Introduction to Negative Numbers

Notes, Examples, Summary, and Practice (with Solutions)

cancel each
other out

$3 + (-8) = -5$

-10   -5   0   3   5   10
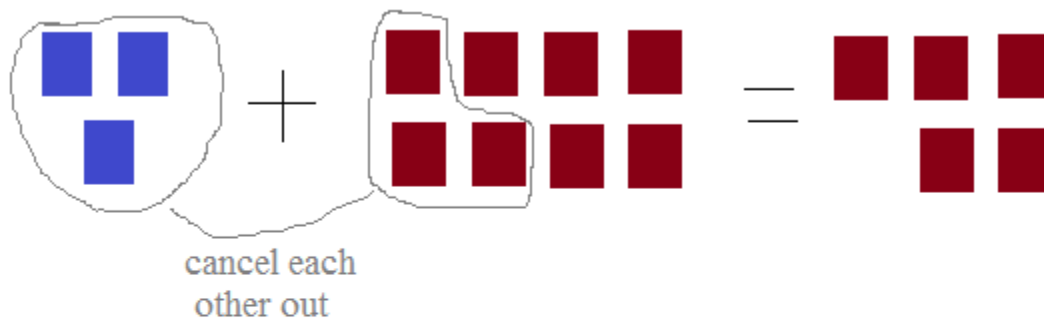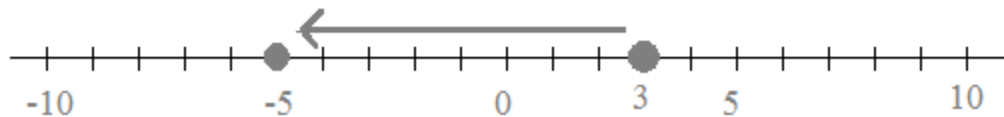
Topics include arithmetic, number lines, word problems, order of operations, and more.

Summary: Working with Negative Numbers

Addition: Positive + Positive = Positive $31 + 22 = 53$

Negative + Negative = Negative $-31 + (-22) = -53$ } Add numbers (and insert sign)

Negative + Positive = <u>Sign of larger number</u> $8 + (-14) = -6$

$-9 + 16 = 7$ } Find the difference (i.e. subtract) and use sign of larger number

$\longrightarrow 16 - 9 = 7$

Subtraction: Positive − Positive = Positive IF 1st number is larger $9 - 5 = 4$

Positive − Positive = Negative IF 2nd number is larger $5 - 9 = -4$ } Find the difference and use correct sign

Negative − Positive = Negative $-8 - 6 = -8 + (-6) = -14$ Add values and use a negative sign

$8 + 6$ (-)

"Negative-negative"   If Negative number <u>follows a minus sign</u>, change to positive. $9 - (-5) = 9 + 5 = 14$

$-7 - (-10) = -7 + 10 = 3$

"Multiplication":   If signs are the same, the result is positive. If signs are opposite, the result is negative.

Positive x Positive = Positive $6 \text{ x } 3 = 18$

Negative x Negative = Positive $-5 \text{ x } (-4) = 20$

Positive x Negative = Negative $2 \text{ x } -7 = 14$
Negative x Positive = Negative $-1 \text{ x } 9 = -9$

"Division":   NOTE: Division is a form of multiplication. $8 \div {}^{-}4$ is the same as $8 \text{ x } \frac{-1}{4}$

So, the multiplication rules apply to division.

Positive $\div$ Positive = Positive $8/4 = 2$

Negative $\div$ Negative = Positive $-9 \div (-3) = 3$ } Same signs, positive

Positive $\div$ Negative = Negative $14 \div (-7) = -2$

Negative $\div$ Positive = Negative $-9 \div 3 = -3$ } Opposite signs, negative

Subtraction and negative number notes

1) "Subtraction" --- $8 + (-3) = 5$ or $8 - 3 = 5$

subtract 3 blocks...
('take away')
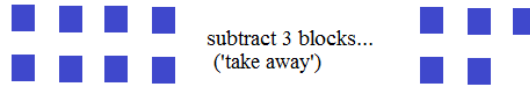
And, what happens if we try to subtract more than we have?

--- $4 + (-6) = -2$ or $4 - 6 = -2$

(cancel out)
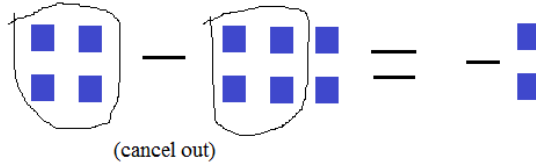
Example: I have 12 dollars in my checking account. If
I take 20 dollars, how much do I have in my account?

Answer: I start with $12. After withdrawing 12, I need 8 more (to reach $20).
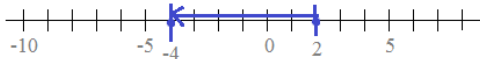So, after taking out 8 more, my account is -8.

2) Placement and location: Movement on the number line.

Left of 0 are negative numbers --- movement to the left is subtraction
Right of 0 are positive numbers -- movement to the right is addition
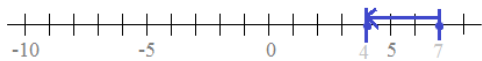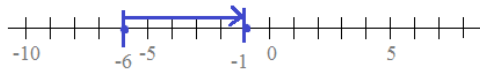

$7 - 3 = 4$ or $7 + (-3) = 4$ start at 7; move 3 units to the left;
land at 4
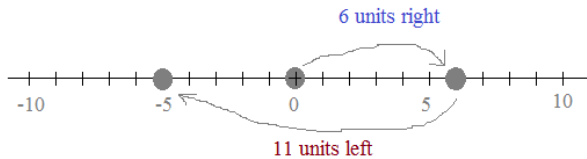
$2 - 6 = -4$ or $2 + (-6) = -4$ start at 2; move 6 units to the left
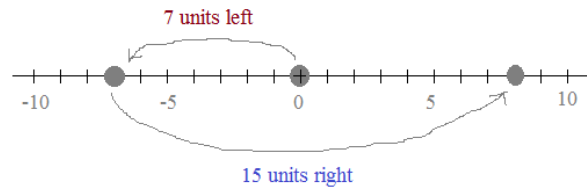(passing over 0); land at -4

$-6 + 5 = -1$ start at -6; move 5 units to the right;
land at -1

Examples: starting at 0

6 units right

$6 - 11 = -5$
or
$(0 +) 6 - 11 = -5$

11 units left

7 units left

$-7 + 15 = 8$
or

$(0 - 7) + 15 = 8$

15 units right

3) General "sign rules"

Same sign, answer keeps the sign... $-2.8 - 2.2 = -5$ both terms are negative
$3.8 + 2.2 = 6$ both terms are positive

If signs are opposite, answer takes
sign of larger number...

$4.5 + (-2.3) = 2.5$ It's positive because $4.5 > 2.3$

$-3.6 + 1.2 = -2.4$ It's negative because $3.6 > 1.2$

$14 - 27 = -13$ the difference between 14 and 27 is 13
(and $|-27| > 14$, so it is negative)

"positive" one unit

"negative" one unit

Examples and Illustrations:

$4 + 7 = 11$

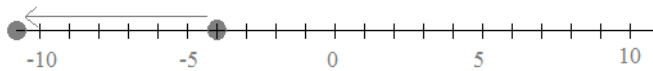Number line: Begin at 4; move 7 units to the right; land at 11

$(-4) + (-7) = -11$

or

$-4 - 7 = -11$

Begin at -4; move 7 units to the left; land at -11

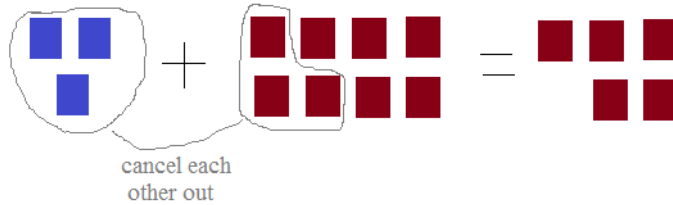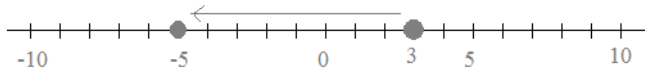*Two negative numbers -- the result is negative..*

$3 - 8 = -5$

or

$3 + (-8) = -5$

or

$-8 + 3 = -5$

Since 8 is greater than 3, the result is negative..

cancel each other out

Begin at 3; move 8 units to the left; land at -5

One negative number, one positive number -- the result is the sign of the larger number.
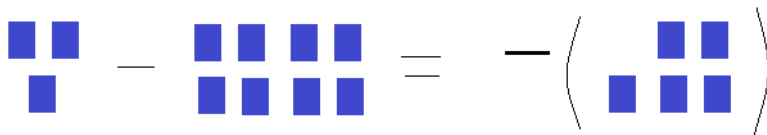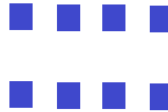
Begin at −8; move 3 units to the right; land at -5
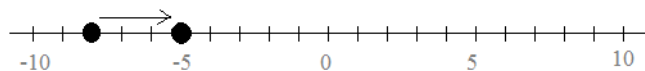
Multiplication and negative number notes

1) "Multiplication" --- $2 \times 4 = 8$ "four groups of two total eight"

   ■ ■ ■ ■
   ■ ■ ■ ■

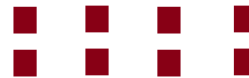 $4 \times 2 = 8$ "two groups of four are eight"

   ■ ■ ■ ■
   ■ ■ ■ ■

So, what happens if one of the numbers is negative?

   $-4 \times 2 = -8$ "two groups of negative four total *negative eight*"

   ■ ■ ■ ■
   ■ ■ ■ ■

   $-2 \times 4 = -8$ "four groups of negative two total negative eight"
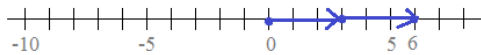
   ■ ■ ■ ■
   ■ ■ ■ ■

   Example: I have 100 dollars in my savings account. If I withdraw 8 dollars each year.
   How much do I have after 5 years?

   $\$100 + (\$-8/\text{year})(5 \text{ years}) = \$100 + (\$-40) = \$60$

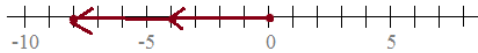2) Placement and location: Movement on the number line.

   Left of 0 are negative numbers --- movement to the left is subtraction
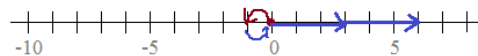   Right of 0 are positive numbers -- movement to the right is addition

$2 \times 3 = 6$	start at 0; move 3 units to the right 2 times.
$-2 \times 4 = -8$	start at 0; move 2 units to the left 4 times.
$-4 \times 2 = -8$	start at 0; move 4 units to the left 2 times.
$-3 \times (-2) = 6$	rewrite: $-1 \times 3 \times -1 \times 2$
	$-1 \times -1 = 1$ (cancel each other)
	$3 \times 2 = 6$

3) General "sign rules"

   Same signs, answer is *positive...* $3 \times 8 = 24$ both terms are positive
   $(-3) \times (-8) = 24$ both terms are negative

   If signs are *opposite*, answer is *negative...* $2 \times (-5) = -10$ "positive times negative"
   $-4 \times 3 = -12$ "negative times positive"

   NOTE: *Division is related to multiplication.* $(eg. -12 \div 4 = -12 \times 1/4 = -3)$ *Therefore, the same rules apply!*
   Same signs ----> result is positive.... Opposite signs ----> result is negative....

Positive / Negative Numbers "cheat sheet"

Negative / Positive Number Rules

Multiplication/Division

1) Same signs $\longrightarrow$ positive

2) Different signs $\longrightarrow$ negative

Addition/Subtraction

3) Same signs $\longrightarrow$ add, same sign

4) Different signs $\longrightarrow$ subtract, sign of 'larger number'

5) If 'minus negative', then change double negative to positive

Ancient Math History

Mr. Caesar and an unruly class

L. Friedman #CXXVII
mathplane.com 2-28-14
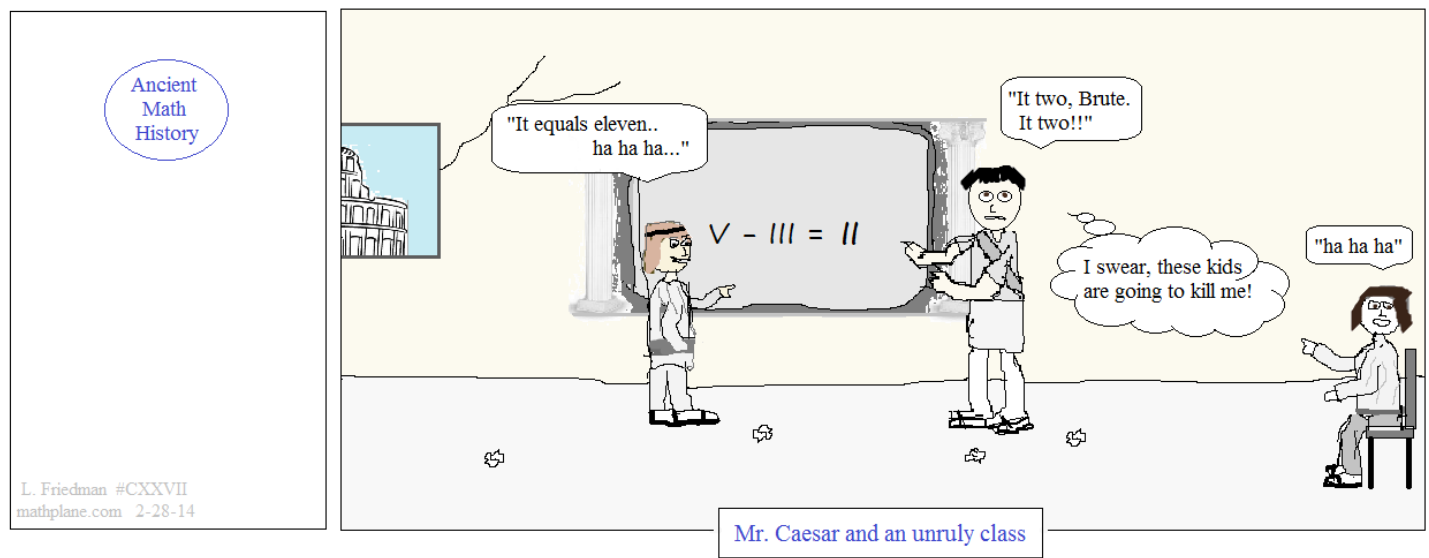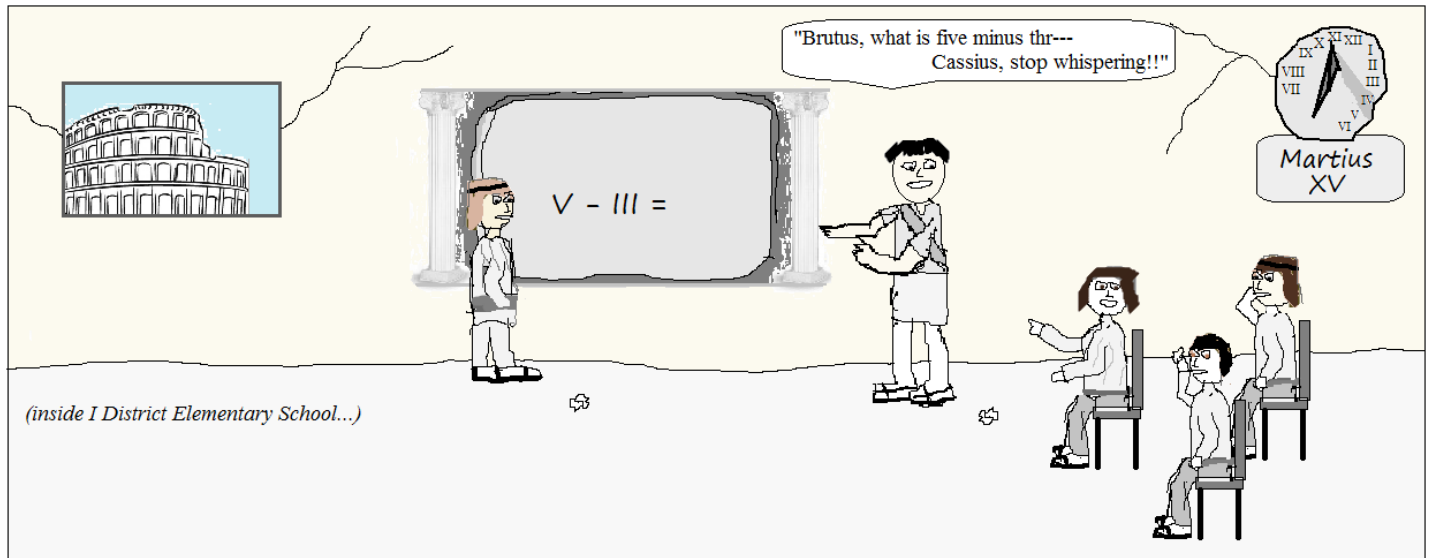
Practice Quiz→

I. Solve

   a) $5 - 9$

   b) $-5 - 13$

   c) $3.62 - 8.44$

   d) $10 + (-16)$

   e) $10 - (-16)$

   f) $\dfrac{-5}{6} + \dfrac{-3}{4}$

II. Let $a = -3$ $b = 2$ $c = -5$

   Answer the following: 1) $a + b$ 2) $a - (-b)$ 3) $-a + 2b + c$

   4) $c + 3b$ 5) $c + 2a$ 6) $-(c + a + 4)$
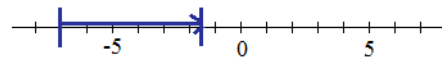
III. Word problems and applications

   A) The temperature inside the freezer is $-10°$F. If the temperature rose 24 degrees, what would it be?
                                        If the temperature fell 6 1/2 degrees, what would it be?

   B) A diver is 22 feet under the surface of the water. If the diver rises 8.8 feet toward the surface, how far
                                        is the diver below the water level?

   C) A running back gains 3 yards on his first run.. Then, he loses 8 yards on his second carry.
                                        What is his average yards per carry?

IV. Number line illustration (optional)

   A) Describe the following with a math expression

   B) Use a number line to represent the following equations.

      1) $-3 + 9 =$

      2) $4 - 7.3 =$

V. Multiplication/Division and more...
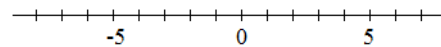
a) 3 x (-4) =

  -3 x (-4) =

  4 x (-3) =

  4 − (-3) =

  -3 − 4 =

  -3 + 4 =

b) 10 x (-5) =

  -5 x (-10) =

  $\dfrac{10}{(-5)} =$

  -10 ÷ 5 =

  -10 − 5 =

  -5 − 10 =

c) 1 + (-1) =

  -1 x (-1) =

  -1 − (-1) =

  -1 + -1 =

  $\dfrac{-1}{-1} =$

Where does the parenthesis go?

Example: 2 + -3 x 4 = -4   Answer: (2 + -3) x 4 = -4   (-1) x 4 = -4

a)   4 + -3 x -4 = -4

b)   -12 ÷ 2 + -4 = 6

c)   8 ÷ -2 + -2 = -6

d)   7 x 3 - -1 = 22

e) $\dfrac{6 \text{ x } 3 - 1}{-2 + 2 \text{ x } 2} = 6$

f) 8 + 0 x (-2) − (-3) = 8

g) 10 + 0 x (-4) + 2 = -38

h) 6 x (-2) + 5 x (4) − (-1) x 7 = 79

Some of the kids learned subtraction...

"Seven friends go to a cabin for the weekend. If only two of them return alive, how many bodies are missing?"

ELM STREET ELEMENTARY

$7 - X = 2$

Freddy
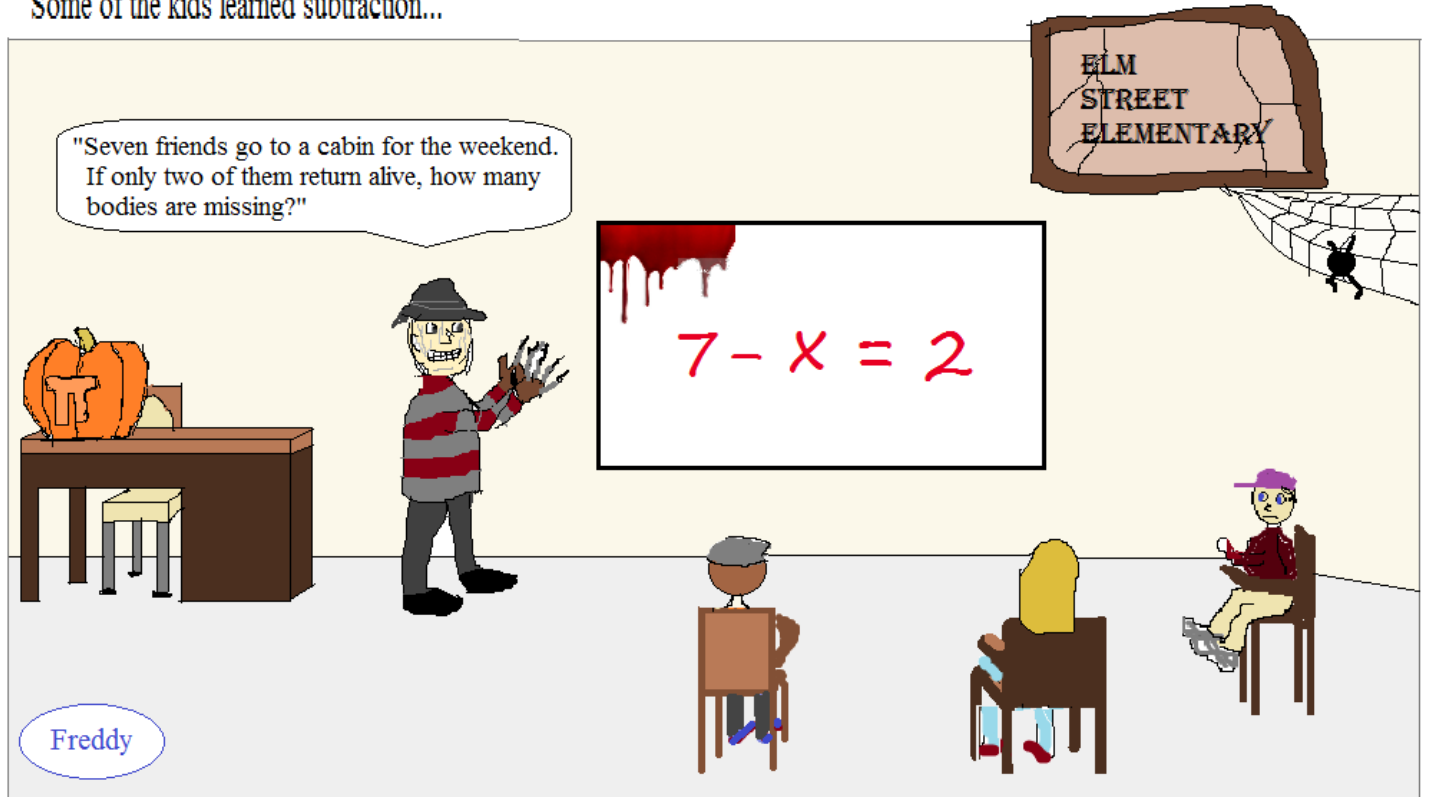
But, most of Mr. Krueger's math class had nightmares....

LanceAF #110
Halloween/2013
mathplane.com

Solutions ->

SOLUTIONS

I. Solve

a) $5 - 9 = \boxed{-4}$

note: $9 - 5 = 4$
   then, when the digits are reversed,
   the answer is the opposite.

b) $-5 - 13 = \boxed{-18}$

note: $5 + 13 = 18$
   then, $-5 + -13 = -18$

c) $3.62 - 8.44$

$$\begin{array}{r} 8.44 \\ -\ 3.62 \\ \hline 4.82 \end{array}$$

therefore, $3.62 - 8.44 = \boxed{-4.82}$

d) $10 + (-16) = \boxed{-6}$

note: since $|-16| > |10|$,
   the sign must be negative

e) $10 - (-16) = \boxed{26}$

$10 + 16 = 26$

note: if double negative,
   you may switch the sign

f) $\dfrac{-5}{6} + \dfrac{-3}{4}$ $\dfrac{-20}{24} + \dfrac{-18}{24} = \dfrac{-38}{24} = \boxed{-1\dfrac{7}{12}}$

II. Let $a = -3$ $b = 2$ $c = -5$

Answer the following:

1) $a + b$

$-3 + 2 = \boxed{-1}$

2) $a - (-b)$

$-3 - (-2) = -3 + 2$
$= \boxed{-1}$

3) $-a + 2b + c$

$-(-3) + 2(2) + (-5) =$
$3 + 4 - 5 = \boxed{2}$

4) $c + 3b$

$-5 + 3(2) = -5 + 6$
$= 6 - 5$
$= \boxed{1}$

5) $c + 2a$

$-5 + 2(-3) = -5 + (-6)$
$\boxed{-11}$

6) $-(c + a + 4)$

$-((-5) + (-3) + 4) =$
$-(-4) = \boxed{4}$

III. Word problems and applications

A) The temperature inside the freezer is $-10°$ F. If the temperature rose 24 degrees, what would it be? $-10 + 24 = \boxed{14°\,F}$

If the temperature fell 6 1/2 degrees, what would it be? $24 - 10 = 14$

$-10 - 6.5 = \boxed{-16.5°\,F}$

B) A diver is 22 feet under the surface of the water. If the diver rises 8.8 feet toward the surface, how far is the diver below the water level?
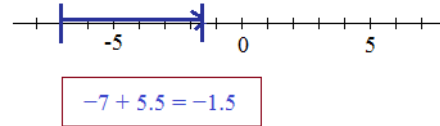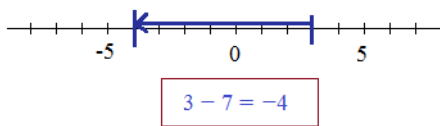
the diver begins at -22 feet.. Then, goes up $+8.8$ feet... The total is -13.2 or $\boxed{13.2 \text{ feet below the water surface}}$

C) A running back gains 3 yards on his first run.. Then, he loses 8 yards on his second carry.
What is his average yards per carry?

$0 +\ \ 3\ -\ 8 = -5$
(total yards)

$\dfrac{-5 \text{ yards}}{2 \text{ carries}} = \boxed{-2.5 \text{ yards/carry}}$

IV. Number line illustration (optional)
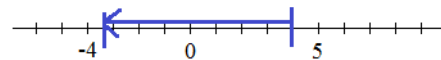
A) Describe the following with a math expression

$\boxed{3 - 7 = -4}$

$\boxed{-7 + 5.5 = -1.5}$

B) Use a number line to represent the following equations.

1) $-3 + 9 = \ 6$

2) $4 - 7.3 = \ -3.3$

SOLUTIONS

V. Multiplication/Division and more...

a) $3 \times (-4) = $ -12

$-3 \times (-4) = $ 12

$4 \times (-3) = $ -12

$4 - (-3) = $ $4 + 3 = 7$

$-3 - 4 = $ $(-3) + (-4) = -7$

$-3 + 4 = $ $4 - 3 = 1$

b) $10 \times (-5) = $ -50

$-5 \times (-10) = $ 50

$\dfrac{10}{(-5)} = $ -2

$-10 \div 5 = $ -2

$-10 - 5 = $ -15

$-5 - 10 = $ -15

c) $1 + (-1) = $ 0

$-1 \times (-1) = $ 1

$-1 - (-1) = $ $-1 + 1 = 0$

$-1 + -1 = $ -2

$\dfrac{-1}{-1} = $ 1

Where does the parenthesis go?

Example: $2 + -3 \times 4 = -4$ Answer: $\left(2 + -3\right) \times 4 = -4$ $(-1) \times 4 = -4$

a) $\left(4 + -3\right) \times -4 = -4$ $(1) \times -4 = -4$

b) $-12 \div \left(2 + -4\right) = 6$ $-12 \div (-2) = 6$

c) $\left(8 \div -2\right) + -2 = -6$ $(-4) + -2 = -6$

d) $7 \times 3 - (-1) = 22$ $7 \times 3 + 1 = 22$ or $(7 \times 3) - -1 = 22$

e) $\dfrac{6 \times (3 - 1)}{-2 + 2 \times 2} = 6$ $\dfrac{6 \times 2}{-2 + 4} = \dfrac{12}{2} = 6$

f) $8 + 0 \times \left((-2) - (-3)\right) = 8$ $8 + 0(1) = 8$
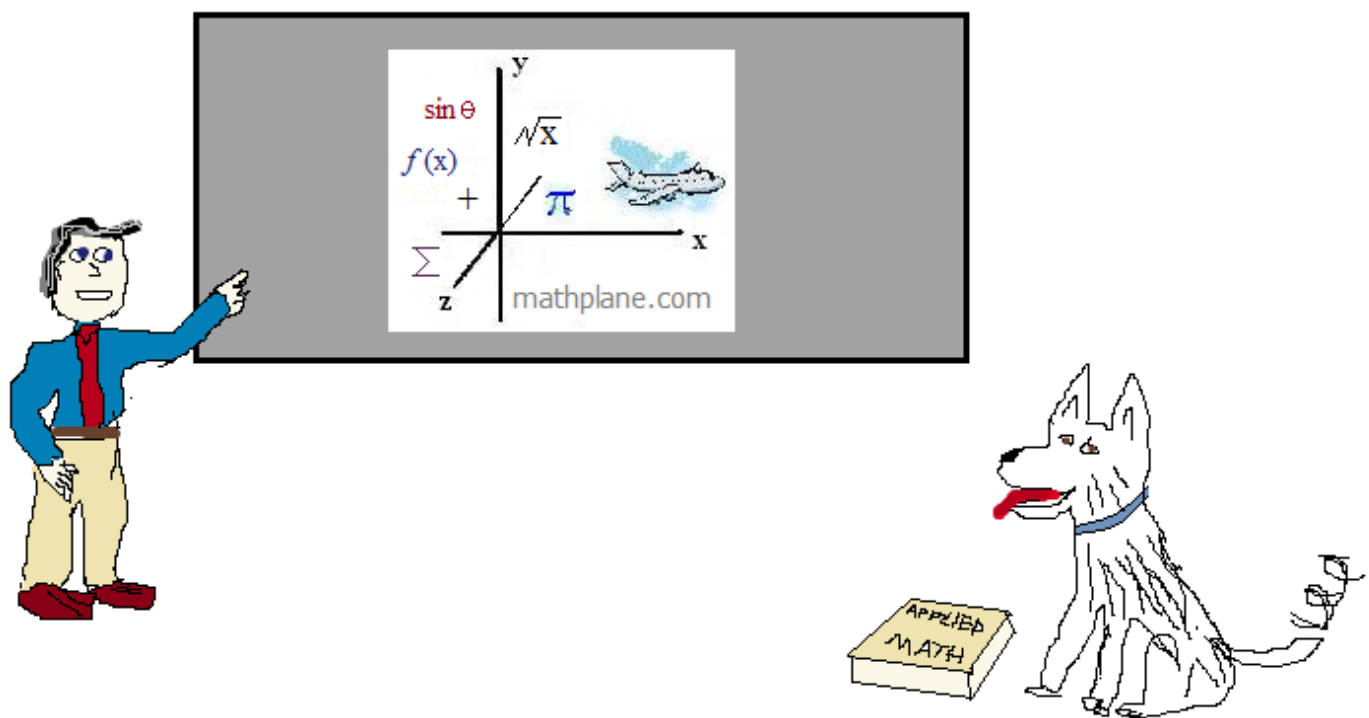
g) $\left(10 + 0\right) \times (-4) + 2 = -38$ $(10) \times (-4) + 2 = -40 + 2 = -38$

h) $6 \times \left((-2) + 5\right) \times (4) - (-1) \times 7 = 79$ $6 \times (3) \times 4 - (-1) \times 7 = $

$72 - (-7) = $

$72 + 7 = 79$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy

Also, at Facebook, Google+, TeachersPayTeachers, and Pinterest

One More Exercise:

Fill in the multiplication table:

X	−3	−5	+7	+9
+12	−36			
−4				
+6				
−8				

Fill in the multiplication table:

X	−3	−5	+7	+9
+12	−36	−60	84	108
−4	12	20	−28	−36
+6	−18	−30	42	54
−8	24	40	−56	−72