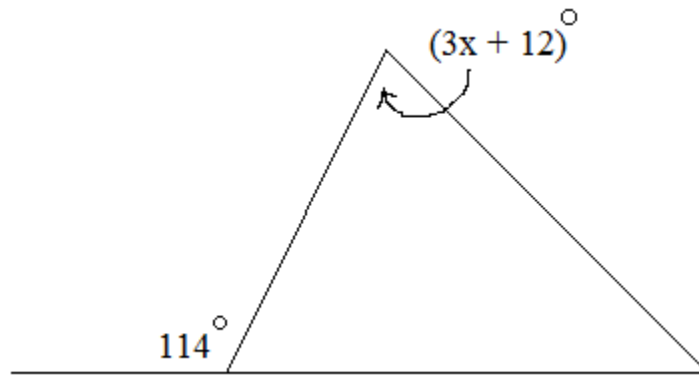
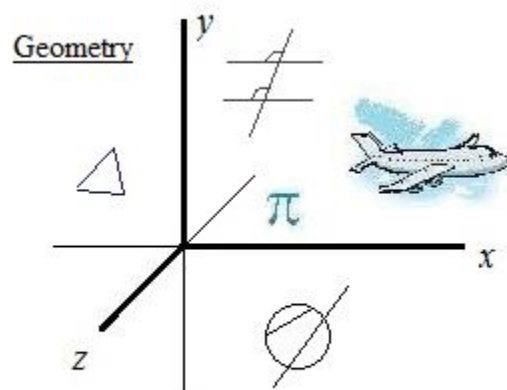


# Geometry Review Test 004

(With Solutions)



*Topics include triangle properties, 2-column proofs, parallel lines, translation & rotation, Always/Sometimes/Never, Quadrilaterals, circles, and more.*



Geometry Review Practice

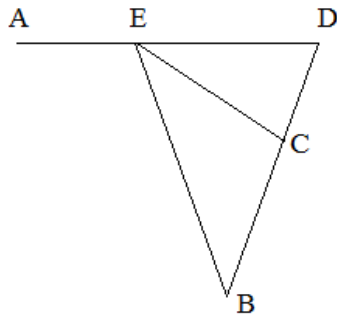
1) Answer the following:

a)  $\vec{CE} \cup \vec{CD}$

b)  $\overleftrightarrow{BC} \cap \overleftrightarrow{ED}$

c)  $\overline{BC} \cap \overline{ED}$

d)  $\vec{AE} \cap \overline{DE}$

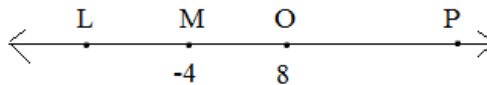


2) If one of 2 supplementary angles is 16 degrees smaller than 3 times the other, then what are the angles?

3) If the perimeter of an equilateral triangle is  $6y + 18$  and one side is  $4y - 14$ , what is the perimeter?

4) The perimeter of a rectangle is 20.  
If the rectangle's length is  $< 4$ , what is the range of possible values of the width?

5) M is the midpoint of  $\overline{LO}$ .  
The ratio of  $LM$  to  $MP$  is 2:5.  
What are L and P?



6) Degrees/Minutes/Seconds (DMS) and Decimal Degrees (DD)

a) Convert  $38\frac{7}{8}^\circ$  into DMS

b) Convert  $118^\circ 12'$  into DD

c)  $90^\circ - 37^\circ 16'' =$

7) For the given conditional statement, "If it is 20 degrees, then it is cold outside."  
State the following and determine if it is false.

a) Inverse

b) Converse

c) Contrapositive

8) Identify the point following the given translation, reflection or rotation

Start point: (4, 3)

*Example:* Reflect over the y-axis.

Answer: (-4, 3)

a) Reflect over the x-axis

b) Translate with the given vector  $\langle 1, -3 \rangle$

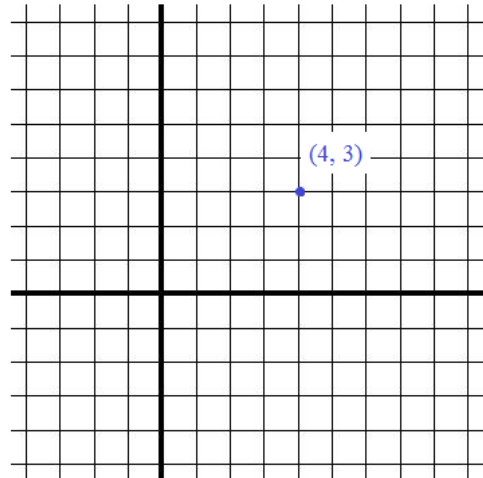
c) Reflect over the origin

d) Rotate 90 degrees clockwise around the origin

e) Reflect over  $x = 3$

f) Rotate 180 degrees around the origin

g) Rotate 90 degrees counter-clockwise around (6, 6)



9) Always/Sometimes/Never? (ASN)

a) Two parallel lines determine a plane.

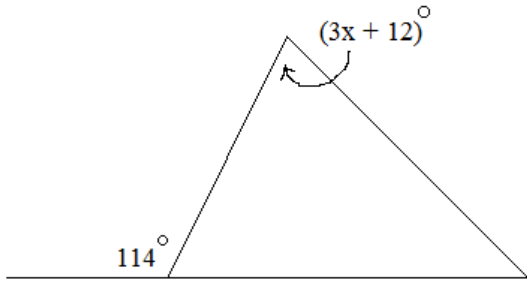
b) Planes that contain skew lines are parallel.

c) Supplements of complementary angles are congruent.

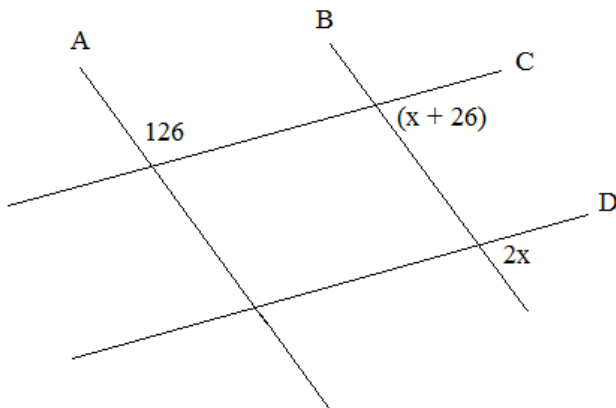
d) If one of the diagonals of a quadrilateral is the perpendicular bisector of the other, then it is a kite.

e) Two triangles are congruent if 2 sides and 1 angle are congruent to corresponding parts of another.

10) What are the restrictions of  $x$ ?



11) If  $C$  and  $D$  are parallel, are  $A$  and  $B$  parallel?



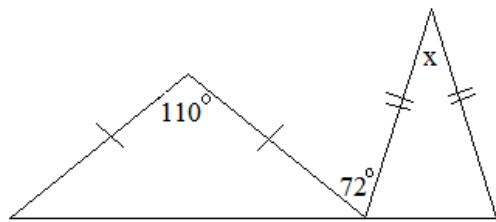
12) If quadrilateral  $PLAY$  has angles

- P: 59 degrees
- L: 37 degrees
- A: 143 degrees
- Y: 121 degrees

Which sides are parallel?

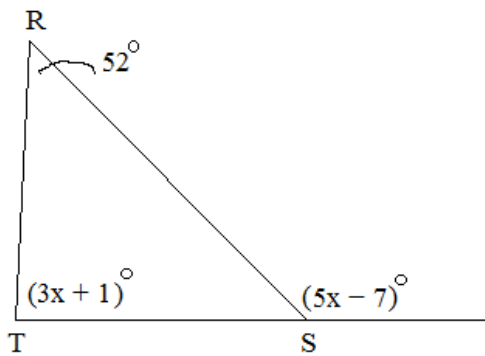
(Optional: Sketch the figure.)

13)



Find  $x$ :

14)

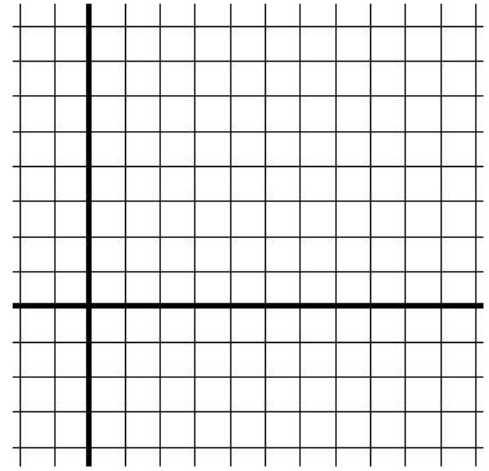


What type of triangle is  $\triangle RST$  ?

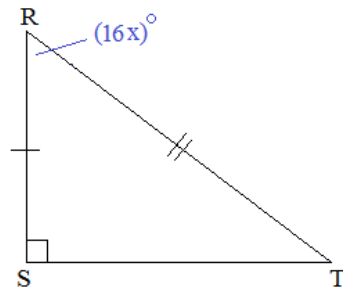
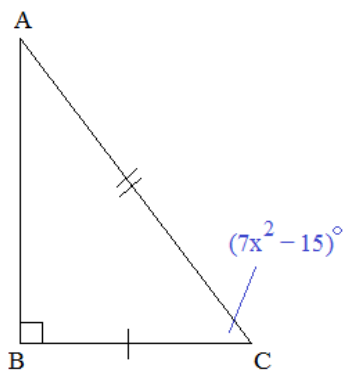
- 15)  $\triangle ABC$  is an isosceles right triangle  
 $M$  is the midpoint of the hypotenuse  $\overline{AB}$

Prove  $\overline{CM}$  is perpendicular to  $\overline{AB}$   
(using a coordinate proof)

16) Identify the quadrilateral QUAD where  $Q(0, 0)$   $U(2, 4)$   $A(8, 1)$   $D(6, -3)$



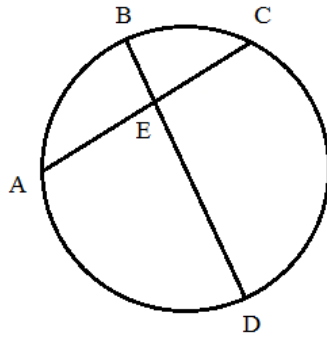
17) Given: The Diagram  
Find: measure  $\angle A$



18) If two sides of a triangle measure 11 and 16, what is the *possible* length of the 3rd side?

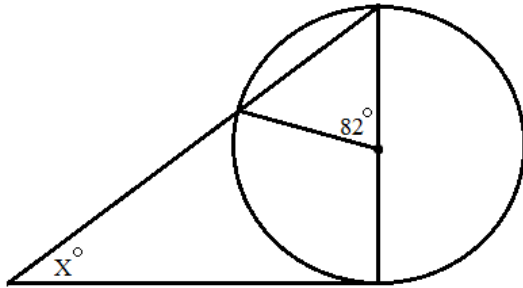
- 19)  $AE = 6$   
 $BE = 4$   
 $CE = 8$

What is  $BD$ ?

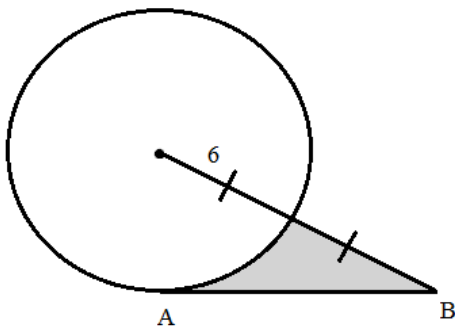


- 20) Find the radius of a circle where a chord is 24" and 9" from the center.

- 21) Find X:



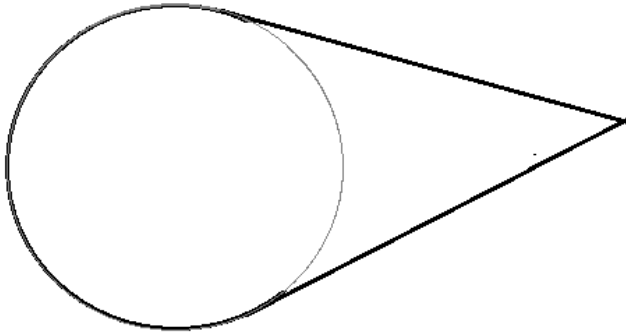
- 22) What is the shaded area?



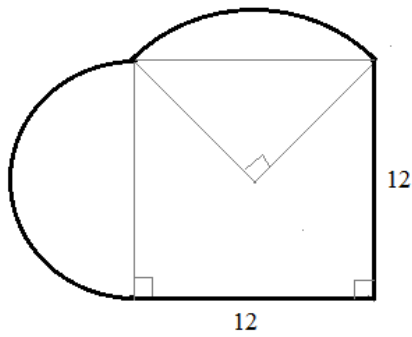
Segment  $\overline{AB}$  is tangent to the circle.  
 The radius of the circle is 6 cm.

23) Find the perimeter of each (bold) figure:

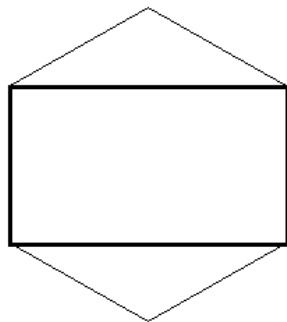
- a) Minor arc intersected by tangents is 120 degrees.  
 Radius of circle is 12.



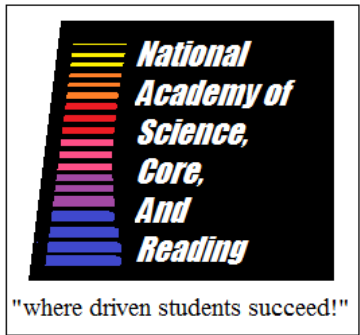
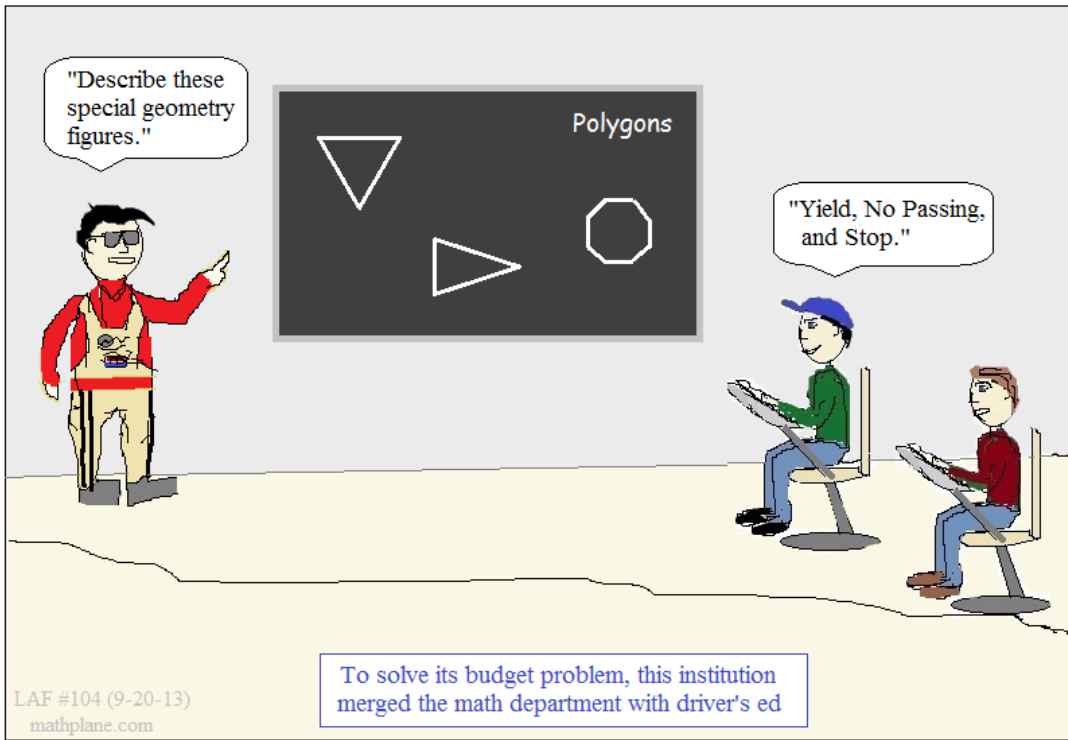
b)



- c) The hexagon is regular with sides 7.  
 What is the rectangle?





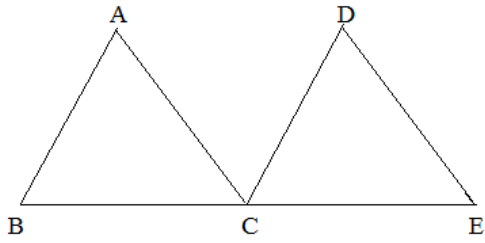


# Geometry Proofs

Geometry Proofs Review

- 1) Given:  $\overline{AB} \parallel \overline{CD}$  ;  $\overline{AB} \cong \overline{CD}$   
 C is the midpoint of  $\overline{BE}$

Prove:  $\overline{AC} \parallel \overline{DE}$

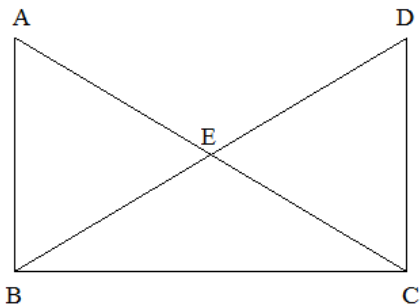


Statements

Reasons

- 2) Given:  $\overline{AB} \perp \overline{BC}$  ;  $\overline{DC} \perp \overline{BC}$   
 $\overline{AC} \cong \overline{DB}$

Prove:  $\overline{AE} \cong \overline{DE}$



Statements

Reasons

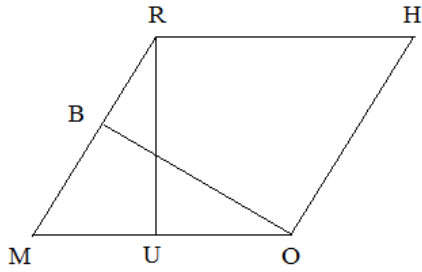
Geometry Proofs Review

3) Given: RHOM is a rhombus

$$\overline{OB} \perp \overline{RM}$$

$$\overline{RU} \perp \overline{MO}$$

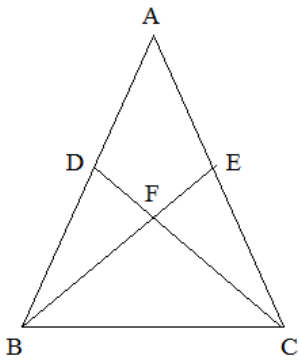
Prove:  $\overline{OB} \cong \overline{RU}$



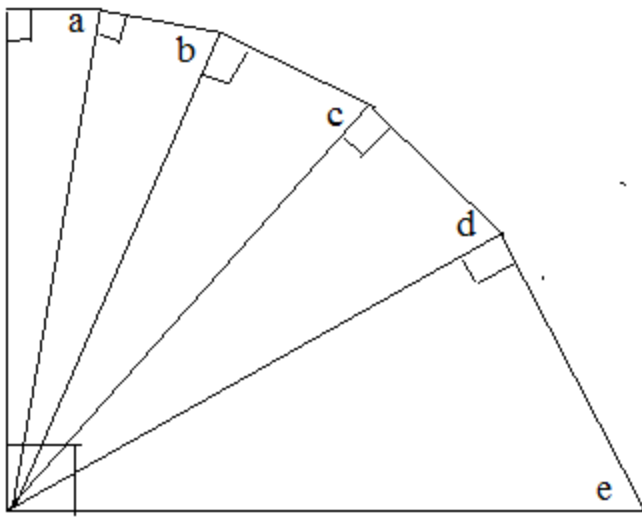
Statements	Reasons

4) Given:  $\overline{AB} \cong \overline{AC}$ ;  $\overline{AD} \cong \overline{AE}$

Prove:  $\triangle FBC$  is isosceles



Statements	Reasons



$$a + b + c + d + e = ?$$

My Fair (Math) Lady

Somewhere in Spain...

LanceAF #73 2-22-13  
www.mathplane.com

PICKERING MATH ACADEMY

".. the ray  $n$  stays mainly in  $d$  plane..."

Professor Higgins

"By George, she's got it!"

"Indeed!"

Eliza Doolittle learns Geometry

**SOLUTIONS-→**

Geometry Review Practice

SOLUTIONS

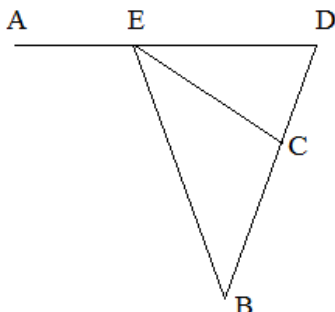
1) Answer the following:

a)  $\vec{CE} \cup \vec{CD}$   $\angle ECD$

b)  $\overleftrightarrow{BC} \cap \overleftrightarrow{ED}$  point D

c)  $\overline{BC} \cap \overline{ED}$   $\emptyset$

d)  $\vec{AE} \cap \vec{DE}$   $\overline{DE}$



2) If one of 2 supplementary angles is 16 degrees smaller than 3 times the other, then what are the angles?

$$16 + (180 - x) = 3x$$

$$196 = 4x$$

$$x = 49$$

$49^\circ$  and  $131^\circ$

3) If the perimeter of an equilateral triangle is  $6y + 18$  and one side is  $4y - 14$ , what is the perimeter?

all sides equal in an equilateral triangle  $3(4y - 14) = 6y + 18$

$$12y - 42 = 6y + 18$$

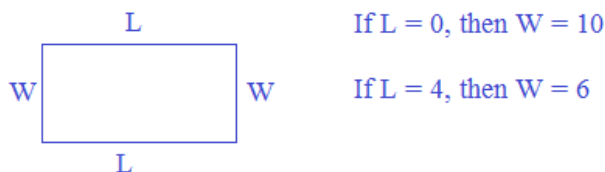
$$6y = 60$$

$$y = 10$$

if  $y = 10$ , then perimeter is  $6(10) + 18 = 78$

4) The perimeter of a rectangle is 20. If the rectangle's length is  $< 4$ , what is the range of possible values of the width?

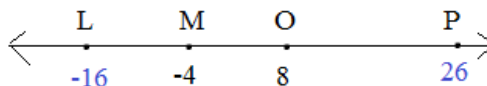
answer:  $6 < W < 10$



5) M is the midpoint of  $\overline{LO}$ . The ratio of LM to MP is 2:5. What are L and P?

M is 12 units from O, so M is 12 units from L... midpoint

$L = -16$



Then,  $\overline{LM} = 12$ , so  $\overline{MP} = 30$  ratio

$$12:30 = 2:5$$

If  $M = -4$ , then  $P = 26$  (distance of 30)

6) Degrees/Minutes/Seconds (DMS) and Decimal Degrees (DD)

SOLUTIONS

a) Convert  $38\frac{7}{8}^\circ$  into DMS       $38.875^\circ$        $.875^\circ \times \frac{60'}{1^\circ} = 52.5'$        $.5' \times \frac{60''}{1'} = 30''$

$38^\circ 52' 30''$

b) Convert  $118^\circ 12'$  into DD       $12' \times \frac{1^\circ}{60'} = .2 \text{ degrees}$

$118.2^\circ$

c)  $90^\circ - 37^\circ 16'' = \begin{array}{r} 89^\circ 59' 60'' \\ - 37^\circ 0' 16'' \\ \hline 52^\circ 59' 44'' \end{array}$

7) For the given conditional statement, "If it is 20 degrees, then it is cold outside." State the following and determine if it is false.

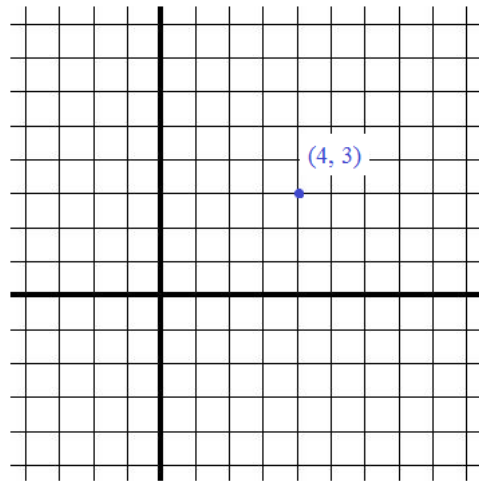
- a) Inverse "If it is cold outside, then it is 20 degrees." FALSE (it could be 22 degrees)
- b) Converse "If it is *not* 20 degrees, then it is *not* cold outside." FALSE (it could be 25 degrees and cold)
- c) Contrapositive "If it is *not* cold outside, then it is *not* 20 degrees." TRUE

8) Identify the point following the given translation, reflection or rotation

Start point: (4, 3)

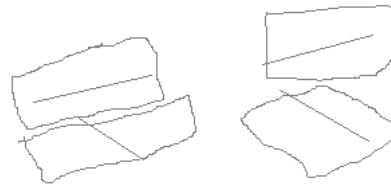
Example: Reflect over the y-axis.  
Answer: (-4, 3)

- a) Reflect over the x-axis      (4, -3)      1 right, 3 down
- b) Translate with the given vector  $\langle 1, -3 \rangle$       (5, 0)
- c) Reflect over the origin      (-4, -3)
- d) Rotate 90 degrees clockwise around the origin      (3, -4)
- e) Reflect over  $x = 3$       (2, 3)
- f) Rotate 180 degrees around the origin      (-4, -3)
- g) Rotate 90 degrees counter-clockwise around (6, 6)      (9, 4)

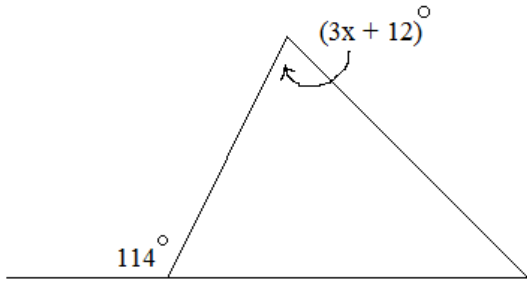


9) Always/Sometimes/Never? (ASN)

- a) Two parallel lines determine a plane. ALWAYS
- b) Planes that contain skew lines are parallel. SOMETIMES
- c) Supplements of complementary angles are congruent. SOMETIMES ( $45^\circ - 45^\circ$ )
- d) If one of the diagonals of a quadrilateral is the perpendicular bisector of the other, then it is a kite. ALWAYS
- e) Two triangles are congruent if 2 sides and 1 angle are congruent to corresponding parts of another. SOMETIMES (angle must be included or right)



10) What are the restrictions of x?



$$-4 < x < 34$$

SOLUTIONS

First restriction is "comparative"...  
(Exterior Inequality Theorem)

$$114 > (3x + 12)$$

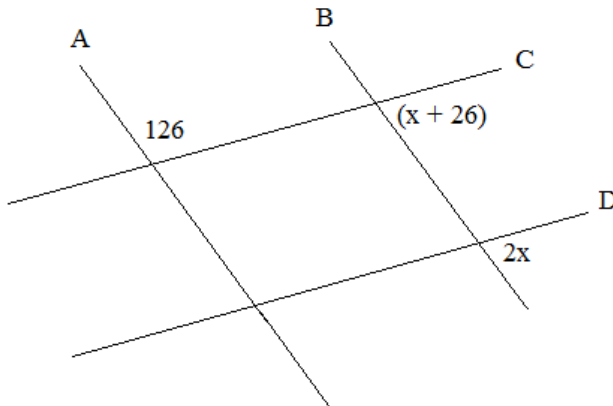
$$102 > 3x \quad x < 34$$

Second restriction is "absolute"...  
(Measure of an angle must be between 0 and 180)

$$3x + 12 > 0$$

$$3x > -12 \quad x > -4$$

11) If C and D are parallel, are A and B parallel?



Since  $C \parallel D$ , then

$$(x + 26) = 2x \quad \text{corresponding angles}$$

$$x = 26$$

Because  $x = 26$ ,

$$x + 26 = 52 \dots$$

so,  $\angle 1 = 52$  vertical angles

$$126 + 52 = 178$$

If same side interior angles  $\neq 180$ ,  
then lines A and B are NOT parallel

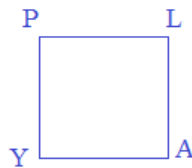
12) If quadrilateral PLAY has angles

- P: 59 degrees
- L: 37 degrees
- A: 143 degrees
- Y: 121 degrees

Which sides are parallel?

(Optional: Sketch the figure.)

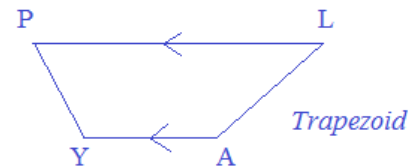
Since the quadrilateral is PLAY,  
the figure will have consecutive vertices  
P - L - A - Y



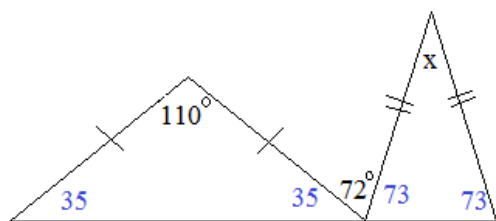
If parallel lines are cut by a transversal,  
then *same side interior angles are supplementary*.

Since  $\angle L$  and  $\angle A$  are supplementary  
and  $\angle P$  and  $\angle Y$  are supplementary

$$\overline{PL} \parallel \overline{YA}$$



13)



$$w + w + 110 = 180 \dots$$

$$x = 35$$

$$35 + 72 + y = 180$$

$$y = 73$$

Find x:  
34°

## SOLUTIONS

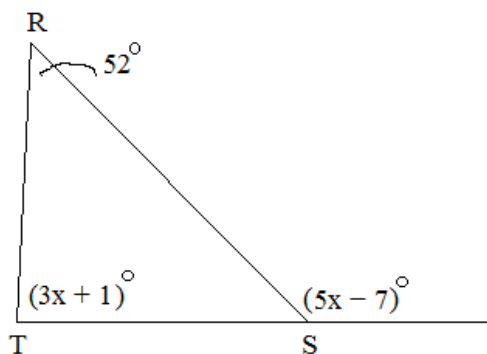
## Geometry Review Practice

- Sum of interior angles of triangle:  $180^\circ$
- If sides are congruent, then opposite angles are congruent
- Sum of angles on line is  $180^\circ$

$$73 + 73 + x = 180$$

$$x = 34$$

14)



What type of triangle is  $\triangle RST$ ?

obtuse, scalene triangle

Exterior Angle Theorem: "exterior angle equals the sum of the remote interior angles"

$$(3x + 1) + 52 = (5x - 7)$$

$$60 = 2x$$

$$x = 30$$

The angles in the triangle are  $52^\circ - 91^\circ - 37^\circ$

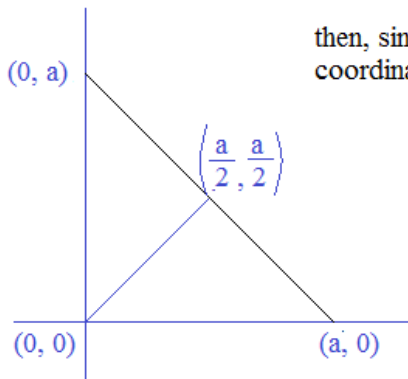
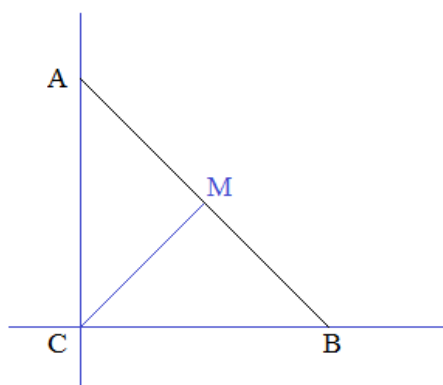
- 15)  $\triangle ABC$  is an isosceles right triangle  
M is the midpoint of the hypotenuse  $\overline{AB}$

Prove  $\overline{CM}$  is perpendicular to  $\overline{AB}$   
(using a coordinate proof)

slope of  $\overline{CM}$  must be opposite reciprocal of slope of  $\overline{AB}$

since it is an isosceles right triangle, the sides are congruent (length a)

then, since M is the midpoint, the coordinates would be  $(a/2, a/2)$



$$\text{slope of } \overline{AB} = \frac{a - 0}{0 - a} = -1$$

$$\text{slope of } \overline{CM} = \frac{a/2 - 0}{a/2 - 0} = 1$$

since slopes are opposite reciprocals, the segments are perpendicular!



16) Identify the quadrilateral QUAD where Q(0, 0) U(2, 4) A(8, 1) D(6, -3)

Step 1: Plot the points (sketch the quadrilateral)

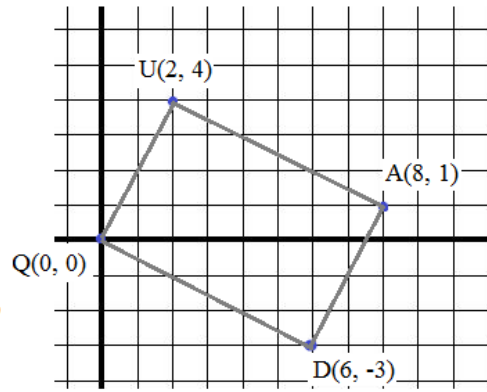
The figure appears to be a rectangle.

Step 2: Find the slopes of each side  
(to determine if any are parallel or perpendicular)

$$\text{Slope} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\begin{aligned} \overline{QU} &= 2 \\ \overline{UA} &= -1/2 \\ \overline{AD} &= 2 \\ \overline{DQ} &= -1/2 \end{aligned}$$

Since opposite sides have identical slopes, they are parallel.  
Therefore, the figure must be a parallelogram....



\*\*\*Then, since the slopes are *opposite reciprocals*, they are perpendicular as well...  
Therefore, the figure must be a rectangle...

Step 3: Check the lengths (to determine if they are congruent)

$$\text{distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\begin{aligned} \overline{QU} &= \sqrt{20} \\ \overline{UA} &= \sqrt{45} \\ \overline{AD} &= \sqrt{20} \\ \overline{DQ} &= \sqrt{45} \end{aligned}$$

Opposite sides are congruent, but not all sides are congruent... So, the figure is a rectangle (but not a square)

17) Given: The Diagram

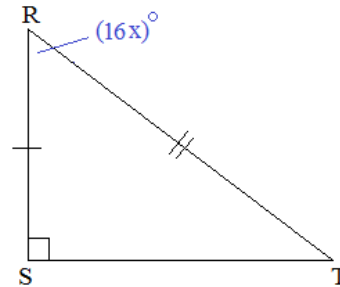
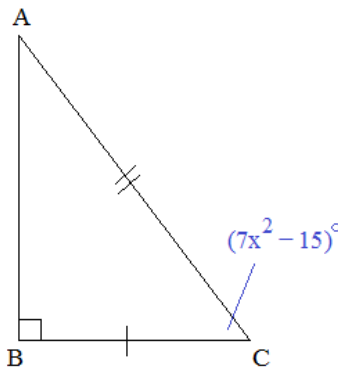
Find: measure  $\angle A$

$$\triangle ABC \cong \triangle TSR$$

(HL - Hypotenuse-Leg Theorem)

$$\text{Then, } \angle R = \angle C$$

CPCTC - Corresponding Parts Congruent  
Triangles are Congruent



$$7x^2 - 15 = 16x$$

$$7x^2 - 16x - 15 = 0$$

$$(7x + 5)(x - 3) = 0$$

$$x = 7/5, 3$$

extraneous

$$\text{If } x = \frac{7}{5}, \text{ then } 7\left(\frac{7}{5}\right)^2 - 15 = \frac{49}{25} - \frac{375}{25} = \frac{-326}{25} \text{ can't have negative angle}$$

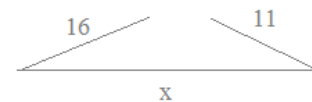
$$\text{If } x = 3, \text{ then } 7(3)^2 - 15 = 48^\circ$$

$$16(3) = 48^\circ$$

$$\text{Since angle } C = 48, \text{ then } \boxed{\text{angle } A = 42^\circ}$$

(complementary angles)

If  $x > 27$  the other sides won't touch..



18) If two sides of a triangle measure 11 and 16, what is the *possible* length of the 3rd side?

If the 3rd side is the largest, then it must be less than 27...

$$\boxed{5 < x < 27}$$

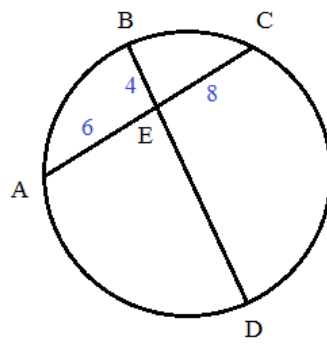
If 16 is the largest side, then the 3rd side must be greater than 5...



SOLUTIONS

- 19)  $AE = 6$   
 $BE = 4$   
 $CE = 8$

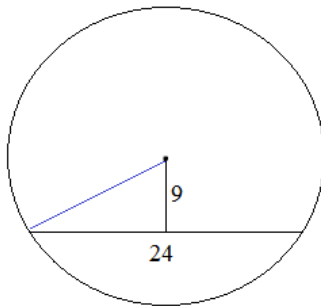
What is  $BD$ ?



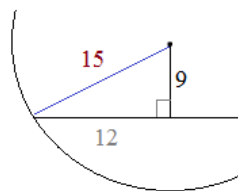
$4 \times (DE) = 6 \times 8$   
 so,  $DE = 12$

Therefore,  $BD = 16$

- 20) Find the radius of a circle where a chord is 24" and 9" from the center.



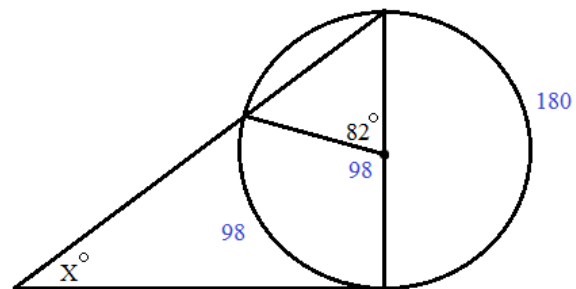
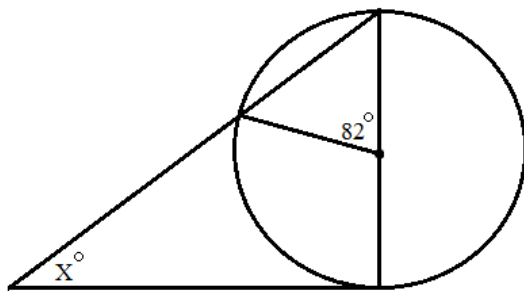
radius is perpendicular bisector of a chord..



3-4-5 or 9-12-15 right triangle..

radius is 15....

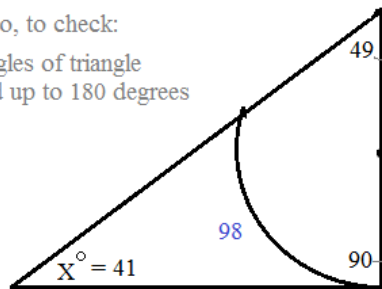
- 21) Find  $X$ :



Since part of the semicircle is 82, the other part (supplement) is 98...  
 Therefore, the arc is 98 (central angle)

Using angle-arc properties:  $\frac{1}{2} (180 - 98) = 41^\circ$

Also, to check:  
 Angles of triangle  
 add up to 180 degrees



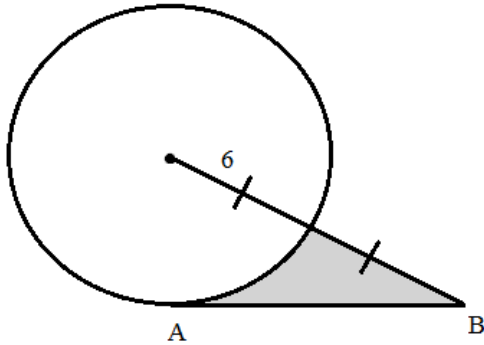
"inscribed angle is 1/2 of arc"

"tangent and radius form a right angle"

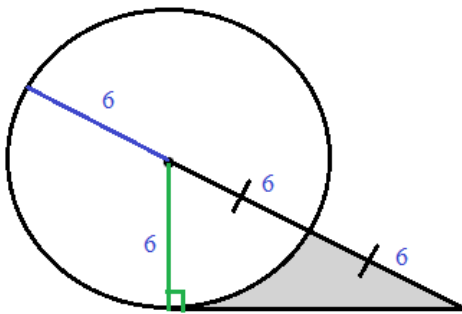
22) What is the shaded area?

SOLUTIONS

Geometry Review Practice

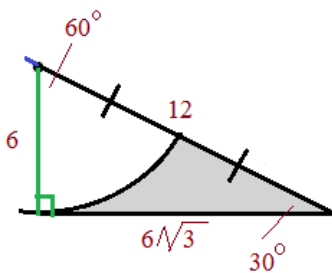


Segment  $\overline{AB}$  is tangent to the circle.  
The radius of the circle is 6 cm.



All radii are congruent...

segment from point of tangency to center is perpendicular



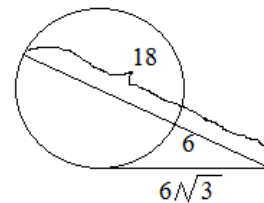
Since hypotenuse is 2x the leg, it is a 30-60-90 right triangle...

Shaded area = (Area of right triangle) - (sector area)

$$\frac{1}{2} (6\sqrt{3} \text{ cm})(6 \text{ cm}) - \frac{60}{360} \pi (6 \text{ cm})^2$$

$$18\sqrt{3} \text{ cm}^2 - 6\pi \text{ cm}^2 = \boxed{12.33 \text{ cm}^2}$$

Also, tangent-secant power theorem:

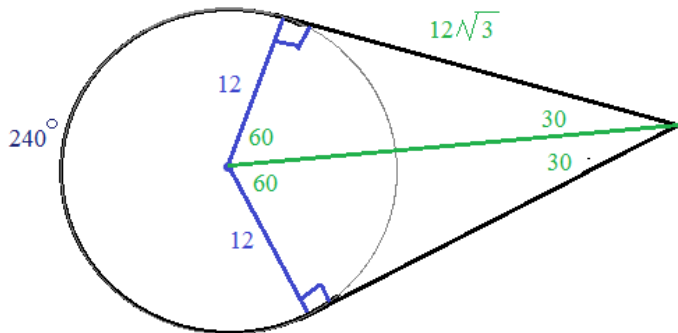


$$6 \times 18 = (6\sqrt{3})^2$$

SOLUTIONS

23) Find the perimeter of each (bold) figure:

- a) Minor arc intersected by tangents is 120 degrees.  
Radius of circle is 12.



Tangents are perpendicular to radii, and tangents that meet at point outside the circle are congruent.

Bisect the central angle ---> congruent 30-60-90 right triangles

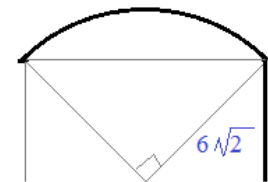
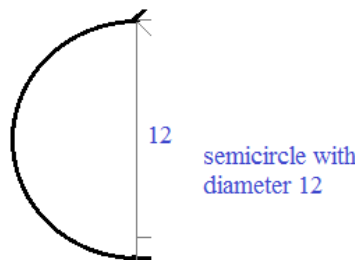
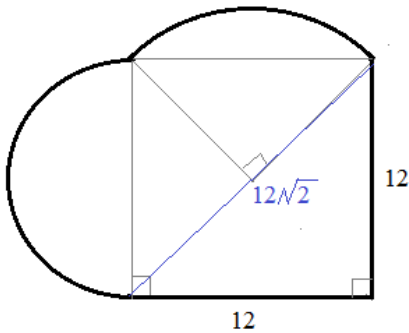
Length of each tangent is  $12\sqrt{3}$

Since minor arc is 120 degrees, the major arc is 240 degrees. And, the arc length is

$$\frac{240}{360} 2\pi(12) = 16\pi$$

$$\text{Perimeter} = 24\sqrt{3} + 16\pi$$

b)



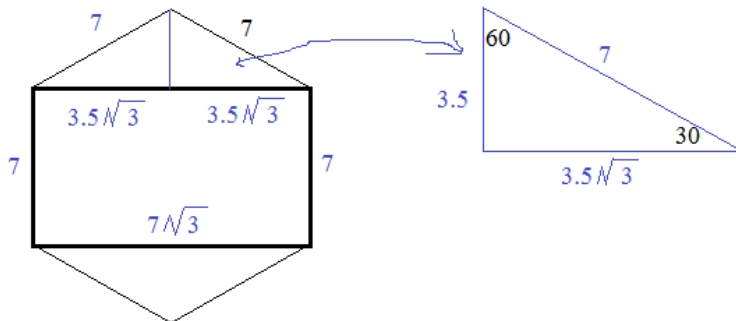
1/4 of a circle with radius  $6\sqrt{2}$

$$\frac{1}{2} \pi (\text{diameter}) = 6\pi$$

$$\frac{90}{360} \cdot 2\pi(\text{radius}) =$$

$$\text{The perimeter is } 12 + 12 + 6\pi + 3\sqrt{2}\pi = 56.2$$

- c) The hexagon is regular with sides 7.  
What is the rectangle?



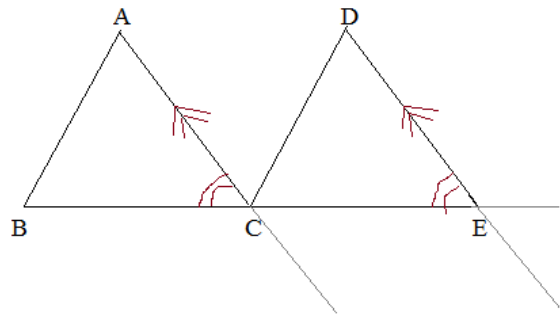
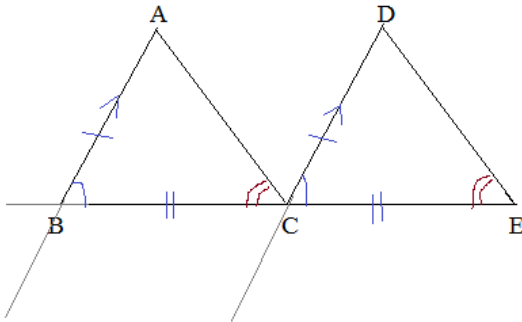
The perimeter is

$$7 + 7\sqrt{3} + 7 + 7\sqrt{3} =$$

$$14 + 14\sqrt{3}$$

- 1) Given:  $\overline{AB} \parallel \overline{CD}$  ;  $\overline{AB} \cong \overline{CD}$   
 C is the midpoint of  $\overline{BE}$

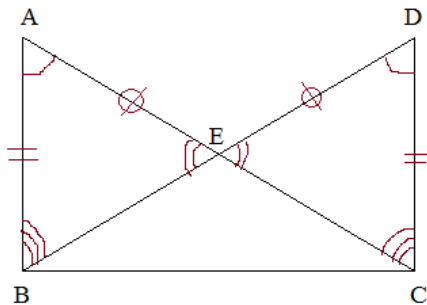
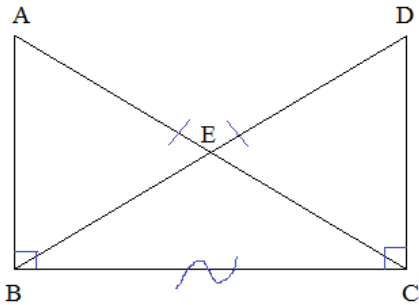
Prove:  $\overline{AC} \parallel \overline{DE}$



Statements	Reasons
1) $\overline{AB} \parallel \overline{CD}$	1) Given
2) $\angle B \cong \angle DCE$	2) If parallel lines cut by transversal then corresponding angles $\cong$
3) $\overline{AB} \cong \overline{CD}$	3) Given
4) C is midpoint of $\overline{BE}$	4) Given
5) $\overline{BC} \cong \overline{CE}$	5) Definition of midpoint
6) $\triangle ABC \cong \triangle DCE$	6) Side-Angle-Side (SAS) (3, 2, 5)
7) $\angle ACB \cong \angle E$	7) Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
8) $\overline{AC} \parallel \overline{DE}$	8) If corresponding angles are $\cong$ , then lines are parallel (converse of above theorem)

- 2) Given:  $\overline{AB} \perp \overline{BC}$  ;  $\overline{DC} \perp \overline{BC}$   
 $\overline{AC} \cong \overline{DB}$

Prove:  $\overline{AE} \cong \overline{DE}$



Statements	Reasons
1. $\overline{AB} \perp \overline{BC}$ ; $\overline{DC} \perp \overline{BC}$	1. Given
2. $\angle ABC$ and $\angle DCB$ are right angles	2. Definition of perpendicular
3. $\angle ABC \cong \angle DCB$	3. All right angles are congruent
4. $\overline{AC} \cong \overline{DB}$	4. Given
5. $\overline{BC} \cong \overline{BC}$	5. Reflexive property
6. $\triangle ABC \cong \triangle DCB$	6. HL (Hypotenuse-leg) (3, 4, 5)
7. $\overline{AB} \cong \overline{DC}$	7. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)
8. $\angle A \cong \angle D$	8. CPCTC
9. $\angle AEB \cong \angle DEC$	9. Vertical Angles Congruent
10. $\angle ABE \cong \angle DCE$	10. No Choice Theorem (If 2 angle pairs congruent, then 3rd angle pair congruent)
11. $\triangle ABE \cong \triangle DCE$	11. AAS (Angle-Angle-Side) (8, 9, 7) or, ASA (Angle-Side-Angle) (8, 7, 10)
12. $\overline{AE} \cong \overline{DE}$	12. CPCTC

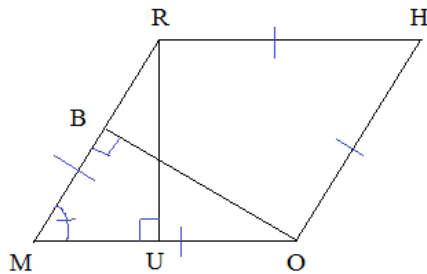
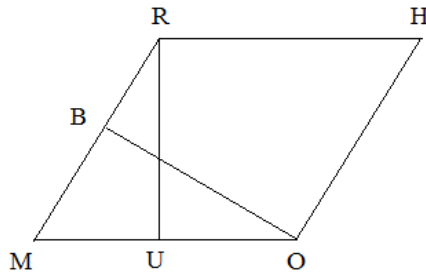
SOLUTIONS

3) Given: RHOM is a rhombus

$$\overline{OB} \perp \overline{RM}$$

$$\overline{RU} \perp \overline{MO}$$

Prove:  $\overline{OB} \cong \overline{RU}$



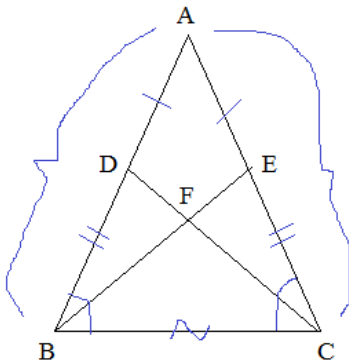
Statements	Reasons
1. RHOM is a rhombus	1. Given
2. $\overline{RM} \cong \overline{MO}$	2. Definition of Rhombus (all sides congruent)
3. $\overline{OB} \perp \overline{RM}$ $\overline{RU} \perp \overline{MO}$	3. Given
4. $\angle OBM$ and $\angle RUM$ are right angles	4. Definition of Perpendicular (perpendicular lines form right angles)
5. $\angle OBM \cong \angle RUM$	5. All right angles congruent
6. $\angle M = \angle M$	6. Reflexive property
7. $\triangle RUM = \triangle OBM$	7. AAS (Angle-Angle-Side) (5, 6, 2)
8. $\overline{OB} \cong \overline{RU}$	8. CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

Strategy: label all the given parts  
Rhombus: all sides congruent

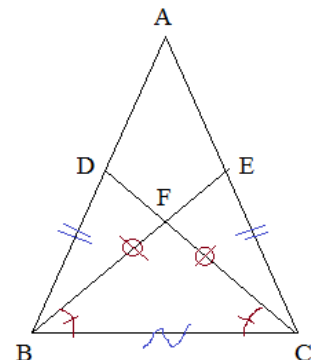
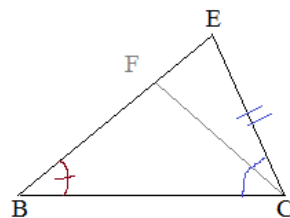
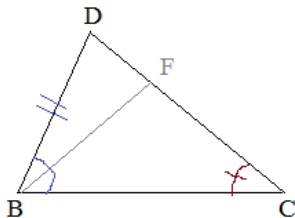
Look for congruent triangles (CPCTC)

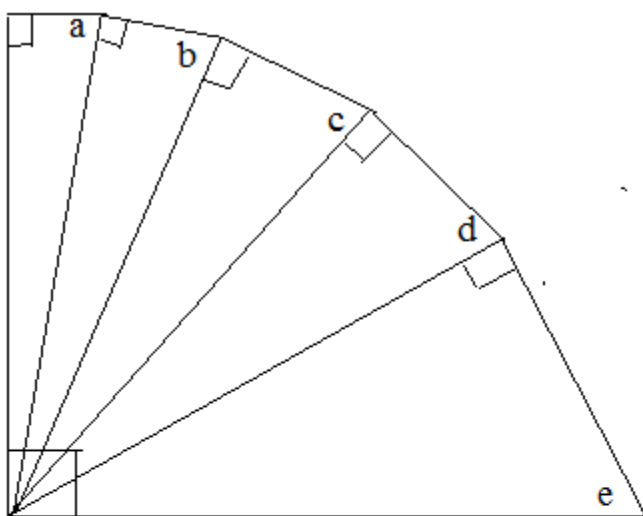
4) Given:  $\overline{AB} \cong \overline{AC}$  ;  $\overline{AD} \cong \overline{AE}$

Prove:  $\triangle FBC$  is isosceles



Statements	Reasons
1. $\overline{AB} \cong \overline{AC}$	1. Given
2. $\angle ABC \cong \angle ACB$	2. If congruent sides, then congruent angles
3. $\overline{AD} \cong \overline{AE}$	3. Given
4. $\overline{DB} \cong \overline{EC}$	4. Subtraction Property
5. $\overline{BC} = \overline{BC}$	5. Reflexive Property
6. $\triangle DBC \cong \triangle ECB$	6. Side-Angle-Side (SAS) (4, 2, 5)
7. $\angle DCB = \angle ECB$	7. CPCTC
8. $\overline{BF} \cong \overline{CF}$	8. If congruent angles, then congruent sides
9. $\triangle FBC$ is isosceles	9. Definition of Isosceles (2 or more congruent sides of triangle)





SOLUTION

$$a + b + c + d + e = ?$$

5 triangles, so the sum of the interior angles must be  $5 \times 180 = 900$

We know there are 5 (small) 90 degree angles..

And, the sum of the lower angles is one (large) 90 degree angle..

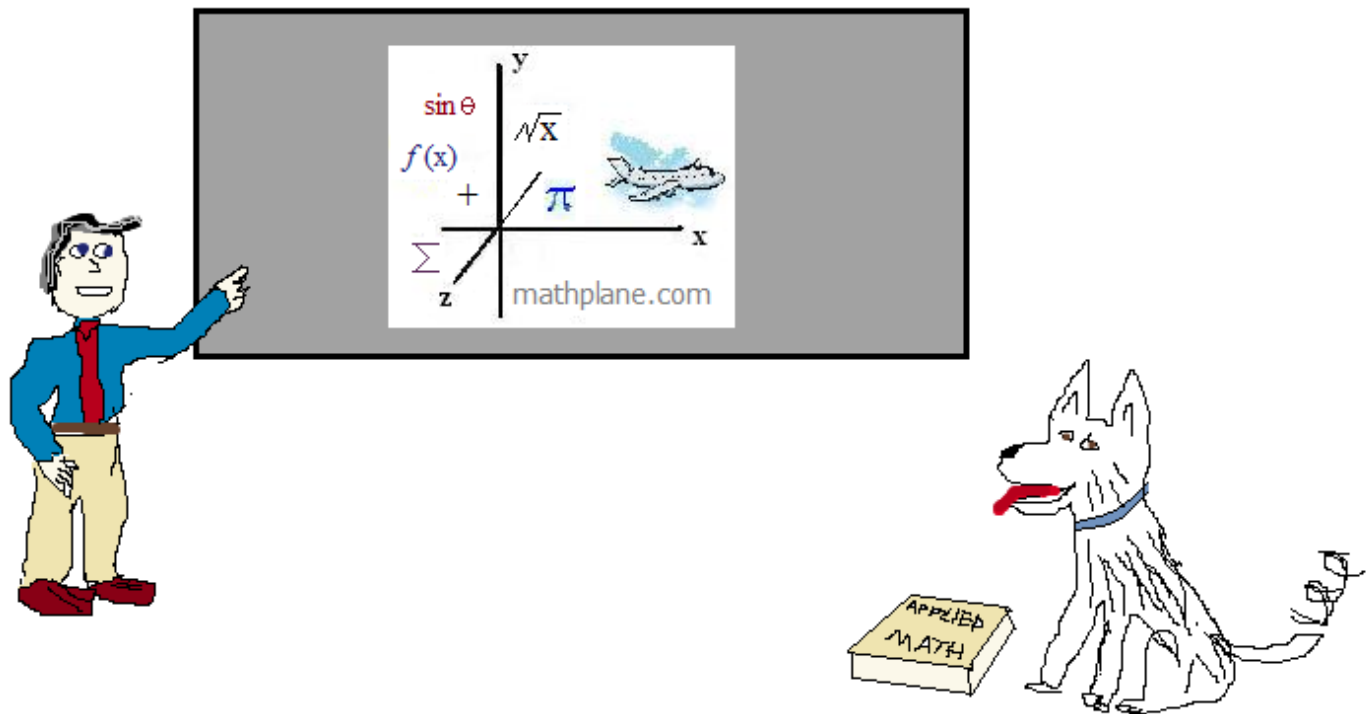
So, we know the 6 angles add up to 540...

therefore, the remaining angles are 360...

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers....



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