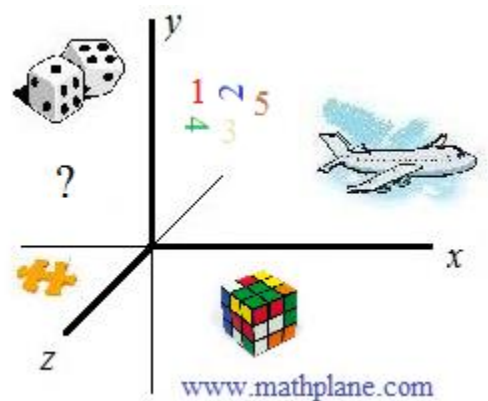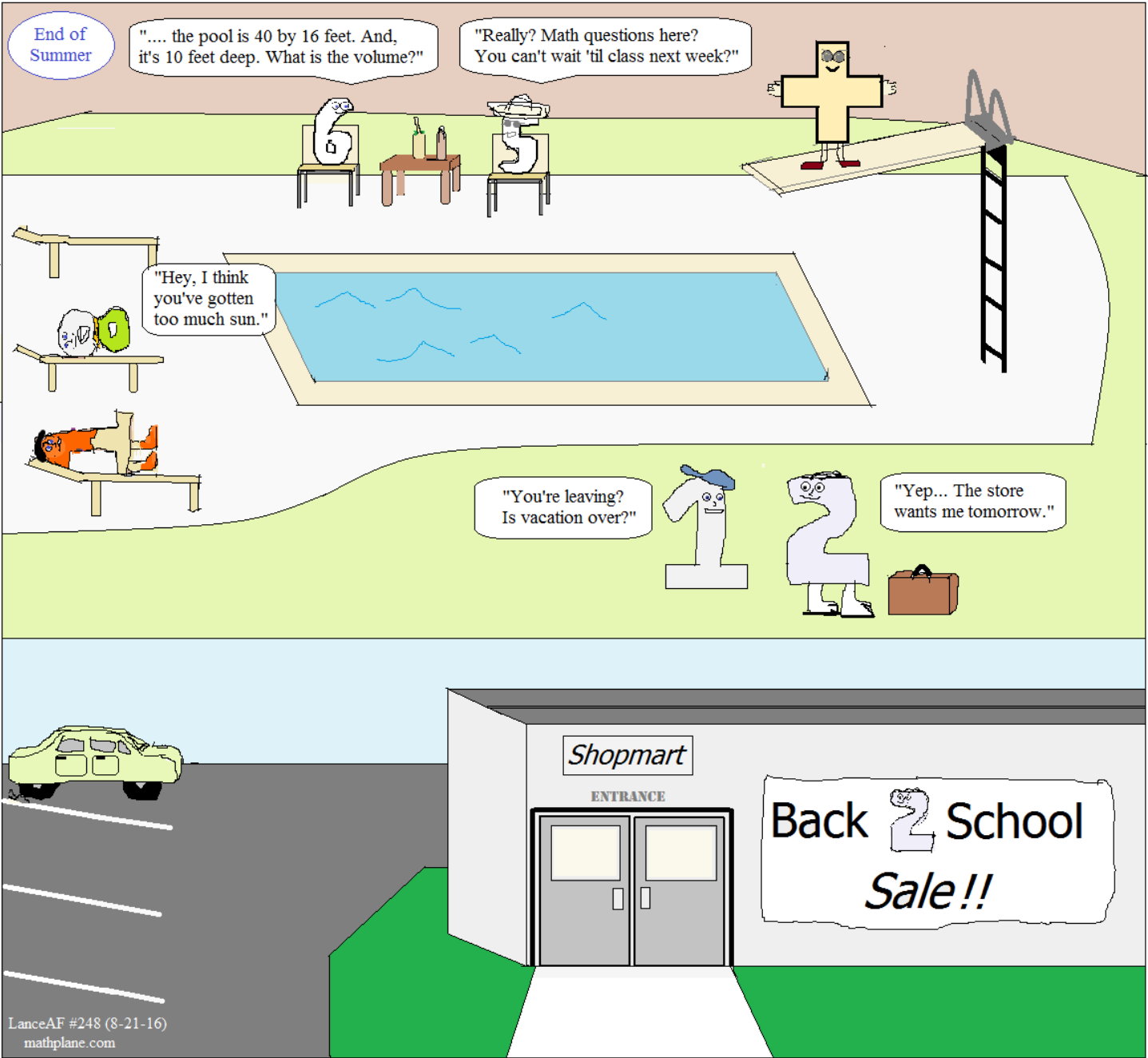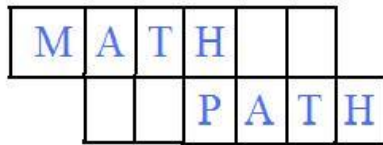# Mathplane Featured Destination:

Math Enrichment Items

- Math Path
- Riddles & Puzzles
- Cartesian Coordinate Cartoon
- Topic: Summation

M	A	T	H	

		P	A	T	H

Directions:

Draw a path connecting the arrow in the upper-left corner to the arrow in the lower-right corner.

Rules:

The path must move between adjacent boxes (i.e. no skipping!)
The path may only travel <u>horizontally or vertically</u>.. (NOT diagonally).
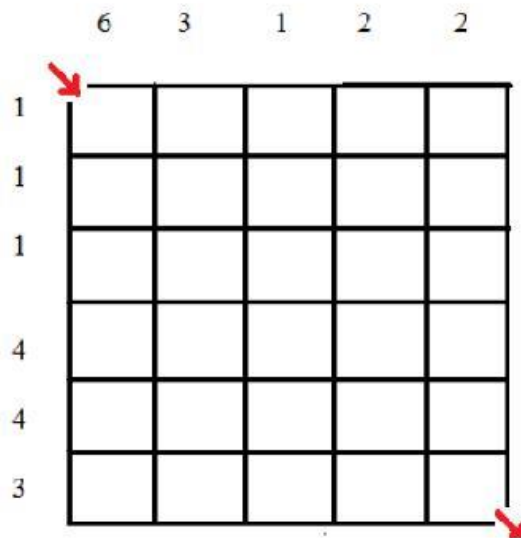The path may not pass through any box more than once...

Hints:

The numbers above the grid represent the number of boxes the path goes through in that column.
The numbers on the left indicate the number of boxes in each row where the path goes through.

Easy

	6	3	1	2	2
1					
1					
1					
4					
4					
3					

M	A	T	H		
		P	A	T	H

Medium

	3	5	1	4	5	2	7
3							
4							
5							
3							
4							
3							
5							
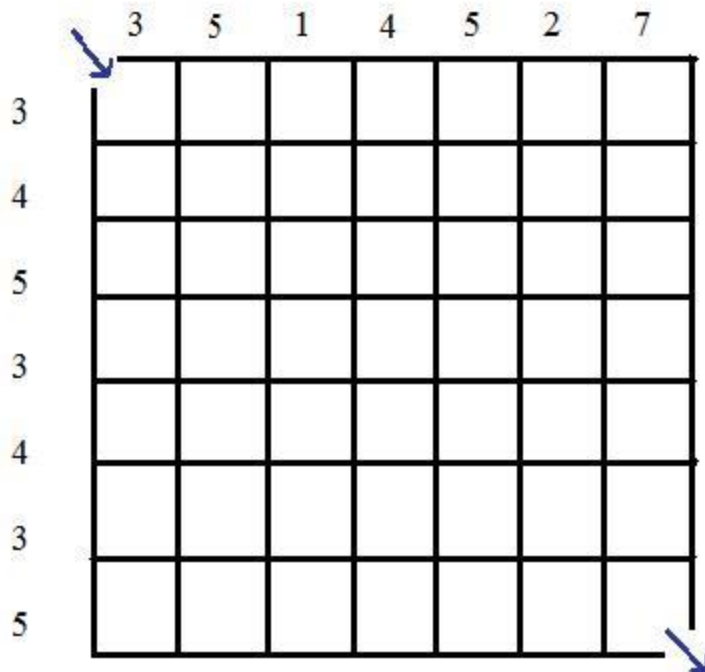
M A T H

P A T H

	8	6	3	4	7	4	3	4	4	7
Hard 10										
3										
8										
3										
1										
10										
6										
4										
5										
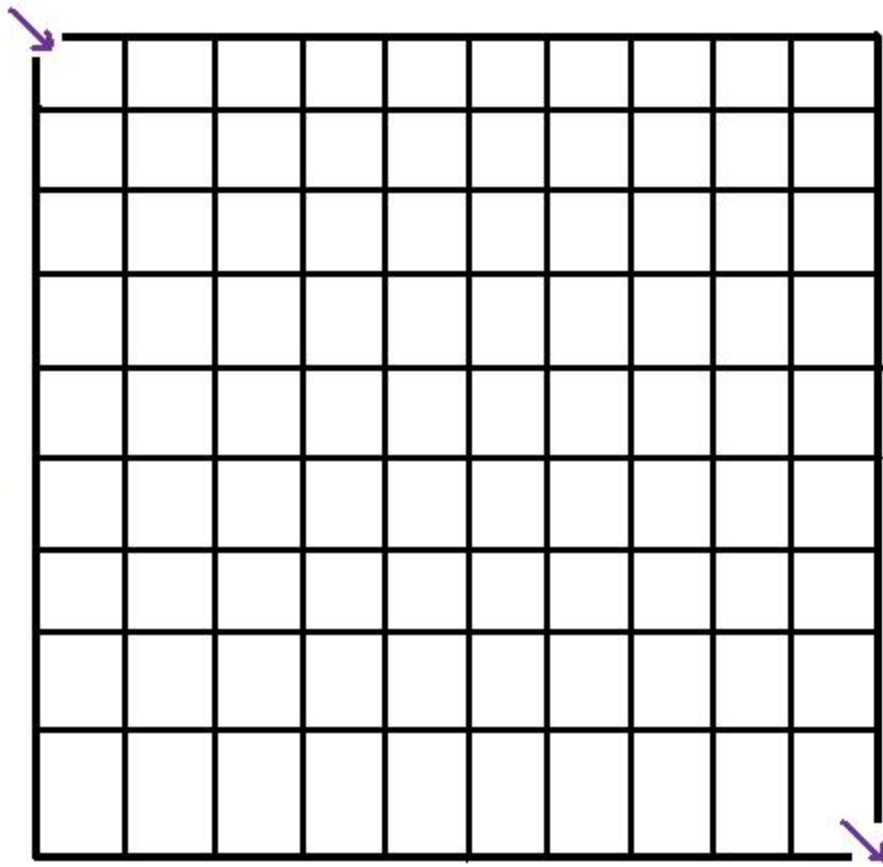
1) What is the largest amount of money you can have in change and still not have change for a dollar?

2) If you were to construct a 7 x 7 checkered square (i.e.7 x 7 chessboard), how many squares (of various dimensions) would there be in total?
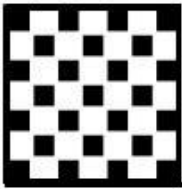
3) What are the next two letters in the following series and why?

W   A   T   N   T   L   I   T   F   S   ___   ___

4) There are 3 boxes. 1 has apples, 1 has oranges, and the other has apples AND oranges.
The boxes are labeled wrong so that *no label is correct*.
Sue opens just one box, and without looking in the box, takes out 1 piece of fruit.
She looks at the fruit and immediately labels all the boxes correctly.
Which box did she open and how did she know!?!?

1) If you say my name, I'll no longer exist.

2) If there are 3 apples, and you take away 2, how many apples do you have?

3) How far can a dog run into the woods?

4) What is the best possible score in bowling, assuming you never throw a strike?

  (note: There are 10 frames in a game...
       A strike is 10 pins PLUS the next 2 rolls...
       A spare is 10 pins PLUS the next 1 roll...)

5) Challenge***

  Explain why   30414093201571337804361260816606064768844331207291027000
                    cannot possibly be 50!   (50 factorial),
                        without actually performing the calculation...

6) Place the digits   9, 4, 7, 6, 5, 1 in the spots below in order to get the *largest* result.


            __ __ x __ __ + __ __ = ?

  6A) Now, position the digits 9, 4, 7, 6, 5, 1 in the spots above to get the *smallest* result.

"Why did the math student get lost?"
(Answer is with the solutions)

1) Draw a line segment connecting (-6, -2) to (2, -2)

2) Draw a ray with endpoint (2, -2) that has a slope of 2/5

3) Construct a circle with diameter of 2 and the center (6, 1)

4) Construct a circle with radius of 1 and the center (11, 3)

5) Draw an isosceles triangle with base 3 and altitude of 5... The median of the base is (2.5, 4)

6) Reflect the image over the y-axis AND shift the triangle down 2 units

7) Shade in the following areas:
(x, y) where $2 < x < 3$ and $0 < y < 4$
and
(x, y) where $-3 < x < -2$ and $-1 < y < 2$

8) Draw line segment #8 connecting (6, 2) to (11, 4)

9) Draw a line segment perpendicular to line segment #8. The length is approximately 2 ½ units and rises from (6, 2)

10) Draw a vertical line segment from (5, 4) to (5, 7)

11) Draw a 1 unit horizontal segment from (5, 7) to (6, 7)

12) Inscribe an ellipse in the region within these points: (6, 7) (6, 8) (8, 7) (8, 8)

13) Draw vertical line segment #13 from (7,7) to line segment #8

14) From the midpoint of segment #13, draw a segment with slope -1 to segment #8 (this should form "an upside down 45 degree angle".)

15) Connect the right endpoint of segment #11 to the bottom of the ellipse.

y-axis

x-axis
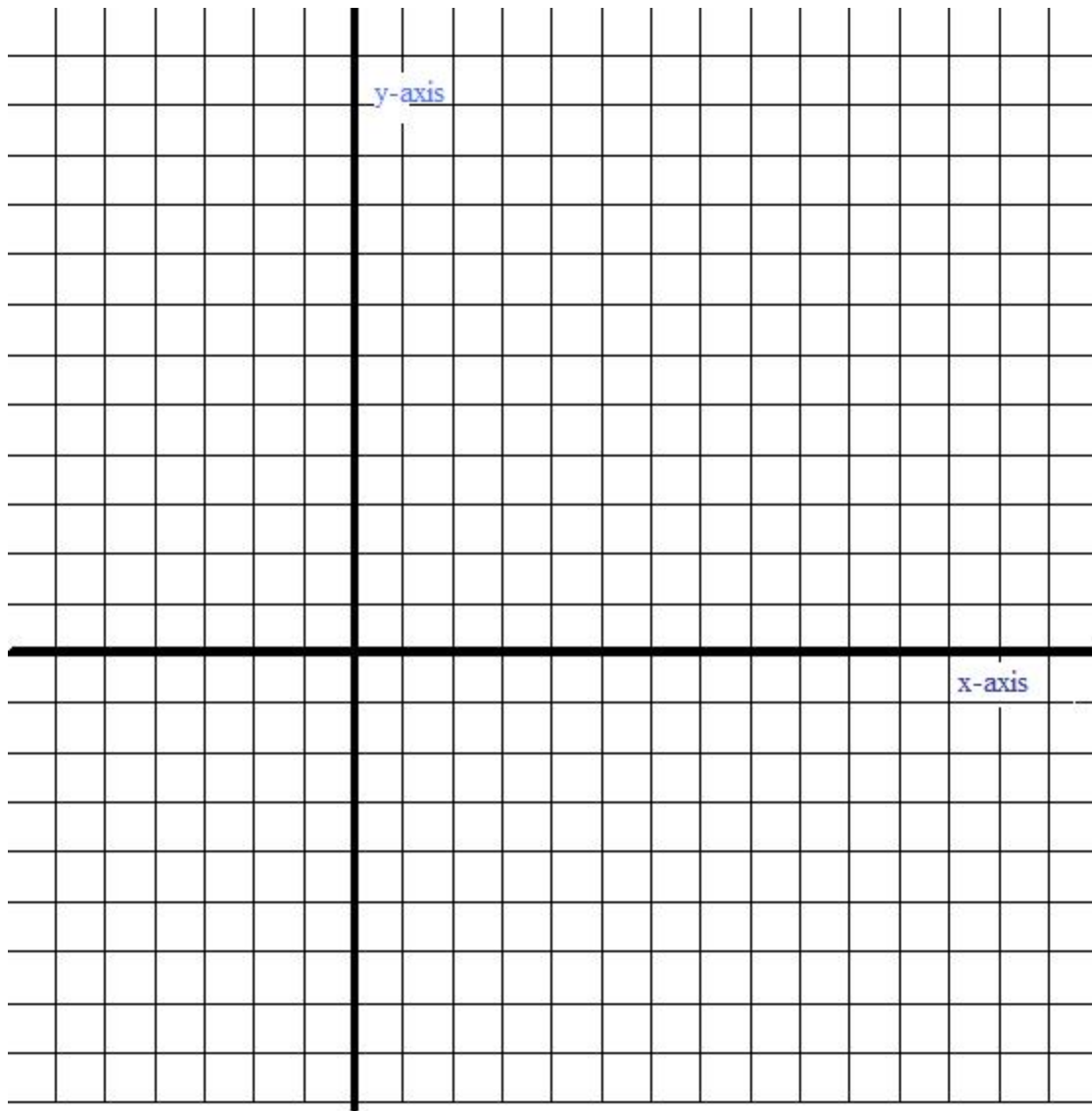
Cartesian Coordinate Cartoons

Note: Each square is
(one unit) x (one unit)

Summation Notes

Definition: The addition of a set of number; the result is their *sum* or *total*

Symbols & Notation: To represent the summation of many similar terms, mathematics use the symbol Σ

This upright capital sigma is defined as

$$\sum_{i=m}^{n} x_i = x_m + x_{m+1} + x_{m+2} + \ldots + x_{n-1} + x_n.$$

i index of summation i, m, n are integers
m lower bound of summation
n upper bound of summation

Example:

$$\sum_{k=2}^{6} k^2 = 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 90$$

Derivation of notation:

$$\sum_{0 \le k < 100} f(k)$$

$f(0) + f(1) + \ldots f(99) + f(100)$

Summation Notes (continued)

1. $\displaystyle\sum_{i=1}^{n} c = c + c + c + \dots + c$ (n times) $= cn$, where c is a constant.

$\displaystyle\sum_{i=1}^{7} 9 = 9 + 9 + 9 + 9 + 9 + 9 + 9 =$

$9 \times 7 = 63$

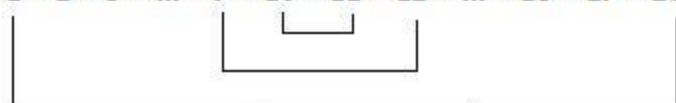$\displaystyle\sum_{i=3}^{6} 5 = 5 + 5 + 5 + 5 = 20$

note: n = 4, because i begins with 3 (rather than 1) and ends with 6...
3, 4, 5, 6 ---> '4 times'

2. $\displaystyle\sum_{i=1}^{n} i = 1 + 2 + 3 + \dots + n = \dfrac{n(n+1)}{2}$

$\displaystyle\sum_{i=1}^{20} i = 1 + 2 + 3 + \dots + 18 + 19 + 20 = \dfrac{20 \times (20+1)}{2}$

$= 420/2 = 210$

Note: $1 + 2 + 3 + \dots + 9 + 10 + 11 + 12 + \dots + 18 + 19 + 20$

each pair adds up to 21
there are 20/2 pairs

$\dfrac{20 \text{ pairs} \times 21}{2} = 210$

3. $\displaystyle\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

$\displaystyle\sum_{i=1}^{8} i^2 = 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = \dfrac{8 \times 9 \times 17}{6}$

$= 204$

4. $\displaystyle\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \dfrac{n^2(n+1)^2}{4}$

$\displaystyle\sum_{i=1}^{6} i^3 = 1 + 8 + 27 + 64 + 125 + 216 = \dfrac{36 \times 49}{4}$

$= 441$

How do you solve when i ≠ 1?

$\displaystyle\sum_{i=4}^{9} i^2 \longrightarrow \sum_{i=1}^{9} i^2 - \sum_{i=1}^{3} i^2$

$\dfrac{9(10)(19)}{6} - \dfrac{3(4)(7)}{6} = 285 - 14 = 271$

Note: $\cancel{1} + \cancel{4} + \cancel{9} + 16 + 25 + 36 + 49 + 64 + 81 = 271$

1) $$\sum_{K=1}^{6} K^2 =$$

2) $$\sum_{r=0}^{7} (2r + 1)^3 =$$

3) $$\sum_{i=1}^{8} 4 =$$

4) $$\sum_{i=1}^{20} 2i =$$

5) $$\sum_{n=4}^{10} 5n + 3 =$$

6) $$\sum_{b=1}^{100} (4 + 3b) =$$

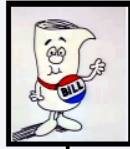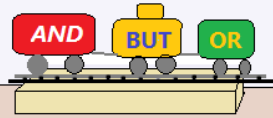7) $$\sum_{c=25}^{150} \left(\frac{1}{(c+4)} - \frac{1}{(c+5)} \right) =$$

8) $$\sum_{r=1}^{3} \sum_{s=1}^{4} rs^2 =$$

9) Give an example of a real world use for summations. (Provide a sample formula)
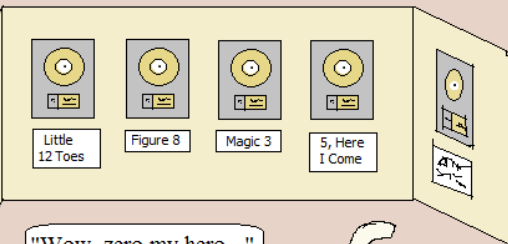
SOLUTIONS-→

M	A	T	H		
		P	A	T	H
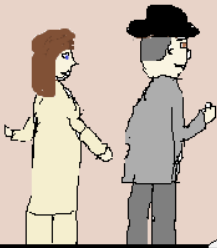
Easy: Step-by-Step Solution

	6	3	1	2	2
1	•	✗	✗	✗	✗
1	•	✗	✗	✗	✗
1	•	✗	✗	✗	✗
4	•				
4	•				
3	•				
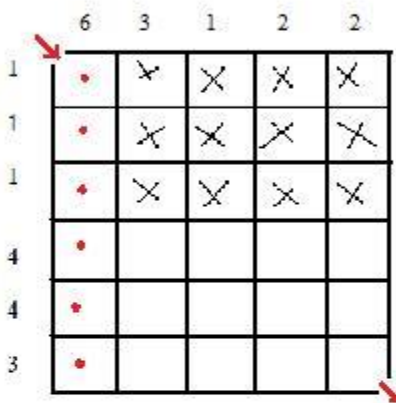
Column 1 has "6"--> fill all boxes
in column 1
Rows 1,2,3 have "1"---> eliminate
remaining boxes

	6	3	1	2	2
1	•	✗	✗	✗	✗
1	•	✗	✗	✗	✗
1	•	✗	✗	✗	✗
4	•	•	•	•	
4	•	•			
3	•	•			
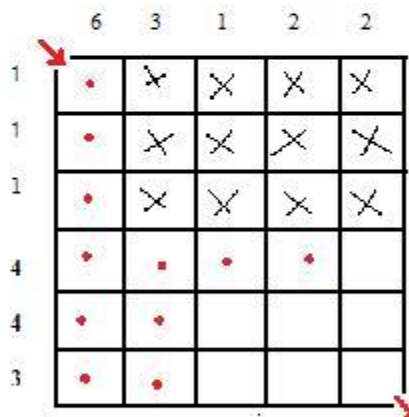
Column 2 has "3" --> fill in
remaining boxes in column 2
Column 3 has "1"---> path
must continue to the right

	6	3	1	2	2
1		✗	✗	✗	✗
1		✗	✗	✗	✗
1		✗	✗	✗	✗
4					✗
4			✗		
3			✗		
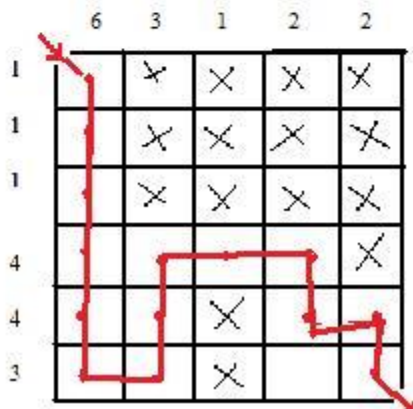
Using the remaining column
and row numbers, you can
confirm the path..

MATH / PATH

MATH (top row)
PATH (bottom row)
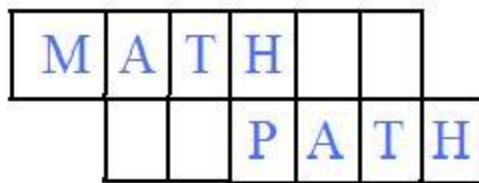
Medium: Step-by-step Solution

Column headers (first grid): 3 5 1 1 5 2 7

Row labels (first grid): 3, 4, 5, 3, 4, 3, 5
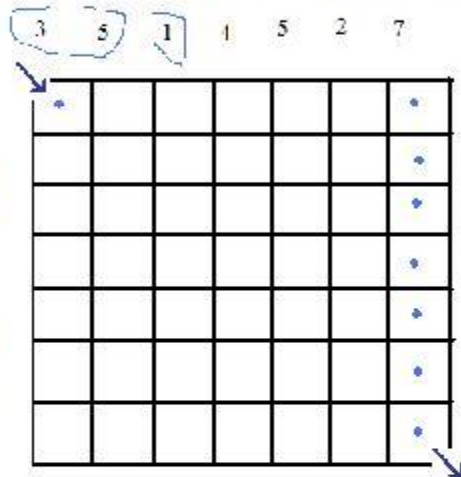
Column 7 is "7"--> fill all boxes

Note: Column 3 is "1"--> columns 1 and 2 MUST BE COMPLETED before crossing over column 3 (because you cannot return)

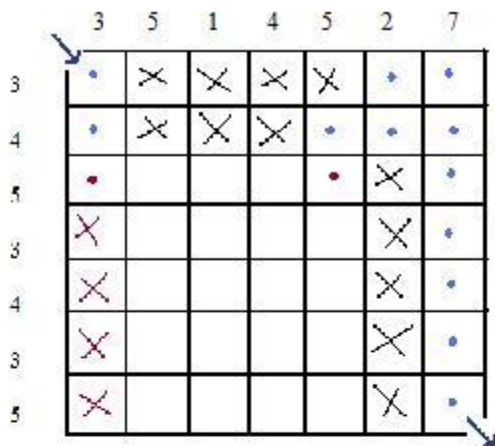Column 6 has "2" & column 7 is entirely filled. Therefore, the top 2 boxes of column 6 must be filled (otherwise, the top right corner cannot be reached)

Column 6 has "2", so remaining boxes are eliminated.

Row 1 has "3", so remaining boxes are eliminated.

For path to continue, the box in row 2/column 1 must be filled.

Column headers (second grid): 3 5 1 4 5 2 7

Row labels (second grid): 3, 4, 5, 3, 4, 3, 5
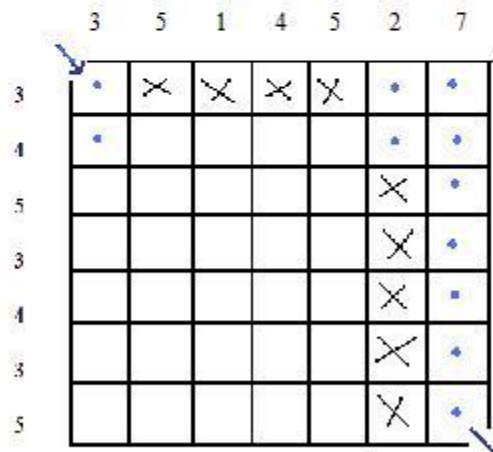
Working back and forth:
add a box in column 5 , row 2
(necessary to extend the path)
eliminate the remaining boxes in row 2
(because row 2 is "4")
Add a box in column 5 (to extend the path)
Add a box in column 1 (to extend the path)
Eliminate more boxes...

Continue working back and forth adding and eliminating boxes according to the shape of the path and the number constraints.
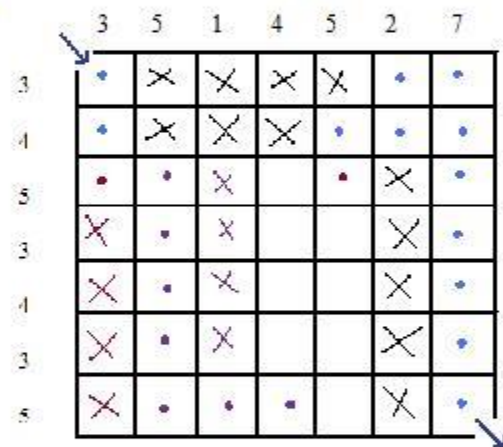
At last, the path meets in the middle. then, connect the boxes!

M A T H

P A T H

Hard: Step-by-step solution

Row 1 is "10"-- fill all the boxes
Row 6 is "10"-- fill all the boxes

Note: Column 1 is "8" (so there will be only 1 open box)

The last 2 boxes in Row 1 are filled, so last box in Row 2 must be filled to connect the corner.

Column 3 is "3" and column 7 has "3" boxes. If either had a filled box in Row 5, then the path could not be completed. They are eliminated.

Similarly, Column 1, Row 5 cannot be filled, because the path can't cross from the top half to the bottom half at that point. It is eliminated.

Note: The middle row is "1". therefore, the top 4 rows must be completed before crossing row 5.

--> The path will double back from column 1 up column 2 to the top. This will eliminate more boxes.

Fill in Column 10, row 3 (to extend the path on the right)

Column 3 is "3", the boxes in Column 3, rows 2,3,4 are eliminated. (otherwise, the path will dead-end in the lower half)

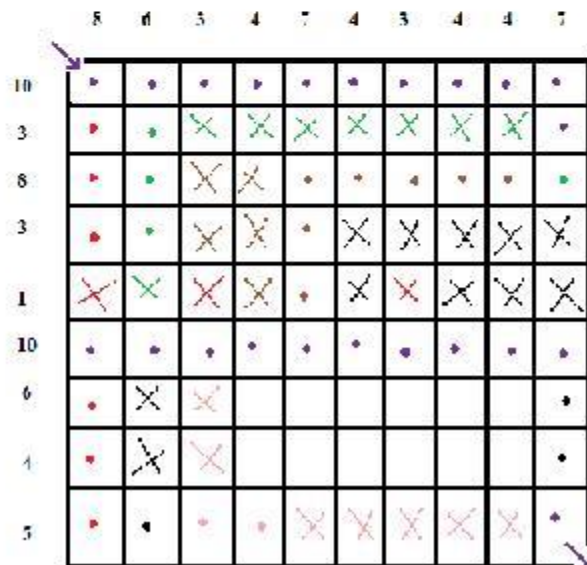Column 4 is "4", so the boxes in Column 4, 2,3,4,5 are eliminated.

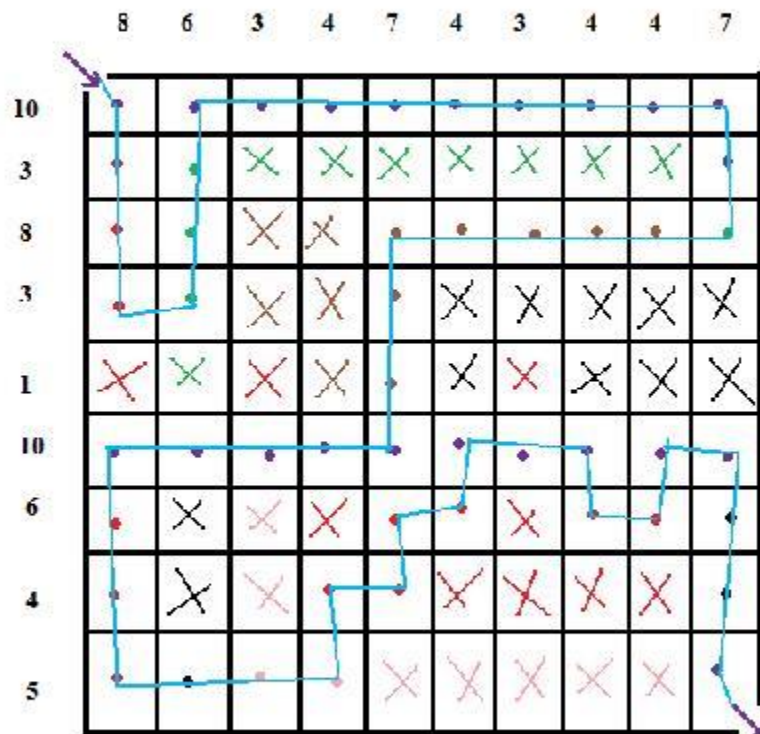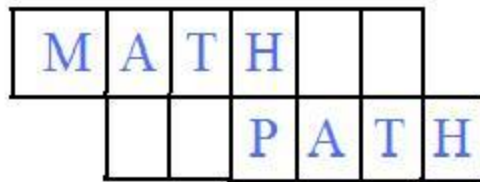The top half is finished, and the path crosses Row 5.
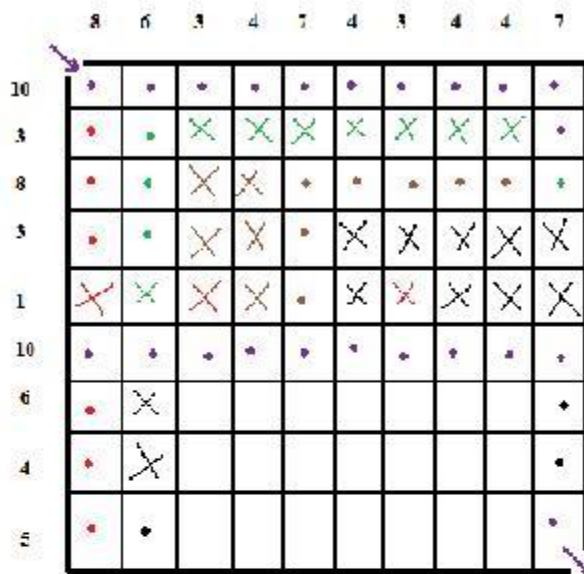
Column 10 is "7", so fill in the remaining boxes

Column 2 is "6", so fill in the bottom box to extend the lower left corner.

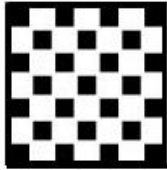Continue filling and eliminating boxes by extending the path and observing the number constraints.

Hard Solution

1) $1.19. Three quarters, four dimes, and four pennies.

2) The following table shows the number of different types of squares on a 7 x 7 chessboard:

Type of square	Number of squares
7 x 7	1
6 x 6	4
5 x 5	9
4 x 4	16
...	...
1 x 1	49

So, total number of squares = $1^2 + 2^2 + 3^2 + 4^2 + ... + 7^2 = 140$.

To gain a better understanding, consider the chessboard above
Note that there is one 7 x 7 square, four 6 x 6 squares -- (upper-left, lower-left, upper-right, & lower-right), etc...

3) A and W (and why) - The pattern is the first letter of every word in the sentence.

4) HINTS:
----"... so that no label is correct"... If you pull an orange out of the box labeled both, then that box must be the oranges...
----Start with the box labeled "Both".. Pull out a piece of fruit... At this point, you can deduce the rest...

EXAMPLE:

box 1 labeled apple
box 2 labeled orange
box 3 labeled both..

Go into box 3... Pull out an apple...
Box 3 must be 'apple'.. (it's not 'both' and it can't be 'orange')
Box 2 is either orange or both... since it began as orange, it must be both (because all were labeled wrong to start)
Box 1 is then orange...

1) Silence

2) You have the 2 you took away.. (there is 1 left over!)

3) Half way.. then, the dog is running out of the woods..

4) The best possible score in bowling is 300. (A strike every roll, which gives 30 in each frame).
The best possible score—without a strike---in a given frame is 19, 10 for the spare and 9 from the
following roll. This is even true for the tenth frame.. Therefore, the answer is $10 \times 19 = 190$.

5) Hint: consider all the numbers that 50 factorial will include!

50 factorial includes, as factors, 10, 20, 30, 40, and 50. Therefore, the value of 50
factorial must end in at least five zeroes. The number given only ends in three
zeroes. The correct value of 50 factorial is close to this, however. It's
30414093201713378043612608166064768844377641568960512000000000000.

6) $\underline{9}\,\underline{5}$ x $\underline{7}\,\underline{6}$ + $\underline{4}$ x$\underline{1}$ = 7224

Put the largest numbers in the tens places.
Then, place the smallest numbers in the second part.
Now, arrange the 5, 6, 7, 9:
Note this comparison--- 95 x76 = 7220   96 x 75 = 7200

6A) It follows that 19 x 47 + 5 x 6 = 923 is <u>NOT</u> the smallest result..
How can we reduce that answer?

First, we should isolate the 7 & 9 to reduce our result...

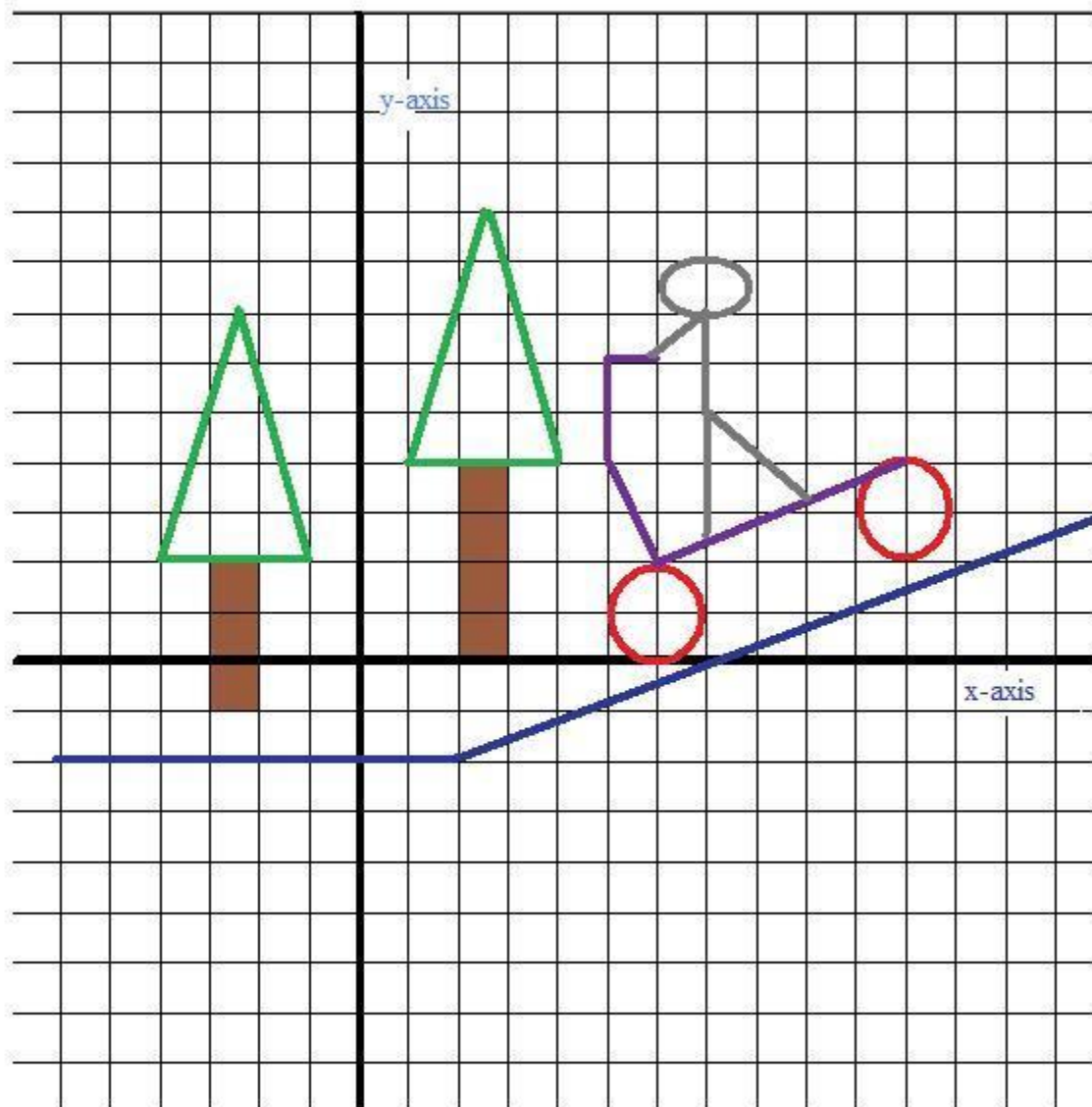Now, switch the 1,4,5,6 combo to the smallest outcome...
14 x 56 is 784
16 x 45 is 720
15 x 46 is 690 (even smaller!)
So, 15 x 46 + 7 x 9 = 753 is the smallest result..

y-axis

x-axis

Why did the math student get lost?

Because he used the wrong sign...
(traffic sign/math sign)

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1) $\displaystyle\sum_{K=1}^{6} K^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 =$

$1 + 4 + 9 + 16 + 25 + 36 = 91$

Also, $\dfrac{n(n+1)(2n+1)}{6} = \dfrac{6(7)(13)}{6} = 91$

SOLUTIONS

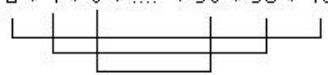2) $\displaystyle\sum_{r=0}^{7} (2r+1)^3 = 1^3 + 3^3 + 5^3 + 7^3 + 9^3 + 11^3 + 13^3 + 15^3 =$

$1 + 27 + 125 + 343 + 729 + 1331 + 2197 + 3375$

$= 8128$

3) $\displaystyle\sum_{i=1}^{8} 4 = 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 32$

$8\,(4) = 32$

4) $\displaystyle\sum_{i=1}^{20} 2i = 2 + 4 + 6 + \dots + 36 + 38 + 40 =$

each pair adds up to 42; there are 10
pairs.. therefore, the total is 420

Also,

$\displaystyle\sum_{i=1}^{20} 2i \quad 2 \sum_{i=1}^{20} i = 2 \times \left(\dfrac{20(20+1)}{2} \right) = 420$

5) $\displaystyle\sum_{n=4}^{10} 5n + 3 = 23 + 28 + 33 + 38 + 43 + 48 + 53 =$

$76 \times 3.5 \text{ pairs} = 266$

Also,

$\displaystyle\sum_{n=4}^{10} 5n + 3 = \left(\sum_{n=1}^{10} 5n + 3 \right) - \left(\sum_{n=1}^{3} 5n + 3 \right) =$

$\left(\displaystyle\sum_{n=1}^{10} 5n + \sum_{n=1}^{10} 3 \right) - (8 + 13 + 18) =$

$5 \times \dfrac{10(10+1)}{2} + 3(10) - 39 = 5 \times 55 + 30 - 39 = 266$

Summation quiz (page 2 solutions)

6) $\displaystyle\sum_{b=1}^{100} (4 + 3b) = \sum_{b=1}^{100} 4 + \sum_{b=1}^{100} 3b = 400 + 15{,}150 = \boxed{15{,}550}$

$4 + 4 + .. + 4 + 4 =$
or, $100(4) = 400$

$3\displaystyle\sum_{b=1}^{100} b = 3\left(\dfrac{100\,(101)}{2}\right) = 15{,}150$

7) $\displaystyle\sum_{c=25}^{150}\left(\dfrac{1}{(c+4)} - \dfrac{1}{(c+5)}\right) = \left(\dfrac{1}{29} - \dfrac{1}{30}\right) + \left(\dfrac{1}{30} - \dfrac{1}{31}\right) + \left(\dfrac{1}{31} - \dfrac{1}{32}\right) + \dots \text{ etc}\dots$

List, notice the pattern, &
regroup. Then, solve!!

$\dfrac{1}{29} + 0 + 0 + 0 + \dots - \dfrac{1}{155} = \boxed{\dfrac{1}{29} - \dfrac{1}{155}}$

8)
$\displaystyle\sum_{r=1}^{3}\sum_{s=1}^{4} rs^2 = \left(1 \times 1^2 + 1 \times 2^2 + 1 \times 3^2 + 1 \times 4^2\right) +$

$\left(2 \times 1^2 + 2 \times 2^2 + 2 \times 3^2 + 2 \times 4^2\right) +$

$\left(3 \times 1^2 + 3 \times 2^2 + 3 \times 3^2 + 3 \times 4^2\right) = \boxed{180}$

Use all possibilities.

$r1s1 + r1s2 + r1s3 + r1s4 =$
$\quad 1 + 4 + 9 + 16 = 30$
$\qquad\qquad\qquad\qquad\qquad +$
$r2s1 + r2s2 + r2s3 + r2s4 =$
$\quad 2 + 8 + 18 + 32 = 60$
$\qquad\qquad\qquad\qquad\qquad +$
$r3s1 + r3s2 + r3s3 + r3s4 =$
$\quad 3 + 12 + 27 + 48 = 90$

9) Give an example of a real world use for summations. (Provide a sample formula)

a) Suppose your education will cost $10,000 this year. Then, your cost will increase by $2000 each year. How much will your 8-year education cost?
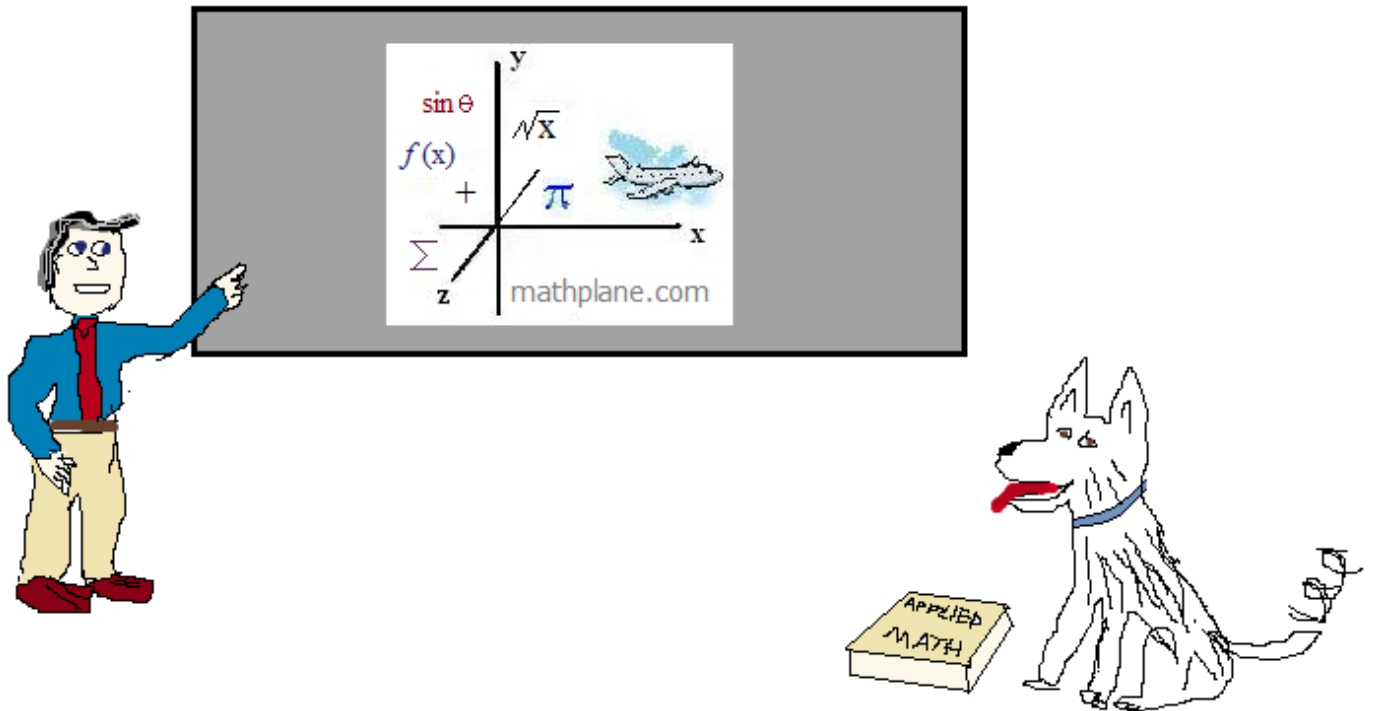b) suppose your education cost is $20,000 and will increase by 10% every year. How much will a 6-year education cost?

a) $\displaystyle\sum_{e=1}^{8} 10{,}000 + 2000(e-1)$

$\$136{,}000$

b) $\displaystyle\sum_{e=1}^{6} 20{,}000\,(1+.10)^{(e-1)}$

$\$154{,}312.20$

Thanks for visiting. (Hope you enjoyed the topics!)

If you have questions, suggestions, or requests, let us know.

Cheers

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