. The following shows the results of a wine tasting survey of 100 people.

96 like wine A,

93 like wine B,

96 like wine C,

92 like A and B,

91 like B and C,

93 like A and C,

90 like all three wines.

(a) Draw a Venn Diagram to represent these data.

Find the probability that a randomly selected person from the survey likes

(b) none of the three wines,

(c) wine A but not wine B,

(d) any wine in the survey except wine C,

(e) exactly two of the three kinds of wine.

Given that a person from the survey likes wine A,

(f) find the probability that the person likes wine C.

---

107. Tetrahedral dice have four faces. Two fair tetrahedral dice, one red and one blue, have faces numbered $0, 1, 2$, and 3 respectively. The dice are rolled and the numbers face down on the two dice are recorded. The random variable R is the score on the red die and the random variable B is the score on the blue die.

(a) Find $\mathrm{P}(R = 3 \text{ and } B = 0)$.

The random variable T is R multiplied by B.

(b) Complete the diagram below to represent the sample space that shows all the possible values of T.

3				
2		2		
1	0			
0				
B \ R	**0**	**1**	**2**	**3**

Sample space diagram of T

(c) The table below represents the probability distribution of the random variable T.

t	0	1	2	3	4	6	9
$P(T = t)$	a	b	$1/8$	$1/8$	c	$1/8$	d

Find the values of a, b, c and d.

Find the values of (d) $E(T)$,

108. On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}, \frac{2}{5}$ and $\frac{1}{10}$ respectively.

(a) Draw a tree diagram to represent this information.

(b) Find the probability that on a randomly chosen day

(i) Bill travels by foot and is late,

(ii) Bill is not late.

(c) Given that Bill is late, find the probability that he did not travel on foot.

109. (a) Given that $\mathrm{P}(A) = a$ and $\mathrm{P}(B) = b$ express $\mathrm{P}(A \cup B)$ in terms of a and b when

(i) A and B are mutually exclusive,

(ii) A and B are independent.

Two events R and Q are such that

$\mathrm{P}(R \cap Q') = 0.15, \quad \mathrm{P}(Q) = 0.35$ and $\mathrm{P}(R \mid Q) = 0.1$

Find the value of

(b) $\mathrm{P}(R \cup Q)$,

(c) $\mathrm{P}(R \cap Q)$,

(d) $\mathrm{P}(R)$.

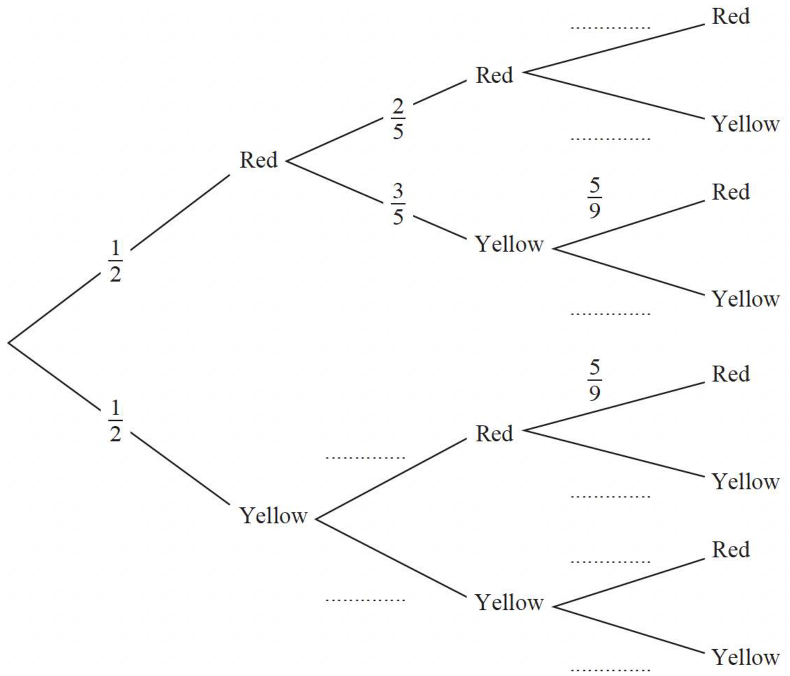
---

110. The bag P contains 6 balls of which 3 are red and 3 are yellow. The bag Q contains 7 balls of which 4 are red and 3 are yellow. A ball is drawn at random from bag P and placed in bag Q. A second ball is drawn at random from bag P and placed in bag Q. A third ball is then drawn at random from the 9 balls in bag Q.

The event A occurs when the 2 balls drawn from bag P are of the same colour.

The event B occurs when the ball drawn from bag Q is red.

(a) Complete the tree diagram shown below.



(b) Find P(A)

(c) Show that P$(B) = \frac{5}{9}$

(d) Show that P$(A \cap B) = \frac{2}{9}$

(e) Hence find P$(A \cup B)$

(f) Given that all three balls drawn are the same colour, find the probability that they are all red.

---

111. Jake and Kamil are sometimes late for school.

The events J and K are defined as follows

$J =$ the event that Jake is late for school

K = the event that Kamil is late for school

$P(J) = 0.25, P(J \cap K) = 0.15$ and $P(J' \cap K') = 0.7$

On a randomly selected day, find the probability that

(a) at least one of Jake or Kamil are late for school,

(b) Kamil is late for school.

Given that Jake is late for school, (c) find the probability that Kamil is late.

The teacher suspects that Jake being late for school and Kamil being late for school are linked in some way.

(d) Determine whether or not J and K are statistically independent.

(e) Comment on the teacher's suspicion in the light of your calculation in (d).

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112. (a) State in words the relationship between two events R and S when $P(R \cap S) = 0$

The events A and B are independent with $P(A) = \frac{1}{4}$ and $P(A \cup B) = \frac{2}{3}$

Find

(b) P(B)

(c) P$(A' \cap \mathrm{B})$

(d) P$(B' \mid \mathrm{A})$

113. Given that

$$\mathrm{P(A)} = 0.35, \quad \mathrm{P(B)} = 0.45 \text{ and } \mathrm{P(A \cap B)} = 0.13$$

find

(a) $\mathrm{P(A \cup B)}$

(b) $\mathrm{P}(A' \mid B')$

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are independent.

(c) Find $P(B \cap C)$

(d) Draw a Venn diagram to illustrate the events A, B and C and the probabilities for each region.

(e) Find $\mathrm{P}([\mathrm{B} \cup \mathrm{C}]')$

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114. In a company the 200 employees are classified as full-time workers, part-time workers or contractors. The table below shows the number of employees in each category and whether they walk to work or use some form of transport.

	Walk	Transport
Full-time worker	2	8
Part-time worker	35	75
Contractor	30	50

The events F, H and C are that an employee is a full-time worker, part-time worker or contractor respectively. Let W be the event that an employee walks to work. An employee is selected at random.

Find

(a) $\mathrm{P(H)}$

(b) $\mathrm{P}([\mathrm{F} \cap \mathrm{W}]')$

(c) $\mathrm{P(W \mid C)}$

Let B be the event that an employee uses the bus.

Given that 10% of full-time workers use the bus, 30% of part-time workers use the bus and 20% of contractors use the bus,

(d) draw a Venn diagram to represent the events F, H, C and B,

(e) find the probability that a randomly selected employee uses the bus to travel to work.