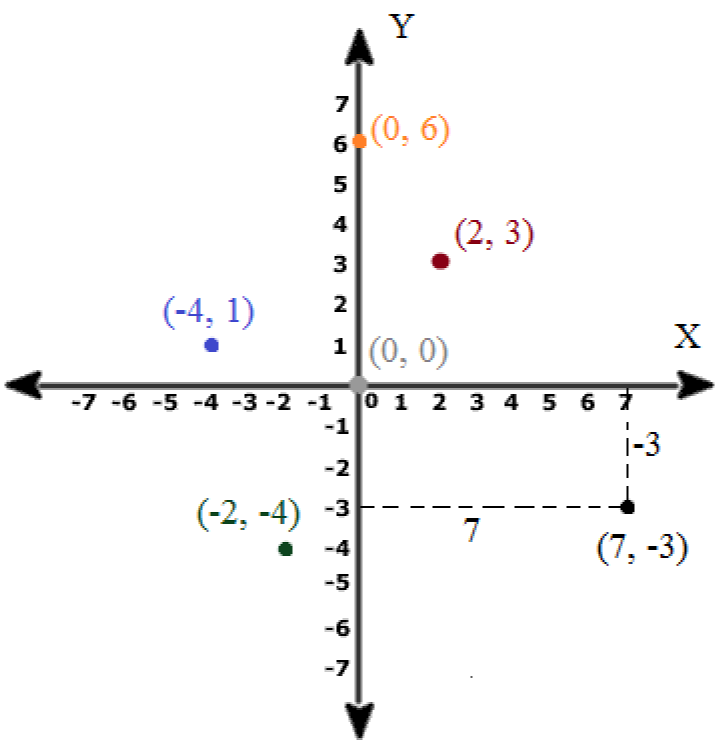
Coordinate Geometry 1

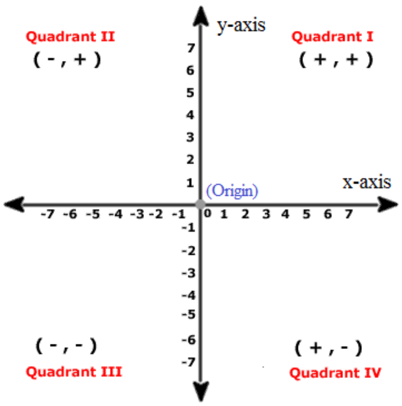
Notes and practice test (w/solutions)



Topics include quadrants, ordered pairs, slope, distance, midpoint, quadrilaterals, Pythagorean Theorem, and more…

Coordinate Geometry Topics and Notes

I. Coordinate Plane (or Cartesian Plane -- named after mathematician Rene Descarte)



- x-axis and y-axis are perpendicular

(x, y)

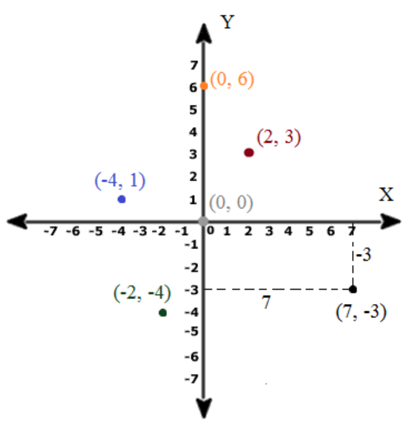
- "Left" of y-axis is negative
  "Right" of y-axis is positive

("left" is negative
"right" is positive)

(x, y)

- "Above" the x-axis is positive
  "Below" the x-axis is negative

("up" is positive
"down" is negative)



- Each point is an "ordered pair"
- Origin is (0, 0)

The first term in the ordered pair is the x value.
(*horizontal* movement from the origin)

The second term in the ordered pair is the y value.
(*vertical* movement from the origin)

II. Slope

$$\text{Slope } m = \frac{\text{"rise"}}{\text{"run"}} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_1 - y_2}{x_1 - x_2}$$

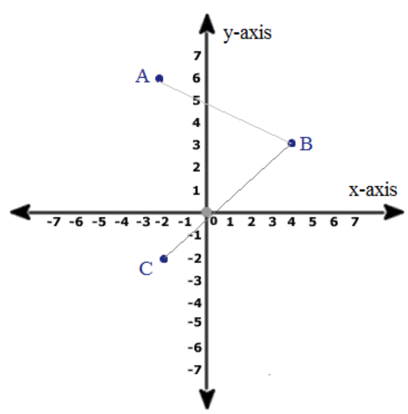
Examples: B = (4, 3) C = (-2, -2)

Slope of $\overline{BC} = \frac{3-(-2)}{4-(-2)} = \frac{5}{6}$ ("positive slope goes upward)

A = (-2, 6) B = (4, 3)

Slope of $\overline{AB} = \frac{6-3}{-2-4} = \frac{-1}{2}$ ("negative" slope goes downward)

Also, slope of $\overline{AC} = \frac{6-(-2)}{-2-(-2)} = \frac{8}{0}$ Undefined!



Vertical lines have undefined slope.
Horizontal lines have 0 slope.

Coordinate Geometry Topics and Notes

III. Linear Equations (Review)

Slope Intercept Form

$y = mx + b$

m: slope; b: y-intercept

Point Slope Form

$y - y_1 = m(x - x_1)$

(x_1, y_1): point; m: slope

Standard Form

$Ax + By = C$

where A, B, and C are integers...

note: the y-intercept b is not the same as the B coefficient of y

Horizontal line (form): $y = b$

Vertical line (form): $x = a$

Using Algebra to verify equivalent linear forms:

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$ Begin with definition of slope....

$$\frac{m}{1} = \frac{y_1 - y_2}{x_1 - x_2}$$ cross multiply...

$$y - y_1 = m(x - x_1)$$ *Point Slope Form!

$$y - b = m(x - 0)$$ substitute y-intercept (0, b)

$$y = mx + b$$ *Slope Intercept Form!

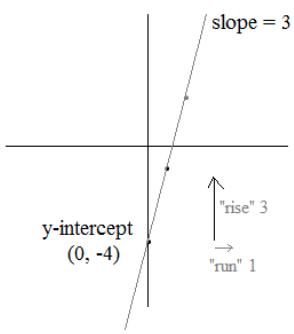
Important

Parallel lines have the *same* slope

Perpendicular lines have *negative reciprocal* slopes

Examples:

1) Graph $y = 3x - 4$



2) Is $2x + 3y = 6$ parallel to $y = \frac{-2}{3}x + 14$?

$y = \frac{-2}{3}x + 14$ slope intercept form; slope = -2/3

$2x + 3y = 6$ (change to intercept form)

$3y = -2x + 6$

$y = \frac{-2}{3}x + 3$ slope intercept form; slope = -2/3

slopes are the same! parallel lines...

3) What is the y-intercept of $4x - 3y = 12$?
What is the x-intercept?

The y-intercept is the point where the line crosses the y-axis..
Its coordinate is (0, b)

$4(0) - 3(b) = 12$ (substitute (0, b) into the equation)

$-3b = 12$

$b = -4$ $\boxed{(0, -4)}$

The x-intercept is the point where a line crosses the x-axis..
Its coordinate is (?, 0) (substitute (?, 0) into the equation)

$4(?) + 3(0) = 12$

$4(?) = 12$ $\boxed{(3, 0)}$

4) Write the equation of a line with slope 4 that passes through (3, -1).

point slope form: $y - (-1) = 4(x - 3)$

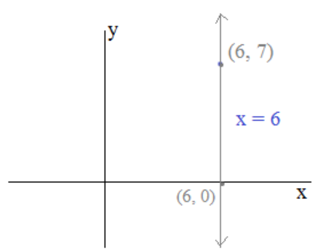
$y + 1 = 4(x - 3)$

slope intercept form: $y + 1 = 4x - 12$

$y = 4x - 13$

standard form: $4x - y = 13$

5) Write the equation of a vertical line passing through (6, 7).



6) Write the equation of a line perpendicular to $y = 3x + 5$ and passing through (2, 4)

The slope of the given line is 3....
therefore, the slope of a perpendicular line is -1/3

So, a line with slope -1/3 passing through (2, 4):

$y - 4 = -1/3(x - 2)$ (pt. slope form)

IV: Midpoint

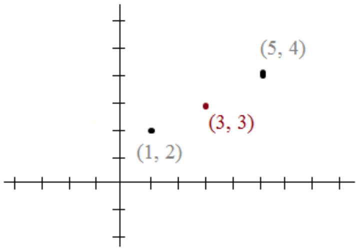
The "half-way point between two locations".
It is equidistant to each point.

The midpoint is similar to the "average"

$$\frac{P_1 + P_2}{2} = \text{Midpoint}$$

The midpoint extends to the Cartesian Plane:

Simply find the midpoint of the X values. And, the midpoint of the Y values.



The midpoint of the X Values:

$$\frac{1+5}{2} = 3$$

The midpoint of the Y Values:

$$\frac{2+4}{2} = 3$$

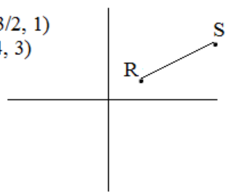
$$\left(\frac{X_1 + X_2}{2}, \frac{Y_1 + Y_2}{2} \right)$$

Midpoint Formula

Examples:

Where does the perpendicular bisector pass through $\overline{RS}$?

R = (3/2, 1)
S = (4, 3)



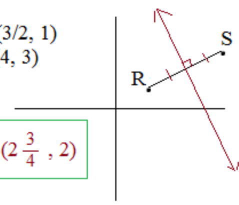
Find the midpoint of $\overline{RS}$:

X coordinate: $\frac{3/2 + 4}{2} = \frac{11/2}{2} = \frac{11}{4}$

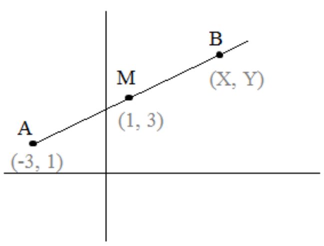
Y coordinate: $\frac{1+3}{2} = 2$

R = (3/2, 1)
S = (4, 3)

$\boxed{M = (2\tfrac{3}{4}, 2)}$



Given AB with midpoint M:
A = (-3, 1) M = (1, 3) What is B?



"Formula" Method

$$\frac{X_A + X_B}{2} = X_M \qquad \frac{Y_A + Y_B}{2} = Y_M$$

$$\frac{-3 + X_B}{2} = 1 \qquad \frac{1 + Y_B}{2} = 3$$

$X_B = 5$ $\boxed{(5, 5)}$ $Y_B = 5$

"Travel" Method

Start at the endpoint. Determine how far you "travel" to the midpoint. Then, add the same amount.

A (-3, 1) $\longrightarrow$ M (1, 3)

X value increased 4 units..
Y value increased 2 units..

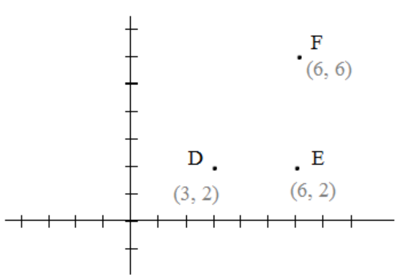
M (1, 3) $\longrightarrow$ B (1 + 4, 3 + 2)

$\boxed{(5, 5)}$

V. Distance

The space between 2 points.
The length of the line segment connecting two points.

Cartesian Plane:



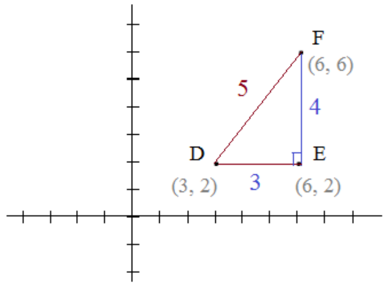
The distance between D and E is 3 units...
(3, 2), (4, 2), (5, 2), and (6, 2)

And, the distance between E and F is 4 units...
(6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

So, what is the distance between D and F?
(And, it is not 7!!)

Pythagorean Theorem

$$a^2 + b^2 = c^2$$



Notice, in this case, that the points can be vertices of a right triangle..

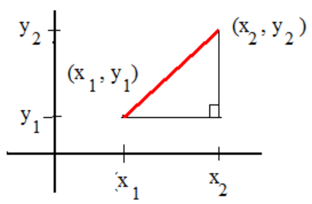
So, $\overline{DE}^2 + \overline{EF}^2 = \overline{DF}^2$

$9 + 16 = 25$

Therefore, the length of $\overline{DF}$ (i.e. distance between D and F) = 5

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Distance Formula



Find the distance between (-2, 5) and (4, 7).

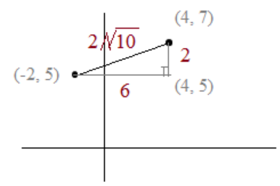
Using Distance Formula:

$$d = \sqrt{(-2-4)^2 + (5-7)^2}$$

$$= \sqrt{(-2-4)^2 + (5-7)^2}$$

$$= \sqrt{36 + 4} = 2\sqrt{10}$$

Using Pythagorean Theorem:

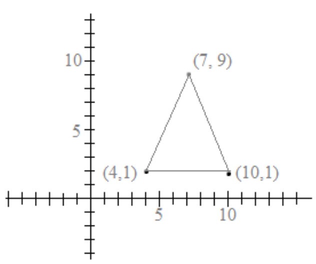


A vertical line drawn from (4, 7) intersects a horizontal line from (-2, 5) at (4, 5).. These form a right triangle!

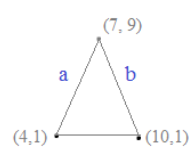
Then, using the pythagorean theorem, the hypotenuse is $2\sqrt{10}$

Examples:

Use coordinate geometry to prove the triangle is isosceles.



Def. of isosceles: triangle with 2 congruent sides.



$$a = \sqrt{(7-4)^2 + (9-1)^2}$$

$$= \sqrt{9 + 64} = \boxed{\sqrt{73}}$$

$$b = \sqrt{(7-10)^2 + (9-1)^2}$$

$$= \sqrt{9 + 64} = \boxed{\sqrt{73}}$$

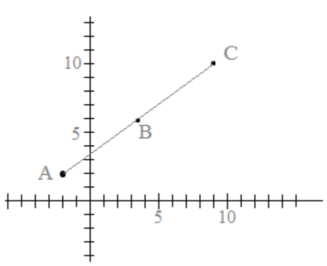
a = b, therefore the triangle is isosceles...

Verify the length of $\overline{AB}$ equals the length of $\overline{BC}$

A = (-2, 2)

B = (3.5, 6)

C = (9, 10)



Method 1: Using Midpoint

Midpoint of $\overline{AC}$

$$\left(\frac{-2+9}{2}, \frac{2+10}{2}\right)$$

$$(3.5, 6)$$

since B is the midpoint of $\overline{AC}$, $\overline{AB} = \overline{BC}$

Method 2: Using Distance

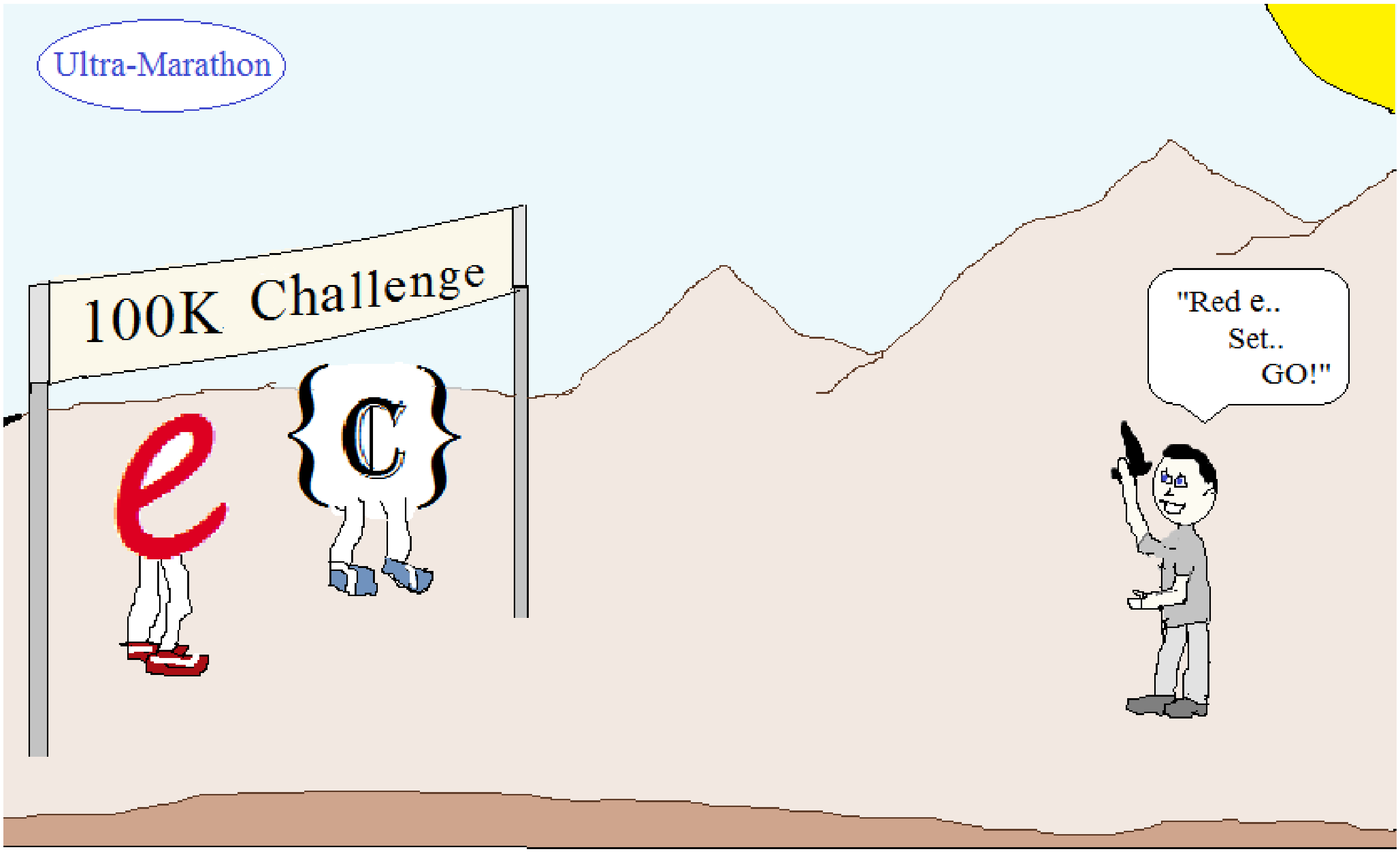
$$d\overline{AB} = \sqrt{(-2-3.5)^2 + (2-6)^2}$$

$$= \sqrt{30.25 + 16} = \boxed{6.80}$$

$$d\overline{BC} = \sqrt{(3.5-9)^2 + (6-10)^2}$$

$$= \sqrt{30.25 + 16} = \boxed{6.80}$$

$$d\overline{AB} = d\overline{BC}$$



Testing the limits of endurance,
these math figures will run on and on...

LanceAF #87 5-24-13
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PRACTICE TEST

Coordinate Geometry Test

Part I:

Identify the Quadrant or Axis of each point.

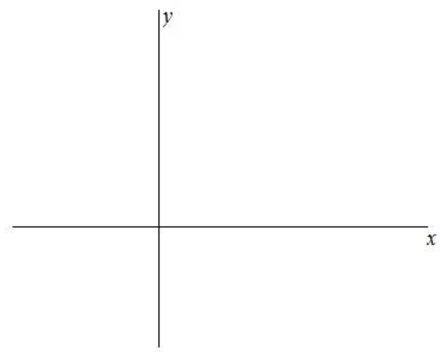
a) (2, -3) b) (-2, 3) c) (1, 210) d) (-21, -3.44) e) (0, 9) f) (9, 0)

Part II:

A = (4, 9)
B = (4, -3)
C = (8, 17)
D = (x, y)

a) What is the Distance between A and B? Midpoint?

b) What is the length of $\overline{AC}$? Midpoint of $\overline{AC}$?

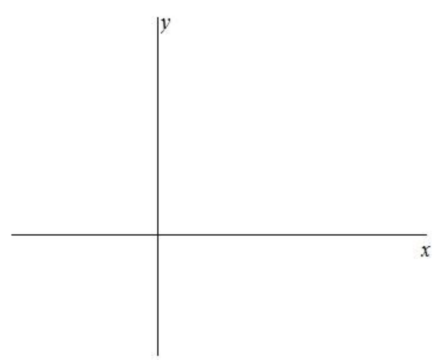


c) If C is the midpoint of $\overline{BD}$, what are the coordinates of D ?

d) What is the slope of $\overline{BC}$? $\overline{AB}$? $\overline{BD}$?

Part III: A rectangle has the following vertices: (1, 1) (7, 1) (1, 4)

a) Where is the 4th vertex?

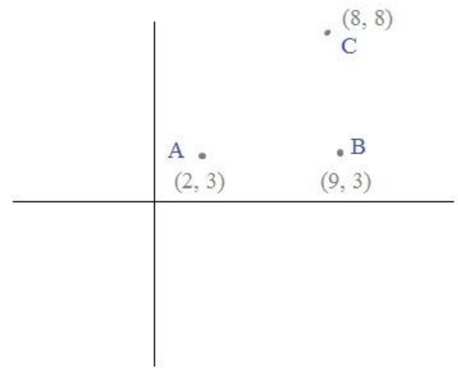


b) What is the perimeter of the rectangle?

c) What is the area of the rectangle?

Coordinate Geometry Test (Continued)

Part IV: A parallelogram has the following vertices: A (2, 3) B (9, 3) C (8, 8)



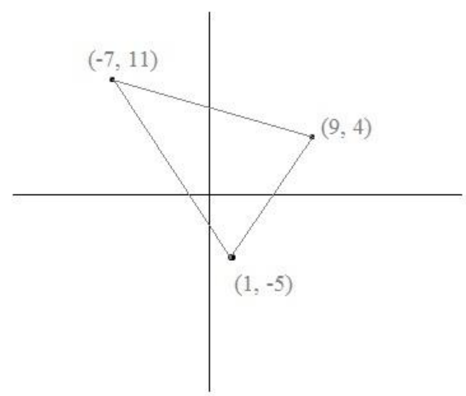
a) What is the 4th vertex of ▱ ABCD ?

b) What is the 4th vertex of ▱ ADBC ?

c) Assuming ▱ ABCD, what is the perimeter? What is the area?

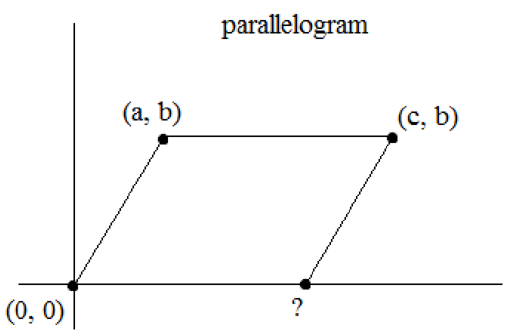
Part V: A triangle has the following vertices: (-7, 11) (9, 4) (1, -5)

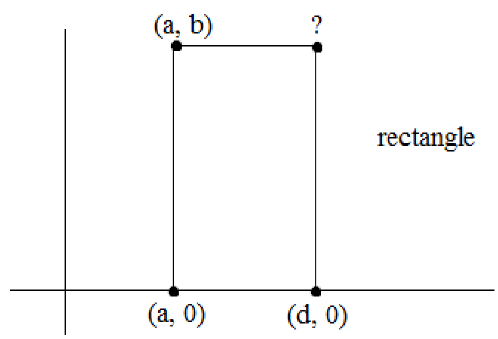
Using Heron's Formula, "Encasement", or any method you prefer, find the area of the triangle

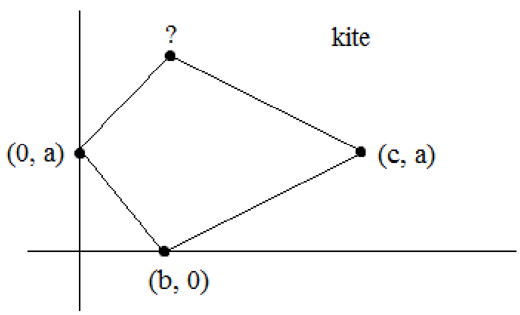


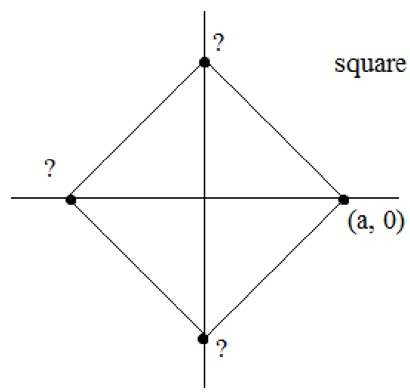
Use a different method to confirm your answer.

Part VI: What are the missing coordinates?

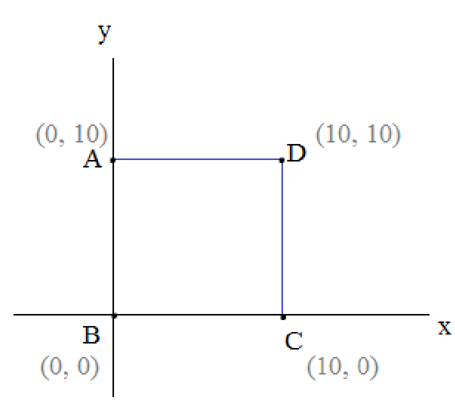


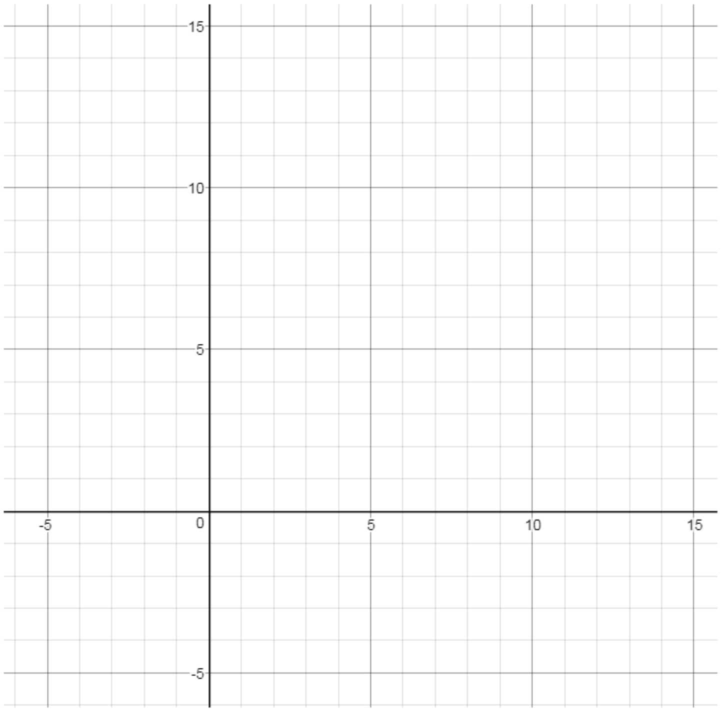






Part VII: When C and D are horizontally shifted to the right, $\overline{AD}$ and $\overline{BC}$ increase 20%.
By what percentage do the *diagonals* increase?





Plot (and connect) the points in the following sequences....

Sequence A: (6, 4) (6, 13) (13, 6) (6, 6)

Sequence B: (2, 5) (1, 6) (1, 0)

Sequence C: (-6, -1) (-3, 2) (-2, 0) (0, -1) (-2, -2) (-6, -3)

Sequence D: (2, 4) (13, 4) (11, 0) (4, 0) (2, 4)

Sequence E: (4, 4) (4, 6) (3, 7) (4, 8) (5, 7) (4, 6)

Sequence F: (-2.5, 0) (-2, -0.5) (-2.5, -1) (-3, -0.5) (-2.5, 0)

Sequence G: (2, 5) (4, 5) (6, 4)

Sequence H: (-4, -2) (-2, -2)

Title:

Title Pieces: Arrange letter pairs in numerical order to reveal title of sketch!

BL 1

D_ 5

KH 3

OC 2

EA 4

GO 6

IS 9

ES 7

_F 8

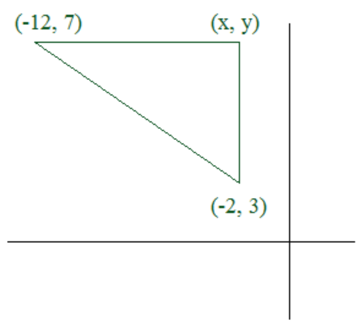
NG 11

HI 10

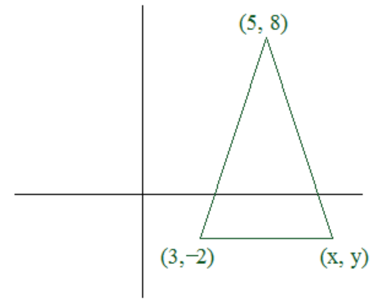
Identify the missing coordinates. Then, find the area of each figure.

Quadrilaterals, Triangles, and Coordinates

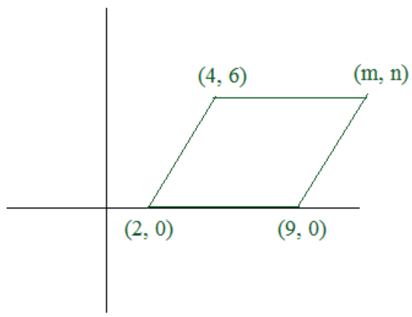
1) Right Triangle



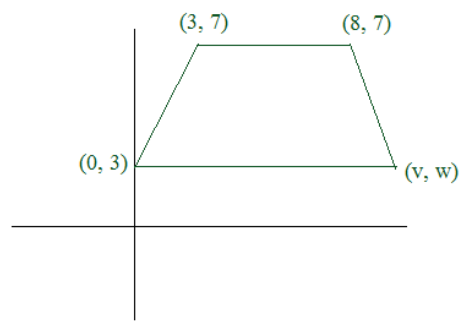
2) Isosceles Triangle



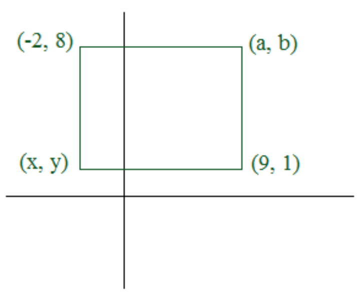
3) Parallelogram



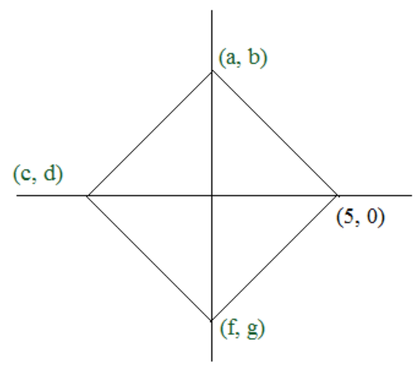
4) Isosceles Trapezoid



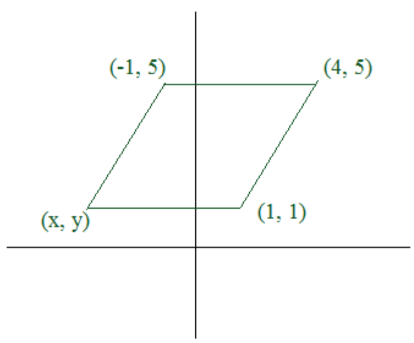
5) Rectangle



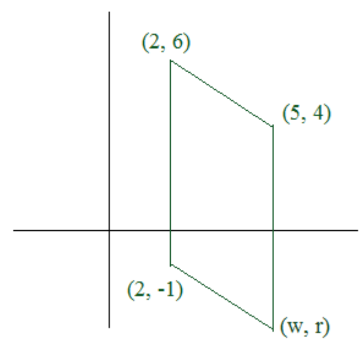
6) Square



7) Rhombus



8) Parallelogram

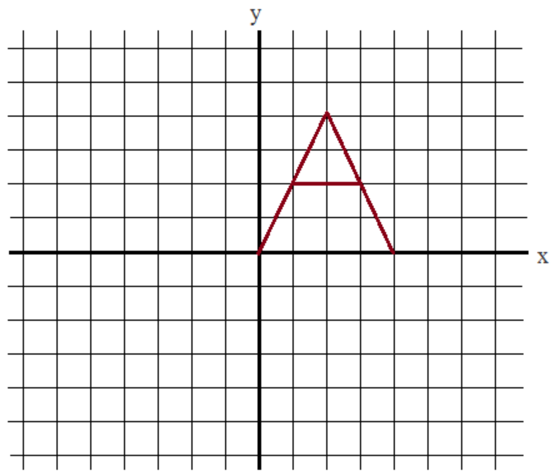


Coordinate Geometry Quiz

Name: *Linus Wright*

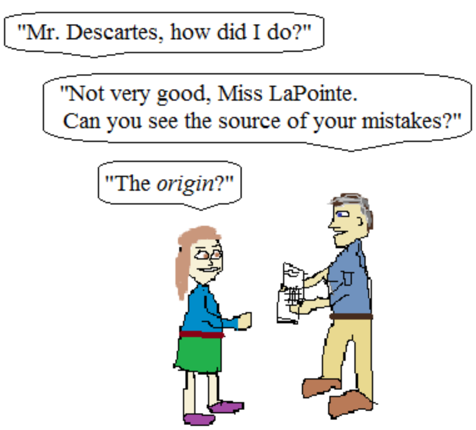
Instructions - Draw the following segments:

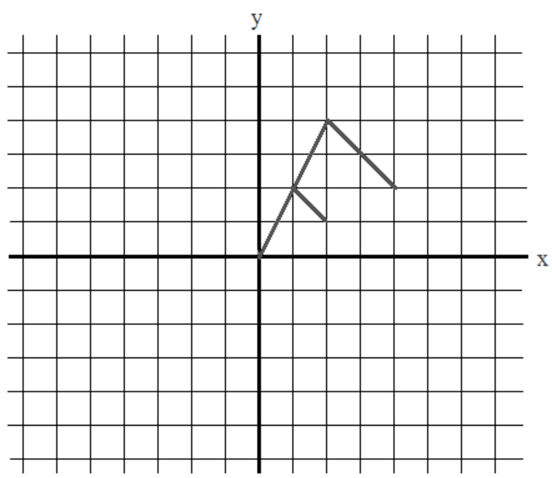
1) $\overline{OA}$ from the origin to (2, 4)
2) $\overline{AB}$ from (2, 4) to (4, 0)
3) midpoint of $\overline{OA}$ to midpoint of $\overline{AB}$



Uncoordinated

Name: *Chloe LaPointe*





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An 'F' on the last quiz?...

.... Chloe didn't know wh(y).

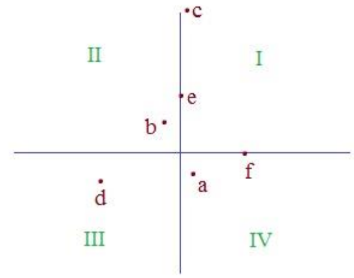
SOLUTIONS

Coordinate Geometry Test

SOLUTIONS

Part I:

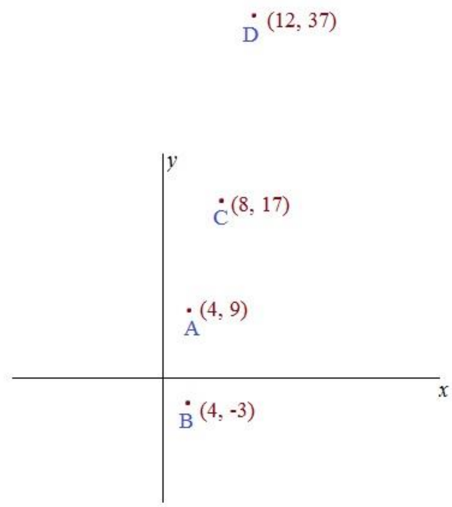
Identify the Quadrant or Axis of each point.



a) (2, -3)	b) (-2, 3)	c) (1, 210)	d) (-21, -3.44)	e) (0, 9)	f) (9, 0)
IV	II	I	III	y-axis	x-axis

Part II:

A = (4, 9)
B = (4, -3)
C = (8, 17)
D = (x, y)



a) What is the Distance between A and B? Midpoint?

distance = 12 units   midpoint: (4, 3)

b) What is the length of $\overline{AC}$? Midpoint of $\overline{AC}$?

Use distance formula to find length of $\overline{AC}$: $\sqrt{(8-4)^2 + (17-9)^2} = \sqrt{80}$

$= 4\sqrt{5}$

Midpoint: $\left(\frac{4+8}{2}, \frac{9+17}{2}\right)$

$= (6, 13)$

c) If C is the midpoint of $\overline{BD}$, what are the coordinates of D ?

From B to C: x goes from 4 to 8 (+ 4)
y goes from -3 to 17 (+ 20)

From C to D: x (+ 4) goes from 8 to 12
y (+ 20) goes from 17 to 37

$\boxed{D = (12, 37)}$

d) What is the slope of $\overline{BC}$? $\overline{AB}$? $\overline{BD}$?

slope $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

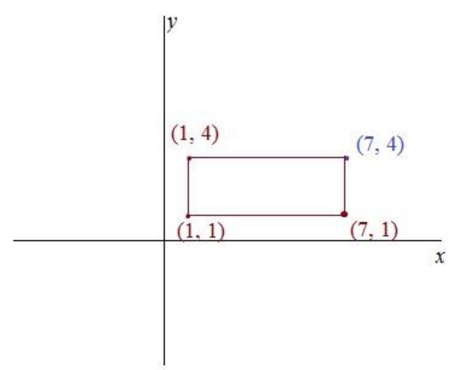
$\overline{BC} = \dfrac{-3 - 17}{4 - 8} = 5$

$\overline{AB}$ is undefined ("no slope")

$m = \dfrac{\text{"rise"}}{\text{"run"}}$

slope of $\overline{BD}$ is also 5

Part III: A rectangle has the following vertices: (1, 1) (7, 1) (1, 4)



a) Where is the 4th vertex?

(7, 4) is the 4th vertex

b) What is the perimeter of the rectangle?

length: 6 units
width: 3 units

perimeter: 18 units

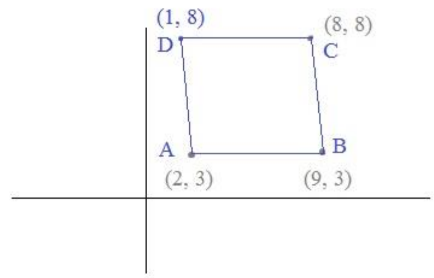
c) What is the area of the rectangle?

area = length x width

area = 18 square units

Coordinate Geometry Test (Continued)

SOLUTIONS



Part IV: A parallelogram has the following vertices: A (2, 3) B (9, 3) C (8, 8)

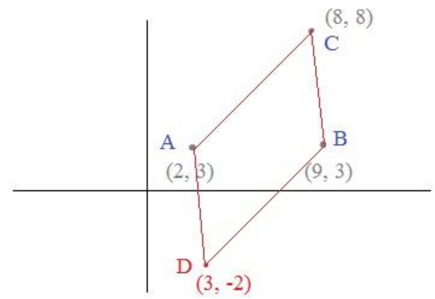
a) What is the 4th vertex of ▱ ABCD ?

Since base $\overline{AB}$ is horizontal and 7 units long, the top of the parallelogram must be horizontal and 7 units long. Therefore, the 4th vertex is (1, 8)

b) What is the 4th vertex of ▱ ADBC ?

Since C ---> B is 1 unit to the right and 5 units down,

A ---> vertex 4 will be 1 unit to the right and 5 units down: (3, -2)



c) Assuming ▱ ABCD, what is the perimeter? What is the area?

measure of $\overline{AB}$ = 7 units...

and $\overline{CD}$ = 7 units..

(distance formula) measure of $\overline{BC} = \sqrt{26}$

$\sqrt{(9-8)^2 + (3-8)^2} = \sqrt{26}$

and $\overline{DA} = \sqrt{26}$

height = 5 units
base = 7 units

Area = bh = 35 sq. units

perimeter = 2l + 2w = $14 + 2\sqrt{26} \cong 24.2$ units

Note: For parallelogram ADBC,
Perimeter ≅ 25.8 units
Area = 35 sq units

Part V: A triangle has the following vertices: (-7, 11) (9, 4) (1, -5)

Using Heron's Formula, "Encasement", or any method you prefer, find the area of the triangle

1) Using Heron's Formula: Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$

$a = \sqrt{16^2 + (-7)^2} = \sqrt{305} \cong 17.46$

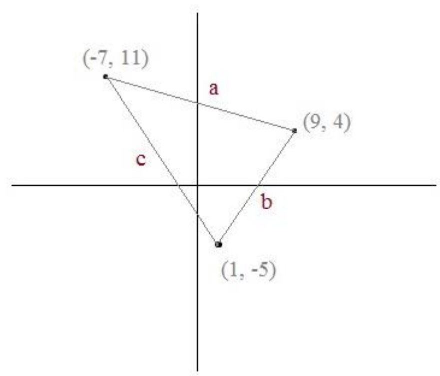
$b = \sqrt{8^2 + 9^2} = \sqrt{145} \cong 12.04$

$c = \sqrt{8^2 + (-16)^2} = \sqrt{320} \cong 17.89$

$s \cong \dfrac{47.39}{2} \cong 23.69$

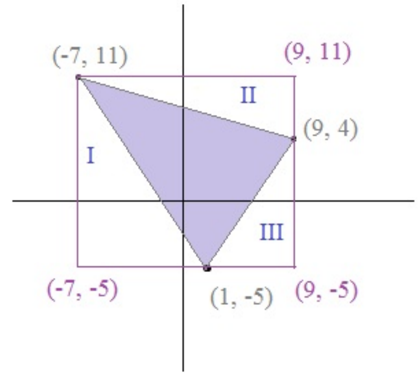
$A \cong \sqrt{23.69\,(23.69 - 17.46)(23.69 - 12.04)(23.69 - 17.89)}$

$\cong \sqrt{23.69\,(6.23)(11.65)(5.8)} \cong$ 99.86 square units



Use a different method to confirm your answer.

2) "Encasement" ('Encase' the triangle with a rectangle; find the area of surrounding right triangles.. Then, subtract area of right triangles from rectangle)



Area of rectangle: l x w = 16 x 16 = 256

Area of I : 1/2 bh = 1/2 (8)(16) = 64

Area of II: 1/2 bh = 1/2 (16)(7) = 56

Area of III: 1/2 bh = 1/2 (8)(9) = 36

256
-64
-56
-36

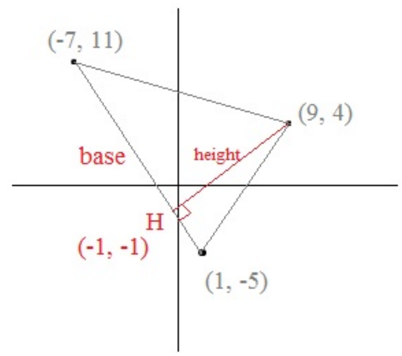
100 (inner triangle) square units

3) Area of a Triangle: 1/2 bh

(use distance formula to find base)

$\triangle_{base} = \sqrt{(-7-1)+(11-(-5))} = \sqrt{320} \cong 17.89$

(to find the height, we need to find the distance from (9, 4) to H)



slope of line through (-7, 11) and (1, -5) is $\frac{11-(-5)}{-7-1} = -2$

Since the height is perpendicular to the base, its slope is $\frac{1}{2}$

Therefore, equation of a line through H and (9, 4) is

y = mx + b
4 = 1/2(9) + b
b = -1/2

$y = \frac{1}{2}x - \frac{1}{2}$

And, equation of base is
y = mx + b
11 = -2(-7) + b
b = -3

$y = -2x - 3$

(set equations equal to each other to find intersection H)

$\frac{1}{2}x - \frac{1}{2} = -2x - 3$

x - 1 = -4x - 6
5x = -5
x = -1 y = -1

A = 1/2bh
1/2(17.89)(11.18)
100 sq units

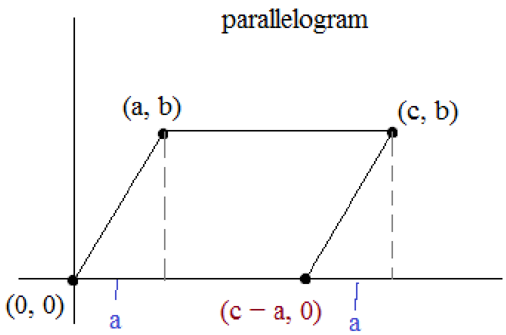
Height is distance from (-1, -1) to (9, 4)

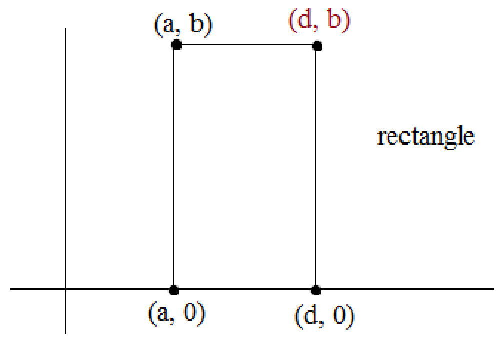
$\sqrt{(-1-9)^2 + (-1-4)^2}$

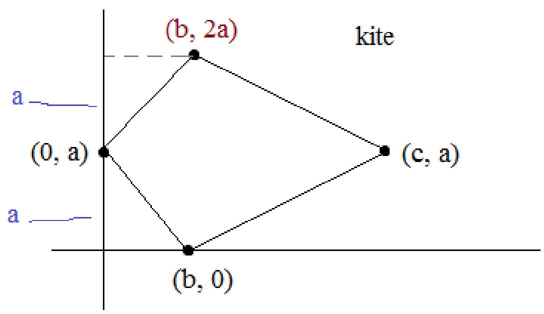
$\sqrt{100 + 25} \cong 11.18$

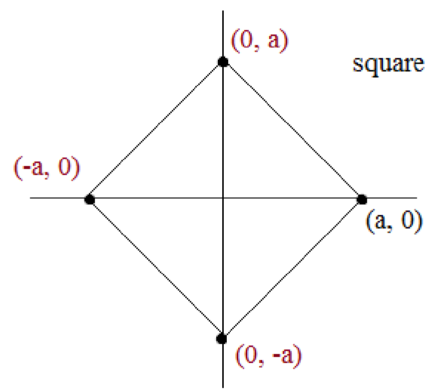
Part VI: What are the missing coordinates?

SOLUTIONS



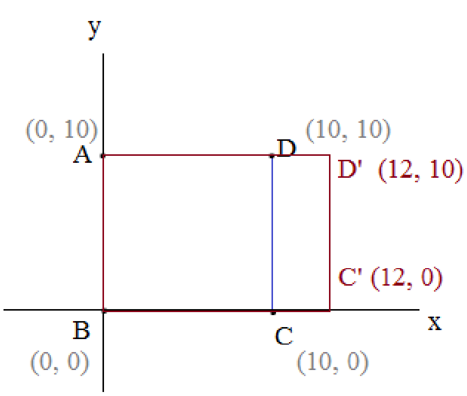




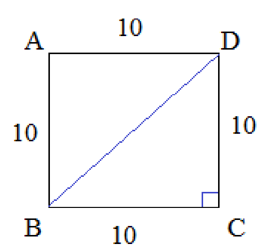


Part VII: When C and D are horizontally shifted to the right, $\overline{AD}$ and $\overline{BC}$ increase 20%.
By what percentage do the *diagonals* increase?

To increase the lengths by 20%, they must go from 10 to 12...

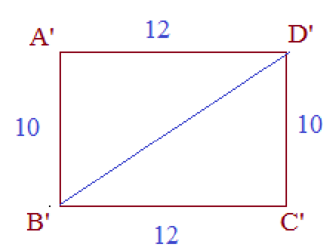


The new rectangle has vertices
(0, 0) (0, 10) (12, 10) (12, 0)



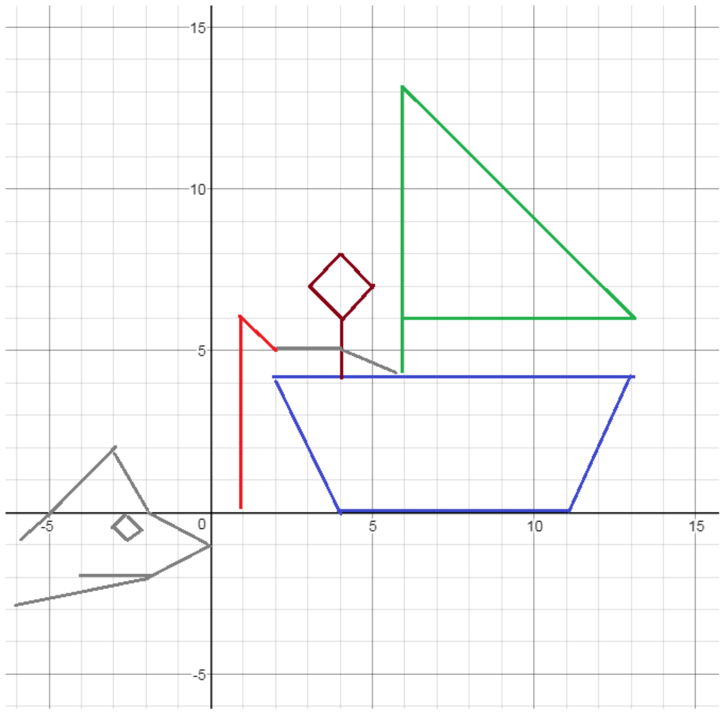
Pythagorean Theorem
(or 45-45-90 right triangle)

diagonal = $10\sqrt{2}$ = 14.14



diagonal = $\sqrt{100 + 144}$ = 15.62

diagonals increased by approximately 10.5%



Plot (and connect) the points in the following sequences....

Sequence A: (6, 4) (6, 13) (13, 6) (6, 6)

Sequence B: (2, 5) (1, 6) (1, 0)

Sequence C: (-6, -1) (-3, 2) (-2, 0) (0, -1) (-2, -2) (-6, -3)

Sequence D: (2, 4) (13, 4) (11, 0) (4, 0) (2, 4)

Sequence E: (4, 4) (4, 6) (3, 7) (4, 8) (5, 7) (4, 6)

Sequence F: (-2.5, 0) (-2, -0.5) (-2.5, -1) (-3, -0.5) (-2.5, 0)

Sequence G: (2, 5) (4, 5) (6, 4)

Sequence H: (-4, -2) (-2, -2)

Title:

"BLOCKHEAD_GOES_FISHING!"

Title Pieces: Arrange letter pairs in numerical order to reveal title of sketch!

BL
1

D_
5

KH
3

OC
2

EA
4

GO
6

IS
9

ES
7

_F
8

NG
11

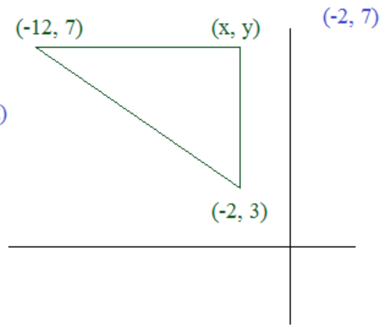
HI
10

Identify the missing coordinates. Then, find the area of each figure.

SOLUTIONS

Quadrilaterals, Triangles, and Coordinates

1) Right Triangle



Area = $\frac{1}{2}$(base)(height)

= $\frac{1}{2}$ (4)(10)

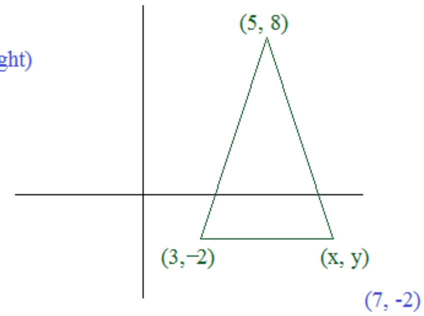
= 20 sq. units

2) Isosceles Triangle

Area = $\frac{1}{2}$(base)(height)

= $\frac{1}{2}$ (4)(10)

= 20



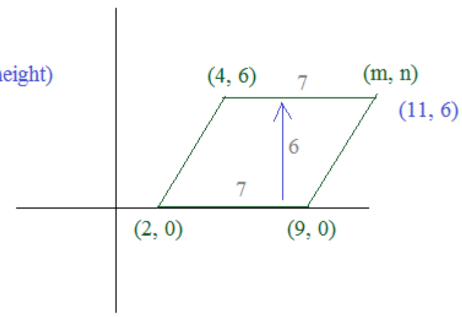
3) Parallelogram

(opposite sides congruent)

Area = (base)(height)

= (7)(6)

= 42

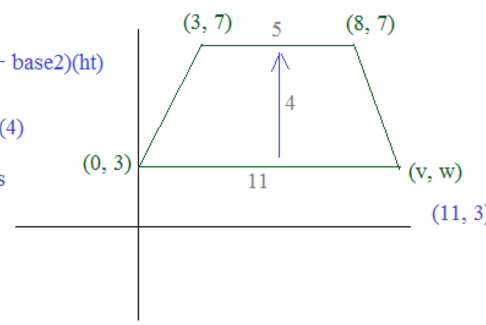


4) Isosceles Trapezoid

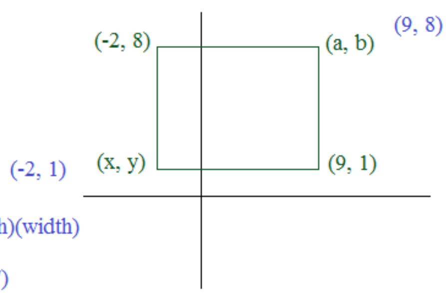
Area = $\frac{1}{2}$(base1 + base2)(ht)

= $\frac{1}{2}$(11 + 5)(4)

= 32 sq. units



5) Rectangle



Area = (length)(width)

= (11)(7)

= 77

6) Square

Area = $(\text{side})^2$

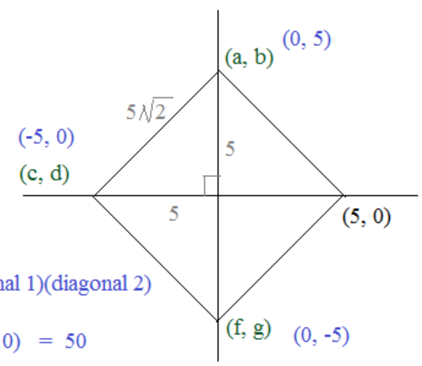
= $(5\sqrt{2})^2$

= 50

Or,

Area = $\frac{1}{2}$ (diagonal 1)(diagonal 2)

= $\frac{1}{2}$ (10)(10) = 50



7) Rhombus

Area = (base)(height)

= (4)(5)

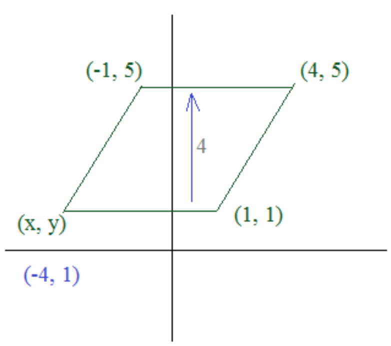
= 20

Or,

Area = $\frac{1}{2}$(d1)(d2)

= $\frac{1}{2}$ $(\sqrt{80})(\sqrt{20})$

= 20

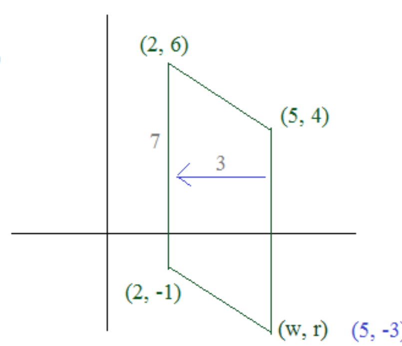


8) Parallelogram

Area = (base)(height)

= (7)(3)

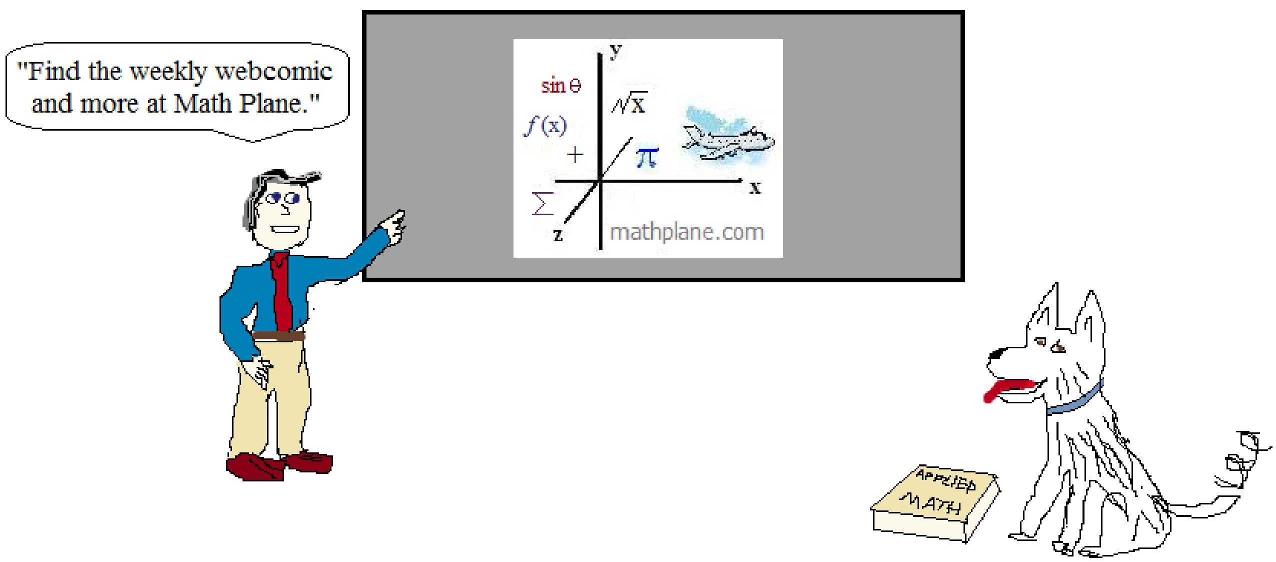
= 21 sq. units



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at Mathplane *Express* for mobile and tablets at mathplane.ORG

One more question:

The following are 3 vertices of a parallelogram:

L (1, 6)   M (8, 6)   P (3, 3)

What is the coordinate(s) of the 4th vertex?

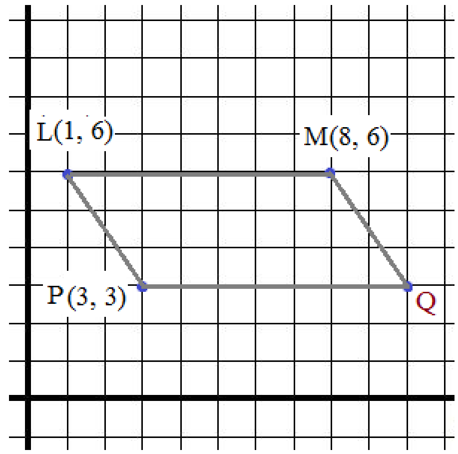
Answer on the next page --→

Coordinate Geometry

The following are the vertices of a parallelogram: L(1, 6) M(8, 6) and P(3, 3).
What is the 4th vertex?

Definition of a parallelogram: "Quadrilateral where opposite sides are parallel"
(also, opposite sides are congruent)

Since the parallelogram is not specified, there are 3 possiblities!

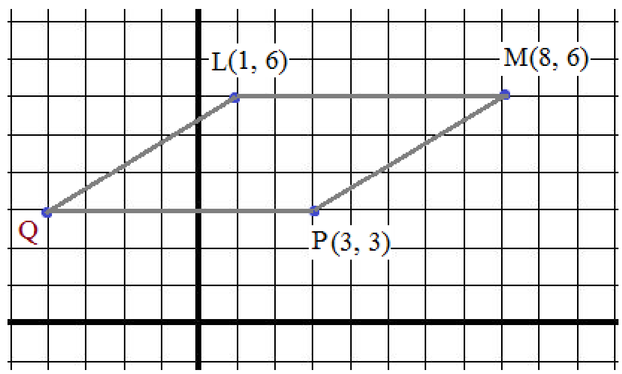


Assume the 4th vertex is Q:

Solution #1: $\overline{LM}$ is a horizontal segment of length 7.
So, $\overline{PQ}$ is a horizontal segment of length 7.

▱ LMQP

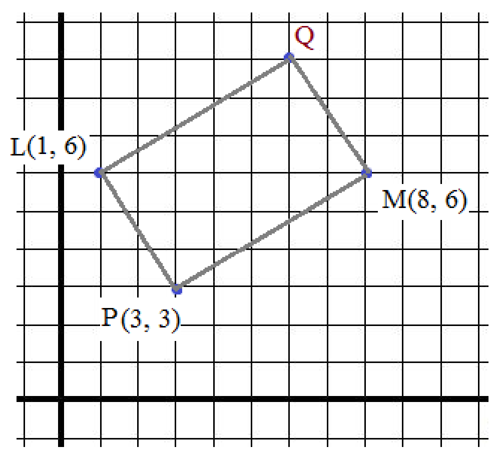
Q is (10, 3)



Solution #2: $\overline{LM}$ is a horizontal segment of length 7.
so, $\overline{QP}$ is a horizontal segment of length 7.
("in the other direction")

▱ LMPQ

Q is (-4, 3)



Solution #3: $\overline{PM}$ is a segment with slope $\frac{3}{5}$ $\frac{\text{"rise"}}{\text{"run"}}$

▱ LQMP

and length $\sqrt{34}$

Therefore, the opposite side (LQ)
must have the same slope and length...

***Starting at (1, 6) -- go up 3 and to the right 5

Q is (6, 9)

LQMP is also a rectangle..