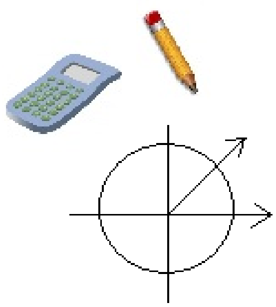
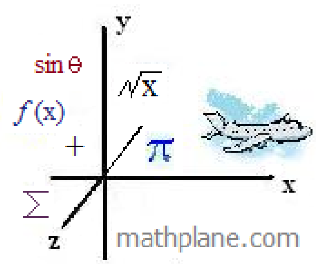
Trigonometry Packet:
Finding Inverse Trig Values





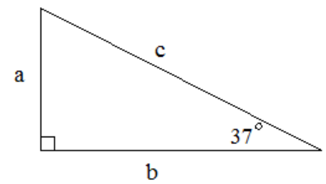
Contents include:

- Notes and Examples
- Practice Test (and Solutions)
- Hidden Message Puzzle (Trig and Right Angle Review)

Finding inverse trigonometry values

Trigonometry Values: When given the angles of a right triangle, you can determine the ratio of the sides.

Example:



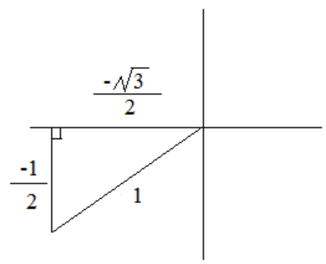
$\text{Sin}(37^\circ) = .602 = \frac{a}{c}$ $\quad$ $\text{Csc}(37^\circ) = 1.66 = \frac{c}{a}$

$\text{Cos}(37^\circ) = .799 = \frac{b}{c}$ $\quad$ $\text{Sec}(37^\circ) = 1.25 = \frac{c}{b}$

$\text{Tan}(37^\circ) = .754 = \frac{a}{b}$ $\quad$ $\text{Cot}(37^\circ) = 1.33 = \frac{b}{a}$

Or, when given an angle from the unit circle, you can identify the trig values.

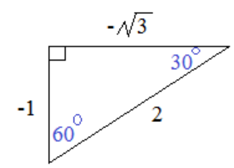
Example:



$\text{Sin}\,\frac{7\pi}{6} = \frac{-1}{2}$ $\quad$ $\text{Csc}\,\frac{7\pi}{6} = -2$

$\text{Cos}\,\frac{7\pi}{6} = \frac{-\sqrt{3}}{2}$ $\quad$ $\text{Sec}\,\frac{7\pi}{6} = \frac{-2}{\sqrt{3}}$

$\text{Tan}\,\frac{7\pi}{6} = \frac{1}{\sqrt{3}}$ $\quad$ $\text{Cot}\,\frac{7\pi}{6} = \sqrt{3}$



Inverse Trigonometry Values: Suppose you are seeking <u>inverse</u> trig values.
(i.e. You are given the ratio; and, looking for the angle.)

Note: Inverse trig functions can be expressed using different notations.
EX: $\text{Sin}^{-1}(1) = \text{ArcSin}(1)$

Examples: Find $\text{Sin}^{-1}\left(\frac{1}{2}\right)$ In other words, "the sine of what angle equals $\frac{1}{2}$?"

What is ArcTan (1) ? In other words, "the tangent of what angle equals 1 ?"

(Answers to the above examples)

$\text{Sin}^{-1}\left(\frac{1}{2}\right) = X$ $\quad$ $X = 30^\circ$ because $\text{Sin}(30^\circ) = \frac{1}{2}$

Also, $X = 150^\circ$ because $\text{Sin}(150^\circ) = \frac{1}{2}$

In fact, $X = 30^\circ + 360^\circ n$ and $150^\circ + 360^\circ n$

$\text{ArcTan}(1) = Y$ $\quad$ $Y = \frac{\pi}{4}$ radians because $\text{Tan}\,\frac{\pi}{4} = 1$

Also, $Y = \frac{5\pi}{4}$ radians because $\text{Tan}\,\frac{5\pi}{4} = 1$

All possible solutions: $Y = \frac{\pi}{4} + \pi n$ radians

Note: Solutions will often be in 'restricted domains' of trig functions -- (see 'principal values'). Or, they may be in a specified range, such as $0^\circ < \theta < 360^\circ$

So, how do we find Inverse Trigonometry Values?!!?

Here is a 4-step method: Finding inverse trig values w/o calculator

1) Draw triangles (on the plane)
2) Label sides (and, don't forget the negatives!)
3) Eliminate incorrect quadrants
4) Solve (find all answers for specified range)

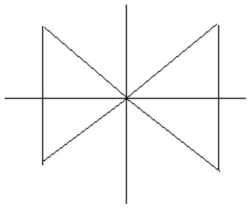
Finding inverse trig values (without a calculator)

Example: Find $\cos^{-1}\left(\frac{1}{2}\right)$ for the range $0^\circ \leq \theta < 360^\circ$

Finding inverse trig values w/o calculator

1) Draw triangles (on the plane)
2) Label sides (and, don't forget the negatives!)
3) Eliminate incorrect quadrants
4) Solve (find all answers for specified range)

Step 1: Draw Triangles

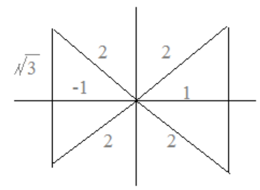


Step 2: Label sides

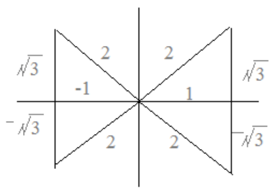
$\text{Cos} = \frac{\text{adjacent}}{\text{hypotenuse}}$ So, for $\frac{1}{2}$ Adjacent sides have length 1
Hypotenuses are 2

Using Pythagoren Theorem,
we see the opposite side is $\sqrt{3}$

(Since the sides are 1 $\sqrt{3}$ 2,
we recognize these are 30-60-90 triangles!)



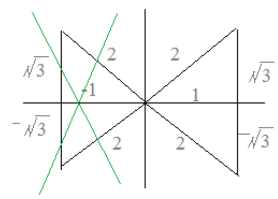
Step 3: Eliminate incorrect Quadrants



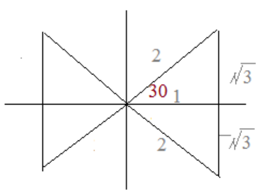
We are seeking angles where

Cos θ is positive $\frac{1}{2}$

So, we eliminate the triangles in
quadrants II and III (because those values are negative!)



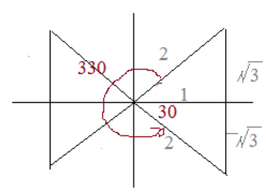
Step 4: Solve



The common reference angle is 30,

so our solution is 30° and 330°

for the range $0^\circ \leq \theta < 360^\circ$



Finding inverse trig values with a calculator (or trig tables)

Example: Find $\sin^{-1}(-.68)$ between 90° and 270°

Finding inverse trig values with a calculator

1) Check the mode (degrees or radians?)
2) Input value and calculate with inverse trig function
3) Use calculator output to solve for specified range
4) Check your answer!

Step 1: Check mode

I check my calculator: degree mode

Step 2: Input value and calculate the inverse function

input: [−] [.] [6] [8]

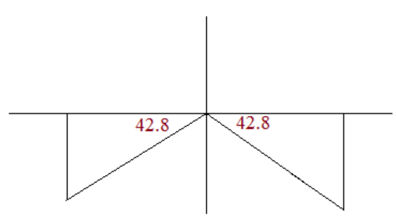
[2nd] [SIN] or [SIN-1]

[=] or [EXE] or [Enter]

output: −42.8436 (degrees)

Step 3: Use calculator output to solve for range

The output is -42.8° but, the specified range is between 90° and 270°



Sine is negative in Quadrants
III and IV..

Using the reference angle 42.8,
the angle between 90 and 270 is

$180 + 42.8 = 222.8^\circ$

Step 4: Check your answer!

input: [2] [2] [2] [.] [8]

[SIN] [=] (in degree mode)

output: -.67944

Finding inverse trig values with specified domains

Example: (Without a calculator) Find X

$\sin X = \frac{-1}{2}$ $\tan X > 0$ $0 \le X < 2\pi$

"Triangle Method"

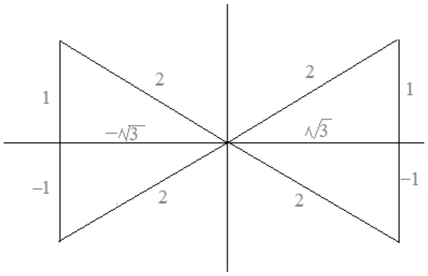
1) Draw triangles (on the plane)
2) Label sides (and, don't forget the negatives!)
3) Eliminate incorrect quadrants
4) Solve (find all answers for specified range)

Step 1:

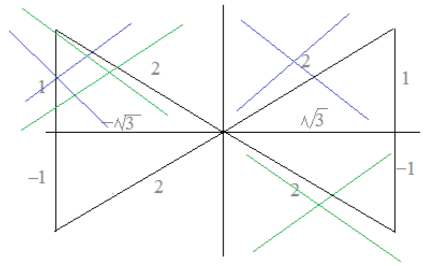
Draw Triangles

Step 2:

Label sides



Step 3: eliminate quadrants



Tan X > 0
(Eliminates Quad II and IV)

Sin X = -1/2
(Eliminates Quad I and II)

X is in Quadrant III

30-60-90 triangle..

reference angle is $\frac{\pi}{6}$

Step 4: Solve

therefore, $X = \frac{7\pi}{6}$

Example: (With a calculator)

For $\cot X = 4$ and $\cos X < 0$ what is X ?

Calculator

1) Check the mode (degrees or radians?)
2) Input value and calculate with inverse trig function
3) Use calculator output to solve for specified range
4) Check your answer!

Step 1: Check mode

Most prefer using degree mode.

Step 2: Input value

using $\cot^{-1}$ input: [4] [2nd] [Cot] [=] or [$\cot^{-1}$] [4] [=]

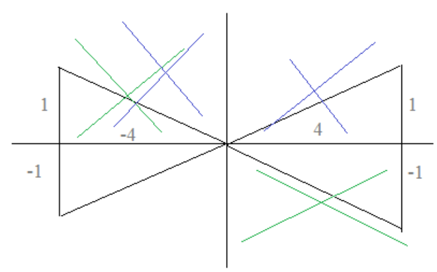
(depends on your calculator keys)

Using the Reciprocal: Tan X = Cot (1/X)

using $\tan^{-1}$ input: [4] [1/x] [2nd] [Tan] [=] or [$\tan^{-1}$] [( ] [1] [÷] [4] [ )] [=]

The output for either: 14.036°

Step 3: Use the calculator output to solve for specified range



$\cot 4 = \cot \frac{4}{1}$ or $\cot \frac{-4}{-1}$

(eliminates quadrants II and IV)

Cos < 0
(eliminates quadrants I and IV)

X is in quadrant III, and the reference angle is 14.04°

X = 180 + 14.04 = 194.04°

Step 4: Check answer!

Does Cot (194.04) = 4 ?

[1] [9] [4] [.] [0] [4] [Cot] [=]

or

[1] [9] [4] [.] [0] [4] [Tan] [1/x]

output: 3.99889 ✓

Inverse Trig Values and functions: Important notes

1) Definition and notation

An inverse is NOT a reciprocal.

The reciprocal of sin(x) is $\frac{1}{\sin(x)} = \csc(x)$

$\text{Sin}^{-1}(x)$ is NOT the reciprocal of sin(x)

$\text{Sin}^{-1}(x) \neq \frac{1}{\sin(x)}$

The inverse of sin(x) implies that

if $y = \sin(x)$, $\sin^{-1}(y) = x$

"the inverse sine of y is x"

Since $x^{-1} = \frac{1}{x}$ it's preferable to use arcsin rather than $\sin^{-1}$

2) "Restricted Domain" and "principal values"

None of the 6 trig functions are 1-to-1. (e.g. sin 30 = 1/2 sin 150 = 1/2 and the sine of all the coterminal angles are 1/2; arcsin(1/2) can be 30, 150, 390, etc... therefore, the inverse is not a function)...

However, each trig function can have its domain restricted, making a valid function.

3) How many answers? It depends on the specified range.

a) Including all coterminal angles (infinite)

example: arctan(1) = X "The tangent of what angle X is 1?"

45 , 225 , 405 , 585 , -135 , etc...

All coterminal angles of 45 and 225

b) Principal values from the restricted domain

Principal values when $x \geq 0$		$x \leq 0$	
$0 \leq \sin^{-1} x \leq \frac{\pi}{2}$	quadrant I	$-\frac{\pi}{2} \leq \sin^{-1} x < 0$	quadrant IV
$0 \leq \cos^{-1} x \leq \frac{\pi}{2}$	quadrant I	$\frac{\pi}{2} < \cos^{-1} x \leq \pi$	quadrant II
$0 \leq \tan^{-1} x < \frac{\pi}{2}$	quadrant I	$-\frac{\pi}{2} < \tan^{-1} x < 0$	quadrant IV
$0 < \csc^{-1} x \leq \frac{\pi}{2}$	quadrant I	$-\frac{\pi}{2} \leq \csc^{-1} x < 0$	quadrant IV
$0 \leq \sec^{-1} x < \frac{\pi}{2}$	quadrant I	$\frac{\pi}{2} < \sec^{-1} x \leq \pi$	quadrant II
$0 < \cot^{-1} x \leq \frac{\pi}{2}$	quadrant I	$\frac{\pi}{2} < \cot^{-1} x < \pi$	quadrant II

c) Depending on the specified range.

examples: 1) Find the arctan(-1) for all angles in the interval $[0^\circ, 360^\circ]$.

Answer: 135° and 315°

2) Evaluate $\sin^{-1}(1/2) = x$ where $\cos(x) < 0$

Answer: 150°

sin 30 and sin 150 are both 1/2
But, since cos(30) > 0, we omit that answer.

4) "Quadrantal inverse values" The values are 0, -1, 1, or undefined

5) "Double Trig Values"

Use order of operations -- i.e. evaluate the content inside the parenthesis first using principal values.

Example: Find $\text{Sin}\left(\cos^{-1}\frac{1}{2}\right)$

step 1: $\left(\cos^{-1}\frac{1}{2}\right) = \frac{\pi}{3}$ (quadrant I)

step 2: $\text{Sin}\,\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Study Break:
Math Snacks

Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

Try with (t), or any beverage...

Also, look for Honey Graham Squares
in the geometry section of your local store...

PRACTICE TEST

Warm-up:

Practice Test: Finding Inverse Trig Values

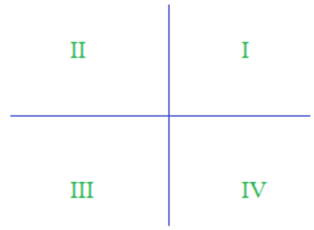
A) Which trig functions are positive in each quadrant?

I:

II:

III:

IV:



Sine
Cosine
Tangent

Cosecant
Secant
Cotangent

B) Determine the quadrant that satisfies the conditions:

1) $\cos\theta < 0$ $\sin\theta < 0$

2) $\sec\theta = 2$ $\csc\theta < 0$

3) $\tan x = 1$ $\sin x < 0$

4) $\tan x < 0$ $\cos > 0$

C) Match the 6 Inverse Trig Functions with their (restricted) domains

$\text{Sin}^{-1}(x)$

$\text{Cos}^{-1}(x)$

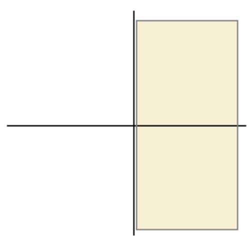
$\text{Tan}^{-1}(x)$

$\text{Csc}^{-1}(x)$

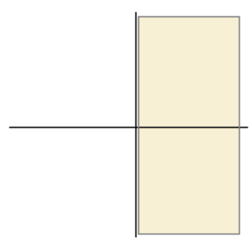
$\text{Sec}^{-1}(x)$

$\text{Cot}^{-1}(x)$

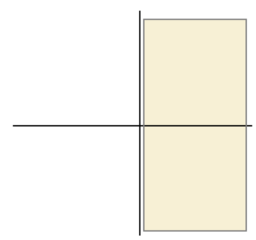
a)



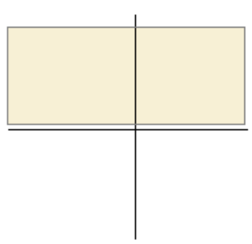
b)



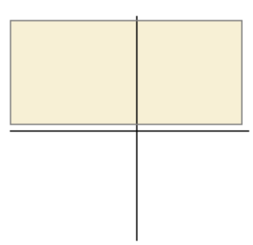
c)



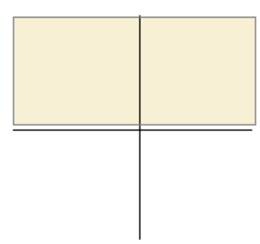
d)



e)



f)



Practice Test: Finding inverse trig values

I. Solve for $0^{\circ} \le X < 360^{\circ}$ (No calculator necessary)

1) $\text{Cos}^{-1}\left(\frac{1}{2}\right) = X$

2) $\arcsin\left(\frac{-1}{2}\right) = X$

3) $\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right) = X$

4) $\text{ArcTan}\,(0) = X$

5) $\arcsin\,(3) = X$

6) $\text{Cot}^{-1}(-1) = X$

II. Determine all possible values, using angles measured in radians.

1) $\arccos\left(\frac{-1}{2}\right)$

2) $\tan^{-1}(1)$

3) $\sin^{-1}(1)$

III. Evaluate for $0^{\circ} \le X < 360^{\circ}$ (Use Calculator)

1) $\text{Arcsin}\,(.37) = X$

2) $\text{Tan}^{-1}\left(\frac{5}{12}\right) = X$

$\text{Arcsec}\,(-2.5) = X$

Practice Test: Finding inverse trig values

IV. Using the restricted domains, find the principal values of the following.

1) $\arccos\left(\frac{\sqrt{3}}{2}\right) =$

2) $\tan^{-1}(1) =$

3) $\sec^{-1}(2) =$

4) $\sin\left(\cos^{-1}\left(\frac{-1}{2}\right)\right) =$

5) $\sec\left(\sin^{-1}(1)\right) =$

6) $\tan\left(\tan^{-1}(Z)\right) =$

V. Given the following functions: $f(x) = \sin^{-1}(x) + 2$ $h(t) = -\cos^{-1}(t)$

(using radians)

Find:

1) $f\left(\frac{\sqrt{3}}{2}\right) =$

2) $h(1/2) =$

3) $f(.35) =$

VI. Additional trigonometry questions

1) If $\tan\theta = \frac{3}{7}$ and $\sin\theta < 0$,

what are the values of the other trig functions?

2) An angle Y is in standard position on the coordinate plane: its initial side is on the x-axis and the terminal side passes through (-5, 3).

What is the measure of angle Y?
(calculator)

What is Sin(Y)? Cos(Y)? Tan(Y)?

Practice Test: Finding inverse trig values

3) Find an equation of a line that makes an angle of 68 degrees with the x-axis and has a y-intercept at -3

4) Find an equation of a line that makes an angle of 40 degrees with the x-axis and has an x-intercept at 4

VII. Finding specific values (with calculator)

Find θ

1) $\sin\theta = -.38$ and $180^\circ < \theta < 270^\circ$

2) $\tan\theta = 2.3$ and $180^\circ < \theta < 270^\circ$

3) $\sin\theta = .8$ and $90^\circ < \theta < 180^\circ$

4) $\cos\theta = .43$ and $270^\circ < \theta < 360^\circ$

Find x

A) $\cos x = -.74$ and $\pi < x < \frac{3\pi}{2}$

B) $\tan x = -1.9$ and $\frac{\pi}{2} < x < \pi$

C) $\cot x = -.66$ and $\frac{\pi}{2} < x < \frac{3\pi}{2}$

D) $\csc x = 2.5$ and $\frac{\pi}{2} < x < \pi$

Find θ

5) $\sec\theta > 0$ and $\tan\theta = -3.1$

6) $\cos\theta < 0$ and $\tan\theta = -2.6$

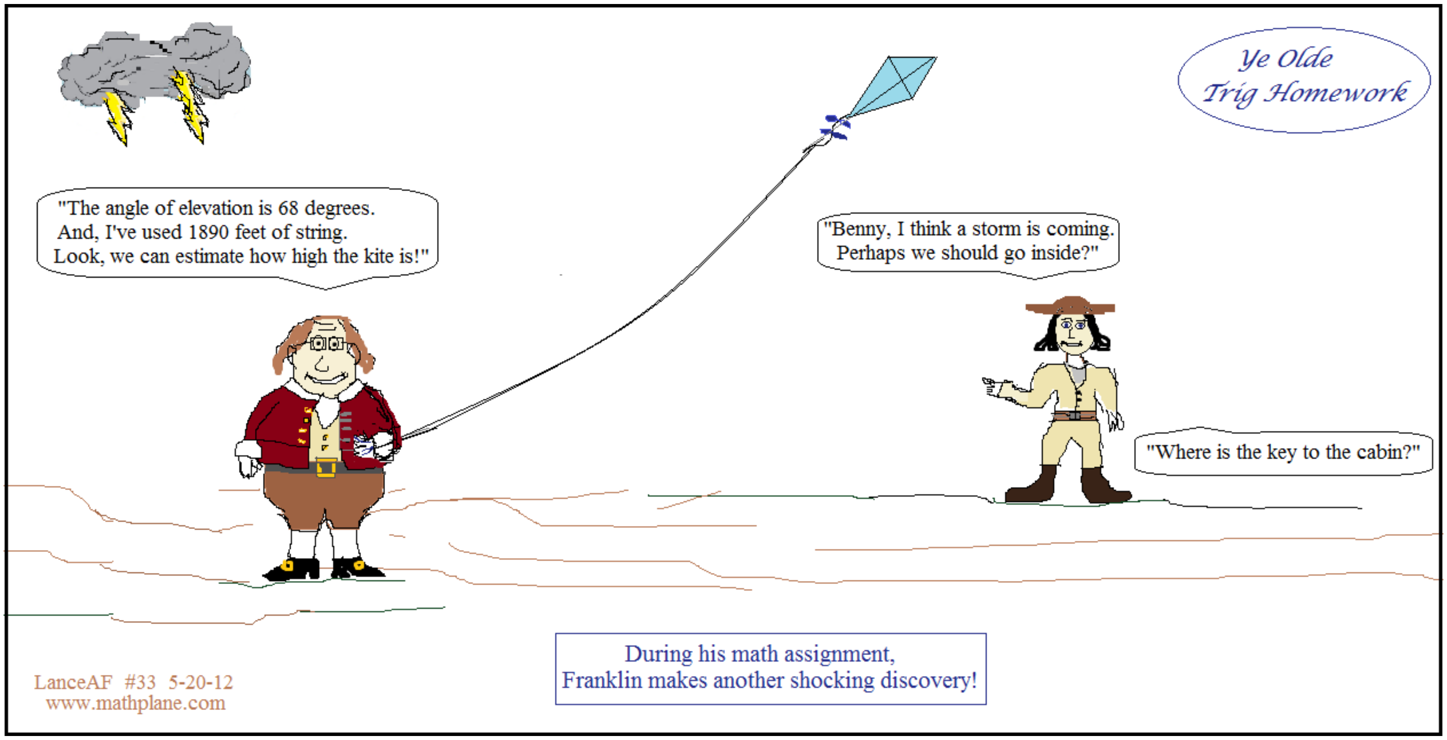
7) $\sin\theta = .7$ and $\tan\theta < 0$

VIII. Solve

For $0 < x < 360^\circ$,

what is $2\sin(x + 42^\circ) = 1$?

$\sec(x - 35^\circ) = 2$?



HIDDEN MESSAGE PUZZLE

An exercise involving Right Triangles and evaluating Trig Functions

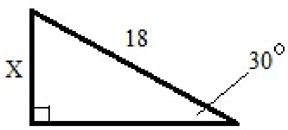
Hidden Term

Clue: "Trig Alert"

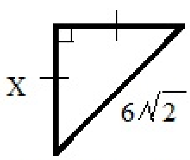
Letter Key:

0	1	2	3	4	5	6	7	8	9
A	C	E	G	I	N	R	S	T	W

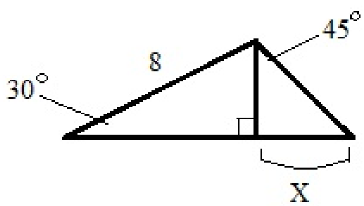
Solve for X. (No calculators!!)

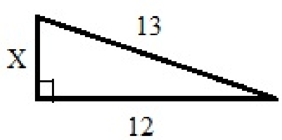
1) 

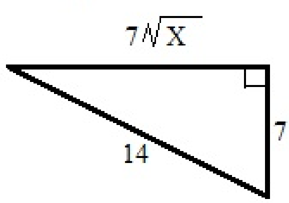
2) X = Cosine 90°

3) 

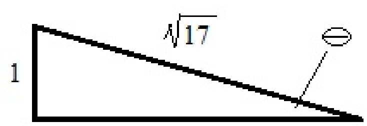
4) Special Right Triangle (sides): 3 - 4 - X

5) 

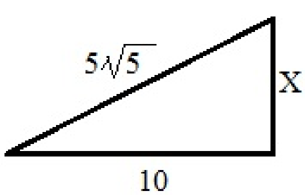
6) 

7) 

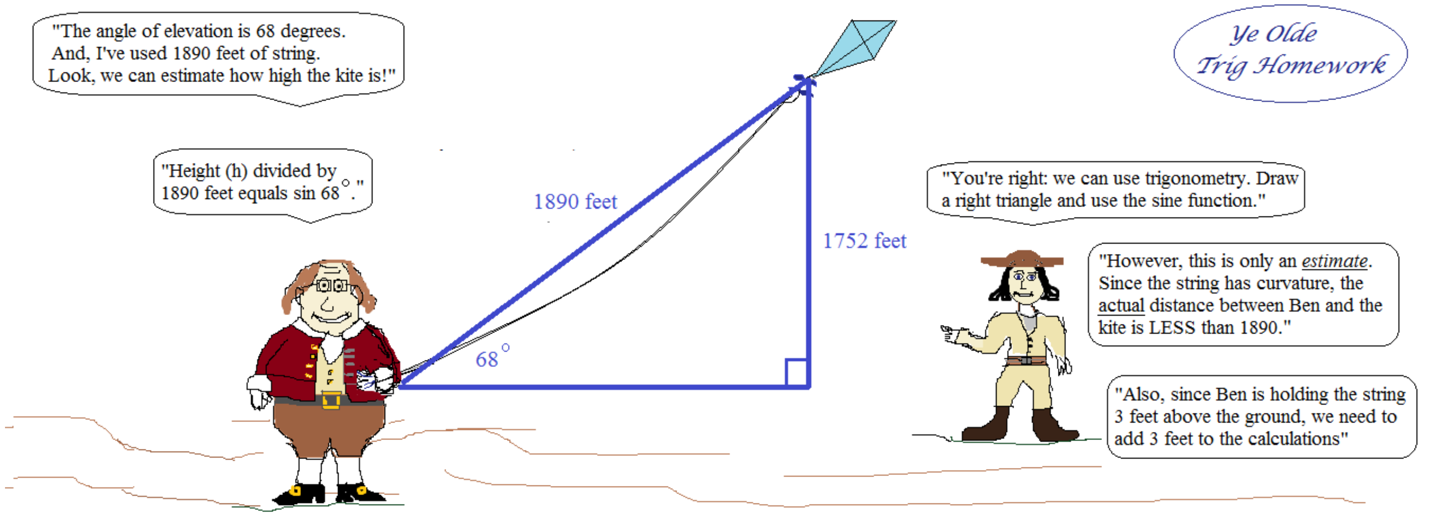
8) Special Right Triangle (sides): X - 24 - 25

9) 

X = Cotangent θ

10) 

11) CSC 30° = X



$$\text{Sine } (68^\circ) = \frac{h}{1890 \text{ feet}}$$

$$.927184 \text{ (1890 feet)} \cong 1752.4 \text{ feet}$$

**Franklin's famous kite experiment occurred in June 1752

What is the approximate height of the kite?

Hint: It's the year of Ben Franklin's famous kite experiment!

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SOLUTIONS

Warm-up:

SOLUTIONS

Practice Test: Finding Inverse Trig Values

A) Which trig functions are positive in each quadrant?

I: Sine, Cosine, Tangent, Cosecant, Secant, Tangent

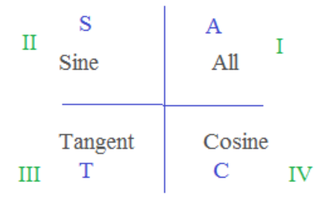
II: Sine, Cosecant

III: Tangent, Cotangent

IV: Cosine, Secant

which trig functions are positive?

All
Students
Take
Calculus



B) Determine the quadrant that satisfies the conditions:

1) $\cos\theta < 0$ $\sin\theta < 0$ Quadrant III
   II or III   III or IV

2) $\sec\theta = 2$ $\csc\theta < 0$ Quadrant IV
   I or IV   III or IV

3) $\tan x = 1$ $\sin x < 0$ Quadrant III
   I or III   III or IV

4) $\tan x < 0$ $\cos > 0$ Quadrant IV
   II or IV   I or IV

C) Match the 6 Inverse Trig Functions with their (restricted) domains

$\text{Sin}^{-1}(x)$

$\text{Cos}^{-1}(x)$

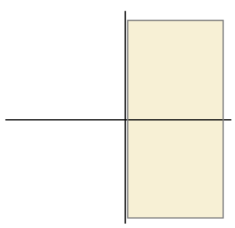
$\text{Tan}^{-1}(x)$

$\text{Csc}^{-1}(x)$

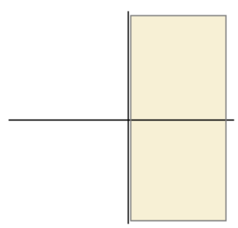
$\text{Sec}^{-1}(x)$

$\text{Cot}^{-1}(x)$

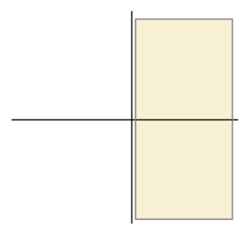
a) $\text{Sin}^{-1}(x)$



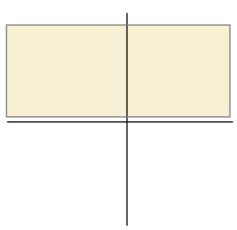
b) $\text{Csc}^{-1}(x)$



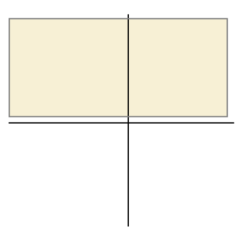
c) $\text{Tan}^{-1}(x)$



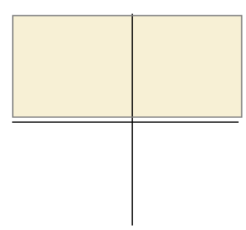
d) $\text{Cos}^{-1}(x)$



e) $\text{Sec}^{-1}(x)$



f) $\text{Cot}^{-1}(x)$



Note: The domains of tangent and cotangent are different. Why?
At 0, cotangent is undefined.

Each area is continuous and includes every positive and negative value.

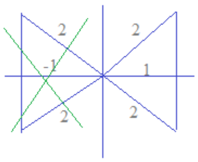
Practice Test: Finding inverse trig values

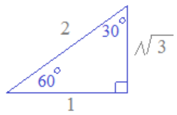
SOLUTIONS

I. Solve for $0^\circ \le X < 360^\circ$ (No calculator necessary)

1) $\text{Cos}^{-1}\left(\frac{1}{2}\right) = X$

$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$





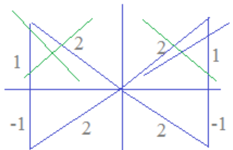
reference angle is 60°

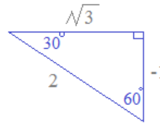
solution must be in quadrants I and IV

60° and 300°

2) $\arcsin\left(\frac{-1}{2}\right) = X$

$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$



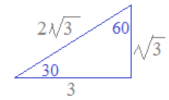


reference angle is 30°

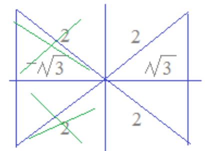
solutions must be in quadrants III and IV

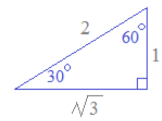
210° and 330°

3) $\sec^{-1}\left(\frac{2\sqrt{3}}{3}\right) = X$



$\sec = \frac{\text{hypotenuse}}{\text{adjacent}}$ $\quad \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}$





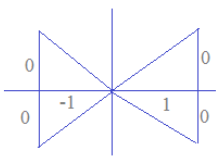
reference angle is 30°

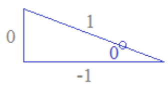
solutions in quadrants I and IV

30° and 330°

4) ArcTan (0) = X

$\tan = \frac{\text{opposite}}{\text{adjacent}}$ or $\frac{y}{x}$



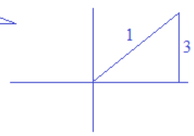


0° and 180°

Quadrantal: y = 0, so lies on the x-axis

5) arcsin (3) = X

No Solutions!



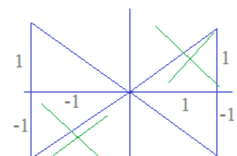
$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$

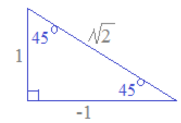
(the hypotenuse cannot be greater than either leg)

sin values are between -1 and 1

6) $\text{Cot}^{-1}(-1) = X$

$\cot = \frac{\text{adjacent}}{\text{opposite}} = \frac{-1}{1} = \frac{1}{-1}$





reference angle is 45°

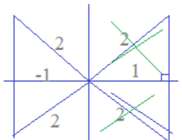
solutions must be in quadrants II and IV

135° and 315°

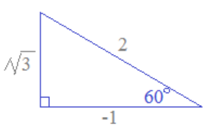
II. Determine all possible values, using angles measured in radians.

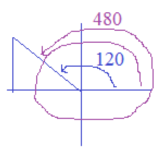
1) $\arccos\left(\frac{-1}{2}\right)$

A negative value of cosine is in quadrants II and III



The reference angle is 60°

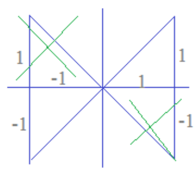




values are 120° and 240° and all the coterminal angles

$\frac{2\pi}{3} + 2\pi n$ and
$\frac{4\pi}{3} + 2\pi n$ where n is any integer

2) $\tan^{-1}(1)$



reference angle is $\frac{\pi}{4}$

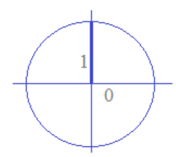
quadrants I and III

$\frac{\pi}{4} + 2\pi n$ and $\frac{5\pi}{4} + 2\pi n$

or, simply $\frac{\pi}{4} + \pi n$

3) $\sin^{-1}(1)$

$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$ $\quad \frac{y}{r}$ (on the unit circle)



quadrantal on the y-axis (between quadrants I and II)

$\frac{\pi}{2} + 2\pi n$

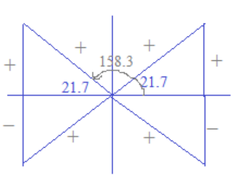
III. Evaluate for $0^\circ \le X < 360^\circ$ (Use Calculator)

1) Arcsin (.37) = X

(calculator in degree mode)

input: .37 inverse sine
output: 21.7 (degrees)

21.7 is the principal value (and reference angle)



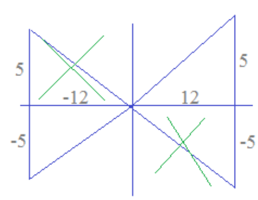
21.7° and 158.3°

sine is positive in quadrants I and II

Note: To check your answer, just find sin(21.7°) and sin(158.3°)

2) $\text{Tan}^{-1}\left(\frac{5}{12}\right) = X$

input: 5 divided by 12 (= .416667)
then, inverse tangent
output: 22.6



tangent is positive in quadrants I and III

22.6° and 202.6°

Arcsec (−2.5) = X

input: -2.5 inverse secant
or
input: 1/-2.5 inverse cosine
output: 113.58°

secant (and cosine) are negative in quadrants II and III

113.58° and 246.42°

note: the reference angle is (180 - 113.58)
66.42°

180 + 66.42 = 246.42°

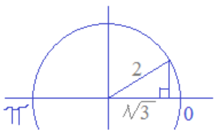
Practice Test: Finding inverse trig values

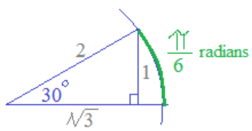
SOLUTIONS

IV. Using the restricted domains, find the principal values of the following.

1) $\arccos\left(\frac{\sqrt{3}}{2}\right) =$

Cosine principal values: $0 \le x \le \pi$



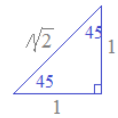


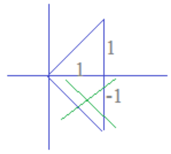
30° or $\frac{\pi}{6}$ radians

2) $\text{Tan}^{-1}(1) =$

Tangent principal values:

in the interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$





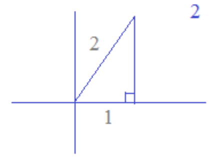
45° or $\frac{\pi}{4}$ radians

3) $\text{Sec}^{-1}(2) =$

arcsec $\frac{2}{1}$ $\frac{\text{hypotenuse}}{\text{adjacent side}}$

Secant principal values: $0 \le x < \frac{\pi}{2}$ if > 1

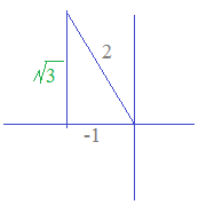
$\frac{\pi}{2} < x \le \pi$ if < -1



$\frac{\pi}{3}$ radians or 60°

4) $\sin\left(\text{Cos}^{-1}\left(\frac{-1}{2}\right)\right) =$

Negative Cosine value: Quadrant II



$\text{Cos}^{-1}\left(\frac{-1}{2}\right) = 120^\circ$

$\sin(120^\circ) = \frac{\sqrt{3}}{2}$

5) $\sec\left(\text{Sin}^{-1}(1)\right) =$

Range of sine values: $-90^\circ \le \theta \le 90^\circ$

The sine of what angle is 1? 90°

Then, what is $\sec(90^\circ)$?

Undefined!!

Note: $\cos(90^\circ) = 0$ Secant is the reciprocal of cosine.. And the reciprocal of 0 is undefined.

6) $\tan\left(\text{Tan}^{-1}(Z)\right) =$

Z

V. Given the following functions: $f(x) = \sin^{-1}(x) + 2$ $\quad h(t) = -\cos^{-1}(t)$

(using radians)

Find:

1) $f\left(\frac{\sqrt{3}}{2}\right) =$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + 2 =$

$\frac{\pi}{3} + 2$

2) $h(1/2) =$

$-\cos^{-1}(1/2) =$

$-\frac{\pi}{3}$

3) $f(.35) =$

$\sin^{-1}(.35) + 2 =$

$\sin^{-1}(.35)$ (in radians) is approx. .357

$.357 + 2 =$ 2.357 radians

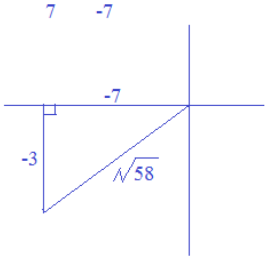
VI. Additional trigonometry questions

1) If $\tan\theta = \frac{3}{7}$ and $\sin\theta < 0$,

what are the values of the other trig functions?

tangent > 0 and sine < 0 -----> quadrant III

$\frac{3}{7} = \frac{-3}{-7}$



$\sin = \frac{-3}{\sqrt{58}}$

$\cos = \frac{-7}{\sqrt{58}}$

$\csc = \frac{-\sqrt{58}}{3}$

$\sec = \frac{-\sqrt{58}}{7}$

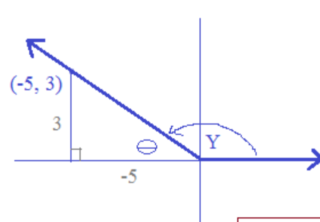
$\cot = \frac{-7}{-3} = \frac{7}{3}$

2) An angle Y is in standard position on the coordinate plane: its initial side is on the x-axis and the terminal side passes through (-5, 3).

What is the measure of angle Y?

(calculator)

What is Sin(Y)? Cos(Y)? Tan(Y)?



First, find the reference angle:

$\tan\theta = \frac{3}{5}$

$\tan^{-1}(\tan\theta) = \tan^{-1}\frac{3}{5}$

$\theta = \tan^{-1}(.60) = 30.96^\circ$

Therefore, angle Y is 149.04°

$\text{Tan}\,Y = -3/5 \quad \text{Sin}\,Y = \frac{3}{\sqrt{34}} \quad \text{Cos}\,Y = \frac{-5}{\sqrt{34}}$

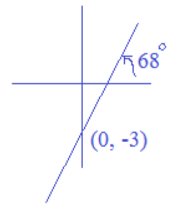
Practice Test: Finding inverse trig values

SOLUTIONS

3) Find an equation of a line that makes an angle of 68 degrees with the x-axis and has a y-intercept at -3

$\tan(68^\circ) = 1.6$

$y = 1.6x - 3$



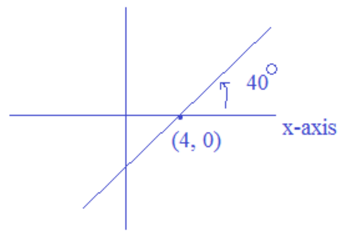
4) Find an equation of a line that makes an angle of 40 degrees with the x-axis and has an x-intercept at 4

$\tan(40^\circ) = .84$ (slope : $\frac{y}{x}$)

$(y - 0) = .84(x - 4)$

or

$y = .84x - 3.36$



VII. Finding specific values (with calculator)

Find θ

1) $\sin\theta = -.38$ and $180^\circ < \theta < 270^\circ$

$\sin^{-1}(-.38) = -22.3$ degrees (quad IV)

so, in quad III, $180 + 22.3 =$ 202.3 degrees

2) $\tan\theta = 2.3$ and $180^\circ < \theta < 270^\circ$

$\tan^{-1}(2.3) = 66.5$ degrees (reference angle)

so, in quad III, $180 + 66.5 =$ 246.5 degrees

3) $\sin\theta = .8$ and $90^\circ < \theta < 180^\circ$

$\sin^{-1}(.8) = 53.1$ degrees (quad I and reference angle)

also, positive in quad II... $180 - 53.1 =$ 126.9 degrees

4) $\cos\theta = .43$ and $270^\circ < \theta < 360^\circ$

$\cos^{-1}(.43) = 64.5$ degrees (principal value)

also, positive in quad IV...

$360 - 64.5 =$ 295.5 degrees

Find x

A) $\cos x = -.74$ and $\pi < x < \frac{3\pi}{2}$

$\cos^{-1}(-.74) = 2.4$ radians (137.7 degrees)

reference angle: $3.14 - 2.4 = .74$

so, in quad III, $3.14 + .74 =$ 3.88 radians (222.3 degrees)

B) $\tan x = -1.9$ and $\frac{\pi}{2} < x < \pi$

$\tan^{-1}(-1.9) = -1.1$ radians $+ \pi K$ $(-62.2^\circ + 180^\circ K)$

so, in quad II, 2.06 radians (117.8 degrees)

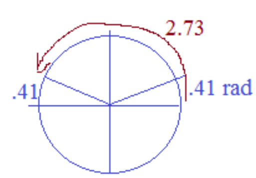
C) $\cot x = -.66$ and $\frac{\pi}{2} < x < \frac{3\pi}{2}$

$\cot^{-1}(-.66) =$ 2.15 radians (123.2 degrees)

or $\tan^{-1}\left(\frac{1}{-.66}\right)$

D) $\csc x = 2.5$ and $\frac{\pi}{2} < x < \pi$

$\csc^{-1}(2.5) = .41$ (23.6 degrees or 156.4 degrees)



in Quad II,

$\pi + .41$ rad $=$ 2.73 rad

Practice Test: Finding inverse trig values

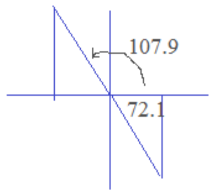
Find θ SOLUTIONS

5) $\sec\theta > 0$ and $\tan\theta = -3.1$

$\tan^{-1}(-3.1) = -72.1^\circ + 360K$

or $107.9^\circ + 360K$

Quad II and IV



since sec > 0 in Quad I or IV,

the solution is in Quad IV....

$-72.1^\circ + 360^\circ K$

where K is any integer

6) $\cos\theta < 0$ and $\tan\theta = -2.6$

$\tan^{-1}(-2.6) = -69^\circ + 360K$

or $111^\circ + 360K$

tangent is negative in Quad II and IV

cosine is negative in Quad II and III

therefore, solution is in Quad II

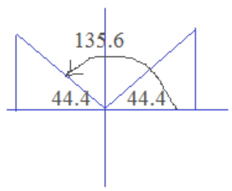
$111^\circ + 360^\circ K$

where K is any integer

7) $\sin\theta = .7$ and $\tan\theta < 0$

$\sin^{-1}(.7) = 44.4^\circ + 360K$

or $135.6^\circ + 360K$



Sine is positive in Quad I and II
Tan is negative in Quad II or IV

solution is in Quad II

$135.6^\circ + 360^\circ K$

where K is any integer

VIII. Solve

For $0 < x < 360^\circ$,

what is $2\sin(x + 42^\circ) = 1$?

Let $A = x + 42$, $2\sin(A) = 1$

$\sin(A) = \frac{1}{2}$

$A = 30^\circ + 360K$ or $150^\circ + 360K$

Therefore, $(x + 42^\circ) = -330, 30, 390, 750, \ldots$

or $(x + 42^\circ) = -210, 150, 510, 870, \ldots$

$x = -372, -12, \boxed{348}, 708, \ldots$

or $x = -252, \boxed{108}, 468, 828, \ldots$

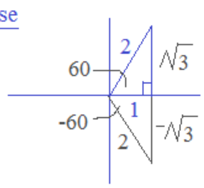
$x = 108^\circ, 348^\circ$

$\sec(x - 35^\circ) = 2$?

Let $B = x - 35$, $\sec(B) = 2$

$B = \sec^{-1}(2) = 60^\circ + 360K$ or $-60^\circ + 360K$

$\sec = \dfrac{\text{hypotenuse}}{\text{adjacent}}$



Therefore, $(x - 35^\circ) = -300, 60, 420, 780, \ldots$

$(x - 35^\circ) = -420, -60, 300, 660, \ldots$

$x = -265, \boxed{95}, 455, 815, \ldots$

or $x = -385, -25, \boxed{335}, 695, \ldots$

$x = 95^\circ, 335^\circ$

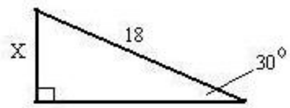
Hidden Term

Clue: "Trig Alert"

Letter Key:

0	1	2	3	4	5	6	7	8	9
A	C	E	G	I	N	R	S	T	W

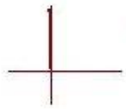
Solve for X. (No calculators!!)

1)


30-60-90 right triangle
so, $2X = 18$
$X = 9$

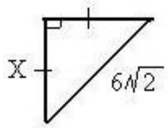
9 → W

2) $X =$ Cosine 90°



$\cos 90 = \frac{0}{1} = 0$

0 → A

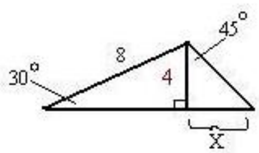
3)


Congruent sides ∴
45-45-90 triangle
$X = 6$

6 → R

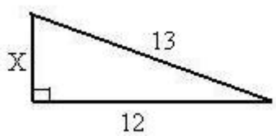
4) Special Right Triangle (sides) : 3-4-X $X = 5$

5 → N

5)


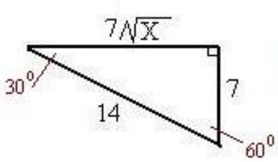
30-60-90 triangle; therefore, small side is 1/2 hypotenuse---> 4
then,
45-45-90 triangle; $X = 4$

4 → I

6)


use pythagorean theorem:
$X^2 + 12^2 = 13^2$
$X = 5$

5 → N

7)


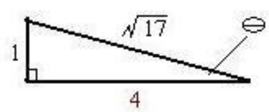
(Since one side is 1/2 of the hypotenuse, it must be a 30-60-90 right triangle)

$X = 3$

3 → G

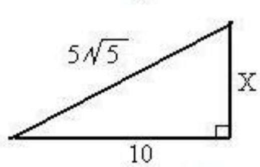
8) Special Right Triangle (sides) : X-24-25 $X = 7$

7 → S

9)


$X =$ Cotangent θ
Other leg is 4 (pythagorean theorem)
Cotangent $= \frac{\text{adjacent}}{\text{opposite}} = \frac{4}{1} = 4$

4 → I

10)


$X^2 + 10^2 = (5\sqrt{5})^2$
$X^2 = 125 - 100$ $X = 5$

5 → N

11) $\csc 30^{\circ} = X$

Cosecant is inverse of Sine so, it is hypotenuse/opposite

CSC 30 = 2/1
= 2

2 → E

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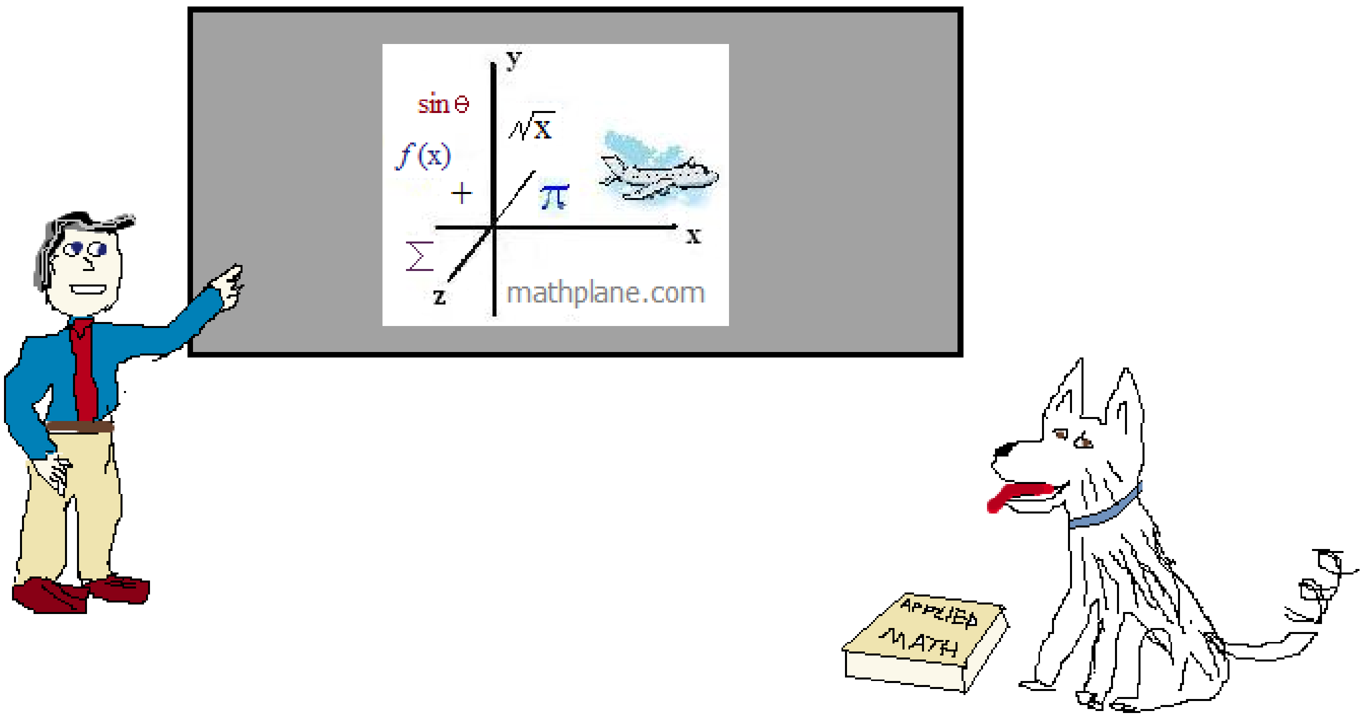
Questions, feedback, suggestions, or requests?   Contact us!

***If you want the original files in .png, .pdf, or .doc format, let us know.

Lance@mathplane.com



Also, at Facebook, Google+, TeachersPayTeachers, and Pinterest

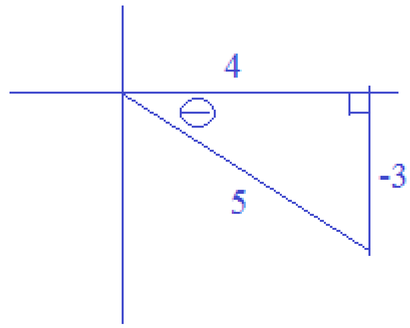
ONE MORE QUESTION:

$$\sin(\cos^{-1}(\tan(\arcsin(-3/5)))) =$$

ANSWER -→

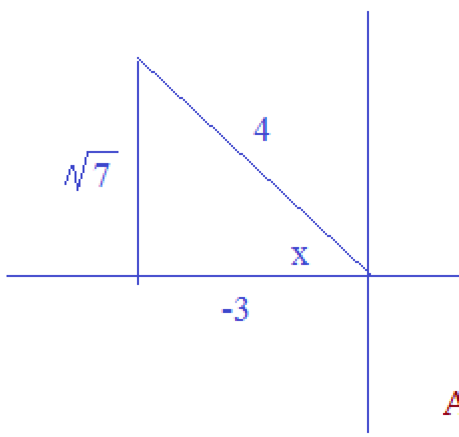
$\sin(\cos^{-1}(\tan(\arcsin(-3/5)))) =$

$\arcsin(-3/5) = \theta$



so, $\tan(\theta) = -3/4$

$\cos^{-1}(-3/4) = x$



therefore,

$$\sin(x) = \boxed{\frac{\sqrt{7}}{4}}$$

ANSWER