Calculus Test 2

Test number 2: Calculus

Topics Covered

1. Calculus AA SL syllabus

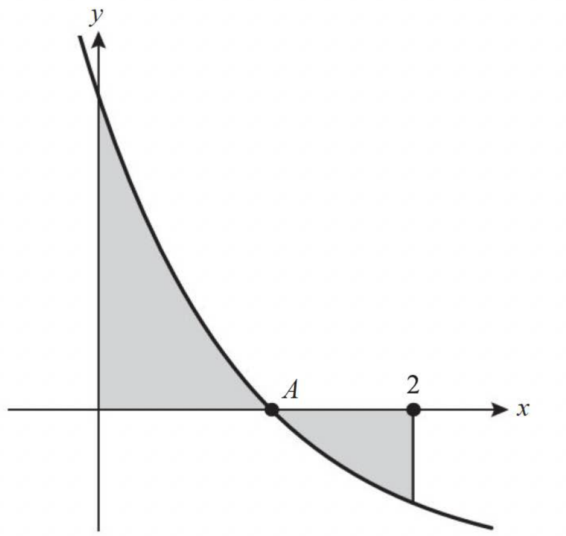
Made and compiled for students of
Chemyst Tuition Centre
5 tank road Singapore
ibmath.sg

1 GDC questions are with *

1. Consider the function $f(x) = \frac{e^{3x}}{kx}, k \neq 0$.
a) Find the x-coordinate of the stationary point. [3]
b) For what values of k is the stationary point:[2]
i) a local minimum....[2]
ii) a local maximum?....[2]
c) Given that the stationary point has y-coordinate $-\frac{e}{2}$, find k and determine the nature of the stationary point....[3]
d) State the location and nature of the stationary point of $g(x) = -f(2x)$...[2]

---

2. The curve in the diagram has equation $y = 3e^{-x} - 1$.

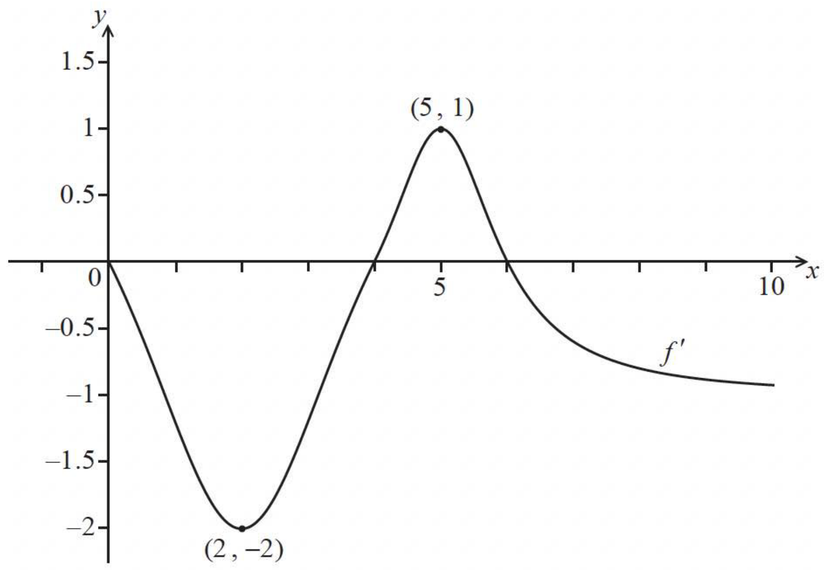


a) Find the exact coordinates of A.[2]
b) Find the shaded area.[3]

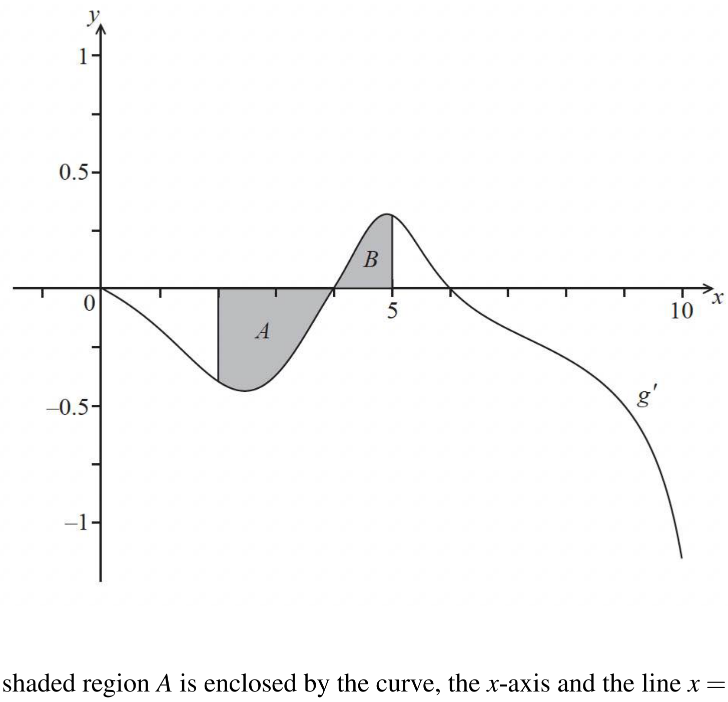
3*. Let $f(x) = \frac{\ln(4x)}{x}$, for $0 < x \leq 5$.

Points P$(0.25, 0)$ and Q are on the curve of f. The tangent to the curve of f at P is perpendicular to the tangent at Q. Find the coordinates of Q. [5]

4.* Consider a function f, for $0 \leq x \leq 10$. The following diagram shows the graph of f', the derivative of f.



(a) The graph of f has a local maximum point when $x = p$. State the value of p, and justify your answer....... [3]
(b) Write down $f'(2)$......[1]
Let $g(x) = \ln(f(x))$ and $f(2) = 3$.
(c) Find $g'(2)$..........[4]
(d) Verify that $\ln 3 + \int_2^a g'(x)\mathrm{d}x = g(a)$, where $0 \leq a \leq 10$.......... [4]
(e) The following diagram shows the graph of g', the derivative of g.



The shaded region A is enclosed by the curve, the x-axis and the line $x = 2$, and has area 0.66 units ^{2}.

The shaded region B is enclosed by the curve, the x-axis and the line $x = 5$, and has area 0.21 units ^{2}. Find $g(5)$.
[4marks]

5. A function f has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where k is a constant. [14 marks]

(a) Find $f''(x)$.

The graph of f has a point of inflexion when $x = 1$.

(b) Show that $k = 3$.

(c) Find $f'(-2)$.

(d) Find the equation of the tangent to the curve of f at $(-2, 1)$, giving your answer in the form $y = ax + b$.

(e) Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$.