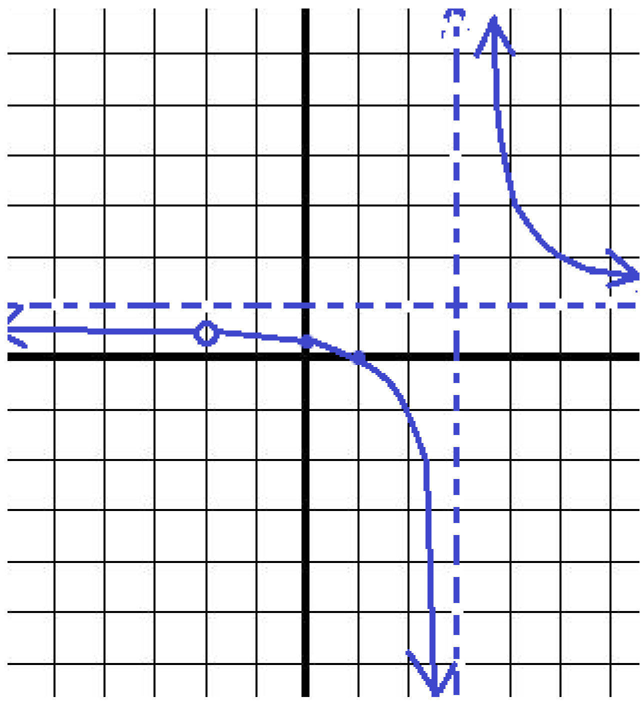
Sketching Rational Expressions

Examples and Practice Test (w/solutions)



Includes Asymptotes, “holes”, intercepts, inequalities, and more…

Sketching Rational Expressions

Four-step approach to sketching rational expression:

Step 1: Find vertical asymptote(s) or "holes"
(all values of x that cannot exist)

Step 2: Find horizontal asymptote
(end behavior of function)

Step 3: Identify the y-intercept
(plug zero into x -- it's the easiest point to find!)

Step 4: Identify any x-intercept(s) or points in other regions of the graph
(this helps shape the sketch)

"Rules for Horizontal Asymptote":

bottom heavy: degree of numerator < degree of denominator
horizontal asymptote: $y = 0$

top heavy: degree of numerator > degree of denominator
no horizontal asymptote

Same: degree of numerator = degree of denominator

$$\text{horizontal asymptote} = \frac{\text{lead coefficient of numerator}}{\text{lead coefficient of denominator}}$$

Note: if degree of numerator is *one more* than degree of the denominator, there will be a *slant asymptote.*

Example: $g(x) = \dfrac{3x - 6}{x + 1}$

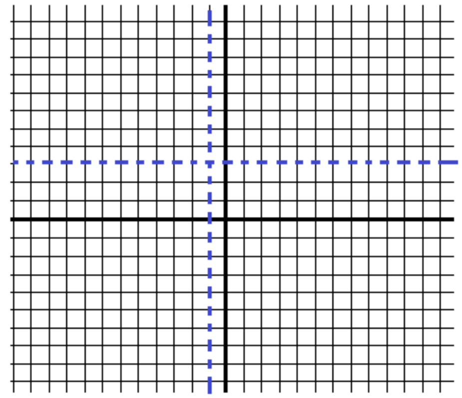
Vertical asymptote: $x = -1$

$g(-1) = \dfrac{-9}{0}$ Undefined...

Horizontal asymptote: $y = 3$

The function will never equal 3
(and the end behavior in both directions is 3)

function is neither top heavy
nor bottom heavy;
coefficients are $3/1 = 3$



At this point, we've established 'boundaries' of the sketch.
We can use this 'frame' to sketch the function..

y-intercept: $(0, -6)$

$g(0) = \dfrac{-6}{1} = -6$

x-intercept: $(2, 0)$

set $\dfrac{3x - 6}{x + 1} = 0$ then, $x = 2$

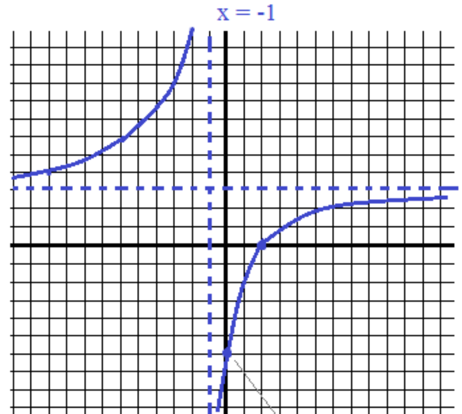
Using these 2 intercepts and the asymptotes,
we can sketch the lower right portion.
(to check your answer, plot other points)

Test point(s) *on the other side* of an asymptote:

$g(-2) = 12$

$g(-4) = 6$

$g(-10) = 4$



Sketching Rational Expressions

Removable discontinuites ("Holes")
-- A hole in the graph
-- A discontinuity that could be 'repaired' by filling it with a point
-- Where the *limit of the function exists*, but doesn't equal the value at that point in the function (eg. it's undefined)

Four-step approach to sketching rational expression:

Step 1: Find vertical asymptote(s) or "holes"
(all values of x that cannot exist)
Step 2: Find horizontal asymptote
(end behavior of function)
Step 3: Identify the y-intercept
(plug zero into x -- it's the easiest point to find!)
Step 4: Identify any x-intercept(s) or points in other regions of the graph
(this helps shape the sketch)

Example: Graph the function

$$f(x) = \frac{x^2 + x - 2}{x^2 - x - 6}$$

Find vertical asymptote(s) or "holes"

factor numerator and denominator

$$\frac{x^2 + x - 2}{x^2 - x - 6} = \frac{(x+2)(x-1)}{(x+2)(x-3)}$$

VA: at x = 3, the function is $\frac{10}{0} \rightarrow$ undefined

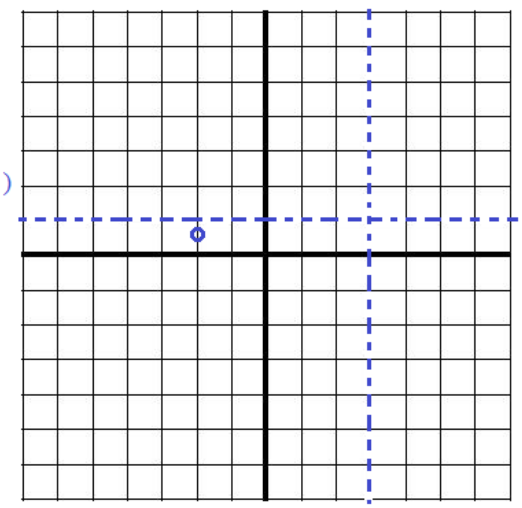
"hole": at x = -2, the function is $\frac{0}{0} \longrightarrow$ indeterminate

Note: excluding the (x + 2), $g(-2)$ would equal $\frac{(-2-1)}{(-2-3)} = 3/5$

VA: x = 3

Hole: $(-2, \frac{3}{5})$

HA: y = 1



Find horizontal asymptote:

Degree of numerator: 2
Degree of denominator: 2

Since they are equal, we look at the lead coefficients:

$$\frac{\text{numerator lead coefficient}}{\text{denominator lead coefficient}} = \frac{1}{1} \longrightarrow y = 1$$

Identify the y-intercept:

$$f(0) = \frac{(0)^2 + (0) - 2}{(0)^2 - (0) - 6} = \frac{1}{3}$$

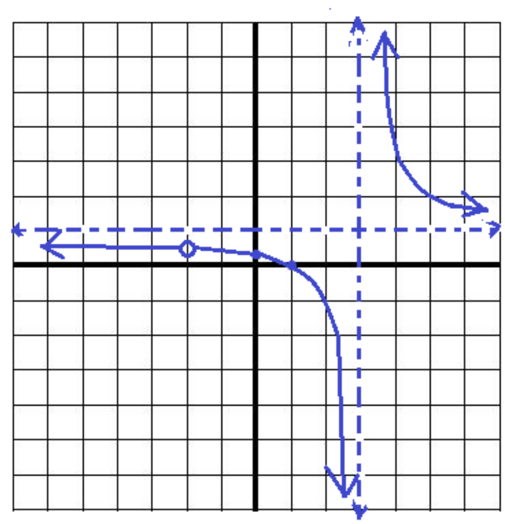
y-intercept: $(0, \frac{1}{3})$

Identify the x-intercept:

find where $f(x) = 0$

x-intercept: (1, 0)

$$\frac{x^2 + x - 2}{x^2 - x - 6} = 0 \qquad \frac{(x+2)(x-1)}{(x+2)(x-3)} = 0 \qquad x = 1$$



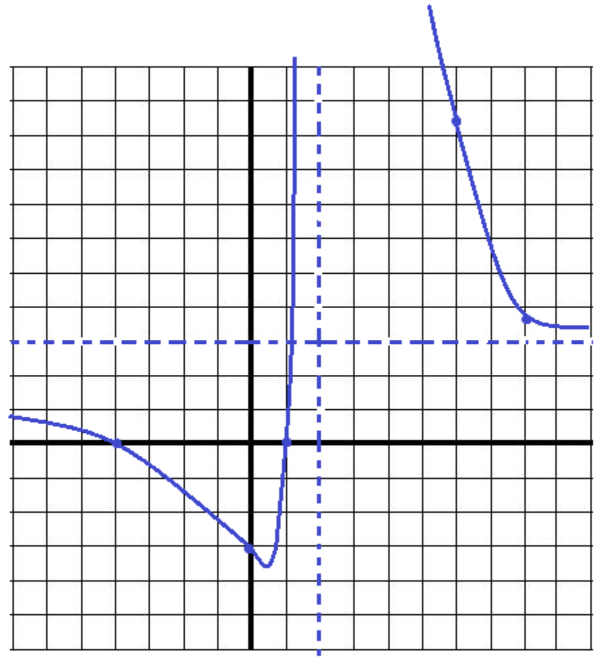
Find a point(s) in the other region(s):

$f(5) = 2$ (5, 2) is a point..

$f(4) = 3$ (4, 3) is a point..

Sketching Rational Expressions

Example: $y = \dfrac{3(x-1)(x+4)}{(x-2)^2}$



Step 1: Vertical Asymptote
(there are no "holes")

Find where the equation is undefined.
(when is denominator = 0?)

$(x-2)(x-2) = 0$ $\quad x = 2$

Step 2: Horizontal Asymptote

Is the equation *top heavy, bottom heavy or same*?

Degree of numerator: 2
Degree of denominator: 2 same

Since they are the same, what are the lead coefficients?

lead coefficient (numerator): $\boxed{3}x^2$
lead coefficient (denominator): $\boxed{1}x^2$ $\quad y = 3$

Step 3: Find the y-intercept (a simple point to find!)

let x = 0: $\dfrac{3(-1)(4)}{(-2)^2} = -3$ $\quad$ (0, -3)

Step 4: Find other points (around the asymptotes)

x-intercepts are
(1, 0) and (-4, 0)

let x = 3, y = 42

let x = 6, y = $9\frac{3}{8}$

let x = 8, y = $3\frac{15}{16}$

let x = 3/2, y = 33

> Note: These are just sketches, demonstrating key intercepts, asymptotes, and end behavior.
> (they may not necessarily be exact)

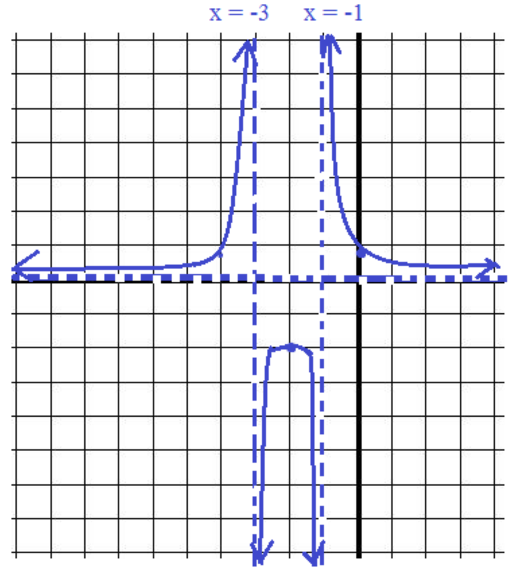
> Note: The function cannot exist at the vertical asymptote. But, it may exist on the horizontal asymptote!

Example: $f(x) = \dfrac{2}{x^2 + 4x + 3}$

Find vertical asymptotes: where is the denominator = 0?

$x^2 + 4x + 3 = (x+1)(x+3)$

At x = -1 or -3, the function is undefined.



Determine horizontal asymptote:

the function is *bottom heavy* -- end behavior and asymptote is the x-axis (y = 0)

Find y-intercept: $(0, \frac{2}{3})$

x-intercepts? since numerator is a constant, there is no x-intercept

Identify points around the asymptotes:

$f(-2) = \dfrac{2}{-1} = -2$

$f(-1/2) = 1.6$

$f(1) = \dfrac{2}{8} = \dfrac{1}{4}$

$f(3) = \dfrac{2}{24} = \dfrac{1}{12}$

$f(-4) = \dfrac{2}{3}$
(above the horizontal asymptote)

$f(-6) = \dfrac{2}{15}$

Sketching Rational Expressions

Slant Asymptote -
-- "oblique" asymptote
-- A linear asymptote that is not parallel to the x-axis
-- Like vertical and horizontal asymptotes, slant asymptotes are lines that a graph approaches.

Example: $y = \dfrac{x^2 - 6x + 5}{x + 4}$

$y = \dfrac{(x-5)(x-1)}{(x+4)}$

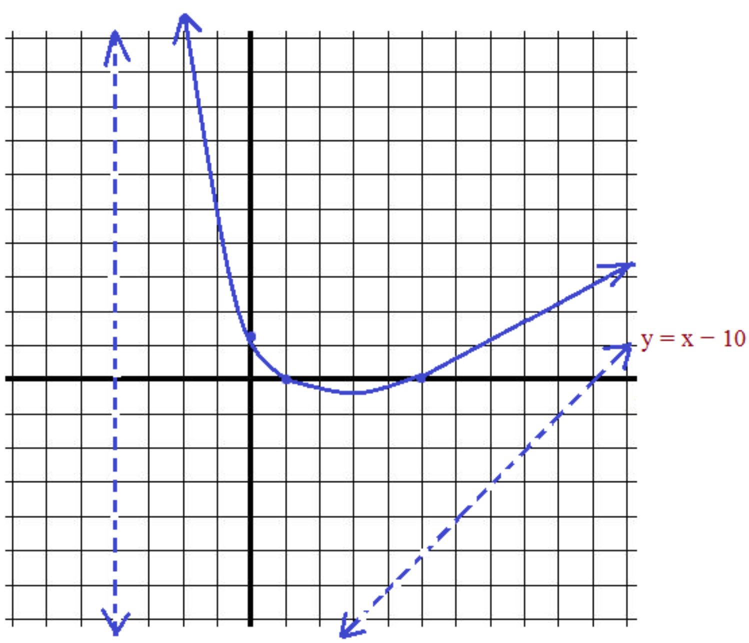
Vertical Asymptote: $x = -4$

Horizontal Asymptote: *top heavy*, so there is none...

Slant Asymptote: $y = x - 10$

y-intercept: $(0, \frac{5}{4})$

x-intercepts: (1, 0) (5, 0)



points include: $(-2, \frac{21}{2})$
$(8, \frac{7}{4})$

and, left of the vertical asymptote,

$(-6, \frac{77}{-2})$

(-9, -28)

(-19, -32)

"Rules for Horizontal Asymptote":

bottom heavy: degree of numerator < degree of denominator
horizontal asymptote: $y = 0$

top heavy: degree of numerator > degree of denominator
no horizontal asymptote

Same: degree of numerator = degree of denominator

horizontal asymptote = $\dfrac{\text{lead coefficient of numerator}}{\text{lead coefficient of denominator}}$

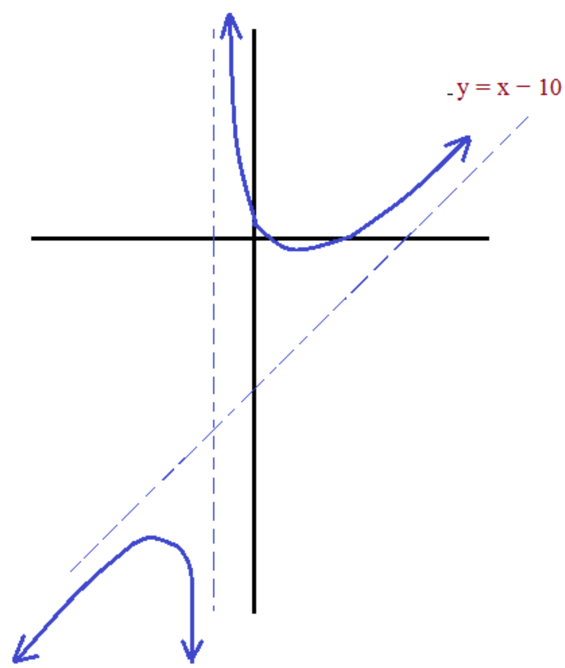
Note: if degree of numerator is *one more* than degree of the denominator, there will be a *slant asymptote.*

Since the degree of the numerator (2) is one greater than the degree of the denominator (1), there is a slant asymptote..

$$\begin{array}{r} x - 10 + \frac{45}{x+4} \\ x+4 \overline{\big) x^2 - 6x + 5} \\ -(x^2 + 4x) \\ \hline -10x + 5 \\ -(-10x - 40) \\ \hline 45 \end{array}$$

Using long division, we can see that as the function goes to infinity,
$\frac{45}{x+4}$ approaches 0

leaving $x - 10$ as the end behavior!



Brilliant mathematicians see things differently...

PRACTICE QUIZ

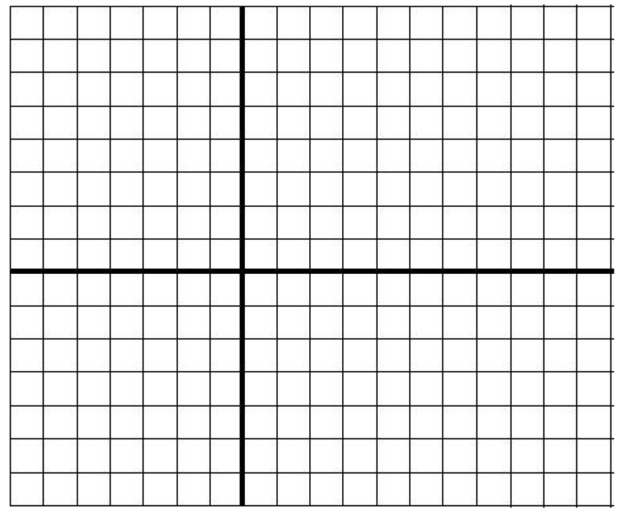
With

SOLUTIONS

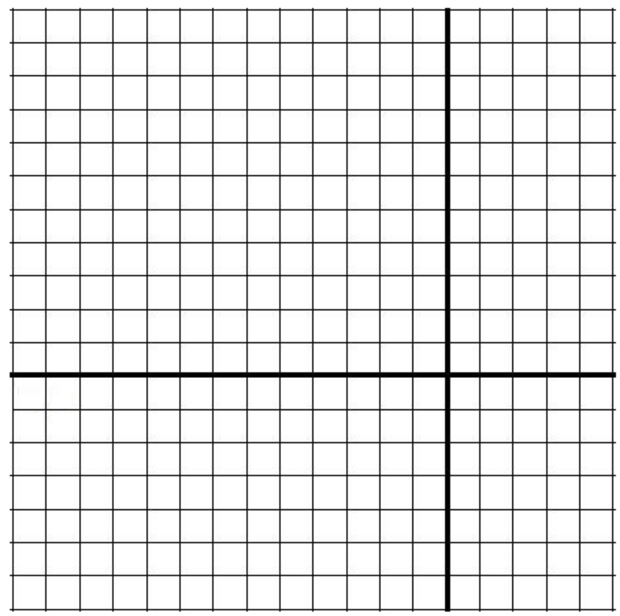
Sketching Rational Expressions

Sketch the following.
Identify any asymptotes, "holes", and intercepts.

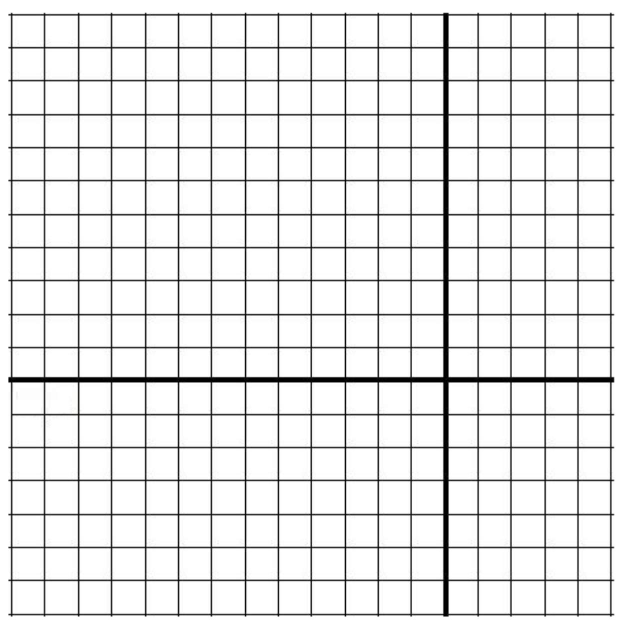
1) $f(x) = \dfrac{x+3}{x-6}$



2) $y = \dfrac{3x^2 - 12}{x^2 + 6x + 8}$



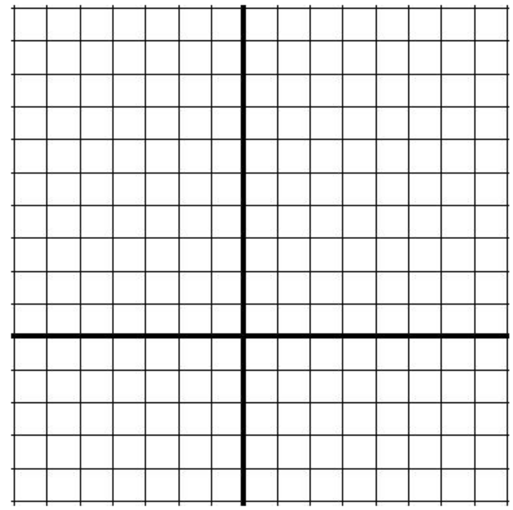
3) $y = \dfrac{3x - 12}{x^2 + 6x + 8}$



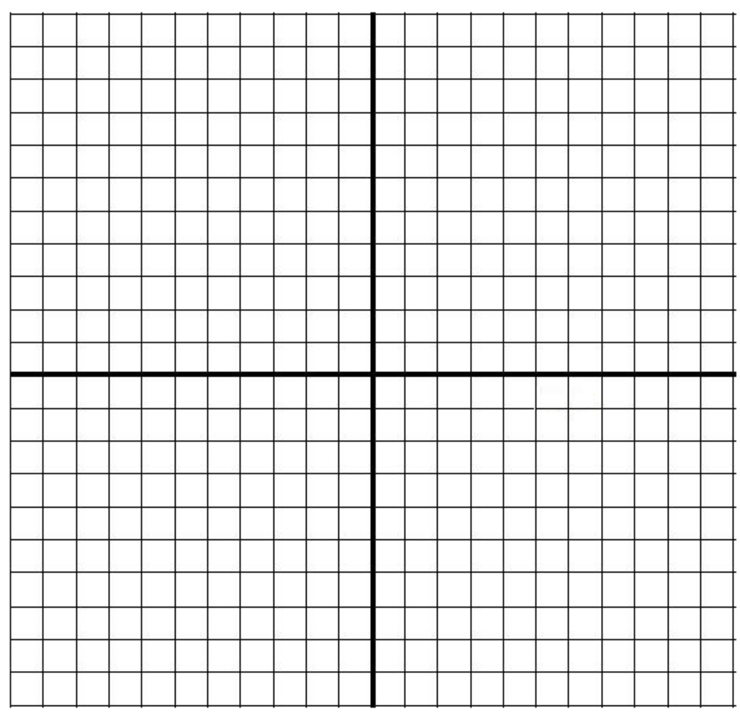
Sketching Rational Expressions

Sketch the following.
Identify any asymptotes, "holes", and intercepts.

4) $f(x) = \dfrac{2}{x^2 - 4} + 3$



5) $y = \dfrac{x^2 - 7x + 10}{x + 1}$



6) $y = \dfrac{(x-3)(x+2)}{(x+2)(x-1)}$

Describe this function's characteristics:

Horizontal asymptote:

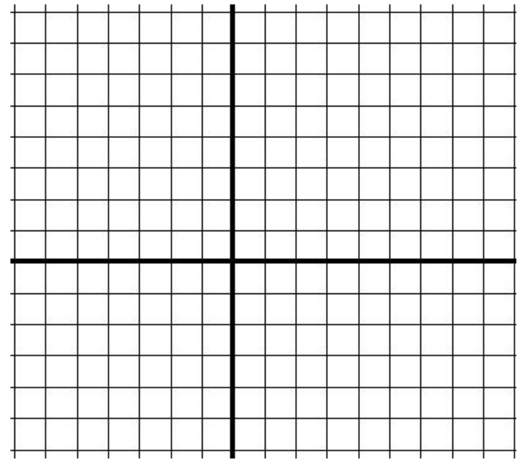
Vertical asymptote(s):

Hole(s)?:

x-intercept(s)?:

y-intercept:

Then graph:



7) Sketch the following rational expression: $f(x) = \dfrac{x^2 - 7x + 10}{3x^2 - 2x - 8}$

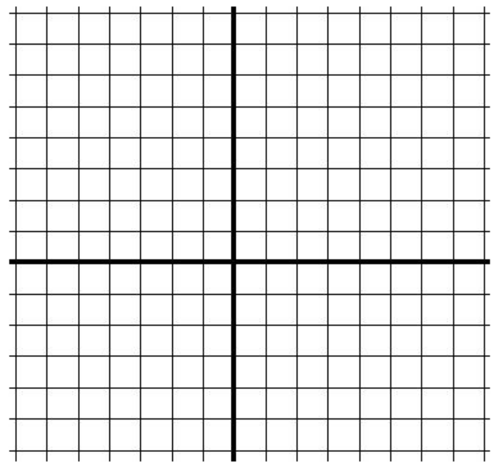
Horizonal Asymptote(s): _______

Vertical Asymptote(s): _______

Hole(s): ________

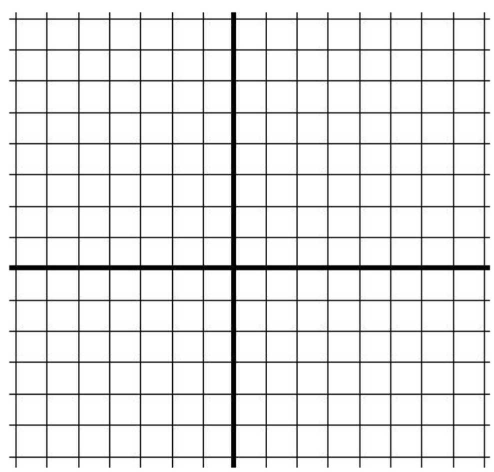
x-intercept(s): _______

y-intercept: _______



8) Sketch the following $g(x) = \dfrac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$

List the horizontal, vertical, and slant asymptotes,
x and y-intercepts



Determining the rational expression

Write a rational function that has the following characteristics:

Zeros: $x = 7 \quad x = 2$

"Hole": $x = 5$

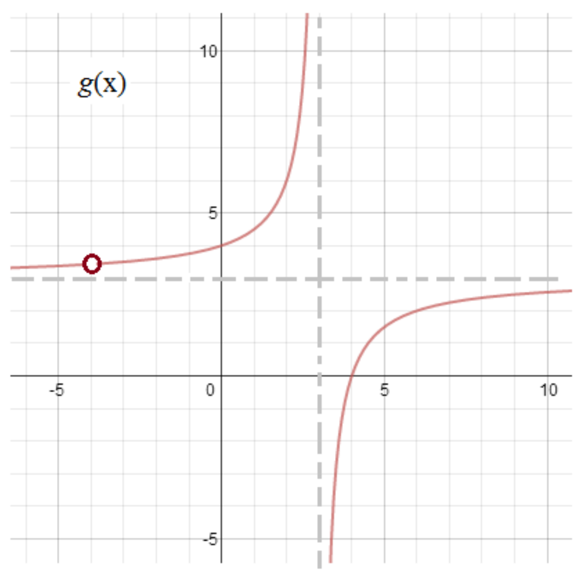
Horizontal Asymptote: $y = 3$

Vertical Asymptotes: $x = 3 \quad x = -4$

$f(x) =$ ____________________

What is a rational expression depicted by the graph?

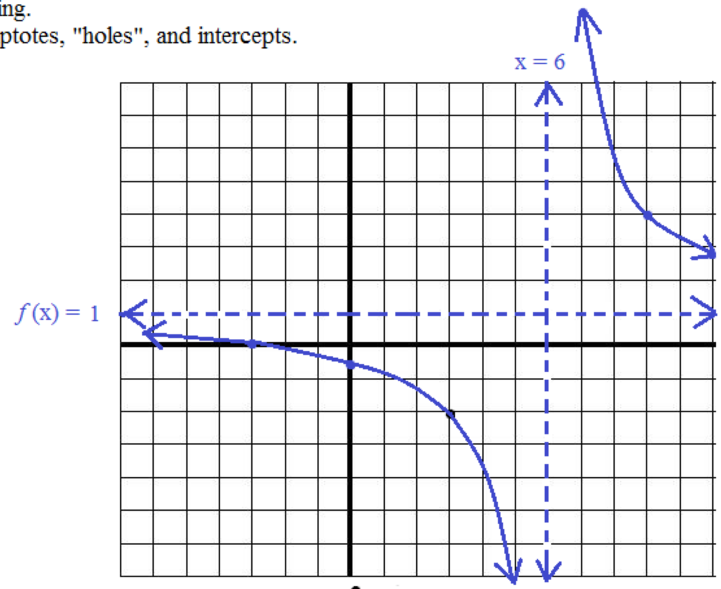
$g(x) =$ ____________________



Sketching Rational Expressions

Sketch the following.
Identify any asymptotes, "holes", and intercepts.

1) $f(x) = \dfrac{x+3}{x-6}$



Vertical Asymptote: x = 6

Horizontal Asymptote: y = 1

(degree of numerator = degree of denominator;
lead coefficients: 1/1 = 1)

y-intercept: $(0, \frac{-1}{2})$

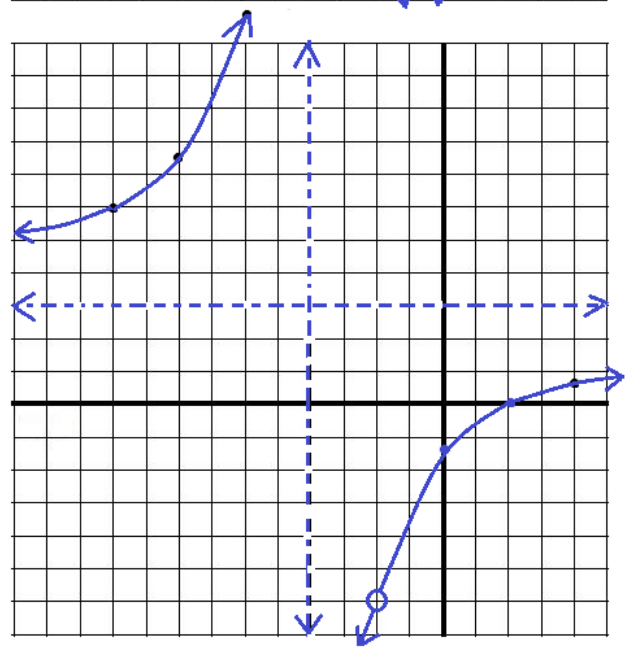
$f(0) = \frac{3}{-6} = \frac{-1}{2}$

x-intercept: (-3, 0)

$f(x) = 0$ when $x = -3$

(3, -2)
(5, -8)
(7, 10)
(9, 4)

2) $g(x) = \dfrac{3x^2 - 12}{x^2 + 6x + 8} = \dfrac{3(x^2-4)}{(x+2)(x+4)} =$



factor $\dfrac{3(x+2)(x-2)}{(x+2)(x+4)}$

VA: $x = -4$

"Hole": at x = -2, the equation is $\frac{0}{0}$

Removing (x + 2), the equation is $\dfrac{3(-2-2)}{(-2+4)} = -6$

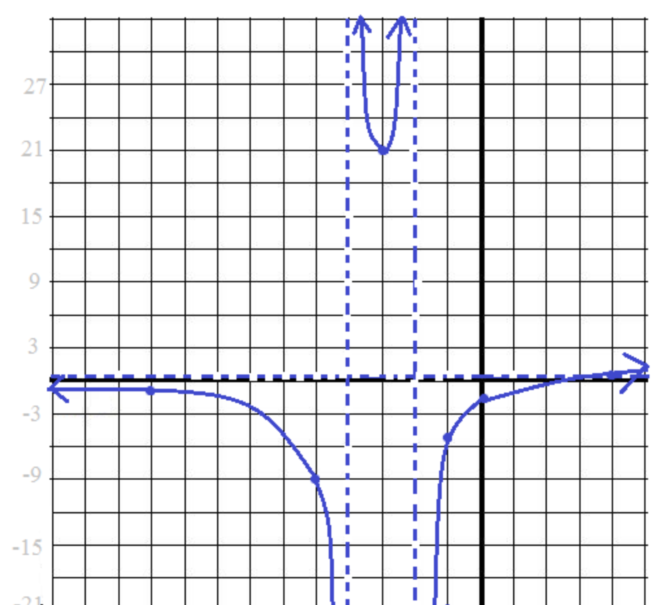
$(-2, -6)$

HA: $g(x) = 3$ (y = 3) degree numerator = deg. of denom.
lead coefficient num/lead coef. den.
= 3/1

x-intercept: (2, 0)
y-intercept: (0, -3/2)

points include: (4, 3/4)
(-6, 12)
(-8, 7 1/2)
(-10, 6)

3) $y = \dfrac{3x - 12}{x^2 + 6x + 8}$



factor each polynomial $\dfrac{3(x-4)}{(x+4)(x+2)}$

Vertical Asymptote: $x = -2$ $x = -4$

Horizontal Asymptote: y = 0

(bottom heavy: degree of numerator < deg. of denominator)

x-intercept: (4, 0) $\dfrac{3x - 12}{(x+4)(x+2)} = 0$ x = 4

y-intercept: (0, -3/2) if x = 0, then y = -12/8 = -3/2

Identify points in other parts of the graph
(around the asymptotes):

at x = 6, y = $\frac{6}{80}$ (6, 3/40)

at x = -1, y = $\frac{-15}{3}$ (-1, -5)

at x = -3, y = $\frac{-21}{-1}$ (-3, 21)

at x = -5, y = $\frac{-27}{3}$ (-5, -9)

at x = -10, y = -42/48 (-10, -7/8)

(note: you can cross the horizontal asymptote...
It describes the end behavior of the function)

Sketching Rational Expressions

Sketch the following.
Identify any asymptotes, "holes", and intercepts.

Solutions

4) $f(x) = \dfrac{2}{x^2 - 4} + 3$

Note: the first part is a rational expression; the second part is a "shift" up 3 units

Vertical Asymptotes: where is $f(x)$ undefined?

$\dfrac{2}{(x+2)(x-2)} + 3$

$x = 2 \quad x = -2$

Horizontal Asymptote: The first part is 'bottom heavy', so its end behavior is 0. Then, we add 3 from the 2nd part..

$y = 3$

$f(x) = 3$

y-intercept: $f(0) = \dfrac{2}{0-4} + 3 \qquad (0, \dfrac{5}{2})$

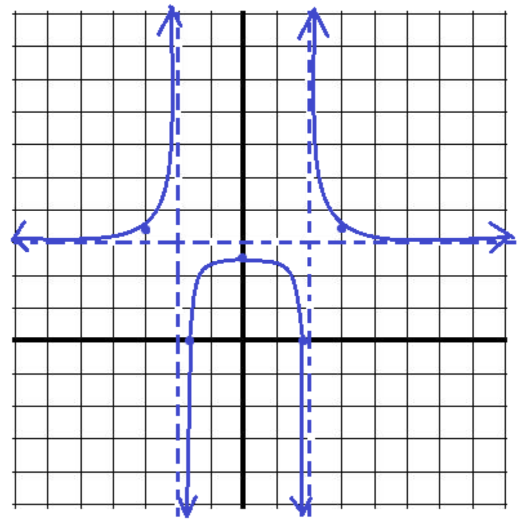
x-intercept: $f(x) = 0$

$\dfrac{2}{x^2 - 4} = -3 \qquad x^2 = \dfrac{2}{-3} + 4$

$x = \sqrt{\dfrac{10}{3}}$ and $-\sqrt{\dfrac{10}{3}}$

Other points:

$(3, 3\frac{2}{5}) \qquad (-3, 3\frac{2}{5})$



5) $y = \dfrac{x^2 - 7x + 10}{x + 1}$

Factor numerator: $\dfrac{(x-2)(x-5)}{x+1}$

VA: $x = -1$

Holes: None

HA: numerator degree: 2
denominator degree: 1
No horizontal asymptote..

Slant Asymptote: YES!

$$\begin{array}{r}
x - 8 + \dfrac{18}{x+1} \\
x + 1 \overline{\big) \; x^2 - 7x + 10} \\
-(x^2 + x) \\
\hline
-8x + 10 \\
-(-8x - 8) \\
\hline
18
\end{array}$$

As x goes to infinity or negative infinity, this term approaches 0

slant asymptote: $y = x - 8$

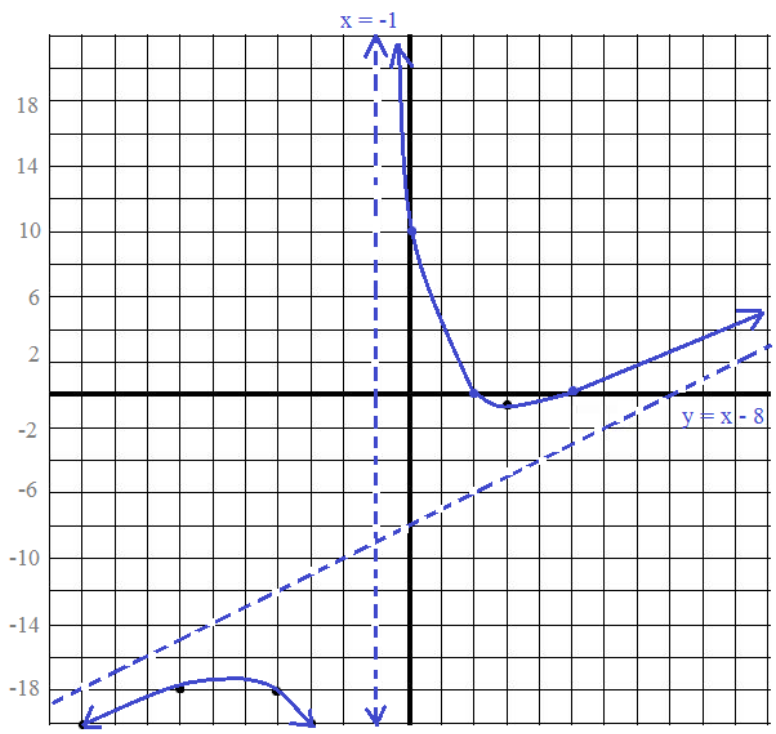
x-intercepts: (2, 0) (5, 0)

y-intercept: (0, 10)

find other points:

x	y
3	-1/2
-3	-20
-4	-18
-7	-18
-10	-20

note: to fit the graph, horizontal spacing is 1 unit
vertical spacing is 2 units



Sketching Rational Expressions

SOLUTIONS

6) $y = \frac{(x-3)(x+2)}{(x+2)(x-1)}$ $\frac{x^2 - x - 6}{x^2 + x - 2}$

Describe this function's characteristics:

Horizontal asymptote: $y = 1$ — degree of numerator: 2, degree of denominator: 2, coefficients: 1/1

Vertical asymptote(s): $x = 1$ — denominator equals zero when $x = 1$

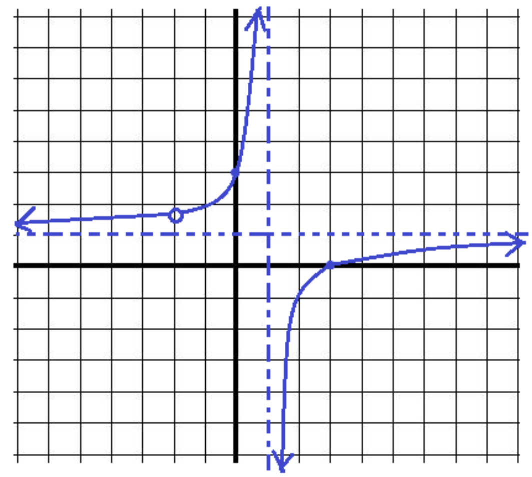
Hole(s)?: $(-2, \frac{5}{3})$ — equation is 'indeterminate' (0/0) when $x = -2$; at $x = -2$ $\frac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$ $y = \frac{-5}{-3}$

x-intercept(s)?: $(3, 0)$ — when $y = 0$, $x = 3$

y-intercept: $(0, 3)$ — when $x = 0$, $y = -6/-2 = 3$

Then graph:

points include $(2, -1)$ and $(5, 1/2)$



7) Sketch the following rational expression: $f(x) = \frac{x^2 - 7x + 10}{3x^2 - 2x - 8} = \frac{(x-2)(x-5)}{(3x+4)(x-2)}$

Horizonal Asymptote(s): $y = \frac{1}{3}$

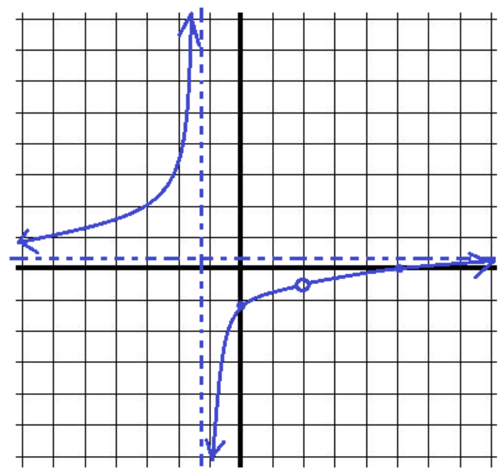
Vertical Asymptote(s): $x = \frac{-4}{3}$

Hole(s): $(2, \frac{-3}{10})$ — 0/0 expression is indeterminate when $x = 2$; substitute 2 into other terms: $\frac{\cancel{(x-2)}(x-5)}{(3x+4)\cancel{(x-2)}} = \frac{-3}{10}$

x-intercept(s): $(5, 0)$

y-intercept: $(0, \frac{-5}{4})$ — $\frac{\cancel{x^2} - \cancel{7x} + 10}{\cancel{3x^2} - \cancel{2x} - 8}$

The x and y intercepts reveal the sketch on the right side of the asymptote... Then, test x = -2 to confirm the left side of the asymptote....



8) Sketch the following $g(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$ $g(x) = \frac{x(x+2)(x-3)}{-3(x-2)(x+3)}$

List the horizontal, vertical, and slant asymptotes x and y-intercepts

top heavy AND the degree of numerator is 1 more than degree of denominator... Slant (or oblique) asymptote..

x-intercepts: $(0, 0)$ $(-2, 0)$ $(3, 0)$

y-intercept: $(0, 0)$

vertical asymptotes: $x = 2$ $x = -3$

horizontal asymptote: none

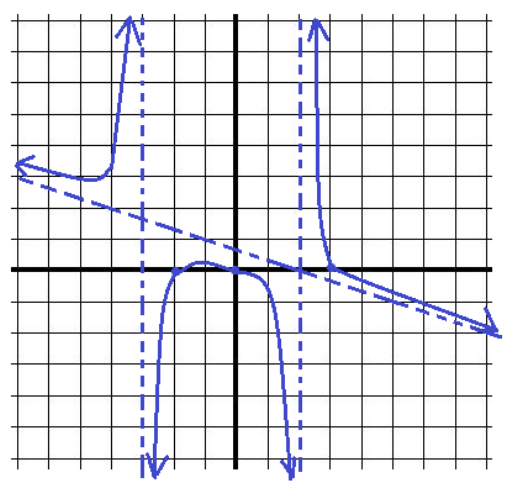
oblique/slant asymptote: $y = \frac{-1}{3}x + \frac{2}{3} + 0$

at $x = -4$, $y = \frac{-56}{-18}$ (above the slant asymptote)

use long division to find slant asymptote

$$\begin{array}{r} -1/3x + 2/3 \\ -3x^2 - 3x + 18 \,\big)\, x^3 - x^2 - 6x \\ -\;(x^3 + x^2 - 6x) \\ -2x^2 + 0 \\ -\;(-2x^2 - 2x + 12) \\ 2x - 12 \end{array}$$

$-1/3x + 2/3 + \frac{2x - 12}{-3x^2 - 3x + 18}$



Determining the rational expression

SOLUTIONS

Write a rational function that has the following characteristics:

Zeros: $x = 7 \quad x = 2$

"Hole": $x = 5$

Horizontal Asymptote: $y = 3$

Vertical Asymptotes: $x = 3 \quad x = -4$

$$f(x) = \frac{3(x-7)(x-2)(x-5)}{(x+4)(x-3)(x-5)}$$

zeros: 7 and 2 ---> $(x-2)(x-7)$

(numerator)

vertical asymptotes: x = 3 and x = -4 ----> $(x-3)(x+4)$

(denominator)

hole at x = 5 ----> $(x-5)$ (numerator AND denominator)

Horizontal asymptote: end behavior is y = 3

a) degrees are the same: (asymptote y = 1)
so, multiply by 3

OR

b) degrees are the same (asymptote y = 1)
add 2

$$f(x) = \frac{(x-7)(x-2)(x-5)}{(x+4)(x-3)(x-5)} + 2$$

What is a rational expression depicted by the graph?

$$g(x) = \frac{3(x-4)(x+4)}{(x-3)(x+4)}$$

Vertical asymptote: x = 3

Hole at x = -4

x-intercept: (4, 0)

$$\frac{(x+4)(x-4)}{(x+4)(x-3)}$$

Horizontal asymptote: y = 3

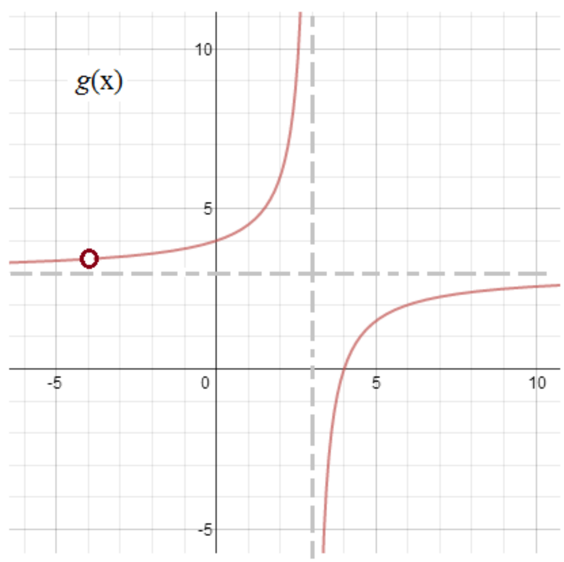
(the degree of numerator: 2
degree of denominator: 2
lead coefficients 3/1)

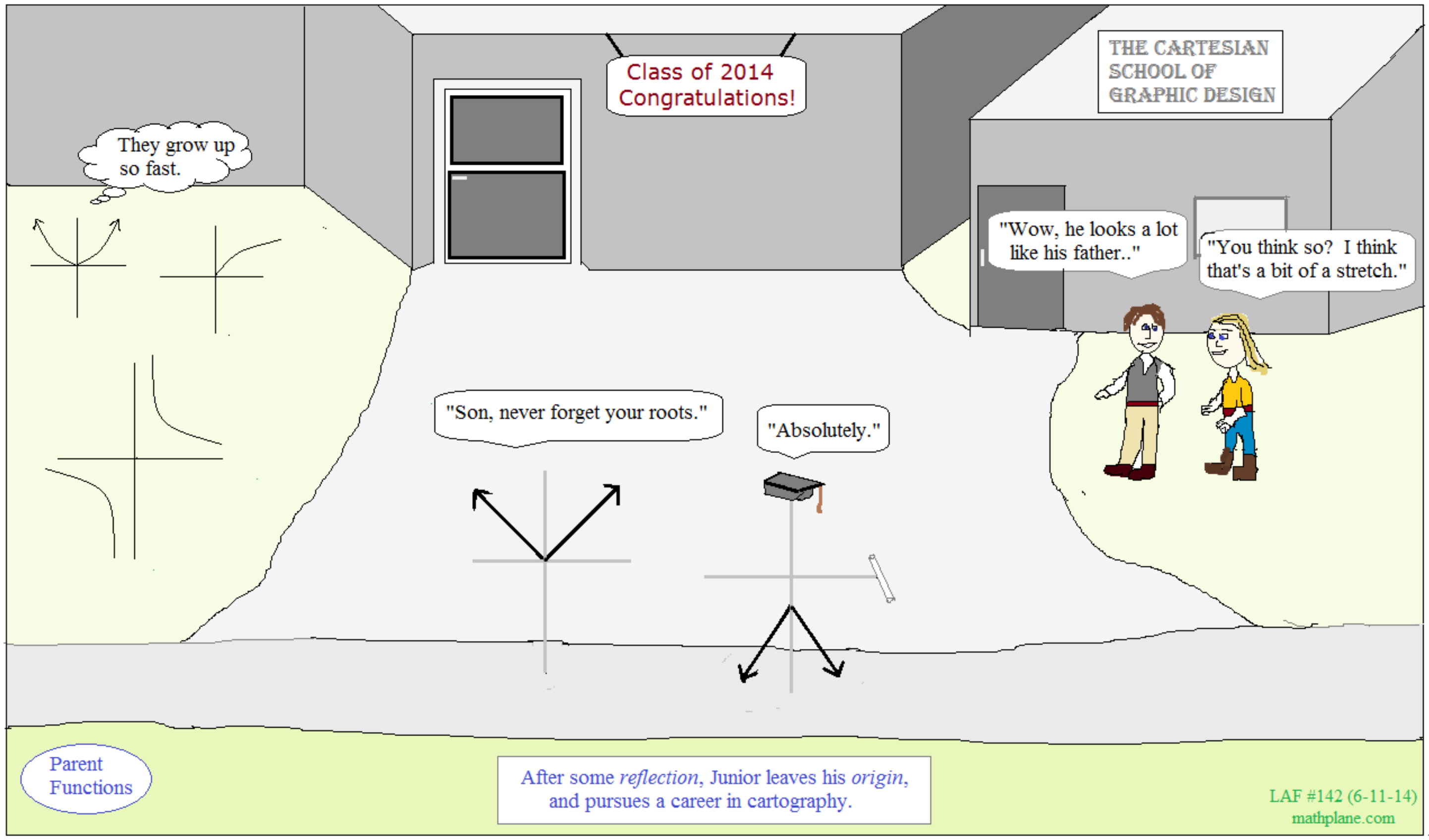
Using (0, 4) in the graph:

$$\frac{(0+4)(0-4)}{(0+4)(0-3)} \cdot a = 4$$

$$\frac{4}{3}a = 4$$

a = 3





EXTRA TOPICS and MORE PRACTICE-→

Sketching Rational Expressions: Extra topics

"Double (Horizontal) Asymptotes"

Example: What is the horizontal asymptote of $y = \dfrac{x + 6}{|x| + 2}$?

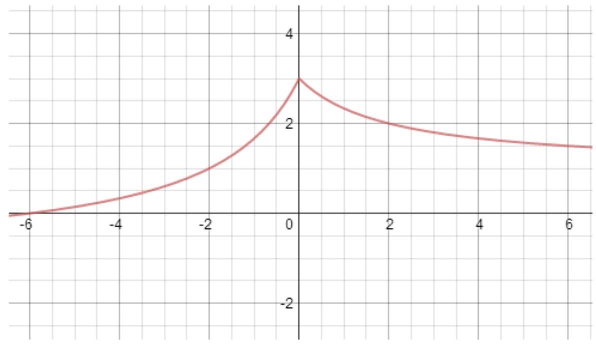
The horizontal asymptote is the "end behavior"....

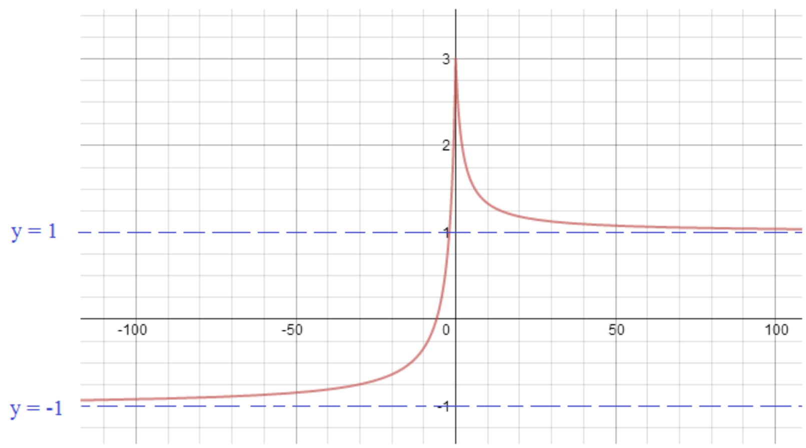
If we plot enough points, a pattern will emerge, and we'll notice that the end behavior is different in each direction!

As x gets larger, the graph approaches 1

And, as -x gets smaller, the graph approaches -1

x	y
-100	-94/102
-10	-1/3
-6	0
-1	5/3
0	3
1	7/3
6	3/2
10	4/3
100	106/102





Ordinarily, to determine horizontal asymptotes, we simply determine the degree of the numerator and the degree of the denominator... Then, look at the coefficients..

HOWEVER, in this case the denominator is NOT a polynomial, because it has a square root as it's lead term...

Why is an absolute value NOT a polynomial?

One explanation: it's not a smooth curve.
But, a better explanation may be this:

$$|x| = \sqrt{x^2} \longrightarrow (x^2)^{1/2}$$

Polynomials DO NOT have fractional exponents...

Sketching Rational Expressions: Extra topics

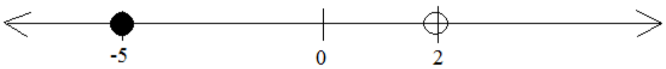
"Sketching Rational Expression Inequalities"

Example a: $\frac{(x+5)}{(x-2)} \leq 0$

Zero: when numerator = 0 $\quad x = -5$

Undefined: when denominator = 0 $\quad x = 2$

(there are no 'holes'/removable discontinuities)

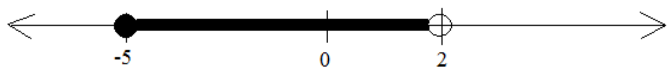


since the expression is < or = , the -5 value is included... (closed circle)
however, since the expression is undefined at 2, that value is excluded... (open circle)

Test -10: $\frac{(-10+5)}{(-10-2)} = 5/12$ ✗

Test 0: $\frac{(0+5)}{(0-2)} = -5/2$ ✓

Test 4: $\frac{(4+5)}{(4-2)} = 9/2$ ✗

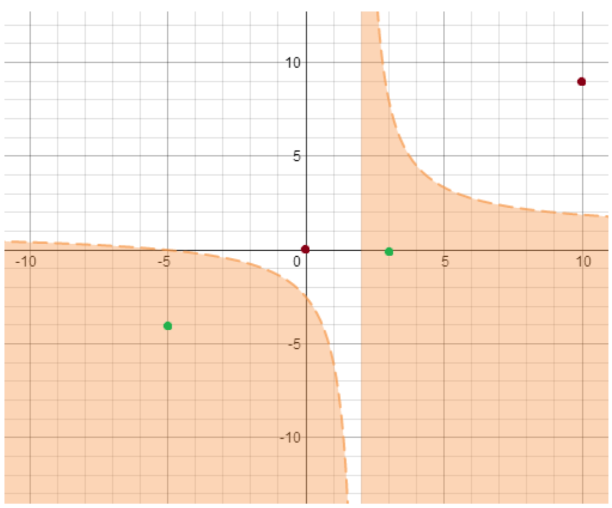


Graphing steps (number line)

1) Find 'critical values' where function is equal
OR where function is undefined

2) Determine if open circle or closed circle
3) Test Regions (and shade)

Example b: $\frac{(x+5)}{(x-2)} > y$



Graphing steps (coordinate plane)

1) Find 'critical values' where function equals 0
OR where function is undefined

2) Recognize dashed lines/curves or solid lines/curves
3) Test regions (and shade)

This function has a zero at -5, and it has a vertical asymptote at x = 2

Since the inequality is <, the curves are 'dashed'.
(and, the vertical asymptote is always 'dashed')

Test (-5, -4): $\frac{(-5+5)}{(-5-2)} > -4$ $\quad 0 > -4$ ✓

Test (0, 0): $\frac{(0+5)}{(0-2)} > 0$ $\quad -5/2 \not> 0$ ✗

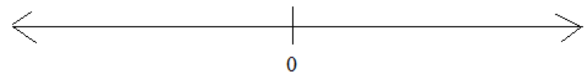
Test (3, 0): $\frac{(3+5)}{(3-2)} > 0$ $\quad 8 > 0$ ✓

Test (10, 9): $\frac{(10+3)}{(10-2)} > 9$ $\quad 13/12 \not> 9$ ✗

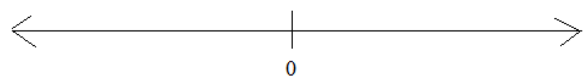
Test the 4 regions separated by the curves and asymptotes...

Solve and Graph each inequality:

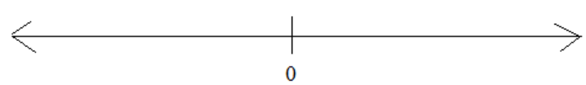
1) $\dfrac{(x+2)^2(x-4)}{x+3} > 0$



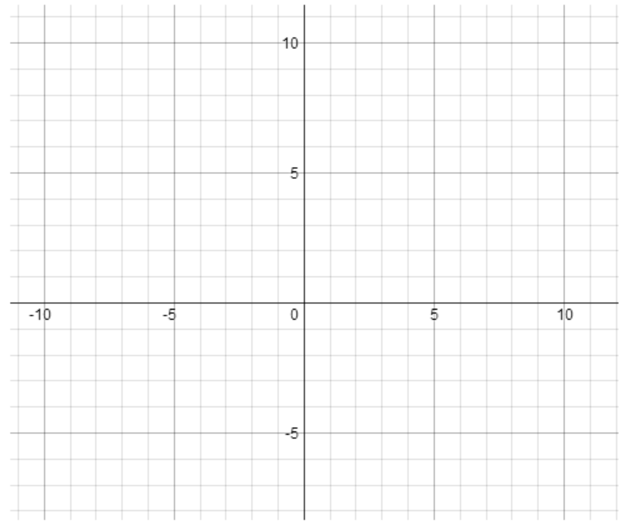
2) $\dfrac{(x+4)}{(x-5)} \geq 8$



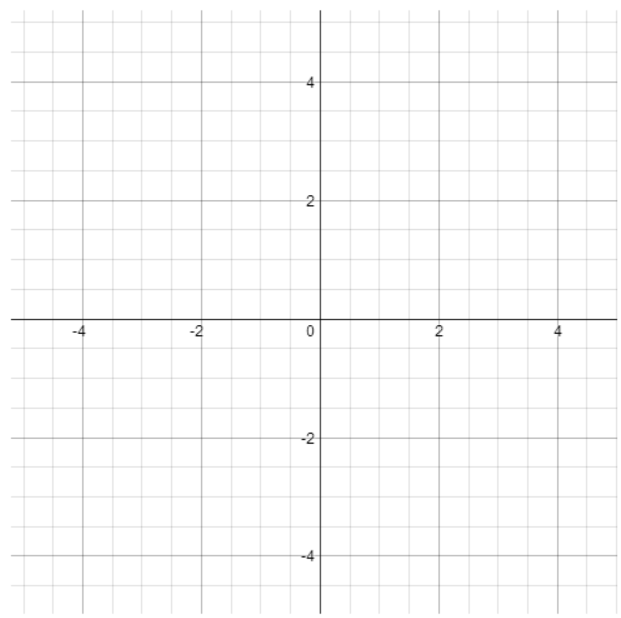
3) $\left(\dfrac{4}{(x+2)} - 2\right)\left(\dfrac{4}{(x-2)} + 2\right) > 0$



4) Sketch $y - 3 \leq \frac{x-4}{x+2}$



5) Sketch $y < \left(\frac{2}{x-2} + 1\right)\left(\frac{2}{x+2} - 1\right)$



Solve and Graph each inequality:

SOLUTIONS

1) $\dfrac{(x+2)^2(x-4)}{x+3} > 0$

Since there is only one variable, we can sketch on a number line...

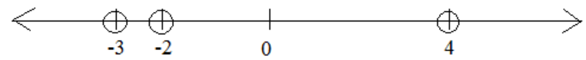
Step 1: Find critical values

At x = -3, the equation is undefined..

At x = -2 or 4, the equation equals zero...

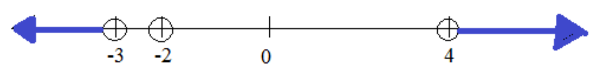
Step 2: 'open' or 'closed' circles

Since the inequality is >, -2 and 4 are open circles and,
since x cannot equal -3, then it is also an open circle



Step 3: Test the 4 regions (pick a point in each)

$x = -5,\ \dfrac{(9)(-9)}{-2} > 0$ $x = -2.5,\ \dfrac{(.25)(-4.5)}{.5} \not> 0$ $x = 0,\ \dfrac{(4)(-4)}{3} \not> 0$ $x = 7,\ \dfrac{(81)(3)}{10} > 0$



$(-\infty, -3) \cup (4, \infty)$

2) $\dfrac{(x+4)}{(x-5)} \geq 8$

To get the critical values,
cross multiply and solve (disregarding the inequality)

$8(x - 5) = (x + 4)$

$8x - 40 = x + 4$

$7x = 44$

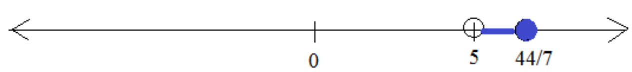
$x = \dfrac{44}{7}$

Then, recognize that x cannot equal 5 (because the rational expression is undefined)

Open or closed circles?

Since the sign is $\geq$ it includes 44/7 (closed)

Since $x \neq 5$, it does not include 5 (open)



Select easy points to test each region

$x = 0$, $-4/5$ not ≥ 8

$x = 6$, $10/1 \geq 8$ ✓

$x = 10$, $14/5$ not ≥ 8

(5, 44/7]

or

$5 < x \leq 44/7$

3) $\left(\dfrac{4}{(x+2)} - 2\right)\left(\dfrac{4}{(x-2)} + 2\right) > 0$

Identify the critical values:

The equation is undefined at x = -2 and x = 2...
And, the equation is equal at x = 0

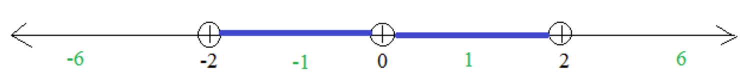
$\left(\dfrac{4}{(x+2)} - 2\right) = 0$ $\left(\dfrac{4}{(x-2)} + 2\right) = 0$

$\dfrac{4}{(x+2)} = 2$ $\dfrac{4}{(x-2)} = -2$

$x = 0$ $x = 0$

Open or closed circles?

All the critical points are open circles, because the inequality is >



Then, test points in each region:

at x = -6, (-3)(3/2) > 0 ? NO
at x = -1, (2)(10/3) > 0 ? YES
at x = 1, (-2/3)(-2) > 0 ? YES
at x = 6, (-3/2)(1) > 0 ? NO

$(-2, 0) \cup (0, 2)$

4) Sketch $y - 3 \leq \frac{x-4}{x+2}$

SOLUTIONS

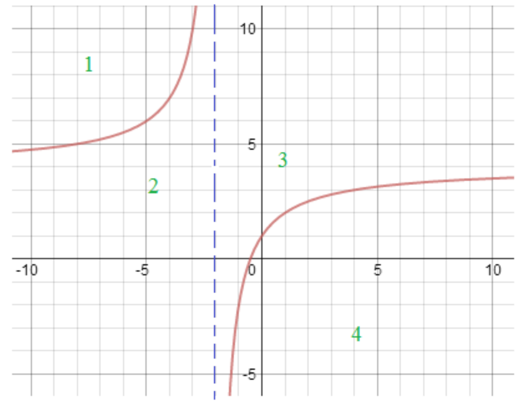
Rewrite the expression and sketch the graph
(ignoring the inequality for now)

$$y = \frac{x - 4}{x + 2} + 3$$

vertical asymptote: $x = -2$

horizontal asymptote: $y = 1/1 + 3$

Since the inequality is < or = , the curves are solid.
and, the asymptote is dashed...



Identify the regions and test points:

region 1: (-10, 7): $7 - 3 \leq \frac{-10 - 4}{-10 + 2}$

$4 \leq 14/8$ NO

region 2: (-5, 2): $2 - 3 \leq \frac{-5 - 4}{-5 + 2}$

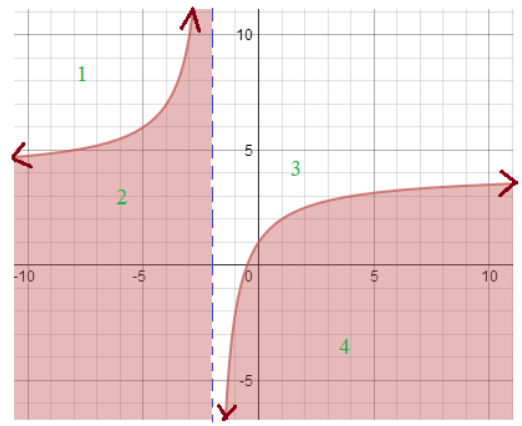
$-1 \leq 3$ YES

region 3: (0, 5): $5 - 3 \leq \frac{0 - 4}{0 + 2}$

$2 \leq -2$ NO

region 4: (4, 0): $0 - 3 \leq \frac{4 - 4}{4 + 2}$

$-3 \leq 0$ YES



5) Sketch $y < \left(\frac{2}{x-2} + 1\right)\left(\frac{2}{x+2} - 1\right)$

Identify the critical points:

at $x = 2$ and $x = -2$, the fractions are undefined...
(vertical asymptotes)

Horizontal asymptote:

Notice, as x gets larger and larger, the fractions approach 0..
or, as x goes to negative infinity, the fractions approach 0...
therefore, the horizontal asymptote is $y = -1$

Plot points:

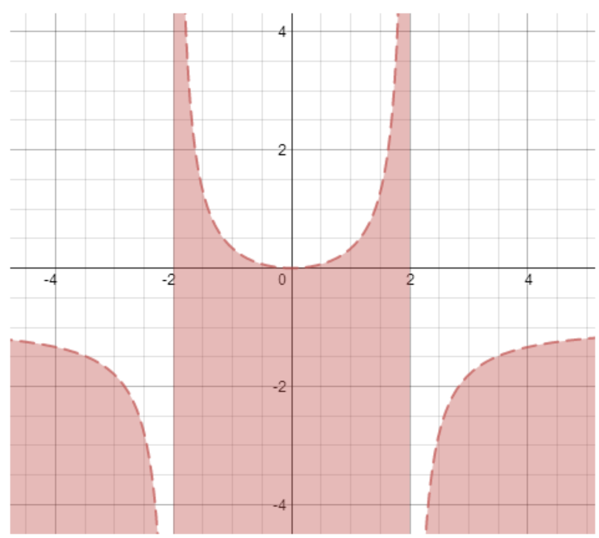
Plot a few points around the asymptotes to get the general shape of the graph.

Dashed lines and shaded regions...

Since the inequality is < (and not equal), the lines are dashed...

Then, test points in each region...

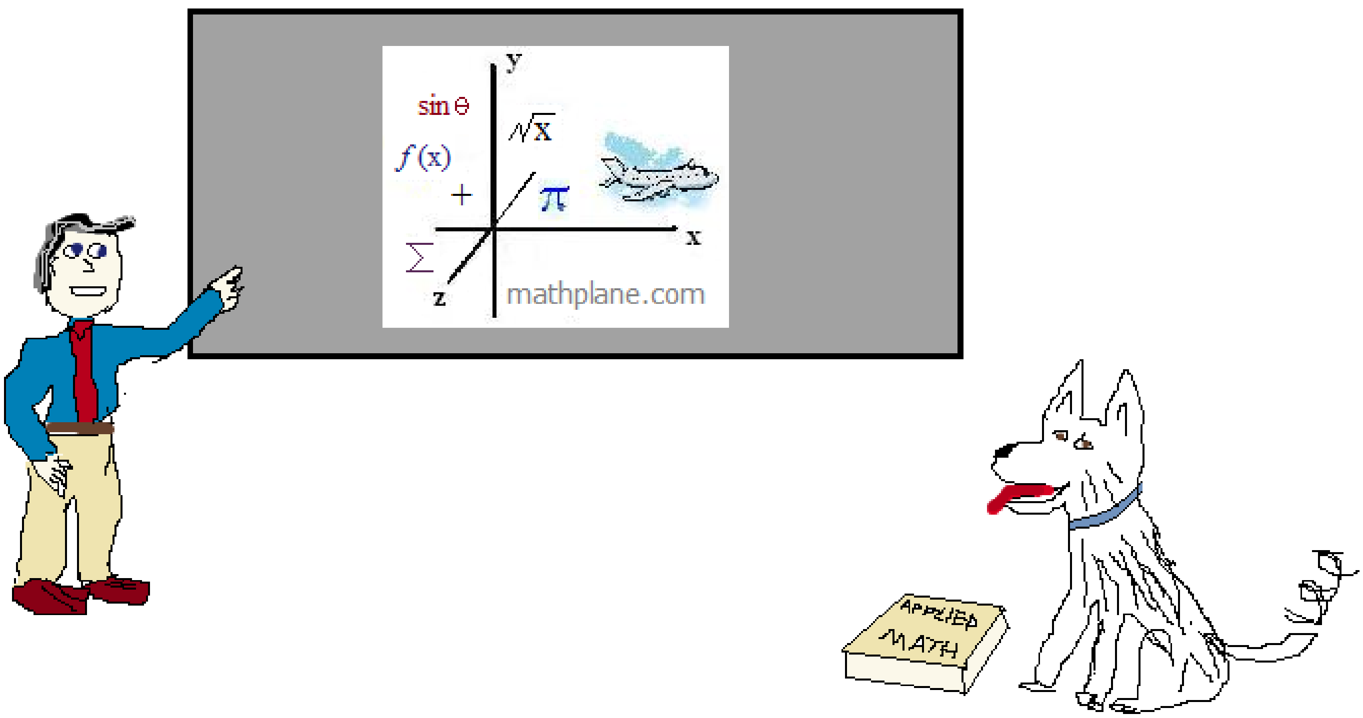
try region 1: (left of -2) (-4, -2)
(optional: check above and below dashed curve)
region 2: (between -2 and 2) (0, -2)
(optional: check above and below the parabola)
region 3: (right of 2) (4, 0)



Thanks for visiting the site. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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