Calculus -- Introduction to Derivatives:

Definitions, Examples, and Practice Exercises (w/ Solutions)

Topics include Product/quotient rule, Chain Rule, Graphing, Relative Extrema, Concavity, and More…

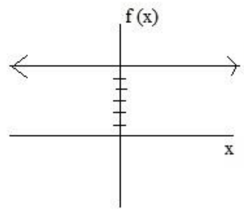
Derivatives: Notes, Rules, and Examples

1) Constant — The derivative of a constant is zero; that is, for a constant c:

$$\frac{d}{dx}(c) = 0$$

Example:

f (x) = 6
f '(x) = 0



(slope at any point on the line is 0)

2) Scalar Multiple (or, constant multiple) — The derivative of a constant multiplied by a function is the constant multiplied by the derivative of the original function:

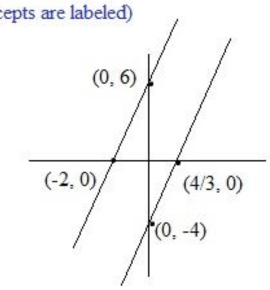
$$\frac{d}{dx}(a \cdot f(x)) = a \cdot \frac{d}{dx}(f(x))$$

Examples:

(2 parallel lines; the intercepts are labeled)

y = 3x + 6
y' = 3

y = 3x - 4
$\frac{dy}{dx} = 3$



(At any point on these lines, the slope is 3)

"Linear" --> f (x) = cx
f '(x) = c

3) Sum/Difference Rule — The derivative of the sum of two functions is the sum of the derivatives of the two functions:

$$\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$$

Example:

f (x) = 7x - 5   f '(x) = 7

u = 7x   u' = 7
v = -5   v' = 0

4) Power Rule

a) Simple version (for monomial)

$$\frac{d}{dx} X^n = n \cdot X^{n-1}$$

Examples:

$y = X^2$
$\frac{dy}{dx} = 2X^1 = 2X$

$f(x) = X^5$
$f'(x) = 5X^4$

$y = \frac{1}{X^3}$

$y' = -3X^{-4} = \frac{-3}{X^4}$

b) General Version of Power Rule

Definition: If $y = [u(x)]^n$, where u is a differentiable function of x, and n is a real number, then

$$\frac{dy}{dx} = n[u(x)]^{n-1}\frac{du}{dx} \qquad \text{or} \qquad \frac{d}{dx}[u^n] = n[u]^{n-1}u'$$

What does it mean? To take the derivative, "take the term in parentheses, *move the exponent in front, subtract one from the exponent, and multiply by the derivative of the term in parentheses."*

Example: $y = (6x^2 - 4x + 7)^3$

$$y' = 3\,(6x^2 - 4x + 7)^2\,(12x - 4)$$

(exponent in front) (subtract one from exponent) (derivative of term in parentheses)

> Note: Basic derivative of monomial uses power rule
>
> Ex: $y = x^3$ $\quad y' = 3(x)^2(1) = 3x^2$

Example: $y = \dfrac{1}{(x^2 + 7x - 11)^3} \longrightarrow y = (x^2 + 7x - 11)^{-3}$

$$y' = -3\,(x^2 + 7x - 11)^{-4}\,(2x + 7)$$

$$= \frac{-3(2x + 7)}{(x^2 + 7x - 11)^4}$$

Example: "Quadratic"

$f(x) = X^2 - 10X + 16$

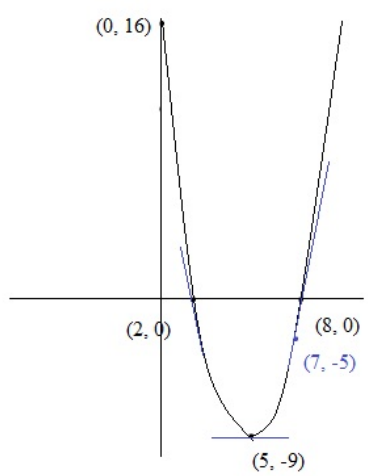
$f'(x) = 2X - 10$

To graph $X^2 - 10X + 16$

y-intercept:
$y = 0 - 10(0) + 16 = 16$
(0, 16)

x-intercepts:
$0 = X^2 - 10X + 16$
$0 = (X-2)(X-8)$
(2, 0) (8, 0)

Axis of symmetry is X = 5
Vertex is (5, -9)



slope at (2, 0) is
2(2) - 10 = -6
slope at X = 7 is
2(7) - 10 = 4

Notice: f '(x) = 2X - 10
Set = 0.. Then, X = 5
Slope at (5, -9) is 0
It's a critical point!

$y = \sqrt{x^2 - 3}$

(find y')

$y = \sqrt{x^2 - 3} = (x^2 - 3)^{1/2}$ Change radical form to exponent form

$y' = 1/2(x^2-3)^{-1/2}\ (2x)$

follow the steps for power rule:
--"move exponent to the front"
--"subtract 1 for new exponent"
--"multiply by derivative of terms in the parentheses"

$y' = \dfrac{1/2\ (2x)}{\sqrt{(x^2 - 3)}}$

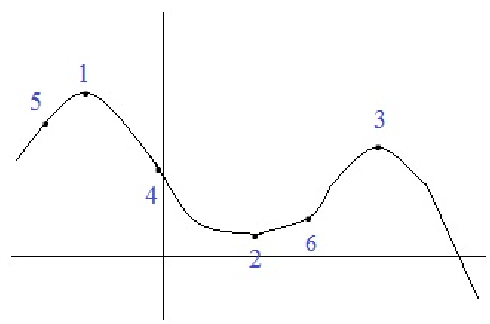
Return to radical form and simplify

$y' = \dfrac{x}{\sqrt{(x^2 - 3)}}$

Critical Values:

First derivative:
- $f'(x) = 0$ critical value (max or min)
- $f'(x) > 0$ slope is positive (function is increasing)
- $f'(x) < 0$ slope is negative (function is decreasing)

Second derivative:
- $f''(x) = 0$ point of inflection
- $f''(x) > 0$ concave up at that point
- $f''(x) < 0$ concave down at that point



1. relative max $f'(x) = 0$; concave down $f''(x) < 0$
2. relative min $f'(x) = 0$; concave up $f''(x) > 0$
3. relative max $f'(x) = 0$; concave down
4. decreasing $f'(x) < 0$; point of inflection $f''(x) = 0$
5. increasing $f'(x) > 0$; concave down $f''(x) < 0$
6. increasing $f'(x) > 0$; concave up $f''(x) > 0$

Examples: Critical Values & Graphing

$f(x) = -X^2 + 10X + 24$

$f'(x) = -2X + 10$

$f''(x) = -2$

Finding critical values:

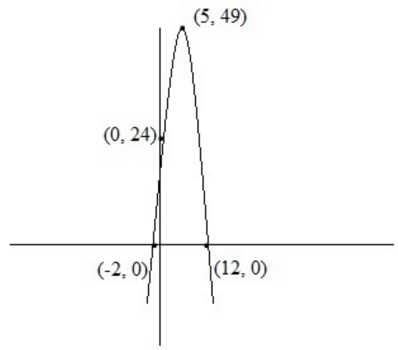
$f'(x) = 0 \quad -2X + 10 = 0 \quad X = 5$

(plug 5 into original equation) (5, 49) is a relative maximum or minimum

$f''(x) = -2 < 0$
The function is concave down.

$f(x) = -X^2 + 10X + 24$

y-intercept: (0, 24)
x-intercepts: (12, 0) (-2, 0)



$y = X^3 + 7X^2 + 16X + 8$

$\frac{dy}{dx} = 3X^2 + 14X + 16$ (first derivative)

$\frac{d^2y}{dx^2} = 6X + 14$ (second derivative)

y intercept is (0, 8)

Critical values: min --> (-2, -4)
max --> (-8/3, -104/27)

Critical Values:

$3X^2 + 14X + 16 = 0$

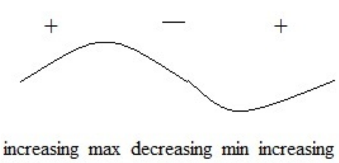
$(3X + 8)(X + 2) = 0$

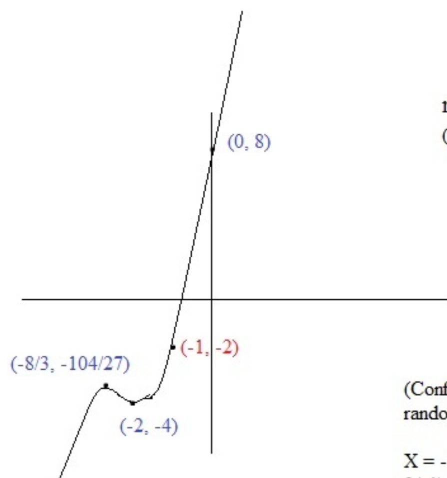
$X = -8/3, -2$

Max or Min?

Method 1: "Test Points"

X	-4	-8/3	-7/3	-2	0
f '(X)	8	0	-1/3	0	16





rough sketch
(using information we found)

(Confirm graph by plotting random points)

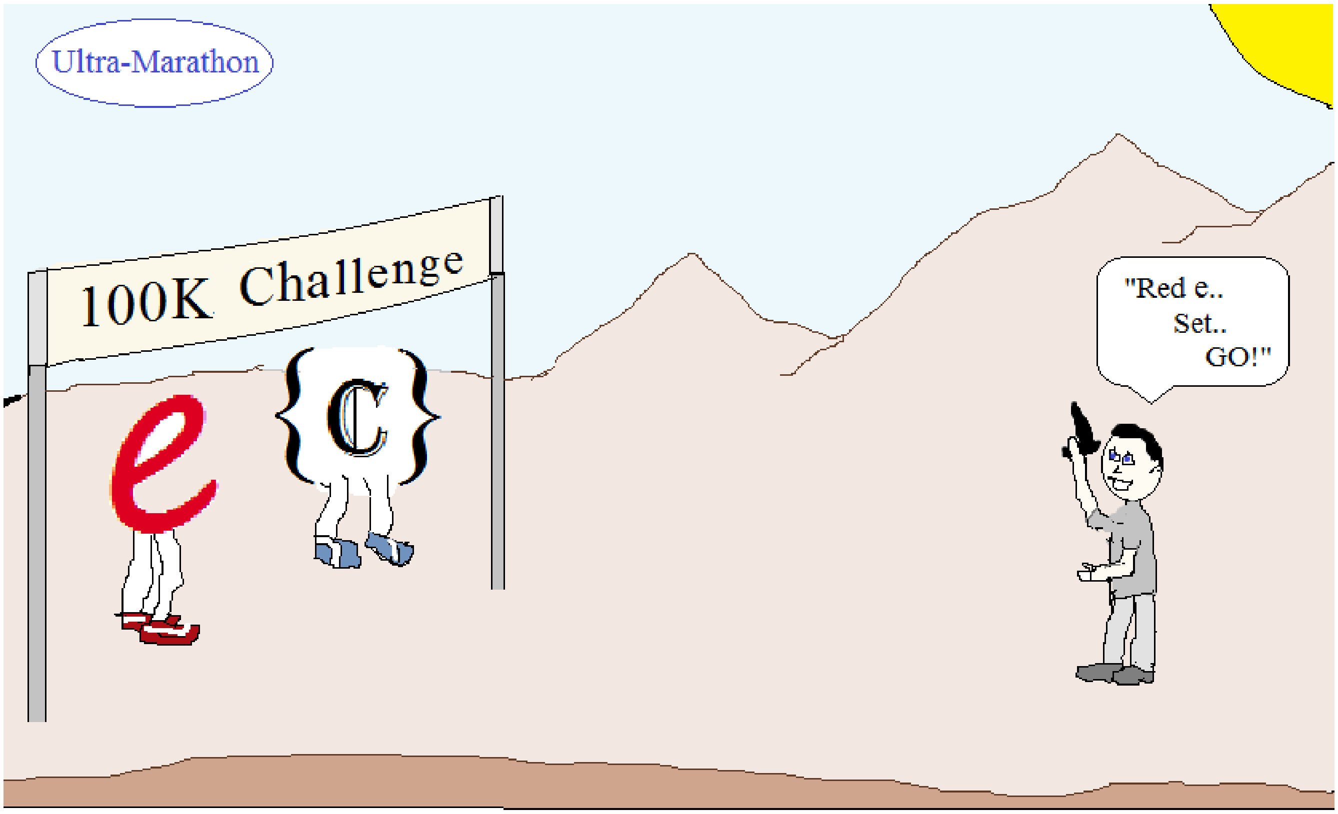
X = -1
f(-1) = -2
(-1, -2)

X = 1
f(1) = 32
(1, 32)

Method 2: "2nd Derivative Test"

Place critical values in 2nd derivative.
If < 0, it's concave down ---> must be a max
If > 0, it's concave up ---> must be a minimum!

$f''(X) = 6X + 14$
$f''(-2) = 2 > 0$ Concave up ---> relative minimum
$f''(-8/3) = -2 < 0$ Concave down --> relative maximum



Testing the limits of endurance,
these math figures will run on and on...

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Derivatives Practice Quiz (and Solutions)

Intro to Derivatives Quick Quiz

I. Find the 1st derivatives

a) $3x + 5$

b) $2x^2 - 6x + 7$

c) $x^2(3x^3 - 8)$

d) $\dfrac{2}{(x+2)^3}$

e) $\sqrt{2x + 5}$

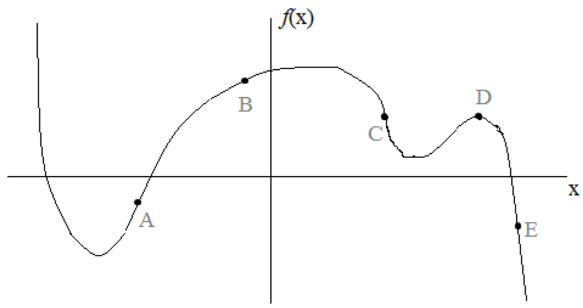
f) $\frac{3}{4}x^3 + \frac{5}{6}x - 12$

II. Find the first and second derivatives; indicate all critical values; graph to confirm your solutions

a) $f(x) = x^2 - 8x + 12$

b) $g(x) = (x + 3)(x^2 - 4x + 3)$

III. Determine the values at the given points. Indicate with <, >, or =



Point A: $f(x)$ 0
$f'(x)$ 0

Point B: $f'(x)$ 0
$f''(x)$ 0

Point C: $f(x)$ 0
$f'(x)$ 0
$f''(x)$ 0

Point D: $f'(x)$ 0
$f''(x)$ 0

Examples:

Point B: $f(x) > 0$

Point E: $f(x) < 0$
$f'(x) < 0$

IV. AROC vs. IROC

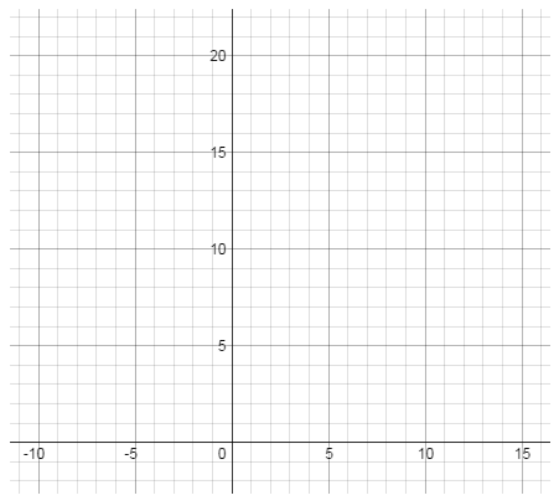
Intro to Derivatives Quick Quiz

For the function $f(x) = x^2 + 4$

a) Find the *average rate of change* (AROC) of the function over the interval [0, 4]

b) Find the *instantaneous rate of change* (IROC) at x = 1

c) Verify the results:
Graph the function $f(x)$, the tangent line through x = 1 and the secant line through x = 0 and x = 4



V. Secant, Tangent and Mean Value Theorem

A secant line goes through the curve $y = x^3 + 2$ at (0, 2) and (2, 10).

a) What is the slope of the secant line?

b) According to the Mean Value Theorem, there exists a point between 0 and 2 with the same instantaneous rate of change. What is that point?

c) What is the equation of the (tangent) line through that point and parallel to the secant?

Intro to Derivatives Quick Quiz

SOLUTIONS

I. Find the 1st derivatives

a) $3x + 5$

$f' = 3$

b) $2x^2 - 6x + 7$

$f' = 4x - 6$

c) $x^2(3x^3 - 8) = 3x^5 - 8x^2$

$f' = 15x^4 - 16x$

d) $\dfrac{2}{(x+2)^3} = 2(x+2)^{-3}$

$f' = 2[-3(x+2)^{-4}]$

$= \dfrac{-6}{(x+2)^4}$

e) $\sqrt{2x+5} = (2x+5)^{\frac{1}{2}}$

$f' = \frac{1}{2}(2x+5)^{-\frac{1}{2}}(2)$

$= \dfrac{1}{\sqrt{2x+5}}$

f) $\frac{3}{4}x^3 + \frac{5}{6}x - 12$

$f' = \frac{9}{4}x^2 + \frac{5}{6}$

II. Find the first and second derivatives; indicate all critical values; graph to confirm your solutions

a) $f(x) = x^2 - 8x + 12$

$f'(x) = 2x - 8$

$f''(x) = 2$

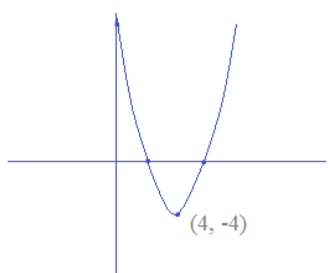
Since $2 > 0$, function is concave up

$f'(x) = 0 \quad 2x - 8 = 0$

$x = 4$

critical value at $(4, -4)$

(relative min.)



x-intercepts: (2, 0) (6, 0)
y-intercepts: (0, 12)

b) $g(x) = (x+3)(x^2 - 4x + 3) = x^3 - 4x^2 + 3x + 3x^2 - 12x + 9$

$= x^3 - x^2 - 9x + 9$

$g'(x) = 3x^2 - 2x - 9$

$g''(x) = 6x - 2$

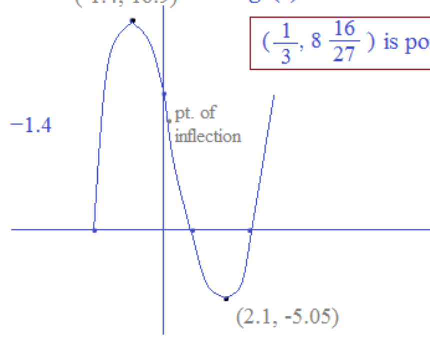
$g'(x) = 0 = 3x^2 - 2x - 9$

$x = \dfrac{2 \pm \sqrt{4 + 108}}{6} = 2.1 \text{ or } -1.4$

critical values:
(2.1, −5.05) (min)
(-1.4, 16.9) (max)

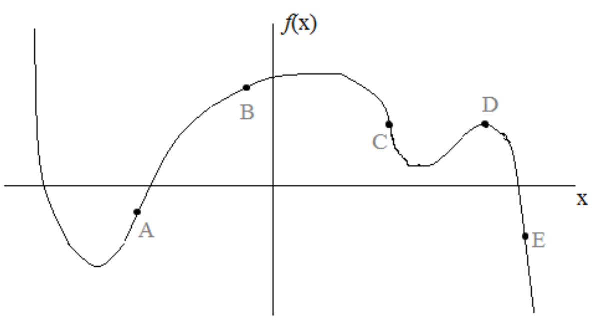
$g''(x) = 0 = 6x - 2 \quad x = 1/3$

$(\frac{1}{3}, 8\frac{16}{27})$ is point of inflection



x-intercepts: (-3, 0) (1, 0) (3, 0)
y-intercept: (0, 9)

III. Determine the values at the given points. Indicate with <, >, or =



Point A: $f(x) < 0$ negative value
$f'(x) > 0$ positive slope

Point B: $f'(x) > 0$ positive slope
$f''(x) < 0$ concave down

Point C: $f(x) > 0$ positive value
$f'(x) < 0$ negative slope
$f''(x) = 0$ point of inflection

Point D: $f'(x) = 0$ relative max (slope is 0)
$f''(x) < 0$ concave down

Examples:

Point B: $f(x) > 0$

Point E: $f(x) < 0$
$f'(x) < 0$

IV. AROC vs. IROC

SOLUTION

Intro to Derivatives Quick Quiz

For the function $f(x) = x^2 + 4$

a) Find the *average rate of change* (AROC) of the function over the interval [0, 4]

AROC is "slope" between 2 points... $f(0) = 4$ (0, 4) $f(4) = 20$ (4, 20) $\text{AROC} = \frac{20 - 4}{4 - 0} = \boxed{4}$

b) Find the *instantaneous rate of change* (IROC) at x = 1

IROC is the "derivative at a point" $f'(x) = 2x + 0$ $f'(1) = \boxed{2}$

c) Verify the results:
Graph the function $f(x)$, the tangent line through x = 1 and the secant line through x = 0 and x = 4

tangent line at x = 1 has slope of 2

$y = 2x + 3$

secant line through x = 0 and x = 4 has a slope of 4:

$y = 4x + 4$

V. Secant, Tangent and Mean Value Theorem

A secant line goes through the curve $y = x^3 + 2$ at (0, 2) and (2, 10).

a) What is the slope of the secant line? $\frac{10 - 2}{2 - 0} = \boxed{4}$

b) According to the Mean Value Theorem, there exists a point between 0 and 2 with the same instantaneous rate of change. What is that point?

The instantaneous rate of change is the derivative: $y' = 3x^2 + 0$

where is the IROC the same as the secant line slope (AROC)?

when $y' = 4$

$4 = 3x^2$

$\frac{4}{3} = x^2$

$\boxed{x = 1.15}$

-1.15 is not in the interval [0, 2]

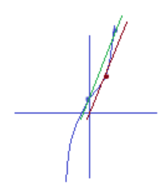
c) What is the equation of the (tangent) line through that point and parallel to the secant?

at x = 1.15, y = 3.54

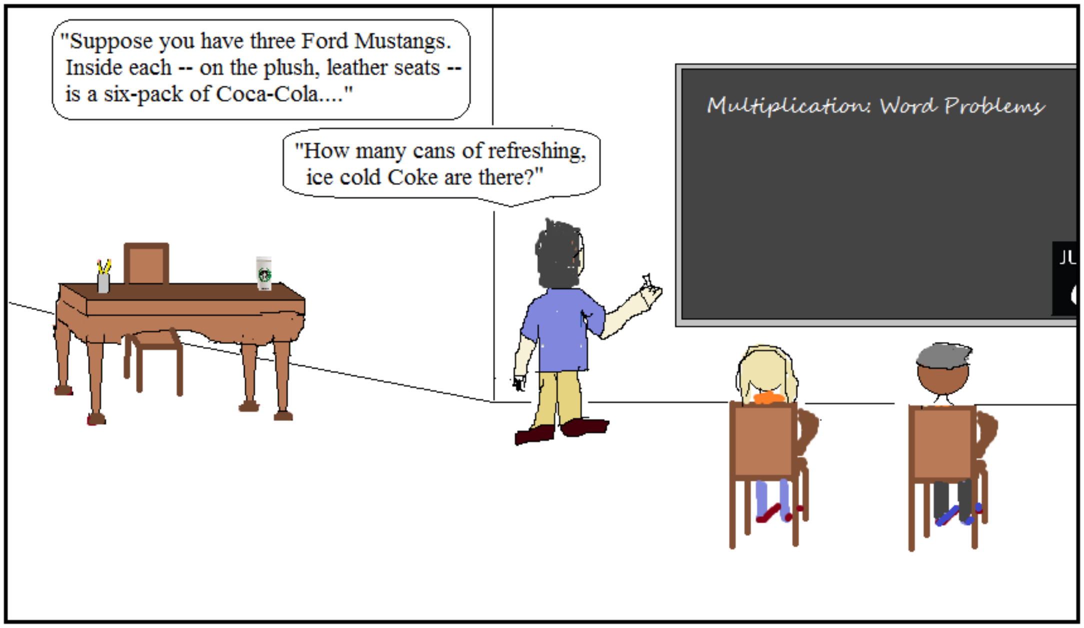
$y = (1.15)^3 + 2$

(1.15, 3.54) and slope is 4

$\boxed{y - 3.54 = 4(x - 1.15)}$



Product Placement



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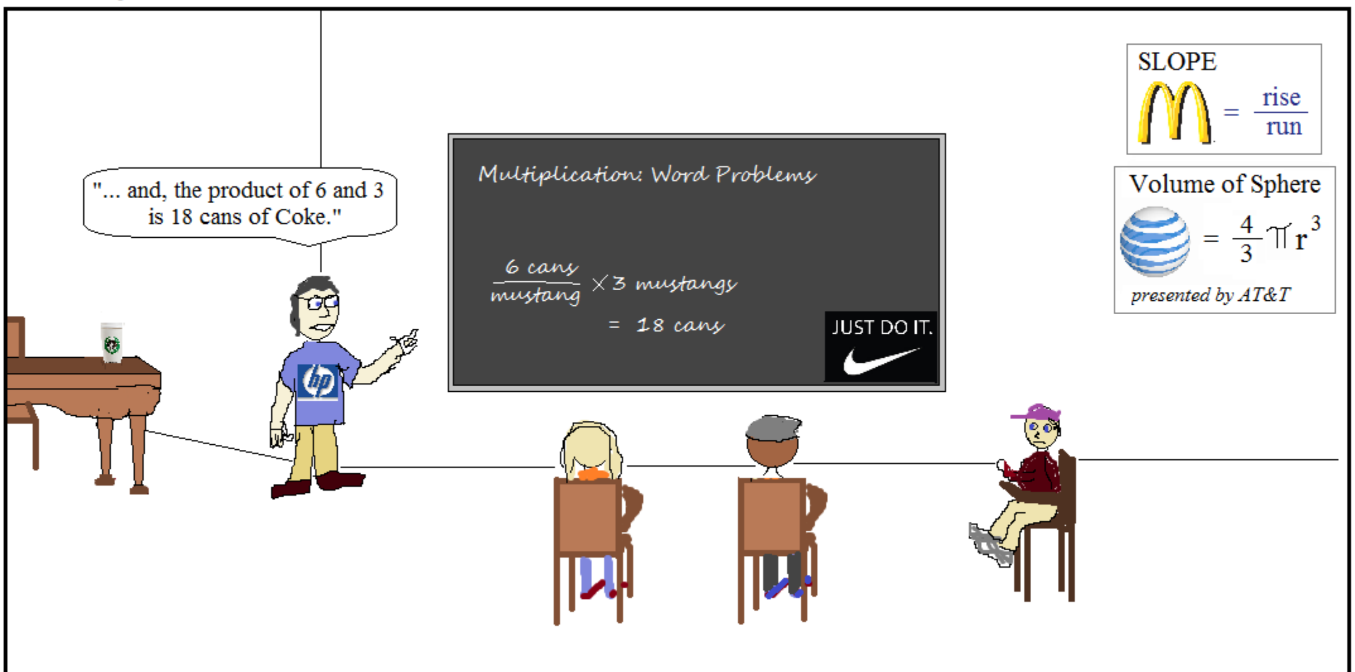

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Product Rule (to find derivatives of the products of 2 (or more) functions)

For functions f and g, the derivative of $f \cdot g$ is

$$f' \cdot g + f \cdot g'$$

or, it may be expressed as

$$\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

"*derivative of first one times the second one, plus derivative of second one times the first one...*"

Example: $f(x) = x^2 + 3x - 7$

$g(x) = 2x + 5$

To find the derivative of $f(x) \cdot g(x)$, use the product rule...

$$f'(x) = 2x + 3 \qquad g'(x) = 2$$

$$f' \cdot g + f \cdot g' = (2x + 3)(2x + 5) + (x^2 + 3x - 7)(2)$$

$$= 4x^2 + 16x + 15 + 2x^2 + 6x - 14$$

$$\boxed{= 6x^2 + 22x + 1}$$

To check, find $f(x)g(x)$:

$$(x^2 + 3x - 7)(2x + 5) = 2x^3 + 5x^2 + 6x^2 + 15x - 14x - 35$$

$$= 2x^3 + 11x^2 + x - 35$$

then, take the derivative of the product:

$$\frac{d}{dx}(f(x)g(x)) = 6x^2 + 22x + 1 \quad \checkmark$$

For 3 functions:

$$\frac{d}{dx}(u \cdot v \cdot w) = \frac{du}{dx} \cdot v \cdot w + u \cdot \frac{dv}{dx} \cdot w + u \cdot v \cdot \frac{dw}{dx}$$

<u>Quotient Rule</u> (to find the derivative of a function that is the quotient of 2 functions)

If $f(x) = \frac{g(x)}{h(x)}$, where $h(x) \neq 0$, then the derivative of $f(x)$ is

$$f'(x) = \frac{g'(x)h(x) - h'(x)g(x)}{[h(x)]^2}$$

> *"derivative of the numerator times the denominator MINUS derivative of the denominator times the numerator, all over the denominator squared."*

Example:

$$y = \frac{2x - 1}{x^3}$$

$$y' = \frac{(2)(x^3) - (3x^2)(2x - 1)}{[x^3]^2}$$

$$y' = \frac{2x^3 - 6x^3 + 3x^2}{x^6} = \frac{-4x^3 + 3x^2}{x^6} = \boxed{\frac{-4x + 3}{x^4}}$$

To check, distribute the denominator:

$$y = \frac{2x}{x^3} - \frac{1}{x^3} = 2x^{-2} - x^{-3}$$

Then, $y' = -4x^{-3} - (-3x^{-4})$

$$= \frac{-4}{x^3} + \frac{3}{x^4}$$

$$= \frac{-4x + 3}{x^4} \checkmark$$

Chain Rule & Power Rule

Definition of Chain Rule: If $y = f(u)$ is a differentiable function of u, and

$u = g(x)$ is a differentiable function of x, then

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

"the change of y with respect to x" "the change of y with respect to u" "the change of u with respect to x"

or $$\frac{d}{dx}[f(g(x))] = f'(u)\,g'(x) = f'(g(x))g'(x)$$

Example: $y = 2u^{10}$ and $u = 2x + 1$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \longrightarrow (20u^9)(2) = 40u^9$$

$$= 40(2x+1)^9$$

(change of y with respect to x)

Example: $y = 3(6x-7)^3$ (find the derivative using the chain rule)

→ Let $u = (6x-7)$ and then $y = 3u^3$

Then, apply chain rule...

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \longrightarrow (9u^2)(6-0) = 54u^2$$

$$= 54(6x-7)^2$$

**This all leads to a special case of the chain rule:

The General Power Rule

The General Power Rule:

Definition: If $y = [u(x)]^n$, where u is a differentiable function of x, and n is a real number, then

$$\frac{dy}{dx} = n[u(x)]^{n-1}\frac{du}{dx} \quad \text{or} \quad \frac{d}{dx}[u^n] = n[u]^{n-1}u'$$

What does it mean? *To find the derivative, "take the term in parentheses, move the exponent in front, subtract one from the exponent, and multiply by the derivative of the term inside the parentheses."*

Example: $y = (6x^2 - 4x + 7)^3$ to find the first derivative...

$$y' = 3\,(6x^2 - 4x + 7)^2\,(12x - 4)$$

"exponent in front" "term in parenthesis" "subtract one from exponent" "derivative of term"

Note: Basic derivative uses power rule:

$y = x^3 \longrightarrow y' = 3(x)^2(1) = 3x^2$

Example:

$$y = \frac{1}{(x^2 + 7x - 11)^3} \longrightarrow y = (x^2 + 7x - 11)^{-3}$$

$$\frac{dy}{dx} = -3(x^2 + 7x - 11)^{-4}(2x + 7)$$

$$= \boxed{\frac{-3(2x + 7)}{(x^2 + 7x - 11)^4}}$$

General Power Rule (continued)

Example: $y = \sqrt{x^2 + 3}$ To find the derivative y':

(change radical to exponential form) $y = (x^2 + 3)^{1/2}$

"move exponent in front" $y' = 1/2$

"times term in parentheses" $y' = 1/2\,(x^2 + 3)$

"subtract 1 for new exponent" $y' = 1/2\,(x^2 + 3)^{-1/2}$

"times derivative of terms in the parentheses" $y' = 1/2\,(x^2 + 3)^{-1/2}\,(2x)$

(Return to radical form and simplify) $y' = \dfrac{1/2\,(2x)}{\sqrt{x^2 + 3}} = \boxed{\dfrac{x}{\sqrt{x^2 + 3}}}$

Example: $y = \sin^2 x + x^5$ To find derivative of y:

$y' = 2(\sin x)^1\,(\cos x) + 5(x)^4\,(1)$

$= 2(\sin x)(\cos x) + 5x^4$

$= \sin 2x + 5x^4$

It's helpful to note:

$\sin^2 x = \sin x \sin x = (\sin x)^2$

$\sin 2x = 2\sin x \cos x$

derivative of sinx is cosx

Practice Exercise (with Solutions)

Differentiation Practice Exercise

Find the derivative of the following:

1) $x^3 - 3x^2 + 10$

2) $\sqrt{x} + \frac{1}{\sqrt{x}}$

3) $\frac{6x + 5}{x^2 + 1}$

4) $(x^3 + 1)^{\frac{1}{2}}$

5) $(2x^2 + 5)(x^2 - 2x + 3)$

6) $(x^2 + \frac{1}{x})^4$

Find the <u>2nd derivative</u> of the following functions:

1) $h(x) = (3x^2 + 4)^3$

2) $g(x) = 5x + 3$

3) $f(x) = \sqrt{x^2 + 9}$

4) $f(x) = (x + 1)(x^2 - 6)$

5) $f(t) = 3t^2 + 16t + \frac{1}{t}$

6) $g(x) = mx^4$

where m is an integer

Using table values and applying derivative rules

Using the values in the table, find the following:

x	$f(x)$	$f'(x)$
0	8	-1
1	-2	3

a) $3f(x)$ at $x = 1$

b) $3f'(x)$ at $x = 1$

c) the derivative of $xf(x)$ at $x = 0$

d) the derivative of $x^2 f(x)$ at $x = 1$

e) the derivative of $\dfrac{f(x)}{x^2 + 2}$ at $x = 0$

f) the derivative of $\dfrac{f(x)}{x}$ at $x = 1$

g) the derivative of $(f(x))^2$ at $x = 0$

Differentiation Practice Exercise

SOLUTIONS

Find the derivative of the following:

1) $x^3 - 3x^2 + 10$

$3x^2 - 3(2x) + 0$

$3x^2 - 6x$

2) $\sqrt{x} + \frac{1}{\sqrt{x}}$

$(x)^{\frac{1}{2}} + (x)^{\frac{-1}{2}}$

$\frac{1}{2}(x)^{\frac{-1}{2}} + \frac{-1}{2}(x)^{\frac{-3}{2}}$

$\frac{1}{2\sqrt{x}} + \frac{-1}{2\sqrt{x^3}}$

$\frac{x}{2\sqrt{x^3}} - \frac{1}{2\sqrt{x^3}}$

$\frac{x-1}{2x^{3/2}}$

3) $\frac{6x+5}{x^2+1}$

use quotient rule:

derivative of numerator: 6
derivative of denominator: 2x

$\frac{6(x^2+1) - 2x(6x+5)}{(x^2+1)^2}$

$\frac{6x^2 + 6 - 12x^2 - 10x}{(x^2+1)^2} = \frac{-2(3x^2 + 5x - 3)}{(x^2+1)^2}$

4) $(x^3 + 1)^{\frac{1}{2}}$

$\frac{1}{2}(x^3+1)^{\frac{-1}{2}} \cdot (3x^2)$

$\frac{3x^2}{2\sqrt{x^3+1}}$

5) $(2x^2+5)(x^2 - 2x + 3)$

use product rule:
derivative of first term: 4x
derivative of second term: 2x - 2

$4x(x^2 - 2x + 3) + (2x - 2)(2x^2 + 5)$

$4x^3 - 8x^2 + 12x + 4x^3 + 10x - 4x^2 - 10$

$8x^3 - 12x^2 + 22x - 10$

$2(4x^3 - 6x^2 + 11x - 5)$

6) $(x^2 + \frac{1}{x})^4$

use chain rule:
$u = (x^2 + 1/x)$
$u' = 2x + (-1/x^2)$

$4(x^2 + \frac{1}{x})^3 (2x + (-1/x^2))$

Find the <u>2nd derivative</u> of the following functions:

1) $h(x) = (3x^2 + 4)^3$

$h'(x) = 3(3x^2+4)^2 (6x)$

$= 18x(9x^4 + 24x^2 + 16)$

$= 162x^5 + 432x^3 + 288x$

$h''(x) = 810x^4 + 1296x^2 + 288$

$18(45x^4 + 72x^2 + 16)$

2) $g(x) = 5x + 3$

$g'(x) = 5$

$g''(x) = 0$

3) $f(x) = \sqrt{x^2+9} = (x^2+9)^{\frac{1}{2}}$

$f'(x) = 1/2(x^2+9)^{\frac{-1}{2}}(2x) = \frac{x}{(x^2+9)^{1/2}}$

$f''(x) = \frac{1 \cdot (x^2+9)^{1/2} - \frac{x}{(x^2+9)^{1/2}} \cdot (x)}{\left((x^2+9)^{1/2}\right)^2} =$

$\frac{(x^2+9) - x^2}{(x^2+9)^{1/2}} \div (x^2+9) = \frac{9}{(x^2+9)^{3/2}}$

4) $f(x) = (x+1)(x^2 - 6)$

expand the function:

$f(x) = x^3 - 6x + x^2 - 6$

$= x^3 + x^2 - 6x - 6$

$f'(x) = 3x^2 + 2x - 6$

$f''(x) = 6x + 2$

5) $f(t) = 3t^2 + 16t + \frac{1}{t}$

$f'(t) = 6t + 16 + \frac{-1}{t^2}$

$f''(t) = 6 + 0 + \frac{-2t(-1)}{t^4}$

$= 6 + \frac{2}{t^3}$

6) $g(x) = mx^4$

where m is an integer

$g'(x) = 4mx^3$

$g''(x) = 12mx^2$

Using table values and applying derivative rules

SOLUTIONS

Using the values in the table, find the following:

x	$f(x)$	$f'(x)$
0	8	-1
1	-2	3

a) $3f(x)$ at $x = 1$

$3 \cdot f(1) = 3 \cdot (-2) = -6$

b) $3f'(x)$ at $x = 1$

$3 \cdot f'(1) = 3 \cdot (3) = 9$

c) the derivative of $xf(x)$ at $x = 0$

product rule: $(1)f(x) + f'(x)(x)$

at $x = 0$: $(1)f(0) + f'(0)(0)$

$8 + 0 = 8$

d) the derivative of $x^2 f(x)$ at $x = 1$

product rule: $(2x)f(x) + f'(x)(x^2)$

at $x = 1$: $2 \cdot f(1) + f'(1) \cdot 1$

$-4 + 3 = -1$

e) the derivative of $\dfrac{f(x)}{x^2+2}$ at $x = 0$

quotient rule: $\dfrac{f'(x)(x^2+2) - (2x+0)f(x)}{(x^2+2)^2}$

at $x = 0$: $\dfrac{(-1)(2) - (0)(8)}{2^2} = -1/2$

f) the derivative of $\dfrac{f(x)}{x}$ at $x = 1$

use quotient rule: $\dfrac{x f'(x) - (1)f(x)}{x^2}$

at $x = 1$: $\dfrac{(1)(3) - (1)(-2)}{(1)^2} = 5$

g) the derivative of $(f(x))^2$ at $x = 0$

power/chain rule: $2 \cdot f(x)^1 \cdot f'(x)$

at $x = 0$: $2 \cdot 8 \cdot -1 = -16$

Thanks for visiting. (Hope the brief notes and practice helped!)

If you have questions, suggestions, or requests, let us know.

Cheers!