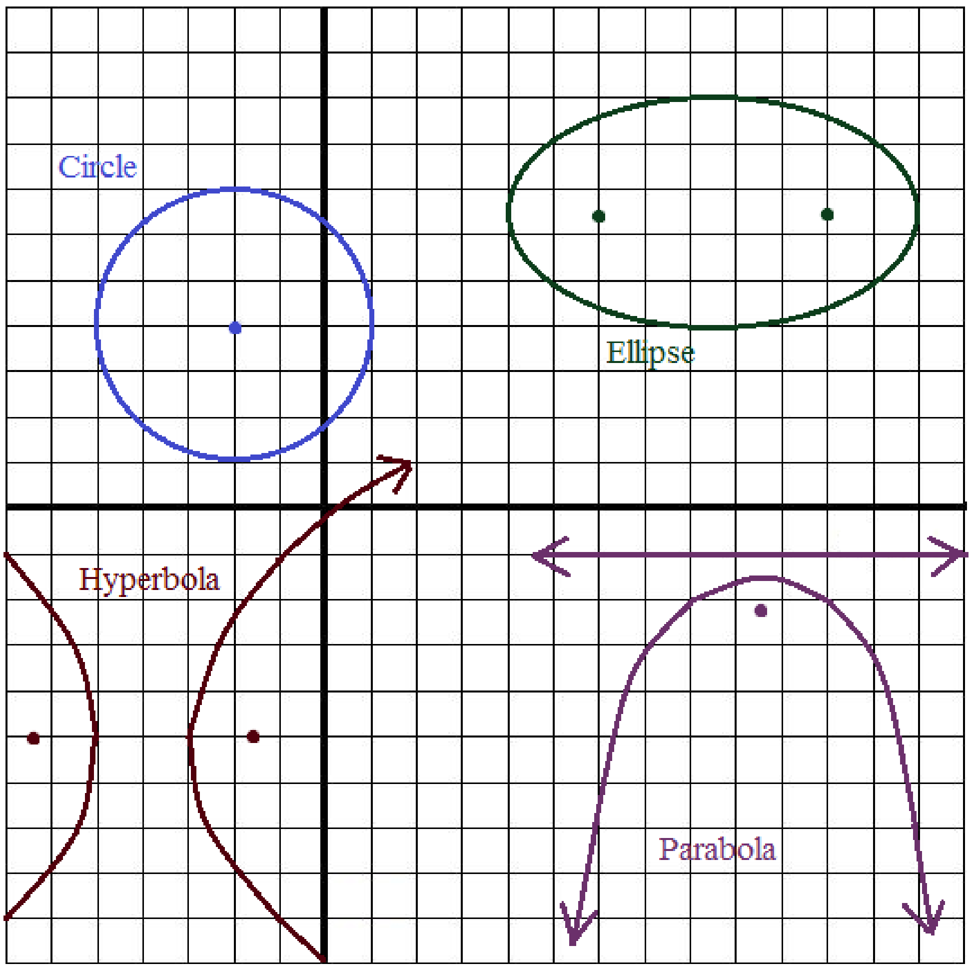
Complete Conics:

Notes, Examples, and Graphs



**Includes 4 Practice Tests (& Solutions)

Table of Contents

PREVIEW

Thanks for checking out this math packet. Inside are conics notes, formulas, graphs, and more. The examples and practice tests are designed to help you learn/review this important Algebra II topic.

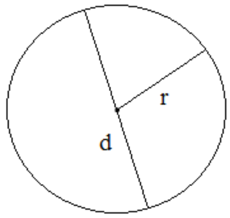
If you have any feedback, questions, or suggestions, let us know!

Lance…

Circle

Notes on Conics

What is it? A set of (coplaner) points equidistant from a given point (the center).



r = radius
d = diameter

d = 2r

Area = πr^2

Circumference = $2\pi r$ or πd

General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
where A = C

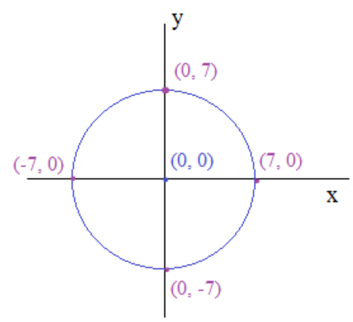
Standard Form: $(x-h)^2 + (y-k)^2 = r^2$

where (h, k) is the center
r is the radius

Example 1: $x^2 + y^2 = 49$

(h, k) = (0, 0)
The center is the origin!

$r^2 = 49$
The radius is 7



Comments:

Standard form: $x^2 + y^2 - 49 = 0$

The coefficients of x^2 and y^2 are equal (both are 1)

Any point on the circle will satisfy the equation. $(0)^2 + (7)^2 = 49$ $(-7)^2 + (0)^2 = 49$

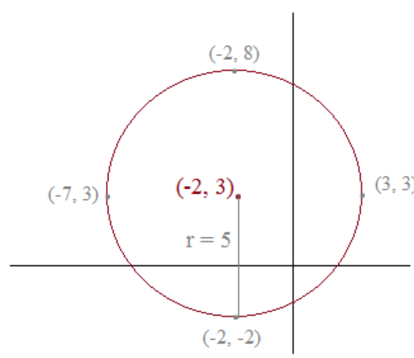
The distance from any point to the center = 7

Example 2: $(x + 2)^2 + (y - 3)^2 = 25$

h = -2
k = 3

The center is (-2, 3)

The radius is 5



Comments:

Because the standard form is $(x - h)^2 + (y - k)^2 = r^2$
the center is (-2, 3), NOT (2, -3)

Since the center is (-2, 3) and the radius is 5,
we know the following points are on the circle:

(-7, 3) 5 units left
(3, 3) 5 units right
(-2, 8) 5 units up
(-2, -2) 5 units down

To check the graph, plug a few points into the equation:

(3, 3) $((3) + 2)^2 + ((3) - 3)^2 = 25$
$5^2 + 0^2 = 25$ ✓

(-2, 8) $((-2) + 2)^2 + ((8) - 3)^2 = 25$
$0^2 + 5^2 = 25$ ✓

Circles, Completing the Square & Inequalities

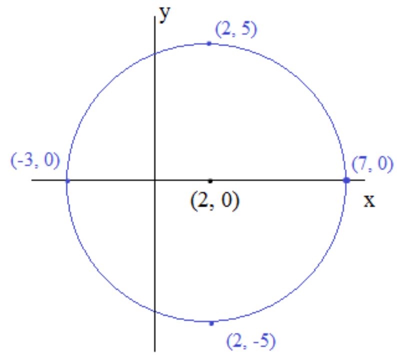
Example 3: $x^2 + y^2 - 4x = 21$

The coefficients of x^2 and y^2 are the same (1), so we know it's a circle.

To find the center, we complete the square. Then, express the equation in standard form.

$$x^2 + y^2 - 4x = 21$$
$$x^2 - 4x + y^2 = 21$$
$$x^2 - 4x + 4 + y^2 = 21 + 4$$
$$(x - 2)(x - 2) + y^2 = 25$$
$$\boxed{(x - 2)^2 + (y - 0)^2 = 25}$$

$h = 2$
$k = 0$
The center is (2, 0)
The radius is 5



To check your solutions, plug points into original equation:

(2, 5): $(2)^2 + (5)^2 - 4(2) = 21$
$4 + 25 - 8 = 21$ ✓

(-3, 0): $(-3)^2 + (0)^2 - 4(-3) = 21$
$9 + 0 + 12 = 21$ ✓

Example 4: $x^2 + y^2 + 6x - 16y + 48 \leq 0$

At first, we'll ignore the < sign and rewrite the equation in standard form.

Step 1: Complete the square:

$$x^2 + 6x + y^2 - 16y = -48$$
$$x^2 + 6x + 9 + y^2 - 16y + 64 = -48 + 9 + 64$$
$$(x + 3)(x + 3) + (y - 8)(y - 8) = 25$$
$$\boxed{(x + 3)^2 + (y - 8)^2 = 25}$$

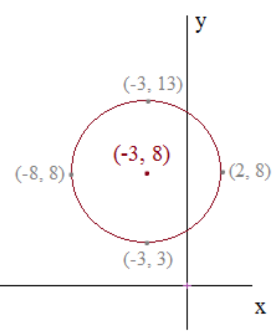
The circle has center (-3, 8) and radius 5

Step 2: Graph and verify circle

$$x^2 + y^2 + 6x - 16y + 48 = 0$$

(2, 8): $(2)^2 + (8)^2 + 6(2) - 16(8) + 48 = 0$
$4 + 64 + 12 - 128 + 48 = 0$
$0 = 0$ ✓

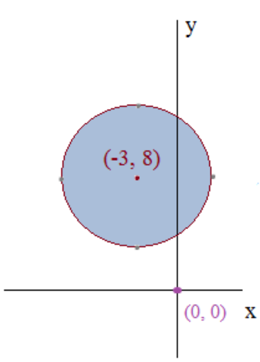
(-3, 13): $(-3)^2 + (13)^2 + 6(-3) - 16(13) + 48 = 0$
$9 + 169 - 18 - 208 + 48 = 0$
$0 = 0$ ✓



Step 3: Test points in original equation to determine the shaded area.

$$x^2 + y^2 + 6x - 16y + 48 \leq 0$$

(0, 0) is outside the circle
(-3, 8) is inside the circle



Test (0, 0): $(0)^2 + (0)^2 + 6(0) - 16(0) + 48 \leq 0$
$0 + 0 + 0 - 0 + 48 \leq 0$ NO

Test (-3, 8): $(-3)^2 + (8)^2 + 6(-3) - 16(8) + 48 \leq 0$
$9 + 64 - 18 - 128 + 48 \leq 0$ YES!

Shaded region will include (-3, 8) and exclude (0, 0)

Find the equation of a line <u>tangent to</u> $x^2 + y^2 - 10x - 8y = -16$ at the point (2, 8)

Step 1: Identify the Conic

Complete the square to put into Standard Form.

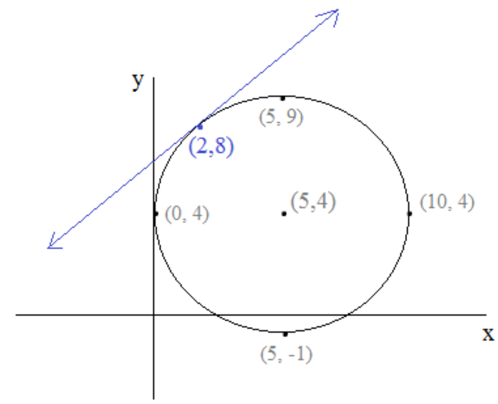
$$x^2 + y^2 - 10x - 8y = -16$$

$$x^2 - 10x + 25 \quad + y^2 - 8y + 16 = -16 + 25 + 16$$

$$(x-5)^2 \quad + \quad (y-4)^2 \quad = \quad 25$$

$$(x-5)^2 + (y-4)^2 = 25$$

Circle with center (5, 4) and radius 5



Step 2: Sketch a graph and verify.

The center is (5, 4)... And, the radius is 5..

So, (5, 9) up
(10, 4) right
(5, -1) down
(0, 4) left
are all 5 units from the center

and, using the distance formula:

$$\sqrt{(5-2)^2 + (4-8)^2} = 5$$

we can verify that (2, 8) is 5 units from the center.

Note: To check your work, substitute a few points into the equation.

(0, 4): $(0)^2 + (4)^2 - 10(0) - 8(4) = -16$

$0 + 16 - 0 - 32 = -16$ ✓

(10, 4): $(10)^2 + (4)^2 - 10(10) - 8(4) = -16$

$100 + 16 - 100 - 32 = -16$ ✓

(2, 8): $(2)^2 + (8)^2 - 10(2) - 8(8) = -16$

$4 + 64 - 20 - 64 = -16$ ✓

All these points fit into the original equation; Therefore, they are all points on the circle!

Step 3: Identify the tangent line

To find the equation of a line, you need a <u>point</u> and the <u>slope</u>.

Point: (2, 8)

What is the slope? Since a line tangent to a circle is perpendicular to the adjoining radius, we need to find the slope of that radius!

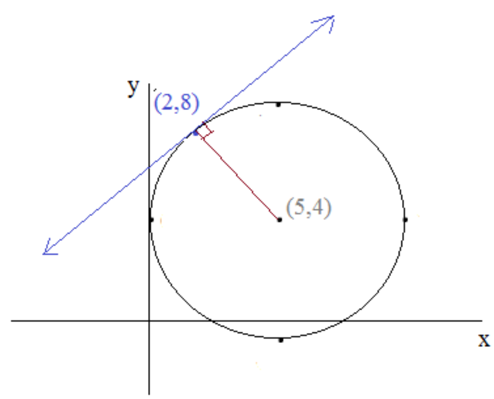
slope of segment connecting (2, 8) and (5, 4)

$$m = \frac{8-4}{2-5} = -\frac{4}{3}$$

Therefore, the slope of a perpendicular line is $\frac{3}{4}$

So, the equation of a line tangent to the circle at (2, 8) is

$$(y-8) = \frac{3}{4}(x-2)$$



Circles Practice Quiz

I. Parts of a circle

Identify the radius, center, and 3 points on each circle:

A) $x^2 + y^2 = 9$

center: _______

radius: _______

3 points: _______

____

____

B) $x^2 + (y - 3)^2 = 4$

center: _______

radius: _______

3 points: _______

____

____

C) $(x - 2)^2 + (y + 1)^2 = 3$

center: _______

radius: _______

3 points: _______

____

____

D) $x^2 + 10x + y^2 = -24$

center: _______

radius: _______

3 points: _______

____

____

E) $3x^2 + 3y^2 + 6x + 12y + 9 = 0$

center: _______

radius: _______

3 points: _______

____

____

II. Equation of a Circle

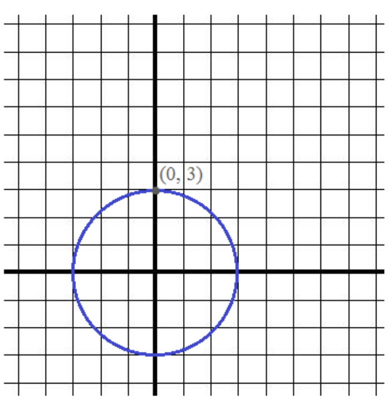
Determine the equation of each circle from the given information.
(Express your answer in both *standard* form and *general* form)

A) radius: 4
center: origin

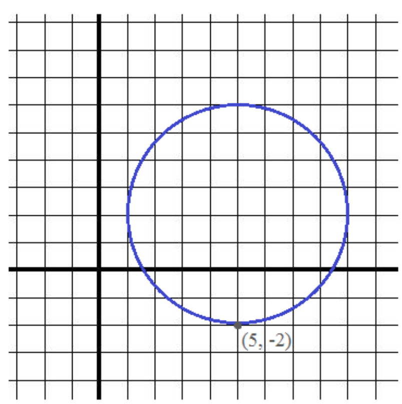
B) radius: 5
center: (2, 4)

C) radius: 7
center: (-4, 0)

D)



E)

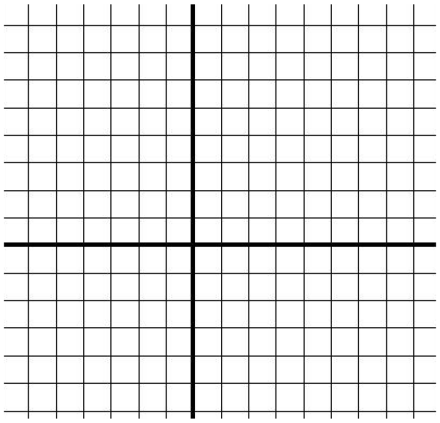


Circles Practice Quiz

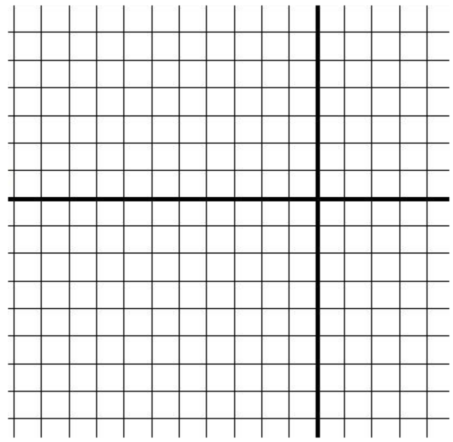
III. Graphing circles

Graph the following circles; label the center, radius, and 2 points

A) $x^2 + (y - 4)^2 = 16$



B) $x^2 + 6x + y^2 + 2y - 15 = 0$



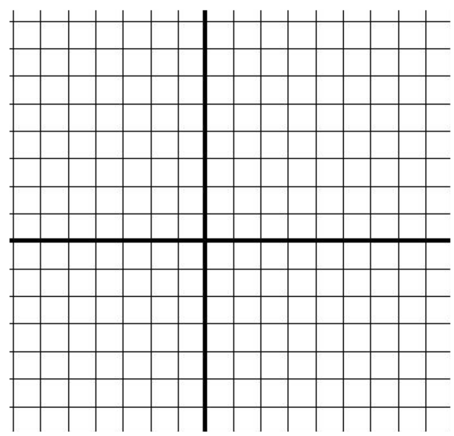
IV. Applications

A) The diameter of a circle has endpoints (2, 4) and (8, 4).
What is the equation of the circle?

B) Given: circle $x^2 + y^2 = 25$

line $x + y = 3$

What are the points of intersection?
Solve algebraically <u>and</u> verify graphically.



C) What is the equation of the line *tangent* to the circle $(x - 3)^2 + (y - 4)^2 = 10$ through the point (4, 1)?

What is the equation of the *normal* through (4, 1) ?

Circles Practice Quiz

SOLUTIONS

Standard form of circle: $(x-h)^2 + (y-k)^2 = r^2$
where (h, k) is the vertex
r is the radius

I. Parts of a circle

Identify the radius, center, and 3 points on each circle:

A) $x^2 + y^2 = 9$

center: (0, 0)

radius: 3

3 points: (0, 3)

(3, 0)

(-3, 0)

(note: there are more than 3 possible points; any (x, y) that is a solution is a point)

B) $x^2 + (y-3)^2 = 4$

center: (0, 3)

radius: 2

3 points: (0, 1)

(2, 3)

(0, 5)

C) $(x-2)^2 + (y+1)^2 = 3$

center: (2, -1)

radius: $\sqrt{3}$

3 points: $(\sqrt{3}+2, -1)$

$(-\sqrt{3}+2, -1)$

$(2, \sqrt{3}-1)$

D) $x^2 + 10x + y^2 = -24$

complete the square to put into standard form:

center: (-5, 0)

radius: 1

3 points: (-5, 1)

(-5, -1)

(-4, 0)

$x^2 + 10x + \quad y^2 = -24$

$x^2 + 10x + 25 + y^2 = -24 + 25$

$(x+5)(x+5) + y^2 = 1$

$(x+5)^2 + y^2 = 1$

E) $3x^2 + 3y^2 + 6x + 12y + 9 = 0$

center: (-1, -2)

radius: $\sqrt{2}$

3 points: (0, -1)

(-2, -1)

(0, -3)

$3x^2 + 3y^2 + 6x + 12y + 9 = 0$ (divide by 3 and rearrange)

$x^2 + 2x + \quad y^2 + 4y + 3 = 0$

$x^2 + 2x + 1 + y^2 + 4y + 4 = -3 + 1 + 4$

$(x+1)(x+1) + (y+2)(y+2) = 2$

$(x+1)^2 + (y+2)^2 = 2$

II. Equation of a Circle

General Form: $Ax^2 + Cy^2 + Dx + Ey + F = 0$

Determine the equation of each circle from the given information.
(Express your answer in both *standard* form and *general* form)

A) radius: 4
center: origin

Standard Form: $(x-0)^2 + (y-0)^2 = 16$

$x^2 + y^2 = 16$

General Form: $x^2 + y^2 - 16 = 0$

B) radius: 5
center: (2, 4)

Standard Form: $(x-2)^2 + (y-4)^2 = 25$

(expand terms and simplify)

General Form: $x^2 - 4x + 4 + y^2 - 8y + 16 = 25$

$x^2 + y^2 - 4x - 8y - 5 = 0$

C) radius: 7
center: (-4, 0)

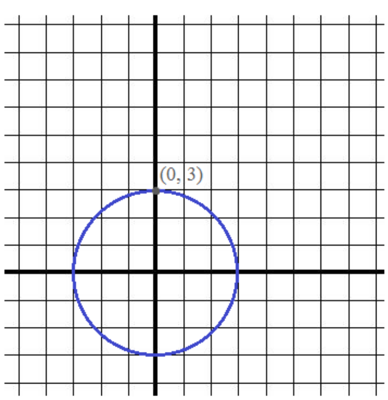
Standard Form: $(x+4)^2 + y^2 = 49$

General Form: $x^2 + 8x + 16 + y^2 - 49 = 0$

$x^2 + 8x + y^2 - 33 = 0$

$x^2 + y^2 + 8x - 33 = 0$

D)



Center: (0, 0)
Radius: 3

Standard Form:

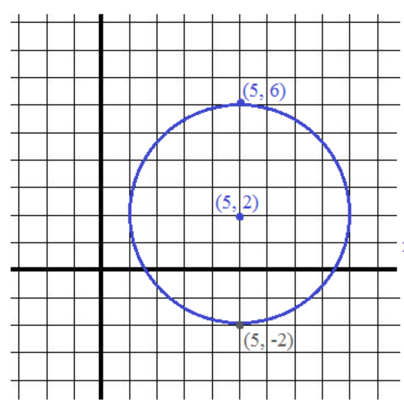
$x^2 + y^2 = 9$

General Form:

$x^2 + y^2 - 9 = 0$

(Note: you may test points to check your equation)

E)



Center: (5, 2)
Radius: 4

Standard Form:

$(x-5)^2 + (y-2)^2 = 16$

General Form:

$x^2 - 10x + 25 + y^2 - 4y + 4 = 16$

$x^2 + y^2 - 10x - 4y + 13 = 0$

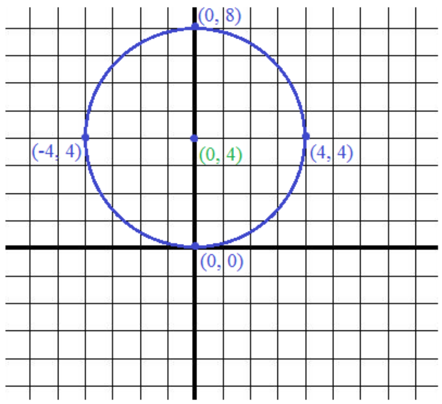
Circles Practice Quiz

SOLUTIONS

III. Graphing circles

Graph the following circles; label the center, radius, and 2 points

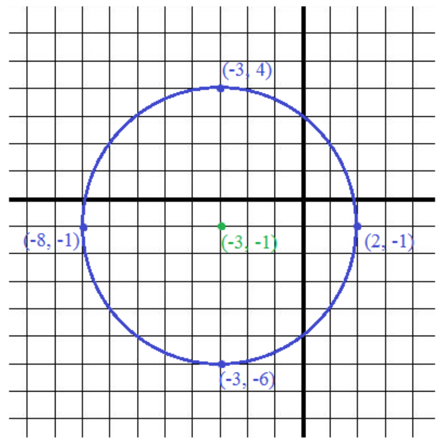
A) $x^2 + (y-4)^2 = 16$



Center: (0, 4)
Radius: 4

$\sqrt{16}$

B) $x^2 + 6x + y^2 + 2y - 15 = 0$



Change to Standard form
(to show center and radius):

$x^2 + 6x + y^2 + 2y - 15 = 0$

$x^2 + 6x + 9 + y^2 + 2y + 1 = 15 + 9 + 1$

$(x+3)^2 + (y+1)^2 = 25$

Center: (-3, -1)
Radius: 5

IV. Applications

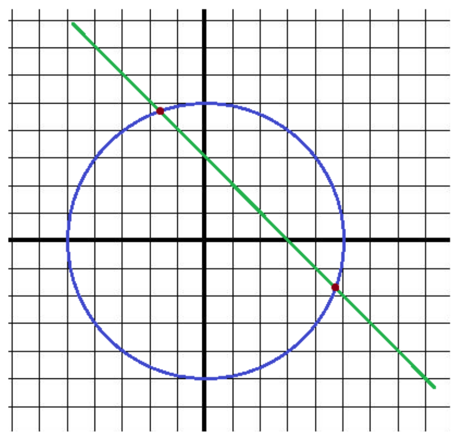
A) The diameter of a circle has endpoints (2, 4) and (8, 4).
What is the equation of the circle?

$(x-5)^2 + (y-4)^2 = 9$

Since the endpoints of the diameter are given, we can use the midpoint to find the center of the circle:
center: (5, 4)
Since the distance between endpoints is 6, the radius is 3

B) Given: circle $x^2 + y^2 = 25$
line $x + y = 3$

What are the points of intersection?
Solve algebraically and verify graphically.



Algebraically, we'll use substitution: $x = 3 - y$

$x^2 + y^2 = 25$

$(3-y)^2 + y^2 = 25$

$9 - 6y + y^2 + y^2 = 25$

$2y^2 - 6y - 16 = 0$

$y^2 - 3y - 8 = 0$

(-1.7, 4.7)	(4.7, -1.7)
approximate pts. of intersection	

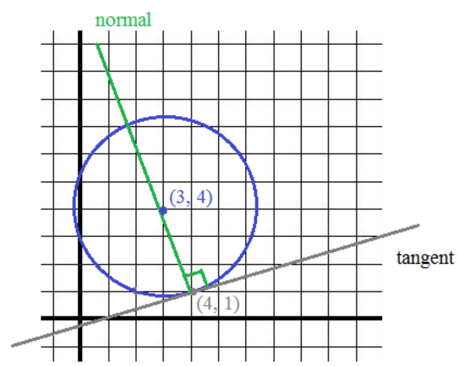
$y = \frac{3 \pm \sqrt{9 - 4(1)(-8)}}{2(1)} = \frac{3+\sqrt{41}}{2}$ and $\frac{3-\sqrt{41}}{2}$

4.7   -1.7

(substitute each y solution to find x)

C) What is the equation of the line *tangent* to the circle $(x-3)^2 + (y-4)^2 = 10$ through the point (4, 1)?

What is the equation of the *normal* through (4, 1) ?



(To find the equation of a line, we need a point and slope)

Point: (4, 1)

Slope? (**A tangent line is perpendicular to the normal; and, the normal goes through the center!)

slope of line through (3, 4) and (4, 1) is -3

(slope of perpendicular line: opposite reciprocal -- 1/3)

Equation of normal: $(y - 1) = -3(x - 4)$

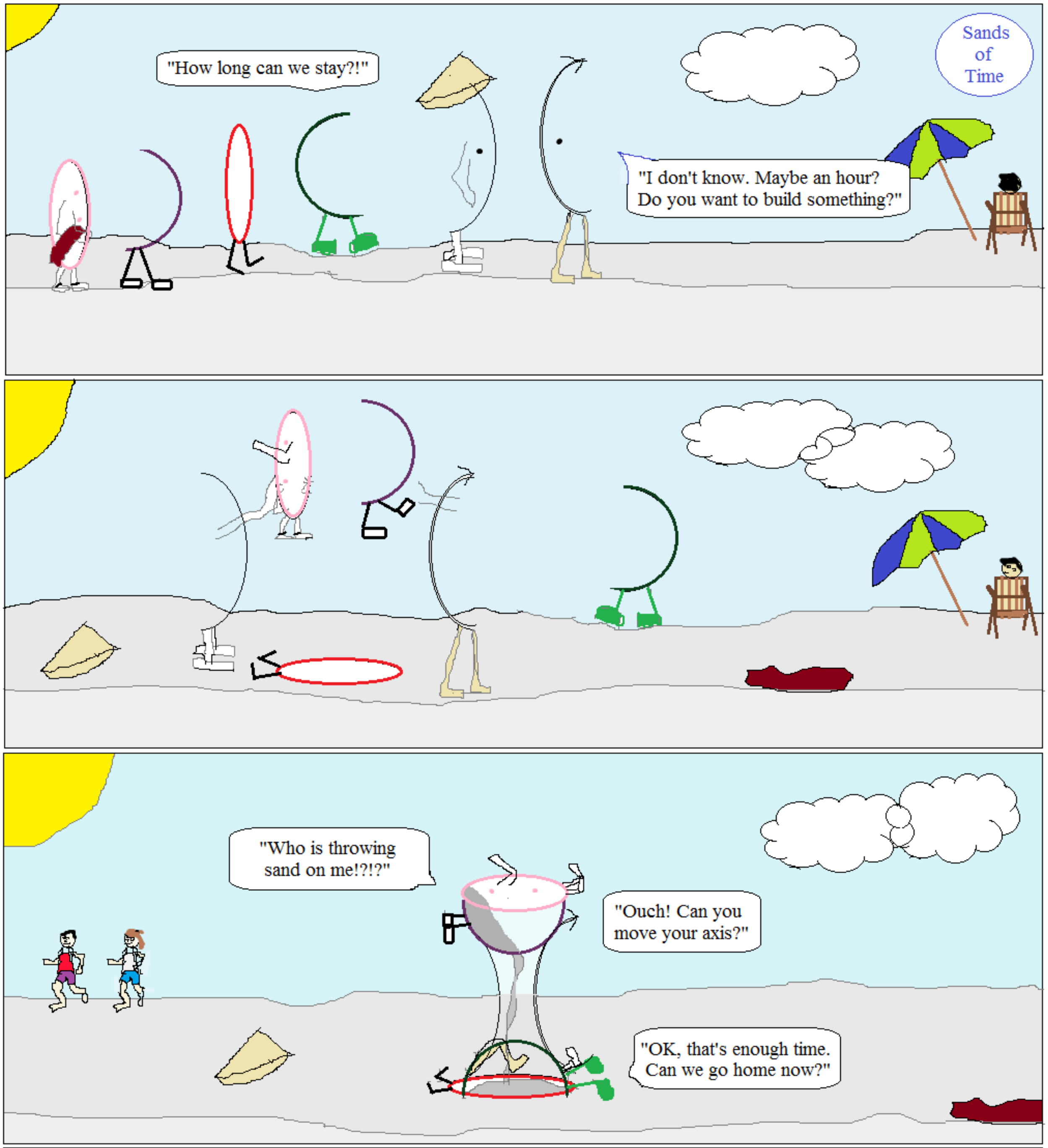
Equation of tangent: $(y - 1) = \frac{1}{3}(x - 4)$

(THIS IS A PREVIEW)

To review ellipses, hyperbolas, and parabolas, purchase the complete conics notes, examples and graphs… (Available from the Mathplane Stores at TES.com and TeachersPayTeachers.com)

Or, visit mathplane.com

Thanks for your support!



Harry Jr. and the remaining Balance of Conicks spend an Hour at Glasse Beach...

LanceAF #151 (8-14-14)
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