Graphing I: Transformations and Parent Functions

Notes, Examples, and practice quiz

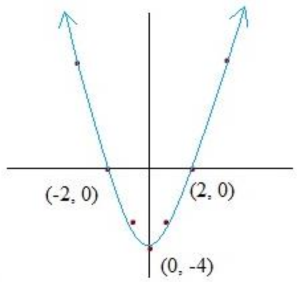
Transformations and Parent Functions

A common way to graph equations is plot points, identify intercepts, and determine the end behavior.

Examples:

$Y = X^2 - 4$

X	Y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

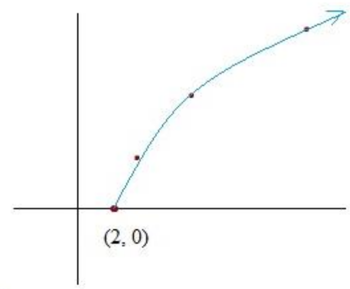


X-intercepts: (-2, 0) and (2, 0)
Y-intercept: (0, -4)
After plotting points, we observe that the end behavior is "up to the left" and "up to the right" (or positive ∞)

Domain: $(-\infty, \infty)$
Range: [-4, ∞]

$f(x) = 3\sqrt{x-2}$

x	f (x)
11	9
6	6
3	3
2	0
0	---



X - intercept: (2, 0)
Y - intercept: NONE
End behavior: "up to the right" and "when going left, stops at X = 2

Domain: [2, ∞)
Range: [0, ∞)

Now, suppose you want to sketch $f(x) = -3(x+14)^2 + 10$

Plotting a few points and identifying intercepts could be difficult and time consuming...

However, using parent functions and transformation techniques can be an effective way to sketch complicated graphs.

"Parent Function" -- A basic function used as a 'building block' for more complicated functions

Common Examples: $f(x) = x^2$ (parabola)

$f(x) = \sqrt{x}$ (square root)

$f(x) = |x|$ (absolute value)

$f(x) = x^3$ (cubic curve)

(Other parent functions include trig functions, logarithms, exponents, greatest integer, and reciprocals)

"Transformation" -- Operations that alter a function (e.g. reflections, translations, stretches, compressions, or rotations)

If $f(x)$ is the parent function,

$a f(b(x - c)) + d$ is the transformed function where

a is the "stretch"
b is the "compression"
c is the "horizontal shift"
d is the "vertical shift"

Transformations and Parent Functions

If $f(x)$ is the parent function,

$a f(b(x - c)) + d$ is the transformed function where

a is the "stretch"
b is the "compression"
c is the "horizontal shift"
d is the "vertical shift"

The "vertical shift" : d

This transformation is the easiest to recognize and utilize. To shift a function up or down (along the y-axis), simply add/subtract the amount at the end of the function.

Compare:

$f(x) = |x|$ (absolute value parent function)

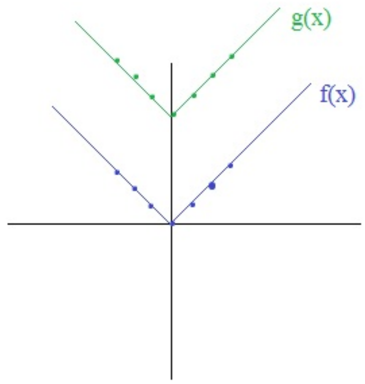
$g(x) = |x| + 7$ ("vertical shift" up 7)

$f(x) = |x|$

x	f (x)
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

$g(x) = |x| + 7$

x	g (x)
-3	10
-2	9
-1	8
0	7
1	8
2	9
3	10



**Note: the vertical shift is the value <u>outside</u> the function.

$f(x + 7) \neq f(x) + 7$ $|x + 7| \neq |x| + 7$

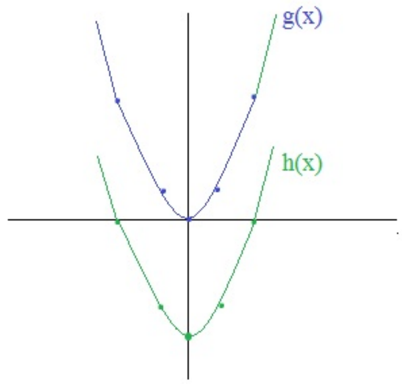
Compare:

$f(x) = x^2$ (parent function)

$h(x) = x^2 - 4$

x	f (x)
-2	4
-1	1
0	0
1	1
2	4

x	h (x)
-2	0
-1	-3
0	-4
1	-3
2	0



(Note: every output of the transformed function h(x) is exactly 4 less than the output of the parent function g(x))

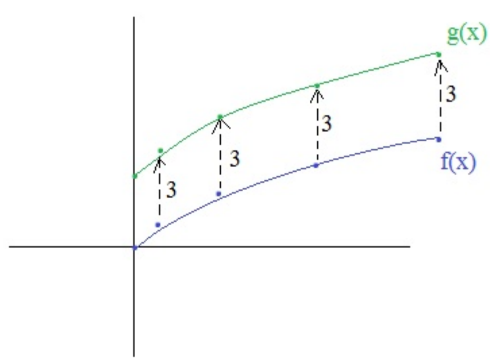
Compare:

$f(x) = \sqrt{x}$ (square root parent function)

$g(x) = \sqrt{x} + 3$

x	f (x)
0	0
1	1
4	2
9	3
16	4

x	g (x)
0	3
1	4
4	5
9	6
16	7



Transformations and Parent Functions

If $f(x)$ is the parent function,

$a f(b(x - c)) + d$ is the transformed function where

- a is the "stretch"
- b is the "compression"
- c is the "horizontal shift"
- d is the "vertical shift"

The "horizontal shift": c

This transformation is very useful. (Similar to a vertical shift), the entire function is simply moved to the right (or left) along the x-axis, determined by the 'c' value.

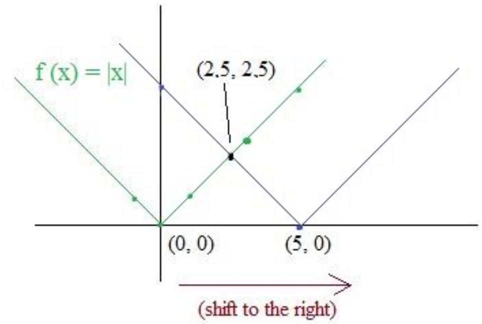
Compare:

$f(x) = |x|$ (absolute value parent function)

$g(x) = |x - 5|$

x	f (x)	g (x)
-1	1	6
0	0	5
1	1	4
3	3	2
5	5	0
7	7	2
9	9	4

Note: the x-intercept of |x| is (0,0) and the x-intercept of |x - 5| is (5,0), verifying a 5 space shift to the right

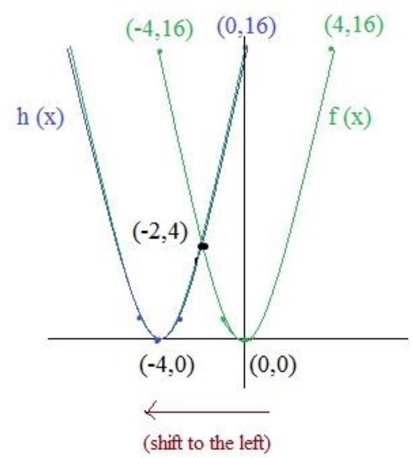


$f(x) = x^2$ (parabola parent function)

$h(x) = (x + 4)^2$

x	f (x)	h (x)
-4	16	0
-3	9	1
-2	4	4
-1	1	9
0	0	16
1	1	25
2	4	36
3	9	49
4	16	64

Note: The output values of h(x) would be the same as f(x) if the inputs were shifted by 4

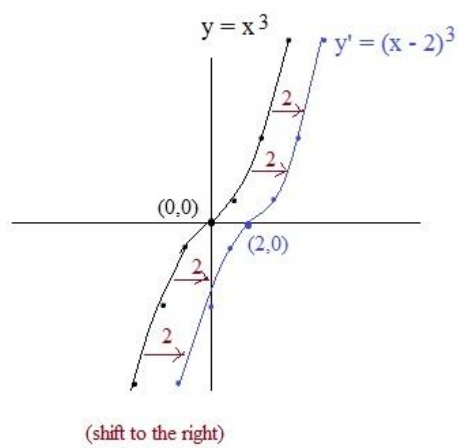


$y = x^3$ (cubic equation)

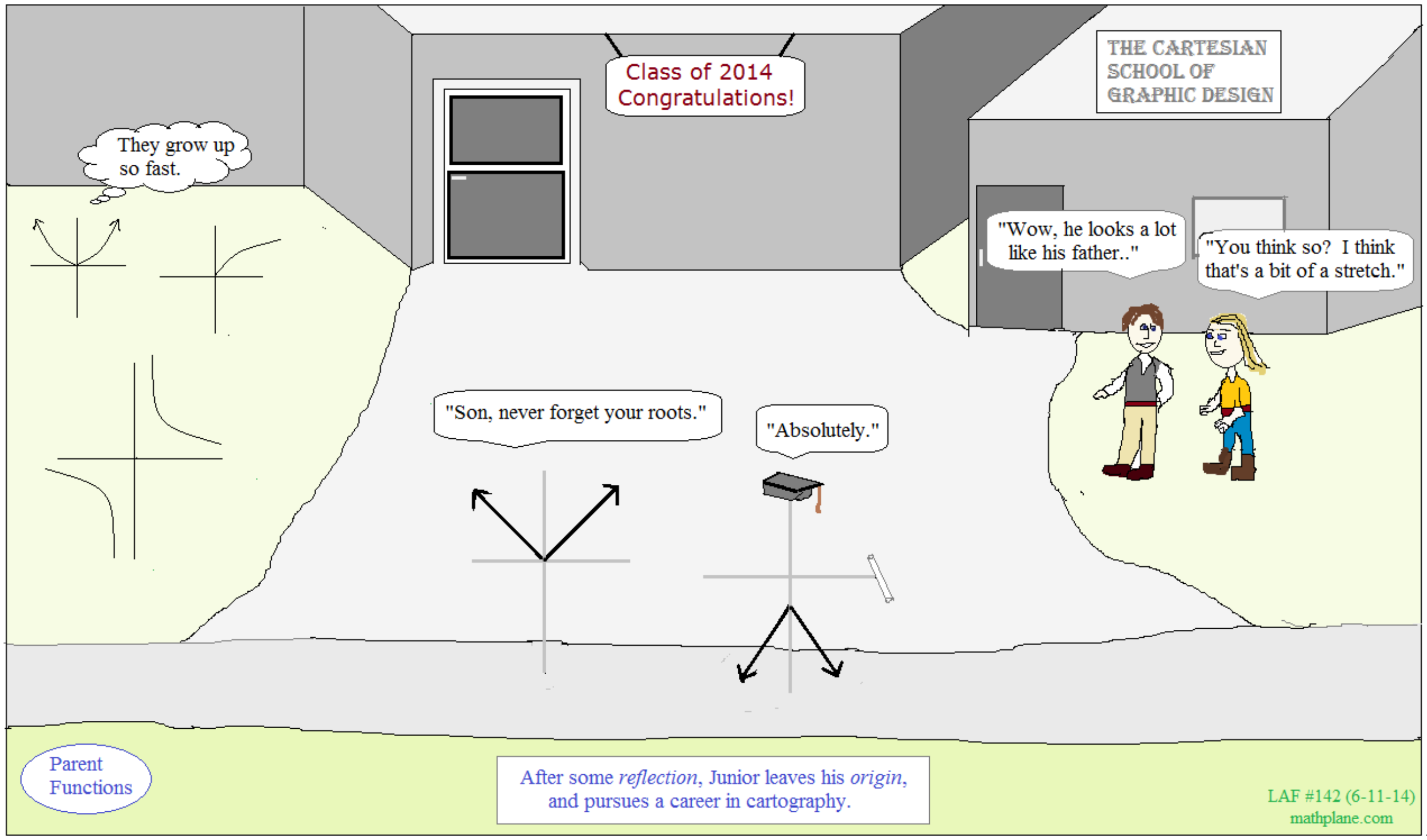
$y' = (x - 2)^3$

x	y	y'
-2	-8	-64
-1	-1	-27
0	0	-8
1	1	-1
2	8	0
3	27	1
4	64	8

Note: the table shows the y' values are the same as the y values when shifted by 2 rows



**As you can see, if the c value is negative, the shift is to the right. And, when the c value is positive, the shift is to the left


Class of 2014
Congratulations!
THE CARTESIAN SCHOOL OF GRAPHIC DESIGN
They grow up so fast.
"Wow, he looks a lot like his father.."
"You think so? I think that's a bit of a stretch."
"Son, never forget your roots."
"Absolutely."
Parent Functions
After some *reflection*, Junior leaves his *origin*, and pursues a career in cartography.
LAF #142 (6-11-14)
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Transformations and Parent Functions

The "horizontal shift": c

The "vertical shift": d

If $f(x)$ is the parent function,

$a f(b(x - c)) + d$ is the transformed function where

a is the "stretch"
b is the "compression"
c is the "horizontal shift"
d is the "vertical shift"

Sketch the following functions:

$f(x) = |x + 6| + 5$

$g(x) = \sqrt{x - 4} - 8$

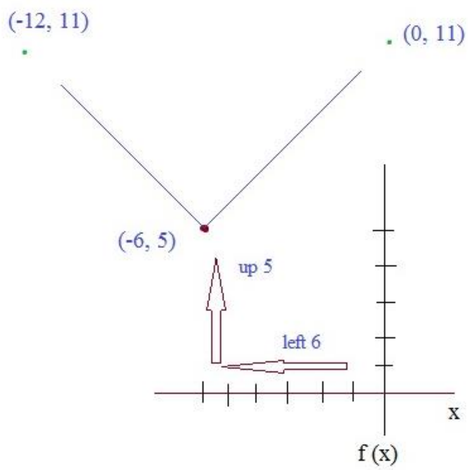
$h(x) = (x + 8)^2 - 12$

Solutions:

$f(x) = |x + 6| + 5$

(the parent function is absolute value $|x|$)

We use a vertical shift "up 5"
and
a horizontal shift "left 6"



To check your sketch, select random points and plug the values into the function.

$x = -6 \quad f(x) = 5$

$f(-6) = |-6 + 6| + 5 = 5$ ✓

$x = 0 \quad f(x) = 11$

$f(0) = |0 + 6| + 5 = 11$ ✓

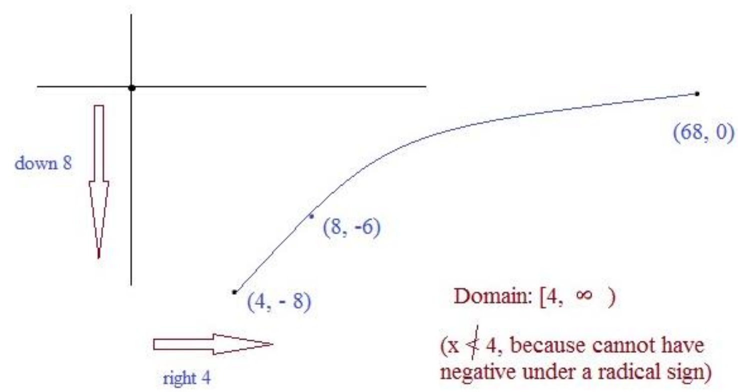
$x = -12 \quad f(x) = 11$

$f(-12) = |-12 + 6| + 5 = 11$ ✓

$g(x) = \sqrt{x - 4} - 8$

(the parent function is square root $\sqrt{x}$)

We observe a vertical shift "down 8"
and
a horizontal shift "right 4"



Domain: $[4, \infty)$

($x \nless 4$, because cannot have negative under a radical sign)

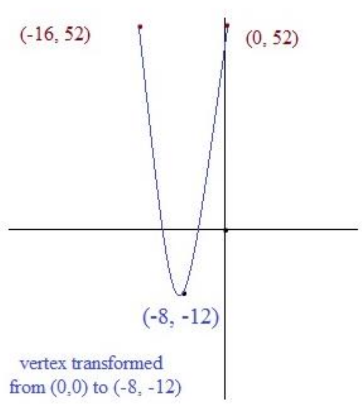
$h(x) = (x + 8)^2 - 12$

(the parent function is x^2)

vertical shift: "down 12"
horizontal shift: "left 8"

y - intercept:

$(0 + 8)^2 - 12 = 52$



Transformations and Parent Functions

If f(x) is the parent function,

$a\,f(b(x - c)) + d$ is the transformed function where

a is the "stretch"
b is the "compression"
c is the "horizontal shift"
d is the "vertical shift"

The "stretch" (or "shrink"): a

This transformation expands (or contracts) the parent function up and down (along the y-axis).
If a > 1, the function's rate of change increased.
If $0 < a < 1$, the function's rate of change is decreased.
(**For $-a$, the function changes direction)

Compare:

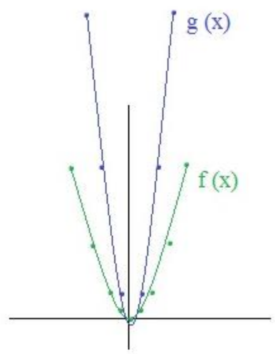
$f(x) = x^2$

$g(x) = 4x^2$

note: $4x^2 \neq (4x)^2$

x	f (x)	g (x)
-3	9	36
-2	4	16
-1	1	4
0	0	0
1	1	4
2	4	16
3	9	36

$g(x) = 4 \cdot f(x)$



g (x) is growing 4 times as fast as f (x)

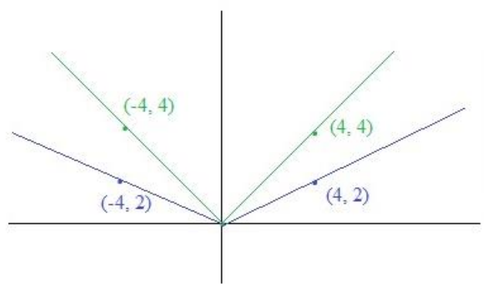
g (x) is a 'stretched' transformation of the parent function

Compare:

$f(x) = |x|$

$h(x) = 1/2\,|x|$

x	f (x)	h (x)
-6	6	3
-4	4	2
-2	2	1
0	0	0
2	2	1
4	4	2
6	6	3



In this example, the output values of h(x) are all 1/2 the value of the parent function's output values.

'Shrink'

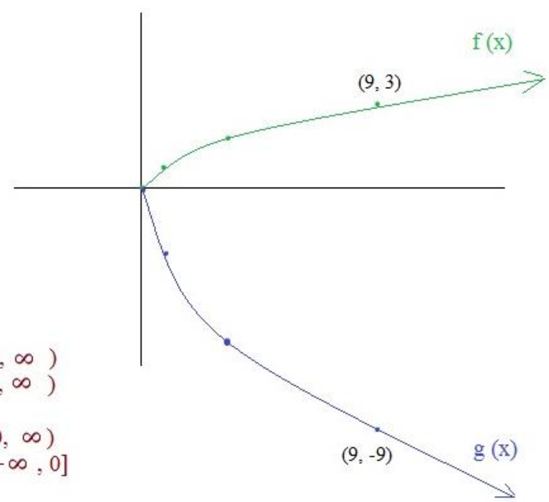
Compare:

$f(x) = \sqrt{x}$

$g(x) = -3\sqrt{x}$

x	f (x)	g (x)
-1	--	--
0	0	0
1	1	-3
4	2	-6
9	3	-9
16	4	-12

Since a > 0 and it is negative, the function is stretched in the other direction.



f (x): domain: $[0, \infty)$
range : $[0, \infty)$

g (x): domain: $[0, \infty)$
range : $(-\infty, 0]$

Transformations and Parent Functions

"Compression" (or "expansion"): b

This transformation compresses (or expands) the parent function lengthwise (along the x-axis).
If $b > 1$, then the function gets compressed (i.e. squeezed)
If $0 < b < 1$, then the function expands wider.
(**For $-b$, the function is flipped over the y-axis)

If $f(x)$ is the parent function,

$a f(b(x - c)) + d$ is the transformed function where

a is the "stretch"
b is the "compression"
c is the "horizontal shift"
d is the "vertical shift"

Compare:

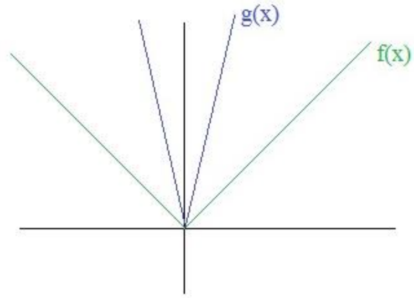
$f(x) = |x|$ (absolute value function)

$g(x) = |4x|$

Note: In this case, $|4x| = 4|x|$
However, in general,
$f(bx)$ is not the same as $b f(x)$
(see the third example)

x	f (x)	g (x)
-3	3	12
-2	2	8
-1	1	4
0	0	0
1	1	4
2	2	8
3	3	12

$b = 4 > 0$
Compressed

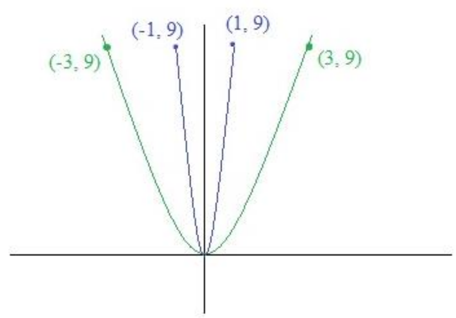


Compare:

$f(x) = x^2$

$h(x) = (3x)^2$

x	f (x)	h (x)
-3	9	81
-2	4	36
-1	1	9
0	0	0
1	1	9
2	4	36
3	9	81



Compare:

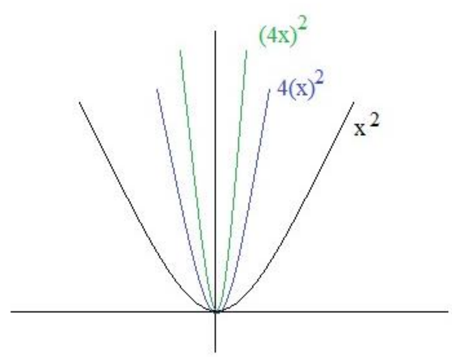
$g(x) = (4x)^2$

transformation: $f(bx)$
("compression")

$h(x) = 4(x)^2$

transformation: $a f(x)$
("stretch")

x	g (x)	h (x)
-3	144	36
-2	64	16
-1	16	4
0	0	0
1	16	4
2	64	16
3	144	36



Transformations and Parent Functions

(Complicated examples)

Sketch the following functions:

$f(x) = 3(x+7)^2 - 6$

$g(x) = -|2x+5| - 4$

$h(x) = \frac{1}{2}\sqrt{x-6} + 5$

If $f(x)$ is the parent function,

$a f(b(x - c)) + d$ is the transformed function where

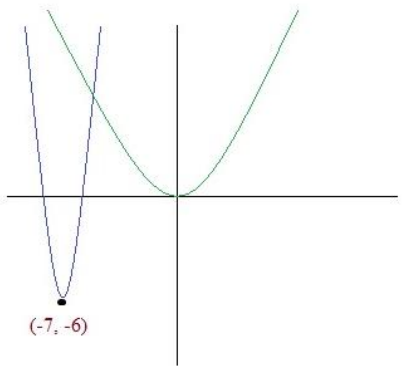
a is the "stretch"
b is the "compression"
c is the "horizontal shift"
d is the "vertical shift"

Solutions:

$f(x) = 3(x+7)^2 - 6$

a = 3 stretch is 3
b = 1 no compression
c = 7 horizontal shift: left 7
d = -6 vertical shift: down 6

parent function: x^2



To check your graph, select some points:

(-7, -6) $3(-7+7)^2 - 6 = -6$

x = -7 f(-7) = -6 ✓

y-intercept: find f(0)

$3(0+7)^2 - 6 = 141$

(0, 141) ✓

x-intercepts: find f(x) = 0

$3(x+7)^2 - 6 = 0$

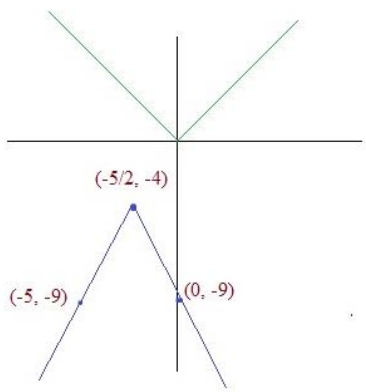
$(x+7)^2 = 2$

$x = -7 \pm \sqrt{2}$ ✓

$g(x) = -|2x+5| - 4$

a = -1 change of direction
b = 2 compression of 2
(***the compression will alter the horizontal shift!!)
c = 5 horizontal shift is 5/2 to the left
d = -4 vertical shift: down 4

parent function: |x|

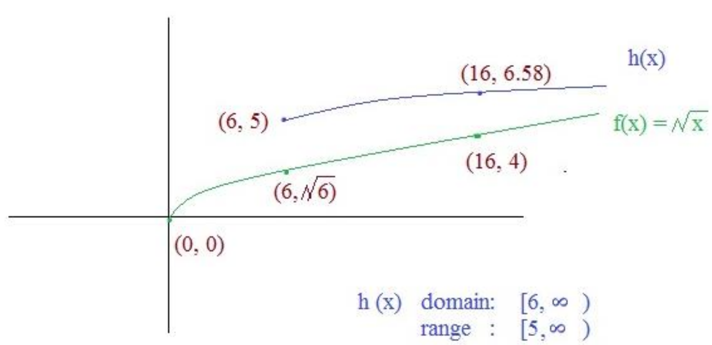


(Plug the points into the function to confirm the sketch)

$h(x) = \frac{1}{2}\sqrt{x-6} + 5$

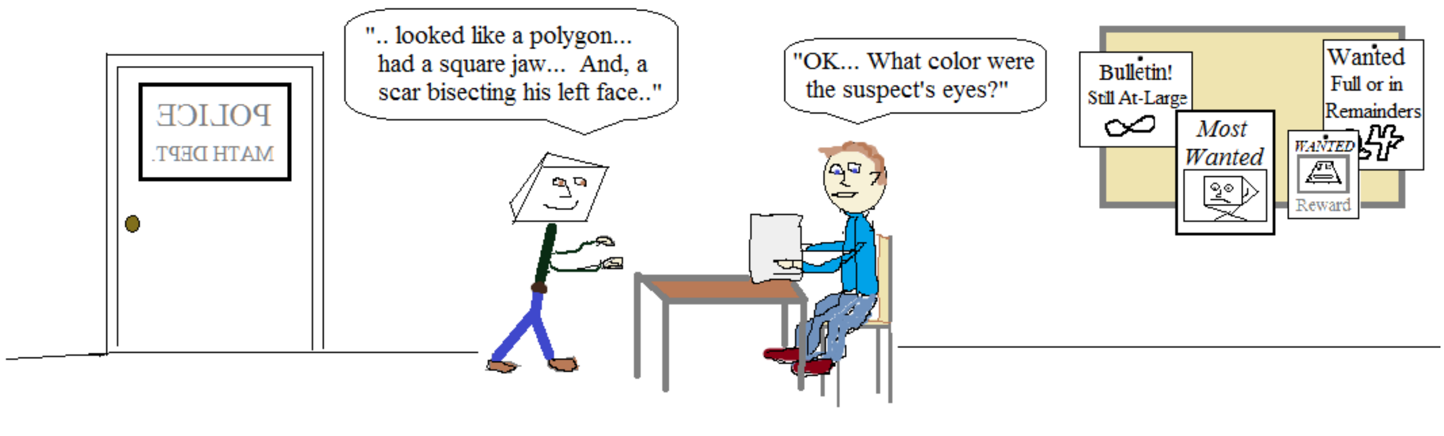
a = 1/2 "shrink" ("flatter")
b = 1 no horizontal compression
c = -6 horizontal shift: right 6
d = 5 vertical shift: up 5

parent function: $\sqrt{x}$



h (x) domain: $[6, \infty)$
range : $[5, \infty)$

Sketch Artist



Using his geometry background,
The Math Guy excels in his new profession.

LanceAF #49A (9-9-12)
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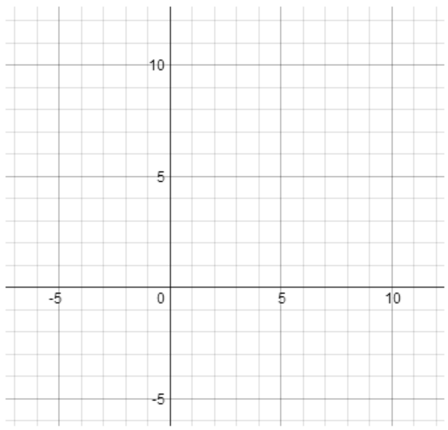
Practice Quiz (w/Solutions)-→

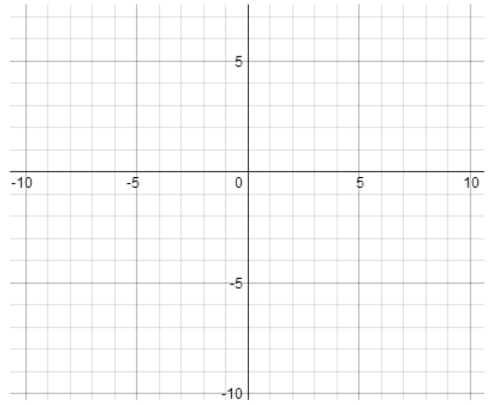
Graphing and Parent Functions Quiz

In the following, a) identify the parent function
b) describe any translations and transformations
c) sketch the functions
d) (optional) determine the domain and range

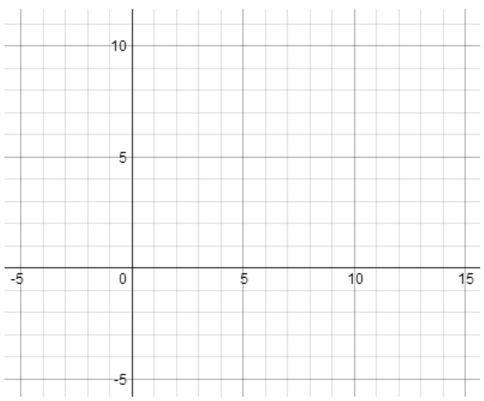
1) $y = |x - 2| + 4$



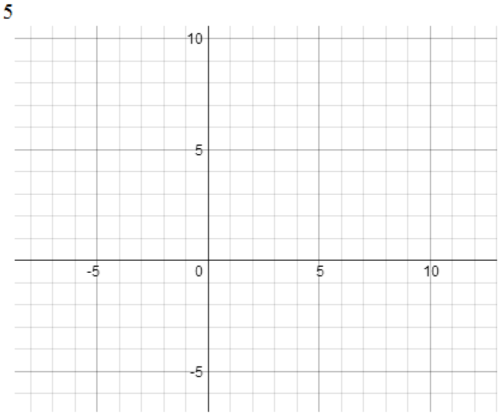
2) $f(x) = -\frac{1}{2}(x + 3)^2$



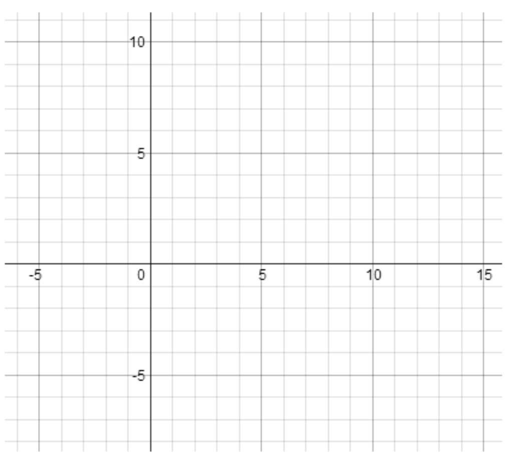
3) $y = 2\sqrt{x - 1} + 3$



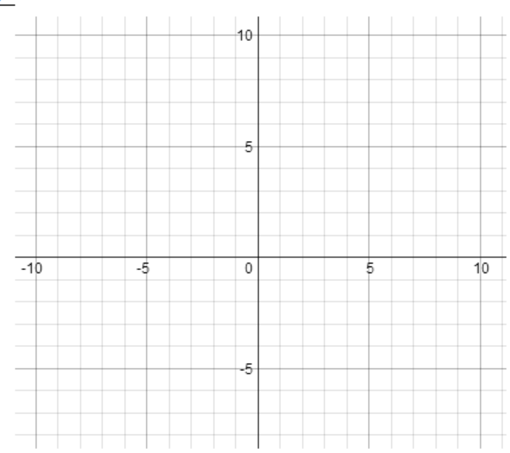
4) $y = -|3x - 3| + 5$



5) $y = -(x - 3)^3 + 3$



6) $g(x) = \frac{(x + 4)^2}{2}$



Graphing and Parent Functions Quiz

SOLUTIONS

In the following, a) identify the parent function
b) describe any translations and transformations
c) sketch the functions
d) (optional) determine the domain and range

If $f(x)$ is the parent function,

$af(b(x - c)) + d$ is the transformed function where

a is the "stretch"
b is the "compression"
c is the "horizontal shift"
d is the "vertical shift"

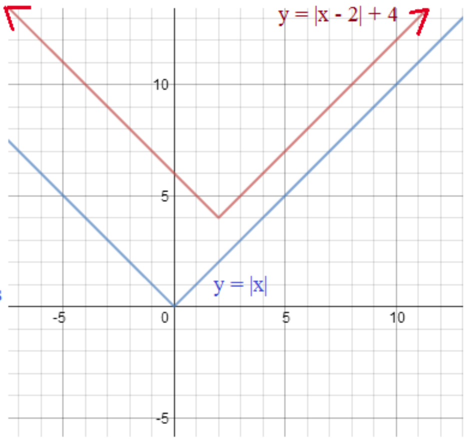
1) $y = |x - 2| + 4$

parent function:
$y = |x|$

horizontal shift (c):
2 units to the right

vertical shift (d):
4 units up

domain: all real numbers
range: $y \geq 4$



2) $f(x) = -\frac{1}{2}(x + 3)^2$

parent function:
$f(x) = x^2$

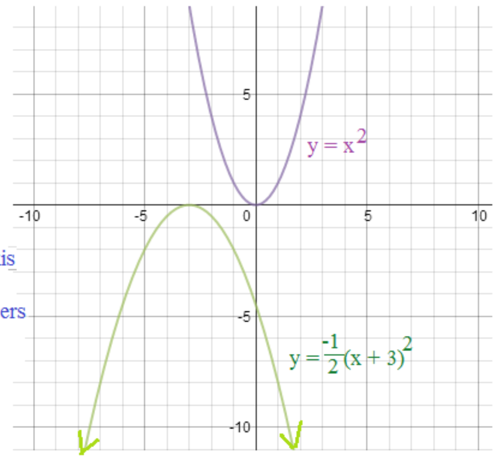
horizontal shift (c):
3 units to the left

amplitude (a): 1/2
(shrink by 2)

reflection over the x-axis

domain: all real numbers
$(-\infty, \infty)$

range: $f(x) \leq 0$
$(-\infty, 0]$



3) $y = 2\sqrt{x - 1} + 3$

parent function:
$y = \sqrt{x}$

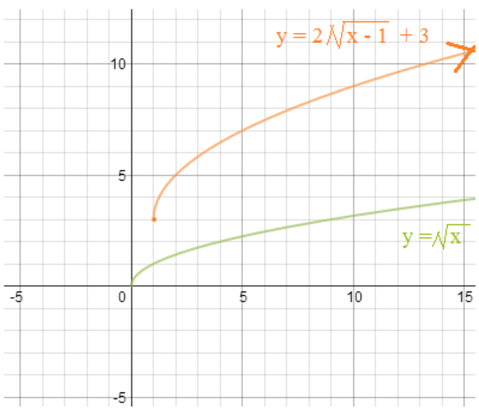
horizontal shift (c):
1 unit to the right

vertical shift (d):
3 units up

amplitude (a):
vertical stretch by 2

domain: $x \geq 1$
(term under radical must be non-negative)

range: $y \geq 3$



4) $y = -|3x - 3| + 5$

**first, rewrite the equation

$y = -|3(x - 1)| + 5$

parent function: $y = |x|$

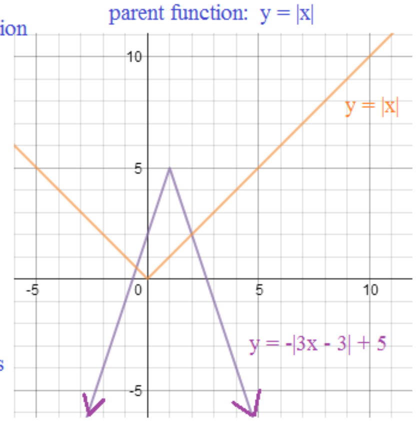
horizontal shift (c):
1 unit to the right

vertical shift (d):
5 units up

reflected over the x-axis

"compression" (b):
1/3 of the width

domain: all real numbers
range: $y \leq 5$



5) $y = -(x - 3)^3 + 3$

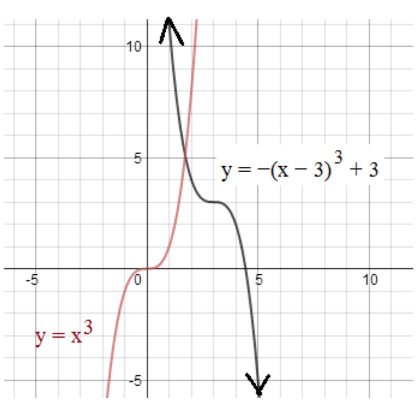
parent function:
$y = x^3$ (cubic)

horizontal shift (c):
3 units to the right

vertical shift (d):
up 3 units

reflected over the x-axis

domain: all real numbers
range: all real numbers



6) $g(x) = \frac{(x + 4)^2}{2}$

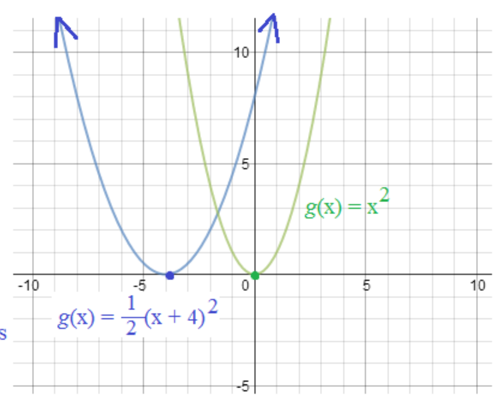
$g(x) = \frac{1}{2}(x + 4)^2$

parent function:
$y = x^2$

horizontal shift (c):
4 units to the left

amplitude (a):
1/2, so it shrinks

domain: all real numbers
range: $g(x) \geq 0$



RELATED TOPIC:

Graphing: Completing the square and transformations

Example: Graph the function $f(x) = x^2 - 2x$

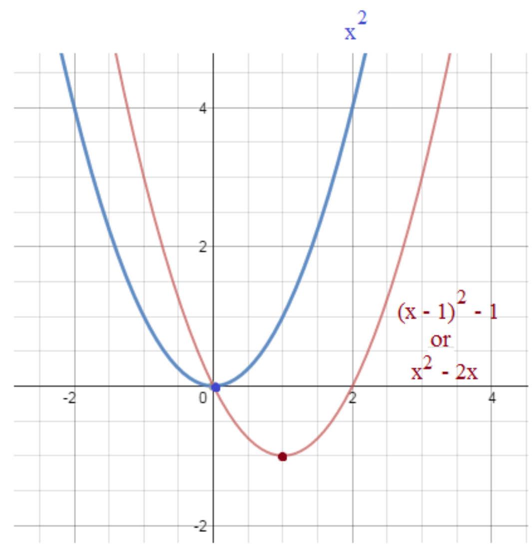
This quadratic does not have a "direct parent function"....
But, if we complete the square:

$$x^2 - 2x + 1 \longrightarrow (x-1)(x-1) = (x-1)^2$$

Then, compare the result with the original function:

$$x^2 - 2x + 1 = (x-1)^2$$

so, $x^2 - 2x = \boxed{(x-1)^2 - 1}$



Now, let's graph:

parent function: x^2

horizontal shift: 1 unit to the right

vertical shift: 1 unit down

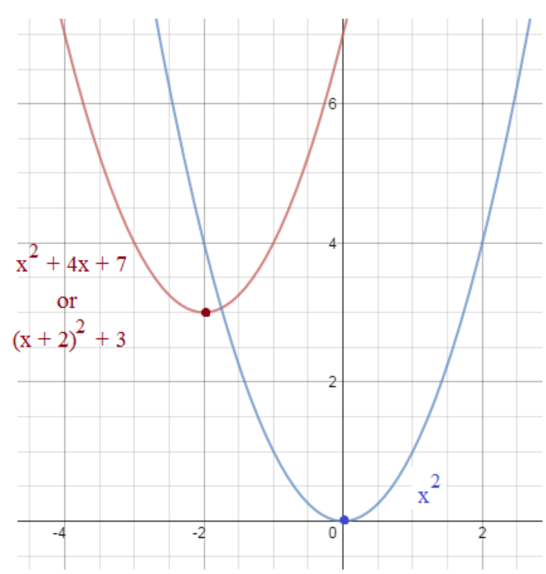
Example: Graph the function $x^2 + 4x + 7$
(by completing the square and using the parent function)

Take the quadratic term and linear term, $x^2 + 4x$, and complete the square

$$x^2 + 4x + 4 \longrightarrow (x+2)(x+2) = (x+2)^2$$

$$x^2 + 4x + 4 = (x+2)^2$$

so, $x^2 + 4x + 7 = \boxed{(x+2)^2 + 3}$



Now, let's graph:

parent function: x^2

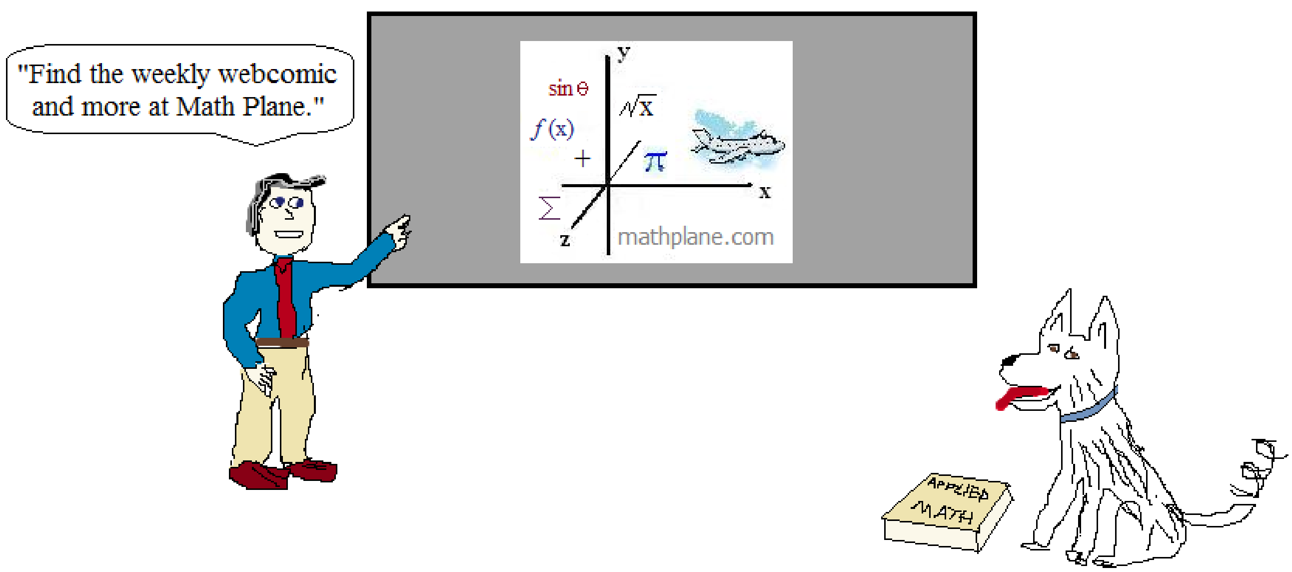
horizontal shift: 2 units to the left

vertical shift: 3 units up

Thanks for visiting the site. (Hope it helped!)

If you have questions, suggestions, or feedback, contact us at mathplane.com

Cheers…



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