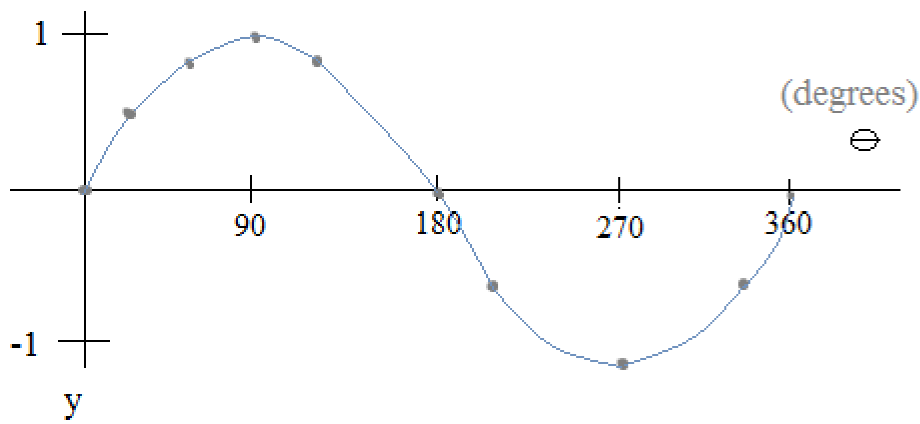
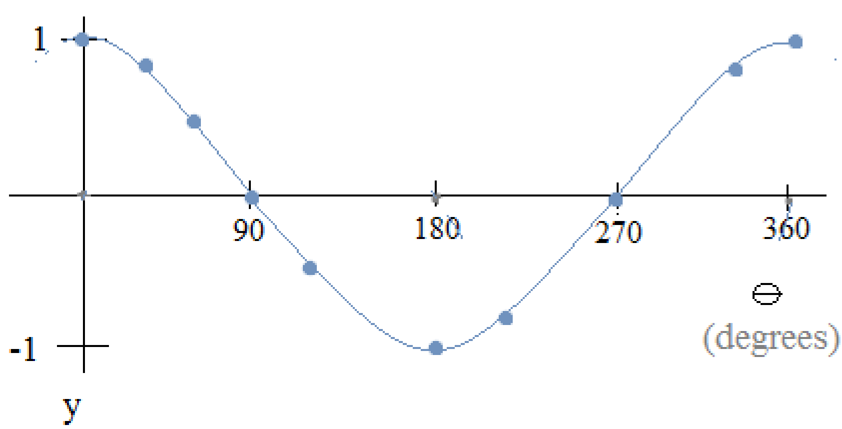
Introduction to Periodic Trig Functions:

Sine and Cosine Graphs



Notes/examples of trig values and the 4 components of trig graphs (amplitude, horizontal (phase) shift, vertical shift, and period).

Includes 2 practice tests (and solutions)



PREVIEW

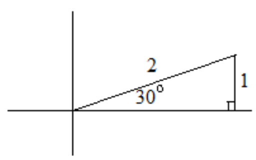
Contents

Periodic Functions: Sinusoidal Graphs

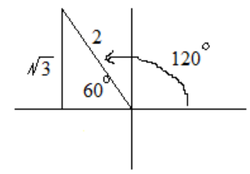
Sketching the Parent Function: $y = \sin(x)$ (radians) $y = \sin\theta$ (degrees)

$\text{Sine} = \dfrac{\text{opposite}}{\text{hypotenuse}}$

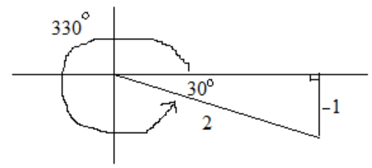
A few examples of common angles:



$\sin 30^\circ = \frac{1}{2}$



$\sin 120^\circ = \frac{\sqrt{3}}{2}$



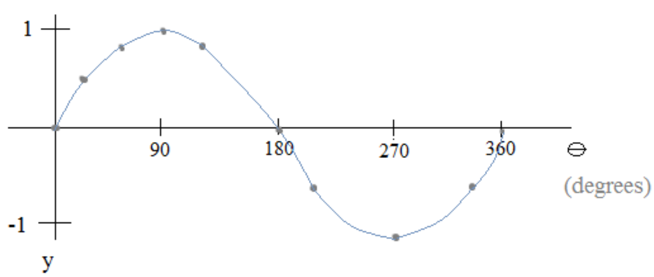
$\sin 330^\circ = \frac{-1}{2}$

The following is a table of chosen values:

$y = \sin\theta$

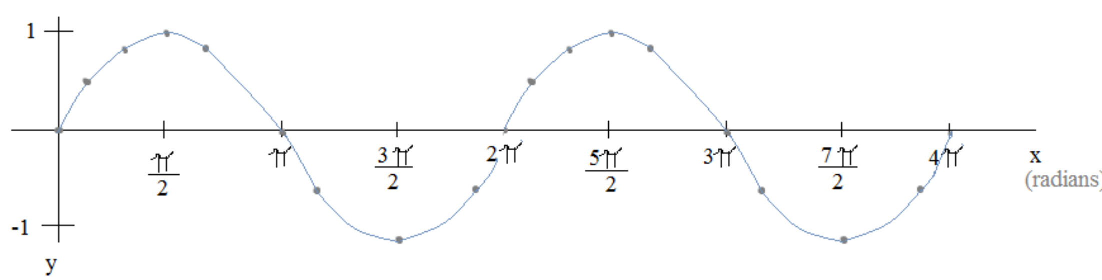
θ	0	30	60	90	120	180	210	270	330	360
y	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	$\frac{-1}{2}$	-1	$\frac{-1}{2}$	0

Then, plot the points...



It's a <u>periodic function</u> -- it will repeat the pattern of y- values at a regular interval of 360°. (The period is 360°)

$y = \sin x$



A cycle is a complete pattern repetition. This graph contains 2 cycles.

The period is 2π (Horizontal length of one cycle)

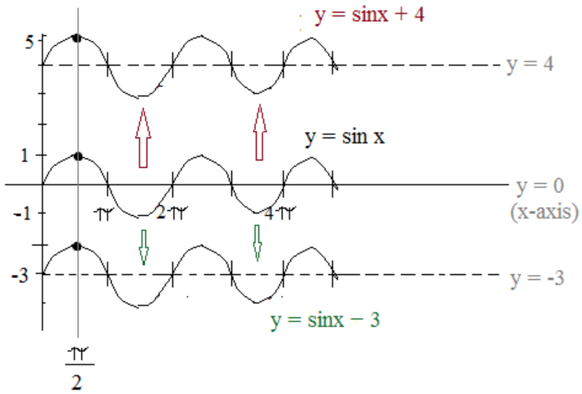
Sine functions: 4 components

$y = \sin(x)$ is the parent function.

$y = A\sin B(x - C) + D$

- A: Amplitude (magnitude)
- B: Period
- C: Horizontal Shift
- D: Vertical Shift

Vertical Shift:



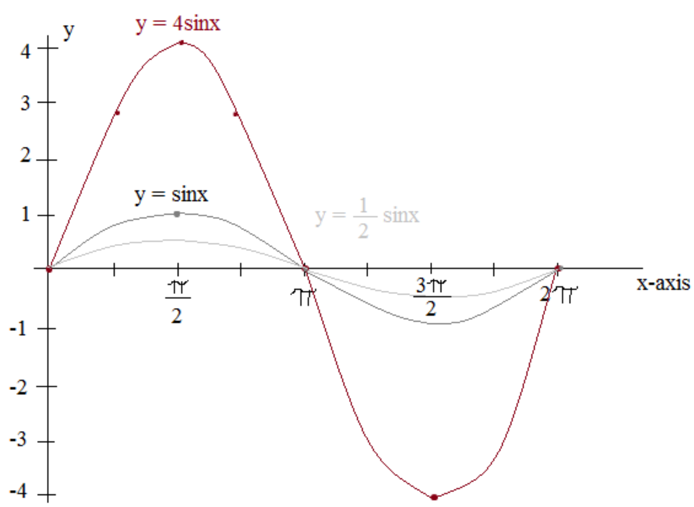
If $x = \frac{\pi}{2}$

$y = \sin x$ —— $\sin\frac{\pi}{2} = 1$

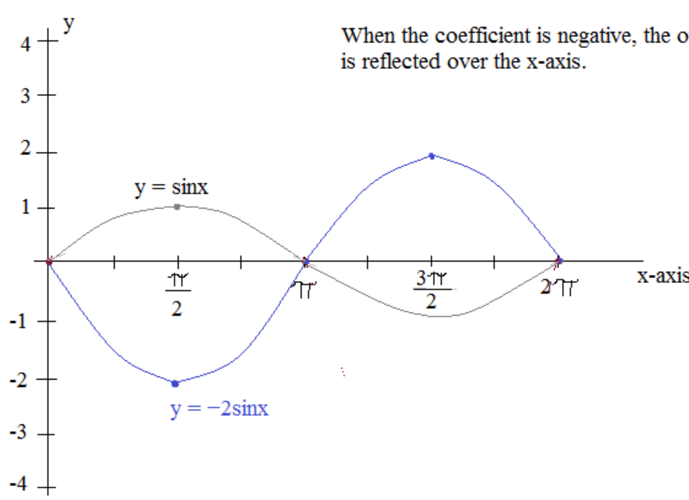
$y = \sin x + 4$ —— $\sin\frac{\pi}{2} + 4 = 5$

$y = \sin x - 3$ —— $\sin\frac{\pi}{2} - 3 = -2$

Amplitude:



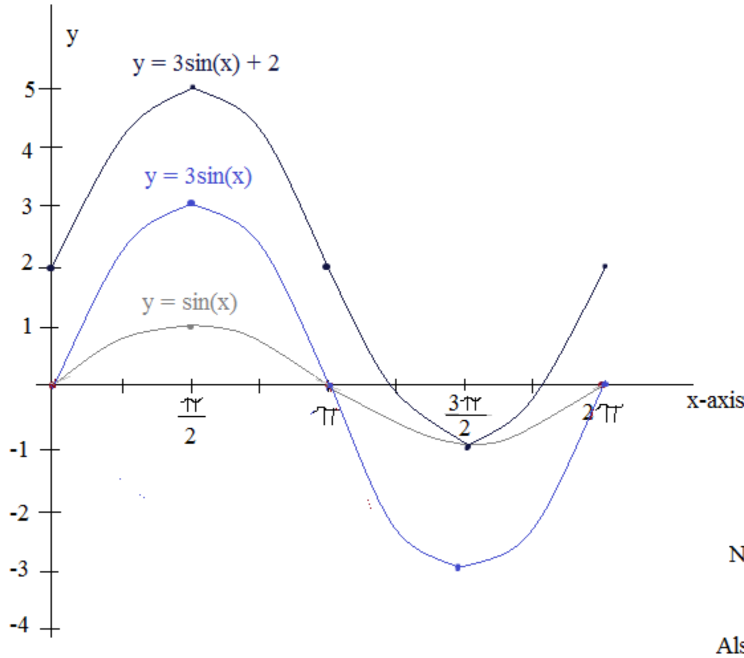
x	sinx	4sinx	$\frac{1}{2}$ sinx	-2sinx
0	0	0	0	0
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$2\sqrt{2}$	$\frac{\sqrt{2}}{4}$	$-\sqrt{2}$
$\frac{\pi}{2}$	1	4	$\frac{1}{2}$	-2
π	0	0	0	0
$\frac{3\pi}{2}$	-1	-4	$-\frac{1}{2}$	2



When the coefficient is negative, the output is reflected over the x-axis.

Sine Functions: Amplitude and Vertical Shift Illustrations

I. Sketch one cycle of the function $f(x) = 3\sin x + 2$



1) Sketch the parent function $y = \sin(x)$

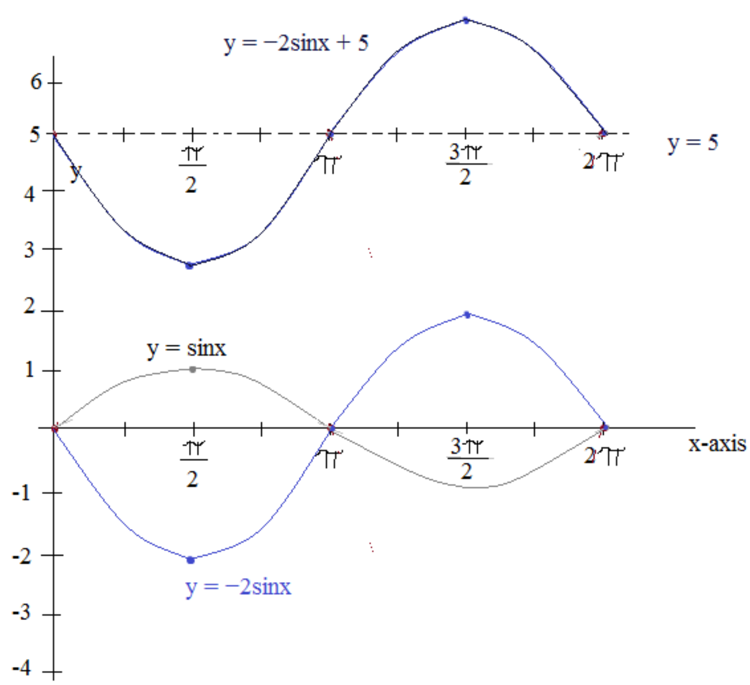
2) *Increase ("stretch") the amplitude* by a factor of 3 $y = 3\sin(x)$

3) *Shift the graph* up 2 units $y = 3\sin(x) + 2$

Notice, $\sin\frac{3\pi}{2} = -1$ $\quad 3\sin\frac{3\pi}{2} + 2 = -1$

Also, sin(x) and 3sin(x) intersect at $0, \pi$, and 2π

II. Sketch $y = -2\sin x + 5$



1) Parent Function $y = \sin x$

2) "Stretch" and "Reflect" $y = -2\sin x$

3) Vertical Shift $y = -2\sin x + 5$

Sine functions: 4 components (continued)

$y = A\sin B(x - C) + D$

A: Amplitude (magnitude)
B: Period
C: Horizontal Shift
D: Vertical Shift

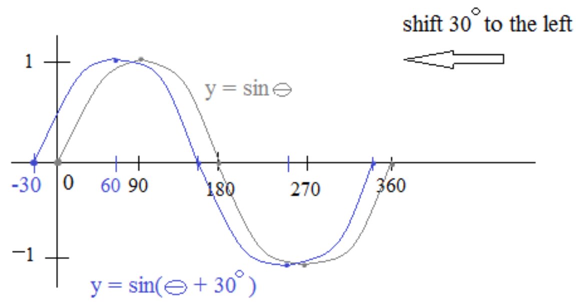
Horizontal ("Phase") Shift:

Question: If $\sin 90^\circ = 1$, then where does $\sin(\theta + 30^\circ) = 1$?

Answer: $\theta = 60^\circ$, because $\sin(60^\circ + 30^\circ) = 1$

Implication: $90^\circ \Rightarrow 60^\circ$ (shift 30° to the left)

Example I: $y = \sin(\theta + 30^\circ)$



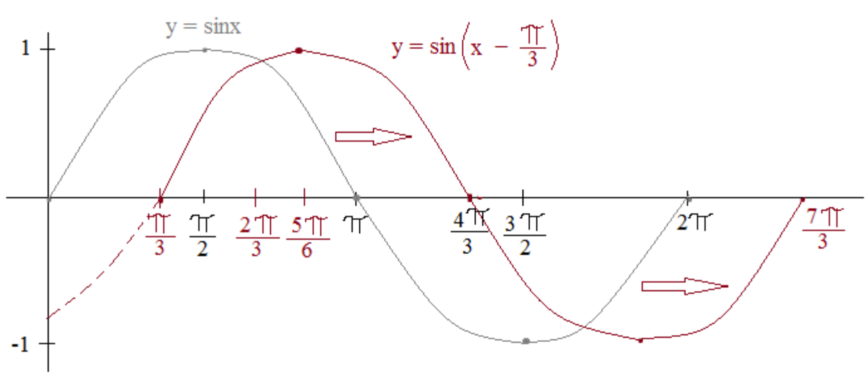
Note: The horizontal shift is the *opposite* direction of the sign.

Compare the values of the parent function $\sin\theta$ to the values of $\sin(\theta + 30^\circ)$

(horizontal shift of 30° to the left)

θ	$\sin\theta$	$\sin(\theta + 30^\circ)$
-30	$\frac{-1}{2}$	0
0	0	$\frac{1}{2}$
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
60	$\frac{\sqrt{3}}{2}$	1
90	1	$\frac{\sqrt{3}}{2}$

Example II: $y = \sin\left(x - \frac{\pi}{3}\right)$



(horizontal shift of $\frac{\pi}{3}$ to the right)

$y = \sin x$ crosses the x-axis at π

and

$y = \sin\left(x - \frac{\pi}{3}\right)$ crosses at $\frac{4\pi}{3}$

$\sin\pi = 0$

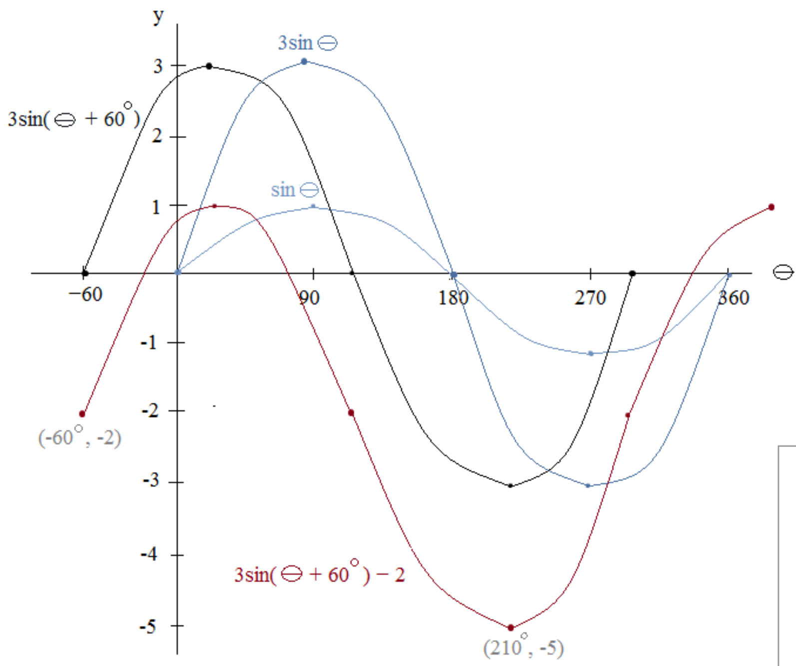
$\sin\left(\frac{4\pi}{3} - \frac{\pi}{3}\right) = 0$

Sine Functions: Amplitude, Horizontal and Vertical Shifts Illustration

$y = A\sin B(x - C) + D$

A: Amplitude (magnitude)
B: Period
C: Horizontal Shift
D: Vertical Shift

Sketch $y = 3\sin(\theta + 60^\circ) - 2$



1) Parent function: $y = \sin\theta$

2) Amplitude (A = 3): $y = 3\sin\theta$
("Stretch by 3x")

3) Horizontal Shift ($C = -60^\circ$): $y = 3\sin(\theta + 60^\circ)$
("Shift 60 degrees to the left")

4) Vertical Shift ($D = -2$) : $y = 3\sin(\theta + 60^\circ) - 2$
("Shift down 2 units")

To check your sketch, plug a few points into the equation:

$y = 3\sin(\theta + 60^\circ) - 2$

$\theta = -60^\circ$ $\quad 3\sin(-60 + 60) - 2 = 3\sin(0) - 2 = -2$ ✓

$\theta = 30^\circ$ $\quad 3\sin(30 + 60) - 2 = 3\sin(90) - 2 = 1$ ✓

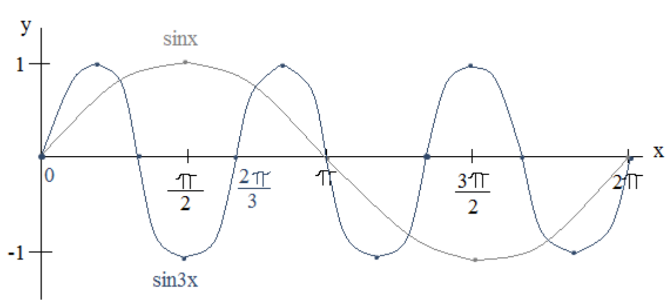
$\theta = 210^\circ$ $\quad 3\sin(210 + 60) - 2 = 3\sin(270) - 2 = -5$ ✓

Period: Horizontal distance required for a periodic function to complete one cycle.

$y = \sin Bx \longrightarrow \text{period} = \dfrac{2\pi}{B}$

Example: $y = \sin 3x$ Period: $\dfrac{2\pi}{3}$

3 cycles between 0 and 2π



For the parent function $y = \sin x$ (where $B = 1$) the period is 2π

As B increases, the period decreases.
In other words, it takes less time to complete one cycle.

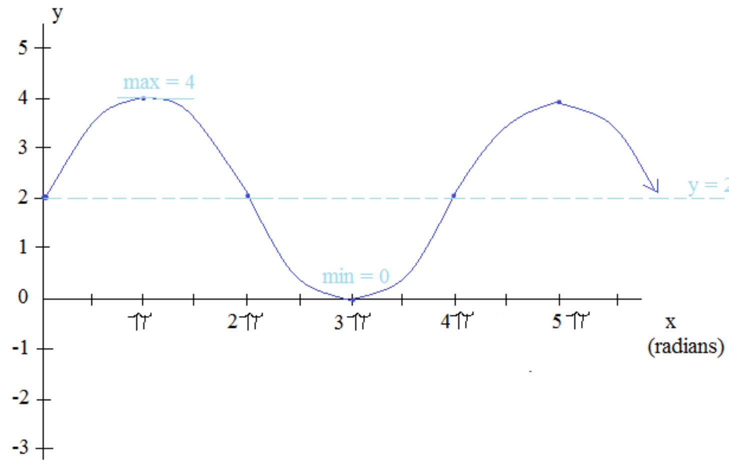
And, for $y = \sin Bx$, as B decreases, the period increases.
In other words, it takes more time to complete one cycle.

Defining Periodic Sine Graphs

$$y = A\sin B(x - C) + D$$

Write equations to describe the graphs below:

Example I:



Use the above general equation as a guide:

D (vertical shift) : The *center* that the sine wave is oscillating over is $y = 2$. Therefore, the vertical shift is up 2 units.

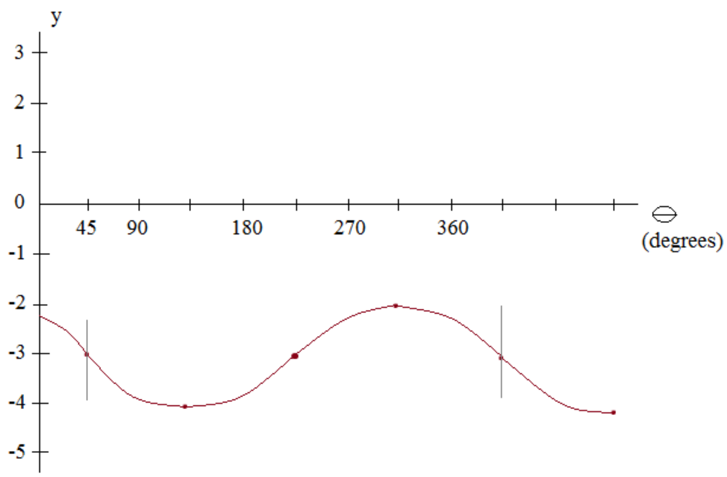
A (amplitude) : The maximum y-value is 4, and the minimum y-value is 0 --- a total span of 4 units. The amplitude is 1/2 of that amount: 2 units

C (horizontal shift) : Since $y = 2$ at 0 radians, there is no horizontal shift.

B (period) : The horizontal distance of one cycle is 4π. Since $\frac{2\pi}{B} = 4\pi$, $B = \frac{1}{2}$

$$y = 2\sin\frac{1}{2}x + 2$$

Example II:



Vertical Shift: The center of the sine wave is $y = -3$. So, $D = -3$

Amplitude: The height from max to min is 2 units. 1/2 of the height is 1

Then, because the *cycle goes down THEN up, it is negative.* $A = -1$

Horizontal Shift: At $y = -3$, the first positive θ value is 45°

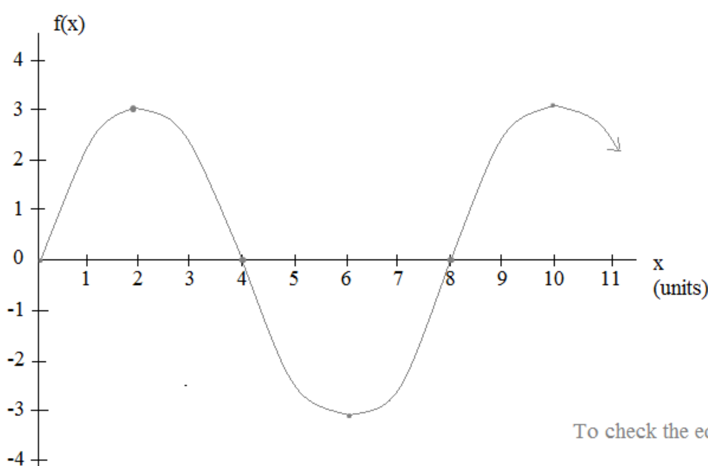
(45° shift to the right)

$C = 45^\circ$

Period: 360 degrees to complete one cycle $B = 1$

$$y = -\sin(\theta - 45^\circ) - 3$$

Example III:



Vertical Shift: None (center is $y = 0$)

Horizontal Shift: None (Cycle begins at (0, 0))

Amplitude ("Stretch"): Magnitude of 3x
$A = 3$

Period: $\frac{2\pi}{B}$ = horizontal distance of one cycle

$$\frac{2\pi}{B} = 8$$

$$B = \frac{\pi}{4}$$

$$f(x) = 3\sin\frac{\pi}{4}x$$

To check the equation, plug in a few points:

(2, 3): $f(2) = 3\sin\frac{\pi}{4}(2) = 3\sin\frac{\pi}{2} = 3$ ✓

(6, -3): $f(6) = 3\sin\frac{\pi}{4}(6) = 3\sin\frac{3\pi}{2} = -3$ ✓

Study Break:
Math Snacks

Preferable to ordinary computer cookies...

Essential part of a well-rounded, academic diet.

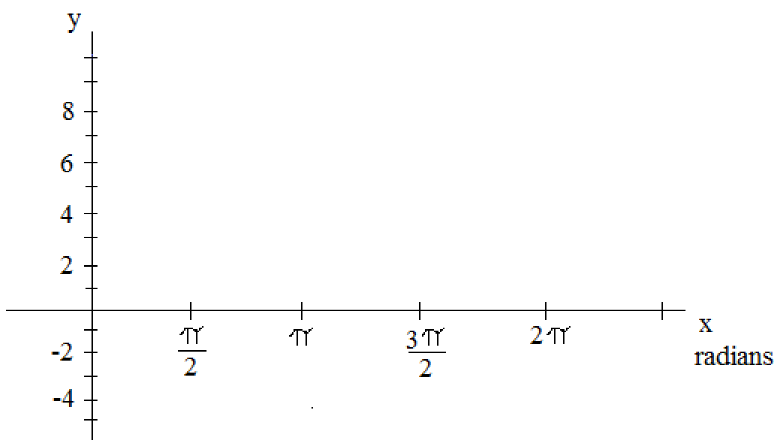
Try with (t), or any beverage...

Also, look for Honey Graham Squares
in the geometry section of your local store...

Sine Function Practice Test

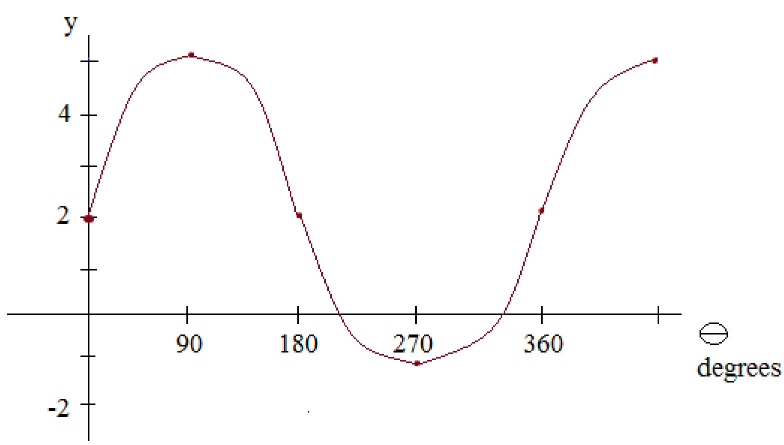
(Next page)

Graph the following function: $4\sin(x - \frac{\pi}{2}) + 3$

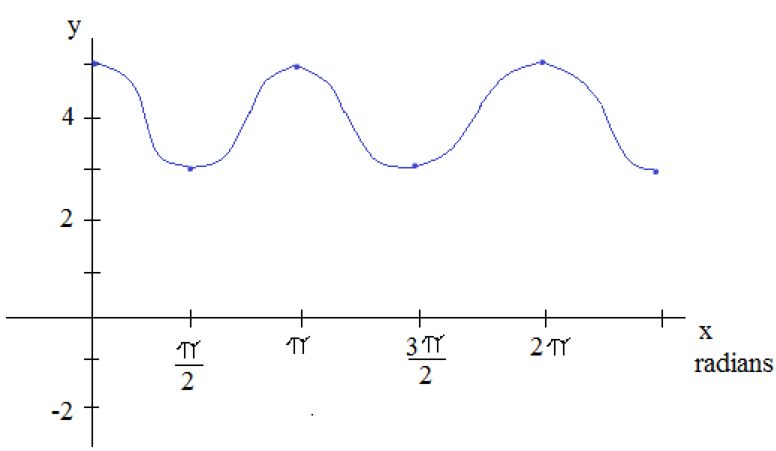


Identify the following sine functions:

1)



2)

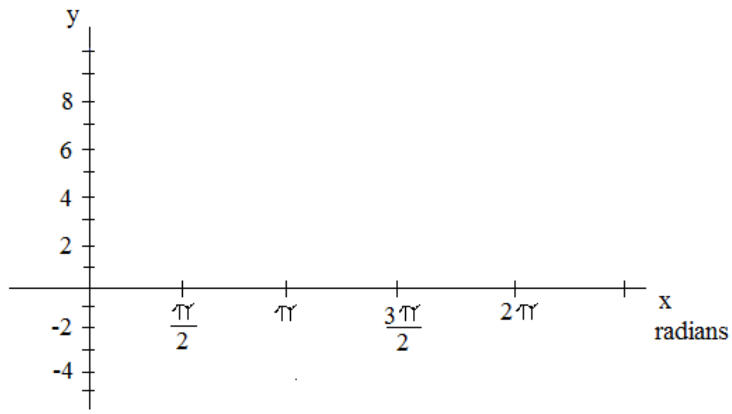


Graph the following Sine Functions. Then, use the given points to check your answers algebraically and graphically.

A) $y = -5\sin x + 3$

Check: $x = \pi$

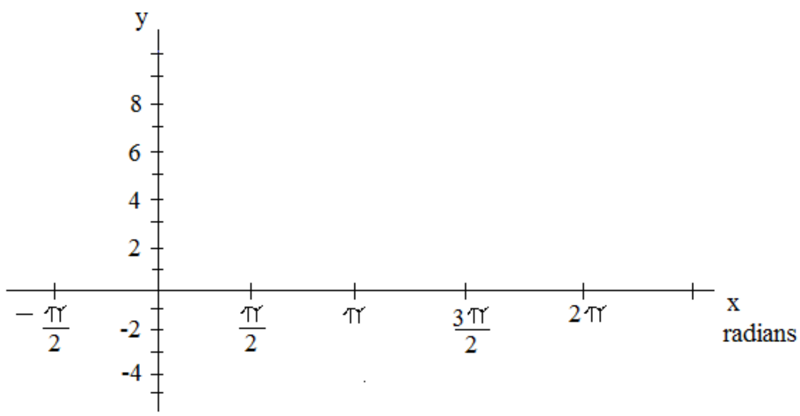
$x = \frac{3\pi}{2}$



B) $y = \sin(2x + \frac{\pi}{2})$

Check: $x = \frac{\pi}{4}$

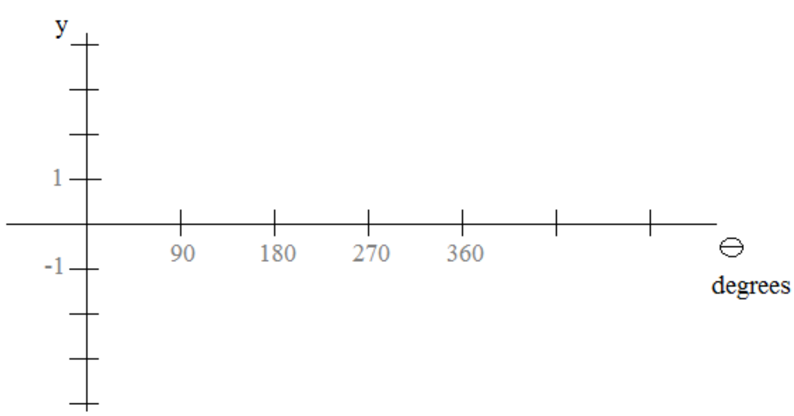
$x = \pi$



C) $y = 3|\sin\theta|$

Check: $\theta = 90^{\circ}$

$\theta = 270^{\circ}$



Characteristics of Sine Function

A) For the graph $y = \sin x$,

1) Domain:

2) Range:

3) x-intercepts:

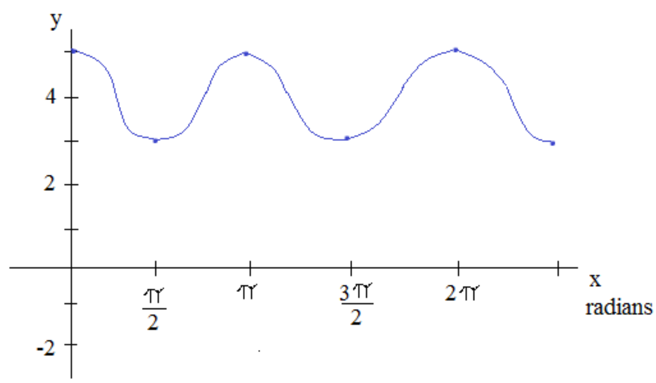
4) y-intercept:

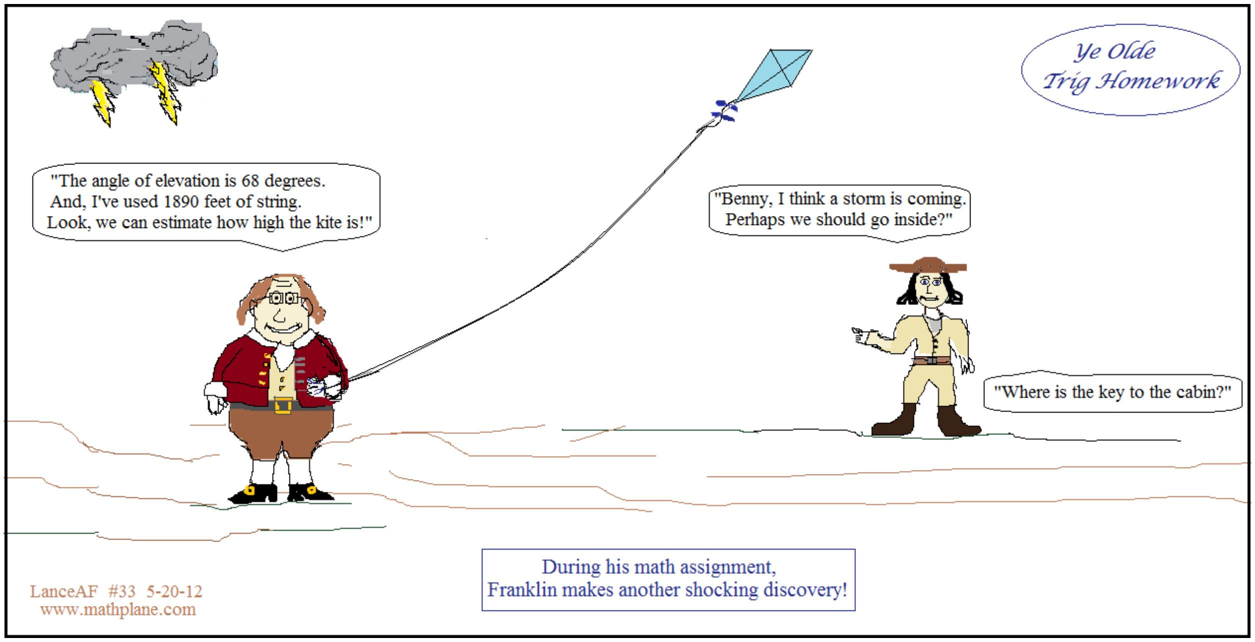
5) the graph of the function is *positive* on the intervals that correspond to which *quadrants*?

B) $f(x) = a\sin b(x - c) + d$

Write an equation where $a < 0$.

Write an equation where $a > 0$.





Practice Test SOLUTIONS

(Next page)

Graph the following function: $4\sin(x - \frac{\pi}{2}) + 3$

$y = A\sin B(x - C) + D$

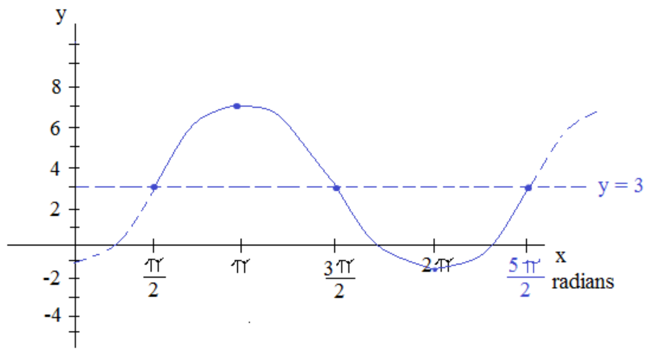
Amplitude (A) $= 4$

Period (B) $= \frac{2\pi}{1}$

1 cycle per 2π

Horizontal Shift (C) $= \frac{\pi}{2}$

Vertical Shift (D) $= 3$



The vertical shift is up 3 units. (The *center* of the sine function will be $y = 3$)

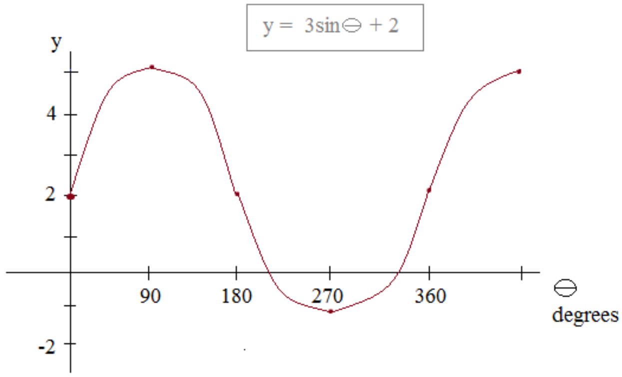
Since the amplitude is 4, the function is stretched by a factor of 4. Therefore, the maximum will be 7 (center 3 + 4) and the minimum will be -1 (center 3 − 4).

The shift is $\frac{\pi}{2}$ to the right. So, we can begin the sketch at $x = \frac{\pi}{2}$ And, the period is 2π

Identify the following sine functions:

1)

$y = 3\sin\theta + 2$



Steps: 1) Identify the *center*

Max y-value: 5 Min y-value: -1

D = 2 Midpoint is 2, so vertical shift is +2

2) Find the amplitude

A = 3 The span of the wave is 6 units (from peak to bottom). The amplitude is 1/2 that value ----> 3

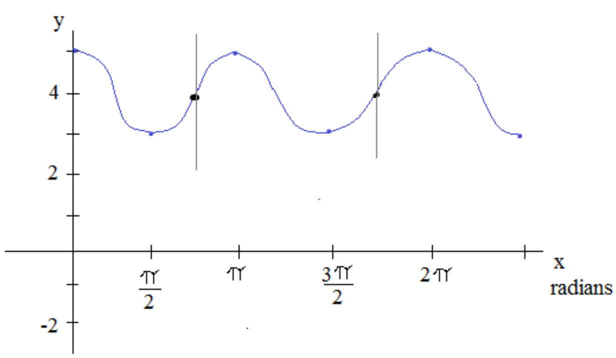
3) Horizontal shift? None, because at 0°, $y = 2$ (the center)

C = 0

4) Period? Since there is 1 cycle from 0° to 360°, the period is 360°.

B = 1

2)



$y = \sin 2(x - \frac{3\pi}{4}) + 4$

D = 4 1) Find the center: max value -- 5, min value -- 3

$y = 4$

A = 1 2) Amplitude ('stretch'): the height of the wave is 2. And, the amplitude is 1/2 that value.

$C = \frac{3\pi}{4}$ 3) Horizontal shift: $\frac{3\pi}{4}$ is a starting point of the cycle.

B = 2 4) Period: the length of one cycle is π. (there are 2 cycles every 2π)

A) $y = -5\sin x + 3$

$y = A\sin B(x - C) + D$

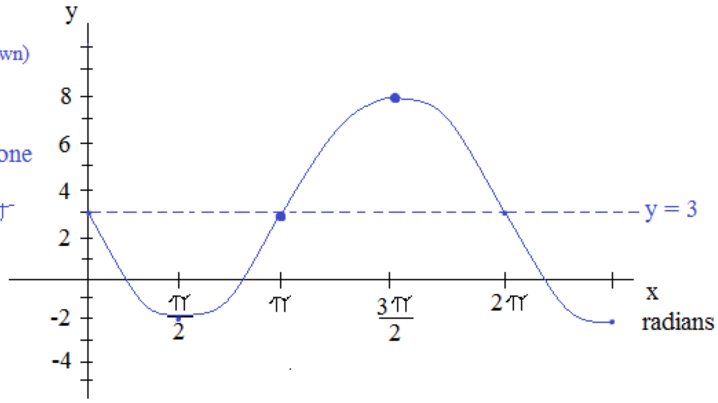
amplitude A = -5
(negative: function faces down)

vertical shift D = 3

horizontal shift C = none

period = $2\pi/B = 2\pi$

(max: 8 min: -2)



Check: $x = \pi$

$x = \frac{3\pi}{2}$

$y = -5\sin x + 3$

$y = -5\sin\pi + 3$

$y = -5(0) + 3 = 3$ ✓

$y = -5\sin\frac{3}{2}\pi + 3$

$y = -5(-1) + 3 = 8$ ✓

B) $y = \sin(2x + \frac{\pi}{2})$ ⟶ change to "standard form"
(factor out the 2)

$y = \sin 2(x + \frac{\pi}{4})$

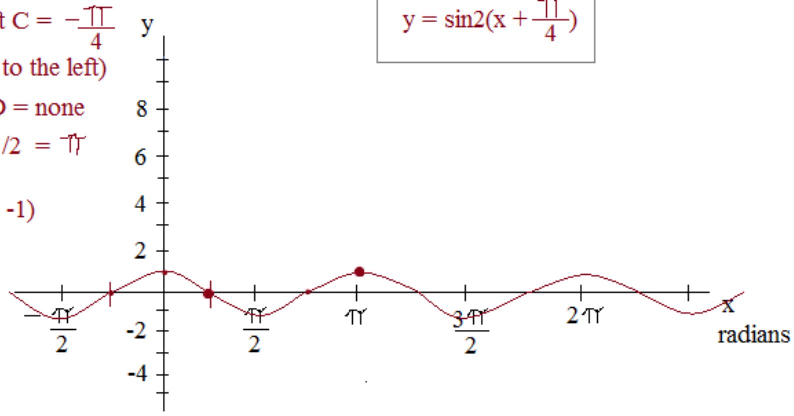
amplitude A = 1

horizontal shift C = $-\frac{\pi}{4}$
(shift to the left)

vertical shift D = none

period = $2\pi/2 = \pi$

(max: 1 min: -1)



Check: $x = \frac{\pi}{4}$

$x = \pi$

$y = \sin(2x + \frac{\pi}{2})$

$y = \sin(2\cdot\frac{\pi}{4} + \frac{\pi}{2})$

$= \sin(4\pi/4) = 0$ ✓

$y = \sin(2\cdot\pi + \frac{\pi}{2})$

$= \sin(5\pi/2) = 1$ ✓

C) $y = 3|\sin\theta|$

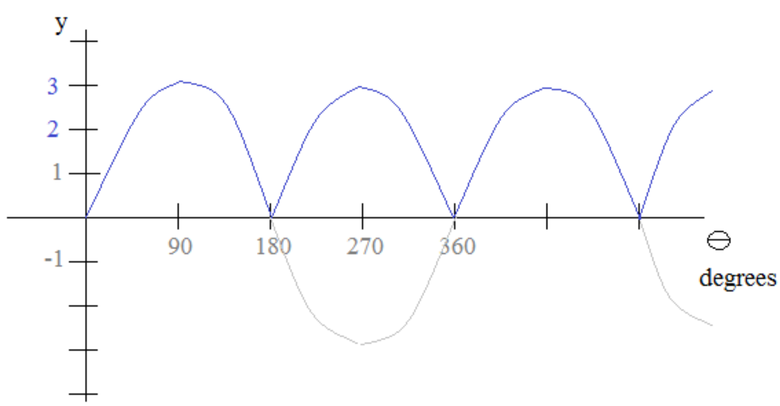
amplitude A = 3

period: 180 degrees

vertical shift: none

horizontal shift: none

(max: 3 min: 0)



Check: $\theta = 90^\circ$

$\theta = 270^\circ$

$y = 3|\sin\theta|$

$y = 3|\sin 90|$

$= 3|1| = 3$ ✓

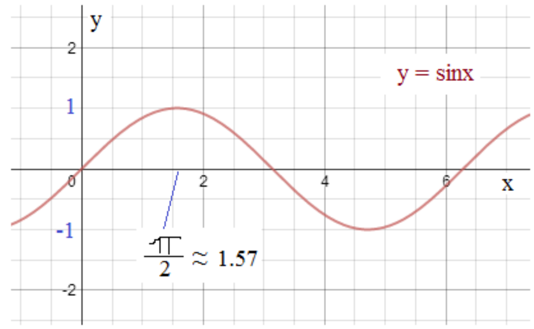
$y = 3|\sin 270|$

$= 3|-1| = 3$ ✓

Characteristics of Sine Function

SOLUTIONS

A) For the graph $y = \sin x$,



1) Domain: all the x-values: all Real Numbers

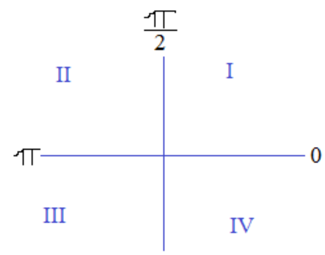
2) Range: all the y-values: [-1, 1] or $-1 \leq y \leq 1$

3) x-intercepts: The points where the function crosses the x-axis: $(\pi k, 0)$ where k is any integer

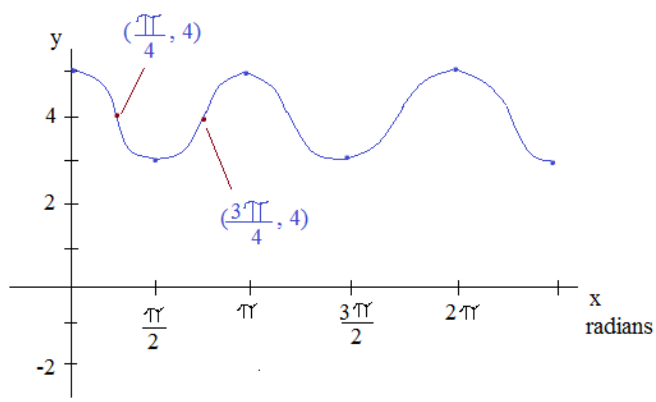
4) y-intercept: The point where the function crosses the y-axis: (0, 0)

5) the graph of the function is *positive* on the intervals that correspond to which *quadrants*?

Sine is positive in quadrants I and II



B) $f(x) = a\sin b(x - c) + d$



Write an equation where a < 0.

If a < 0, then sin function will go down first... Here is one possibility...

$$f(x) = -1\sin 2(x - \frac{\pi}{4}) + 4$$

Write an equation where a > 0.

If a > 0, then sin function will go up first.

Here is one possibility...

$$f(x) = 1\sin 2(x - \frac{3\pi}{4}) + 4$$

(To check equations, test points on the graph)

amplitude is 1 ---- 'a' can be 1 or -1

vertical shift is up 4 --- 'd' will be +4

period is π --- 'b' will be 2

***the horizontal shift will correspond to where the graph starts

Use the following points to write and graph a sinusoidal model.

Identifying Cosine and Sine Functions

maximum $(0, 10)$ and minimum $(2\pi, 0)$

maximum $(\pi, 4)$ and minimum $(0, -2)$

maximum $(\frac{\pi}{4}, 8)$ and minimum $(\frac{\pi}{2}, 2)$

maximum $(2, 22)$ and minimum $(8, 14)$

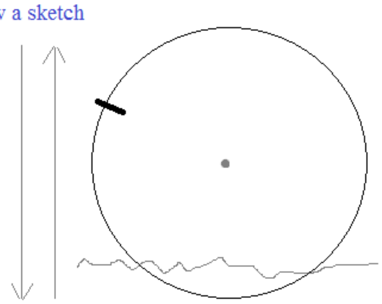
The following trig function models the position of a rung on a waterwheel:

$$y = -20\sin\left(\frac{\pi}{6}t\right) + 16$$

where t = seconds
y = number of feet *above* water level

a) What is the diameter of the wheel?
b) At the top of the wheel, how high is the rung above water level?
c) How many rotations per minute does the wheel make?
d) What percentage of time does a rung spend under water?

Step 1: Draw a sketch



Step 2: Identify the measurements

amplitude: 20 feet (distance from middle to peak)

vertical shift: up 16 feet (position of sine *wave center*)

horizontal shift: None

period: $\frac{2\pi}{B} = \frac{2\pi}{\pi/6} = 12$ seconds

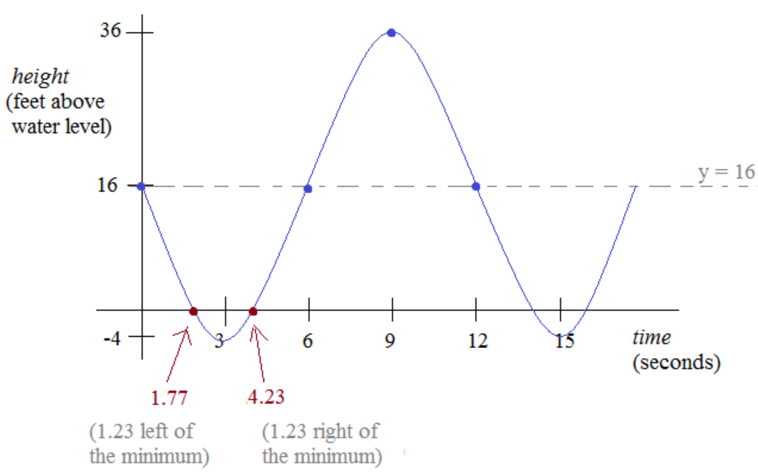
Step 3: Answer the questions

a) Since the amplitude is 20 feet, the **diameter of the wheel is 40 feet**

b) Since the vertical shift is up 16 feet, the new 'wave center' is y = 16.
Therefore, the top of the waterwheel is 16 + 20 = **36 feet above the water level.**
(and, the bottom is -4 feet or, 4 feet under water.)

c) The period (one rotation) is 12 seconds. Therefore, the **wheel rotates 5 times per minute.**

d) To determine when the rung is under water, let's sketch the graph:



To find when the rung is at water level, let y = 0

$$0 = -20\sin\left(\frac{\pi}{6}t\right) + 16$$

$$-16 = -20\sin\left(\frac{\pi}{6}t\right)$$

$$\frac{4}{5} = \sin\left(\frac{\pi}{6}t\right)$$

(radian mode) $\sin^{-1}(.80) = \left(\frac{\pi}{6}t\right)$

$.927 \approx .523t$ $\quad t \approx 1.77$

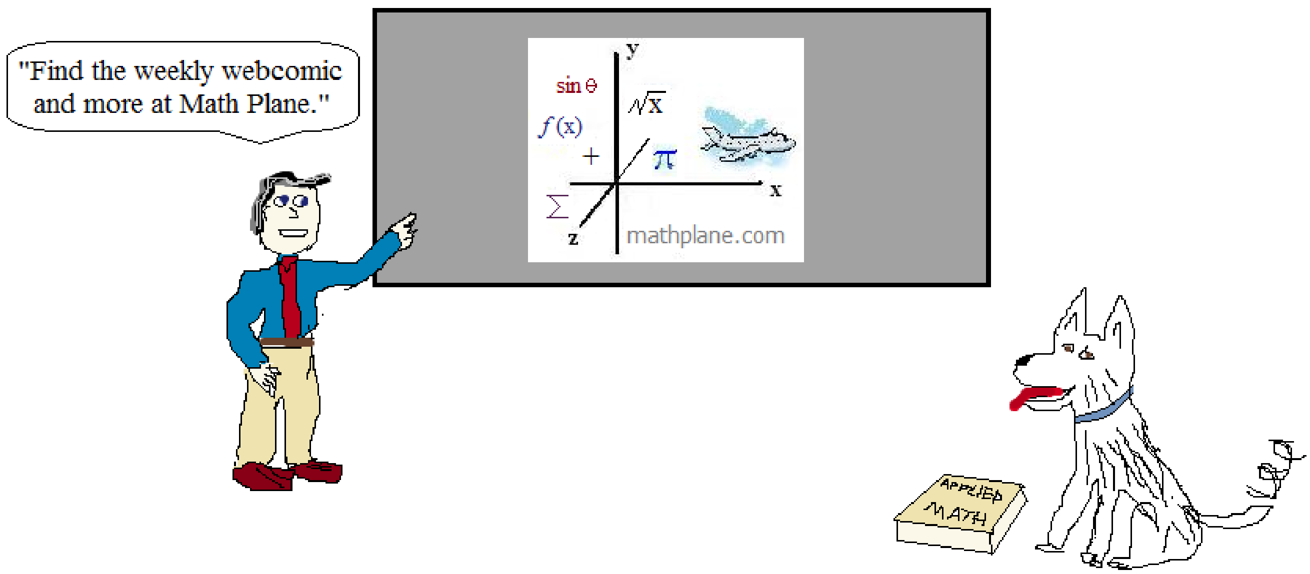
Then, the rung is under water level from 1.77 seconds to 4.23 seconds.

Therefore, every 12 seconds, the rung is under water approximately 2.46 seconds.

$\frac{2.46}{12} = .205$ or **20.5% of the time**

Thanks for checking out this *preview*.

To learn about Cosine Functions and see a few word problems, visit the trig section at mathplane.com. Or, purchase materials at the mathplane stores at TES.com and TeachersPayTeachers.com. All proceeds go to site maintenance and improvement. (Plus, treats for Oscar the dog!). We appreciate your support.



Are you looking for tangent and the reciprocal trig functions?

Check out "Introduction to Periodic Trig Functions 2"

At TeachersPayTeachers, TES, or the mathplane site