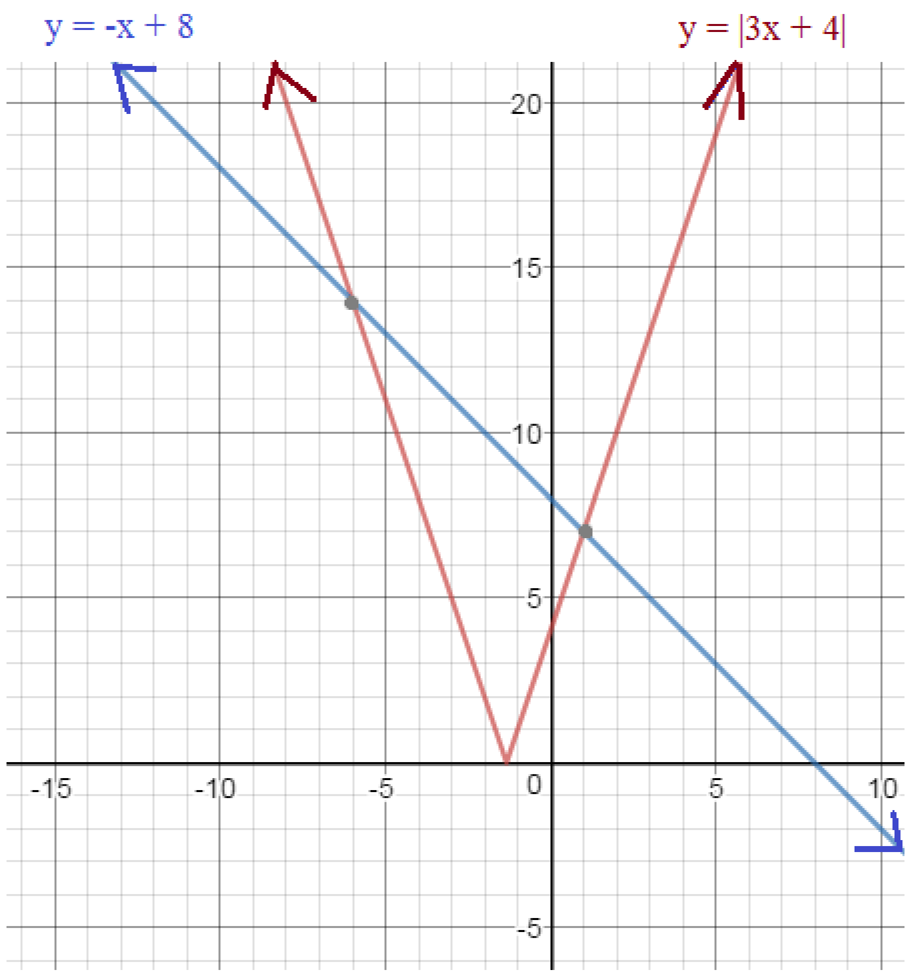
Absolute Value Graphing Topics

Notes, Examples, and Exercises (with Solutions)



Includes graphing, slope, number lines and distance, tables, solving equations, and more…

Mathplane.com

Absolute value: 0, 1, or 2 solutions???

Example: $|2x + 8| = 4$

Solve Algebraically

$|2x + 8| = 4$

"split the absolute value"

$2x + 8 = 4$ OR $2x + 8 = -4$

$2x = -4$ $\quad$ $2x = -12$

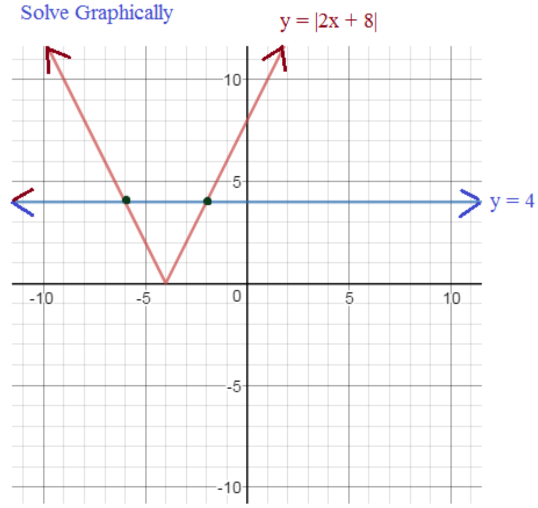
$x = -2$ $\quad$ $x = -6$

Check: $|2(-2) + 8| = 4$ $\quad$ Check: $|2(-6) + 8| = 4$

$|4| = 4$ ✓ $\quad$ $|-4| = 4$ ✓

2 SOLUTIONS: (-2, 4) and (-6, 4)

Solve Graphically



Example: $|x + 5| = 2x$

Solve Algebraically

$|x + 5| = 2x$

"split the absolute value"

$x + 5 = 2x$ OR $x + 5 = -(2x)$

$5 = x$ $\quad$ $3x = -5$

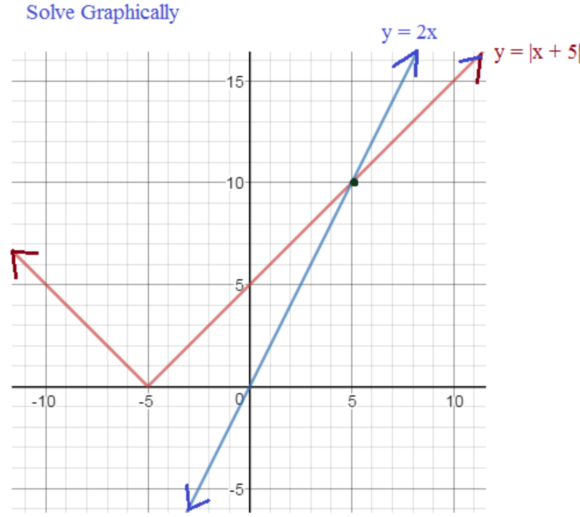
$x = -5/3$

Check: $|(5) + 5| = 2(5)$ $\quad$ Check: $|(-5/3) + 5| = 2(-5/3)$

$|10| = 10$ ✓ $\quad$ $|10/3| \neq -10/3$

1 SOLUTION: (5, 10)

Solve Graphically



Example: $|x + 3| = x - 7$

Solve Algebraically

$|x + 3| = x - 7$

"split the absolute value"

$x + 3 = x - 7$ OR $x + 3 = -(x - 7)$

$3 \neq -7$ $\quad$ $x + 3 = -x + 7$

$2x = 4$

$x = 2$

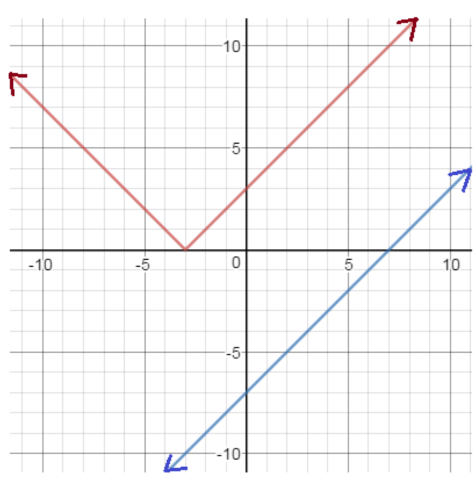
But, if you check:

$|(2) + 3| = (2) - 7$

$5 \neq -5$

NO SOLUTIONS

Solve Graphically



Graphing Absolute Values on the Coordinate Plane

METHOD 1: Using the Parent Function

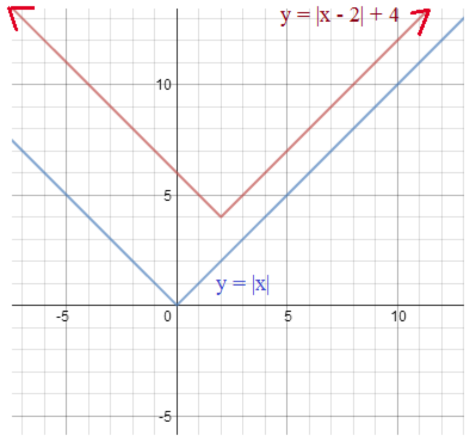
Example: $y = |x - 2| + 4$

parent function:
$y = |x|$

horizontal shift (c):
2 units to the right

vertical shift (d):
4 units up

domain: all real numbers
range: $y \geq 4$



function: $f(x)$

$y = af b(x - c) + d$

a: vertical stretch/dilation

b: horizontal compression

c: horizontal shift

d: vertical shift

Example: $y = -|3x - 3| + 5$

**first, rewrite the equation

$y = -|3(x - 1)| + 5$

horizontal shift (c):
1 unit to the right

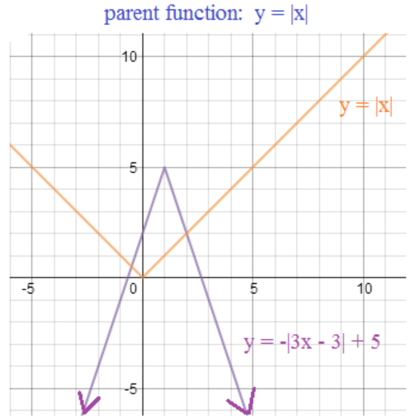
vertical shift (d):
5 units up

reflected over the x-axis

"compression" (b):
1/3 of the width

domain: all real numbers
range: $y \leq 5$

parent function: $y = |x|$



positive 'a' value, then graph opens up...
negative 'a' value, then graph opens down...

METHOD 2: Recognizing the Vertex and Slope

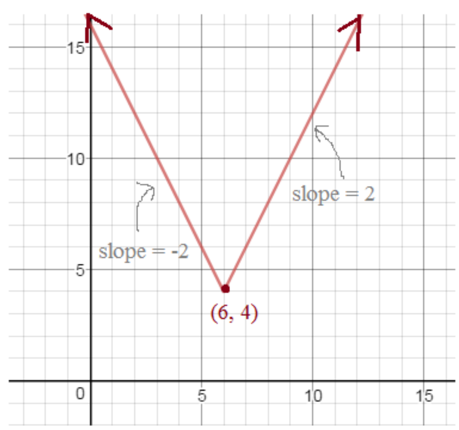
Example: $y = 2|x - 6| + 4$

vertex: (6, 4)

slope: $a = 2$

so, left of the vertex: -2

right of the vertex: 2



$y = a|x - h| + k$

slope: a   vertex: (h, k)

Example: $y = -|2x + 10| + 3$

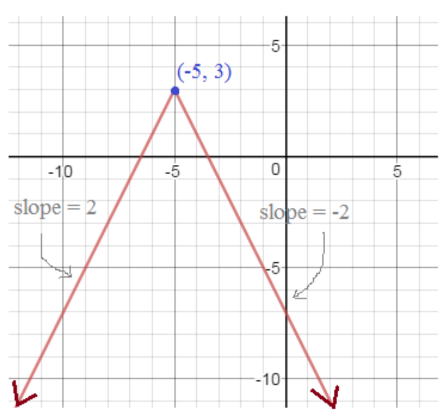
rewrite: $y = -2|x + 5| + 3$

vertex: (-5, 3)

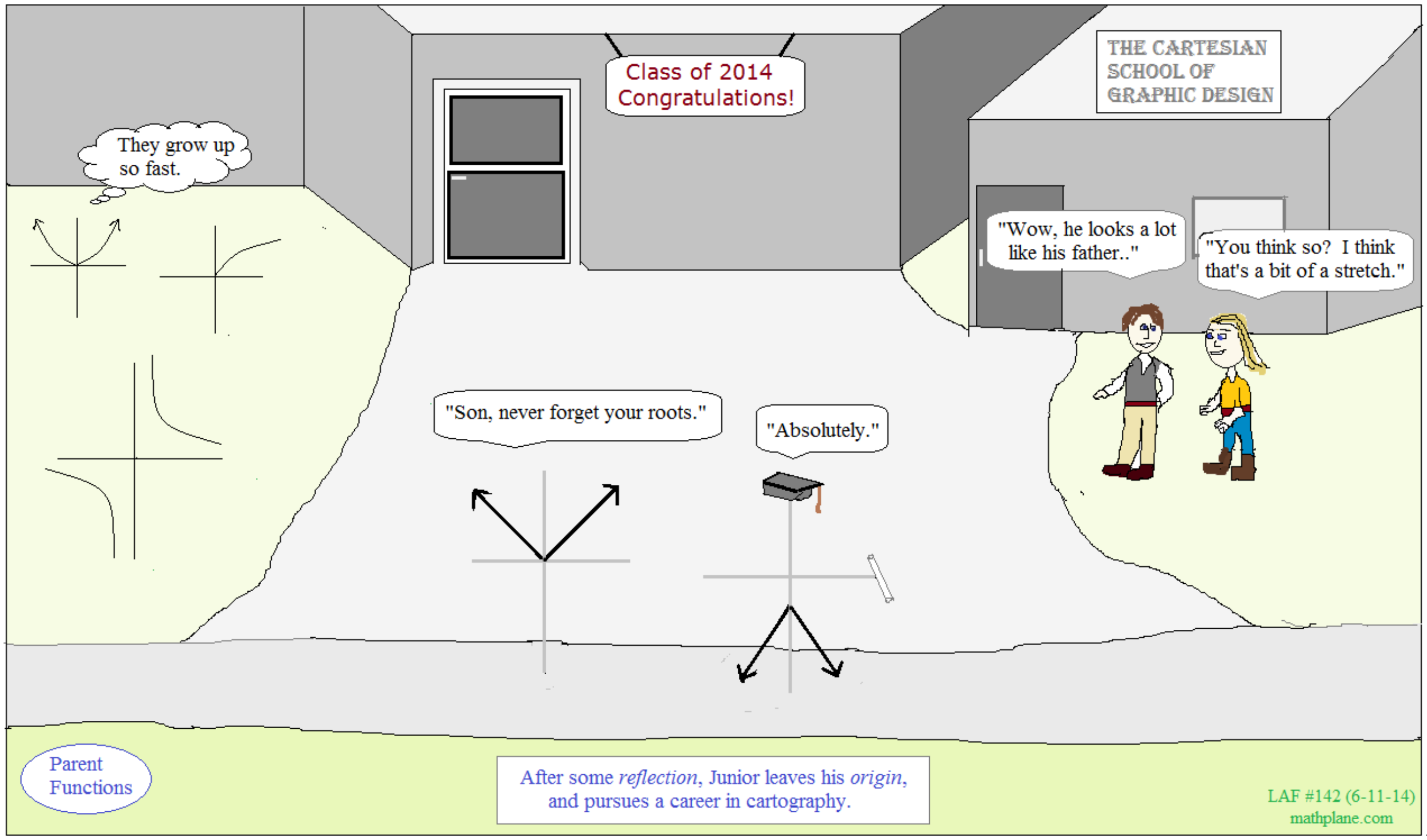
slope: $a = -2$

so, left of the vertex: 2

right of the vertex: -2



(To check your graph, just test a few points...)

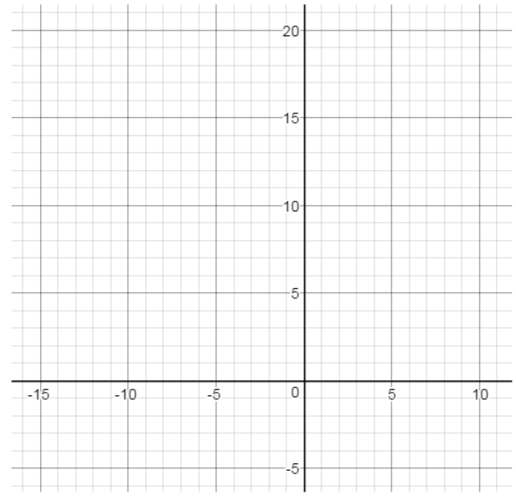

Class of 2014 Congratulations!
THE CARTESIAN SCHOOL OF GRAPHIC DESIGN
They grow up so fast.
"Wow, he looks a lot like his father.."
"You think so? I think that's a bit of a stretch."
"Son, never forget your roots."
"Absolutely."
Parent Functions
After some *reflection*, Junior leaves his *origin*, and pursues a career in cartography.
LAF #142 (6-11-14)
mathplane.com

Exercises-→

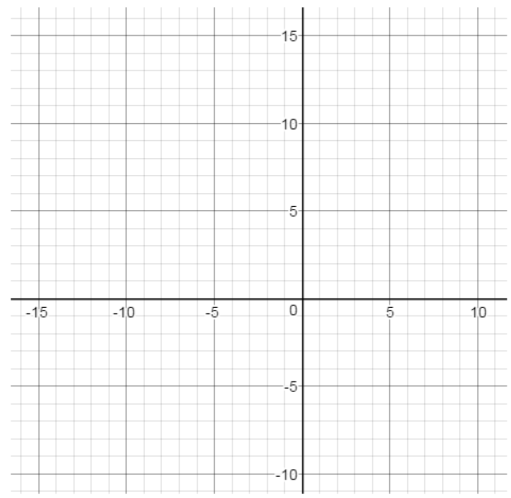
Absolute value equations and the coordinate plane

Solve algebraically and graphically

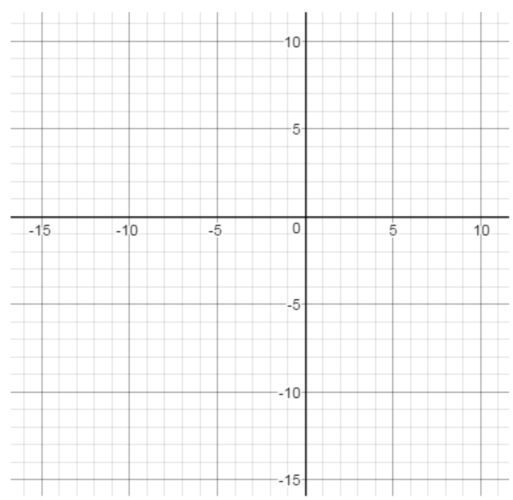
1) $|3x + 4| = -x + 8$



2) $|x + 7| - 4 = -2x$



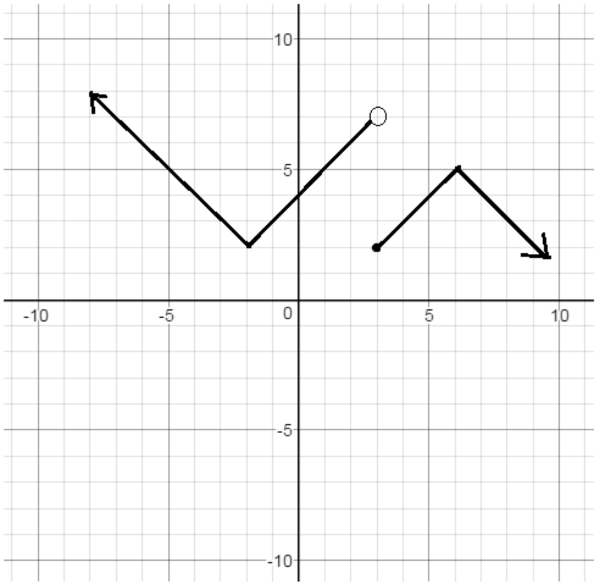
3) $-2|x + 5| = -7$



Piecewise Absolute Value Functions

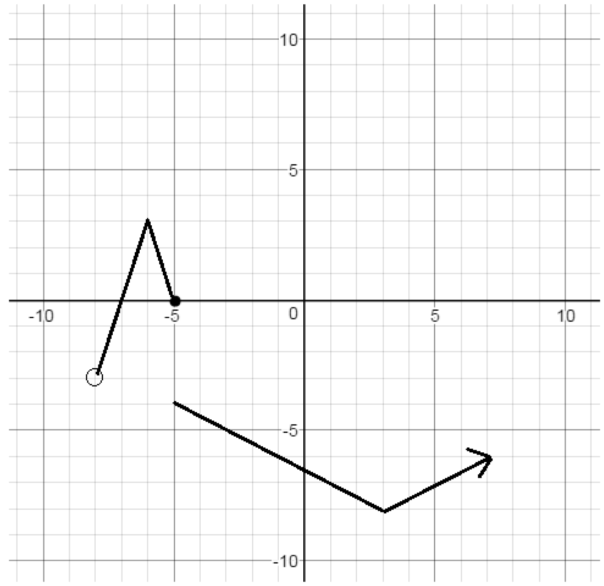
I. Use a minimal number of "pieces" to describe the graphs...

1)



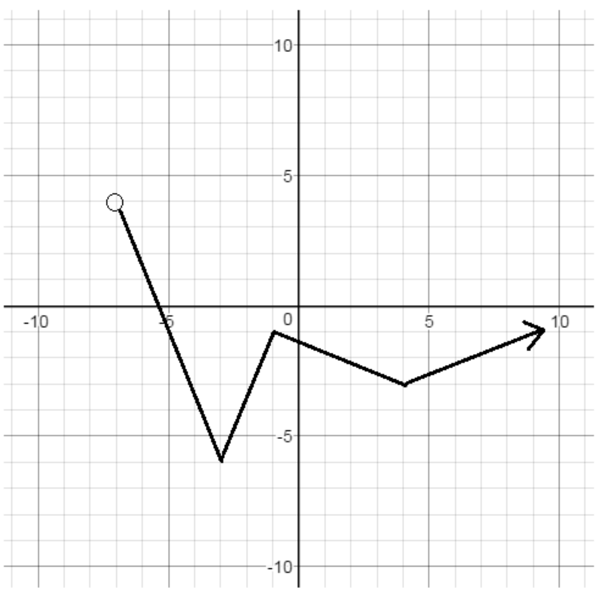
$f(x) = \begin{cases} \\ \end{cases}$

2)



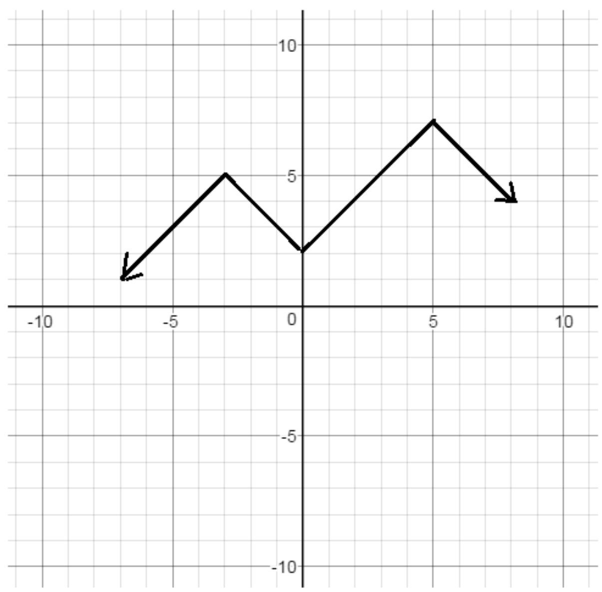
$g(x) = \begin{cases} \\ \end{cases}$

3)



$h(x) = \begin{cases} \\ \end{cases}$

4)



$p(x) = \begin{cases} \\ \end{cases}$

II. Write each equation as a piecewise function (i.e. using linear equations)

Piecewise Absolute Value Functions

1) $y = 3|x + 2| + 4$

2) $y = 5 - |x + 7|$

3) $y = |3x| + 4$

III. Write each absolute value inequality to describe the phrase . Then, graph on the number line.

Absolute Value and Distance

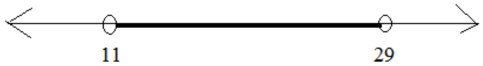
1) all real numbers where "the distance from 5 is at most 12 units away"

2) all real numbers where "the distance from -6 is over 9 units away"

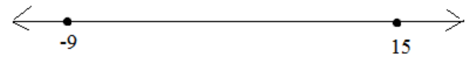
3) all real numbers "within 11 units of 13"

IV. Describe each number line graph/compound inequality with an absolute value equation/inequality.

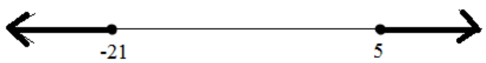
1) $11 < x < 29$



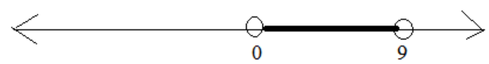
2) $x = -9$ or 15



3) $x \geq 5$ or $x \leq -21$



4) $x < 9$ and $x > 0$



V. Interpreting tables of values

x	*g*(x)
-5	5
-3	1
-1	
0	
1	-7
2	-5
5	
9	

a) *g*(x) is an absolute function. Fill in the blanks.

b) what is the minimum value of *g*(x) ?

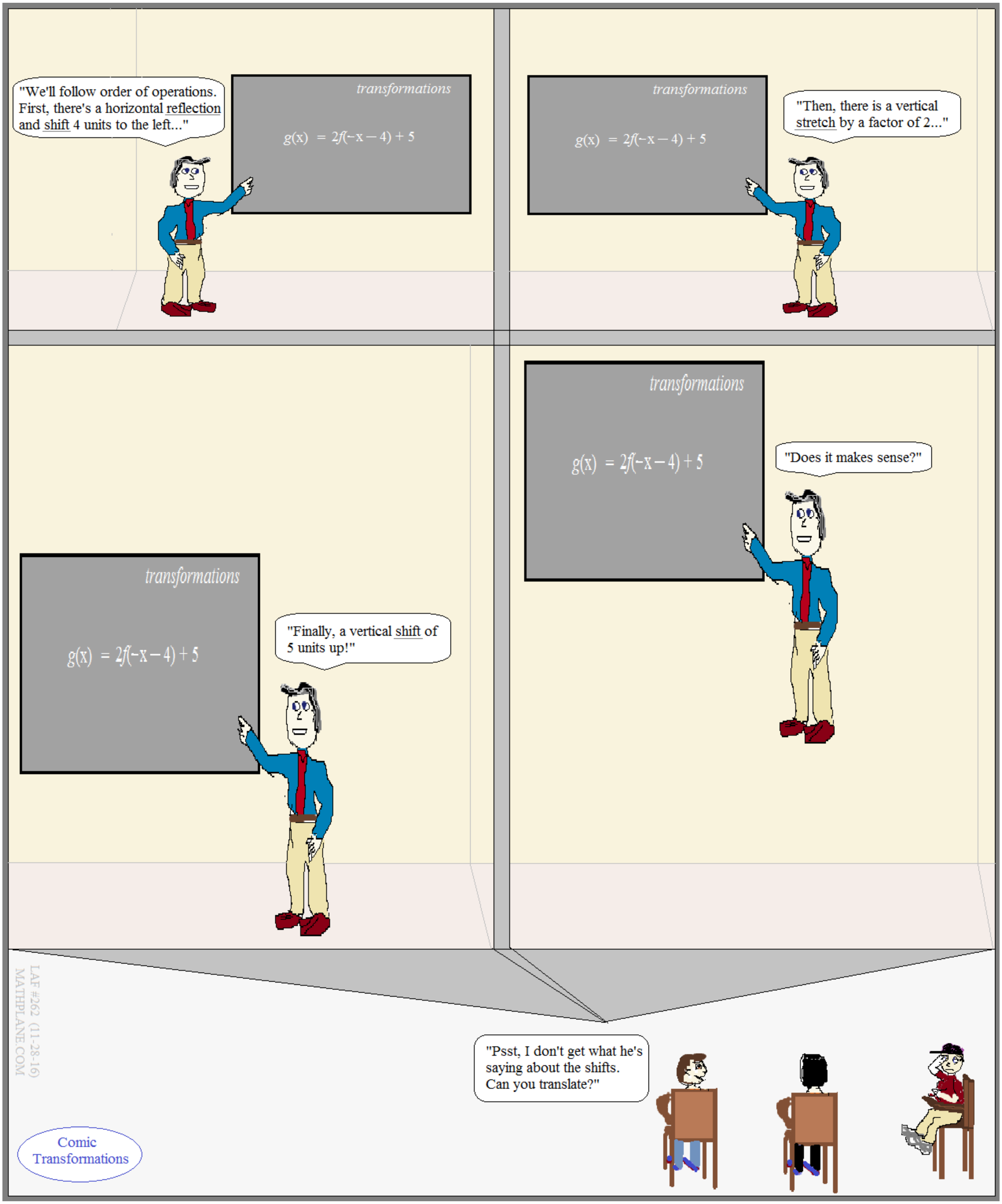
c) what is the equation of the function?

x	*f*(x)
-5	
-4	-7
-3	
-2	-1
-1	
0	-1
1	
2	

a) If this an absolute value function, fill in the rest of the table.

b) What is x when *f*(x) = -10?

c) Write the equation.



transformations
g(x) = 2f(−x − 4) + 5
"We'll follow order of operations. First, there's a horizontal reflection and shift 4 units to the left..."
transformations
g(x) = 2f(−x − 4) + 5
"Then, there is a vertical stretch by a factor of 2..."
transformations
g(x) = 2f(−x − 4) + 5
"Finally, a vertical shift of 5 units up!"
transformations
g(x) = 2f(−x − 4) + 5
"Does it makes sense?"
"Psst, I don't get what he's saying about the shifts. Can you translate?"
LAF #262 (11-28-16)
MATHPLANE.COM
Comic Transformations

SOLUTIONS-→

Absolute value equations and the coordinate plane

Solve algebraically and graphically

1) $|3x + 4| = -x + 8$

Algebraically

"Split the absolute value"

$3x + 4 = -x + 8$ OR $3x + 4 = -(-x + 8)$

$4x = 4$ $\qquad$ $3x + 4 = x - 8$

$x = 1$ $\qquad$ $x = -6$

Check for extraneous answers

If $x = 1$: $\qquad$ If $x = -6$:

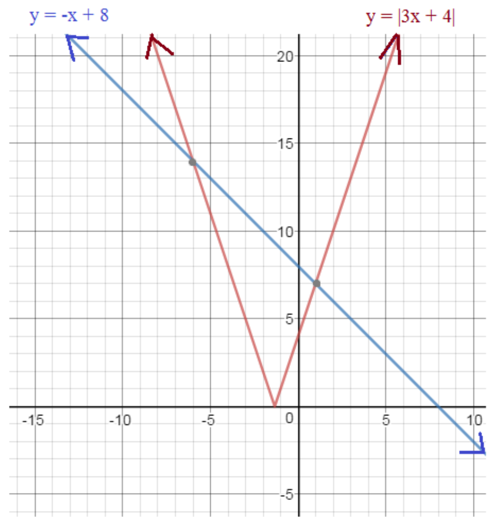
$|3(1) + 4| = -(1) + 8$ $\qquad$ $|3(-6) + 4| = -(-6) + 8$

$|7| = 7$ ✓ $\qquad$ $|-14| = 14$ ✓

TWO SOLUTIONS: $x = 1, -6$

Intercepts at

(1, 7) and (-6, 14)



2) $|x + 7| - 4 = -2x$

Algebraically

"Isolate the absolute value"

$|x + 7| = -2x + 4$

"Split the absolute value"

$x + 7 = -2x + 4$ OR $x + 7 = -(-2x + 4)$

$3x = -3$ $\qquad$ $x + 7 = 2x - 4$

$x = -1$ $\qquad$ $x = 11$

check for extraneous solutions

if $x = -1$: $\qquad$ if $x = 11$:

$|(-1) + 7| = -2(-1) + 4$ $\qquad$ $|(11) + 7| - 4 = -2(11)$

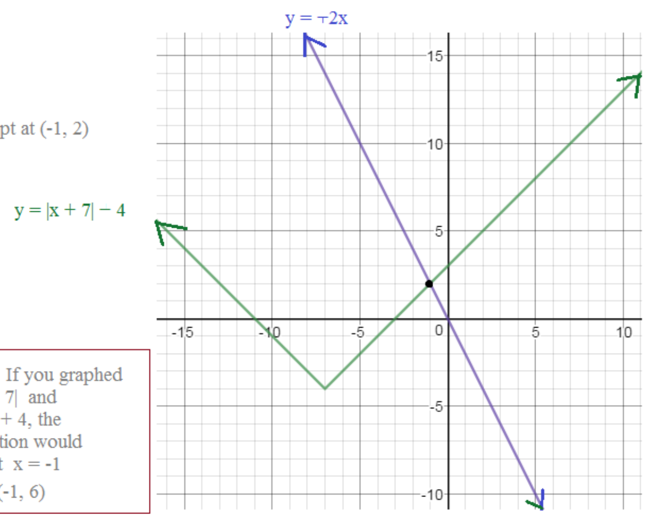
$|6| = 6$ ✓ $\qquad$ $|18| \neq -18$

1 SOLUTION: $x = -1$

Intercept at (-1, 2)

NOTE: If you graphed $y = |x + 7|$ and $y = -2x + 4$, the intersection would occur at $x = -1$

(-1, 6)



3) $-2|x + 5| = -7$

Algebraically

"Isolate the absolute value"

$|x + 5| = \frac{-7}{-2}$

"Split the absolute value"

$x + 5 = 7/2$ OR $x + 5 = -7/2$

$x = -3/2$ $\qquad$ $x = -17/2$

check for extraneous solutions

if $x = -3/2$: $\qquad$ if $x = -17/2$:

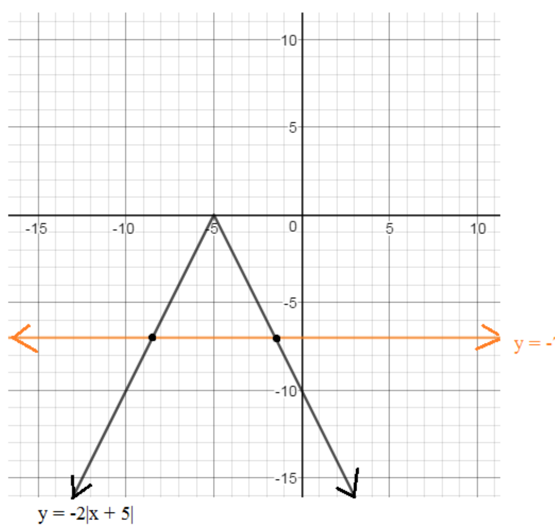
$-2|(-3/2) + 5| = -7$ $\qquad$ $-2|(-17/2) + 5| = -7$

$-2|-7/2| = -7$ ✓ $\qquad$ $-2|-7/2| = -7$ ✓

2 SOLUTIONS: $x = -3/2, -17/2$

Intercepts at

(-3/2, -7) and (-17/2, -7)

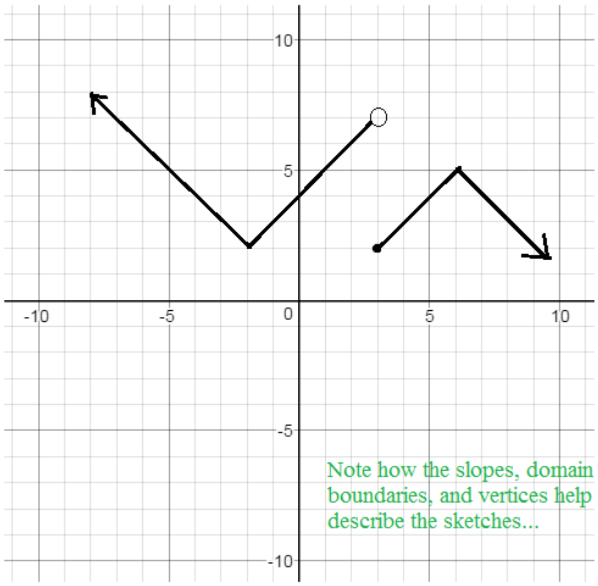


Piecewise Absolute Value Functions

I. Use a minimal number of "pieces" to describe the graphs...

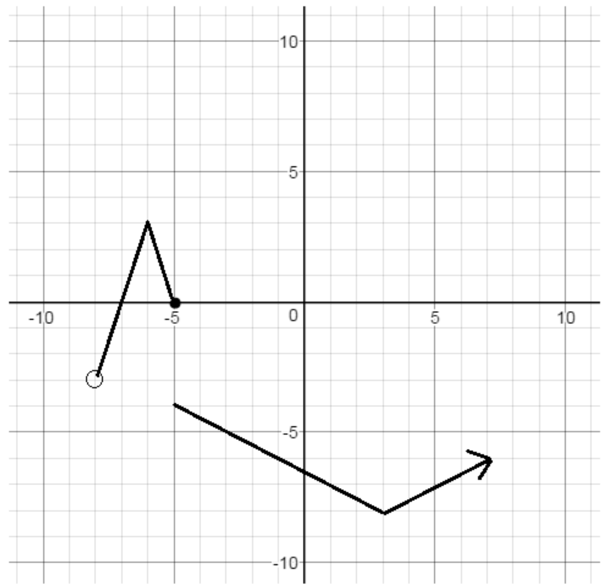
SOLUTIONS

1)



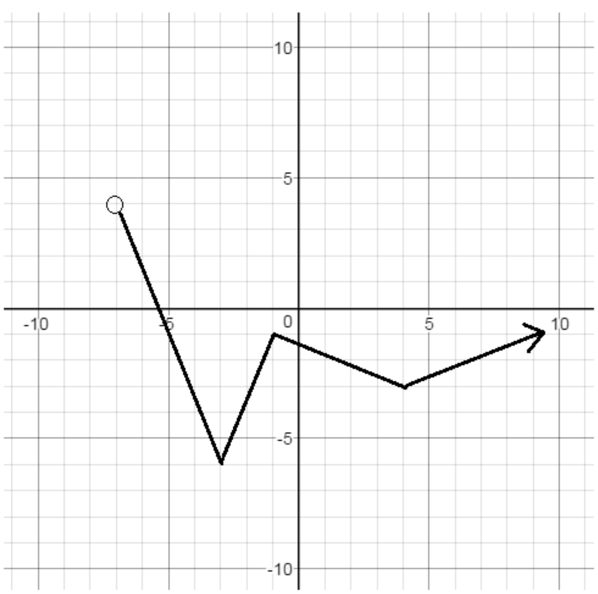
$$f(x) = \begin{cases} |x+2| + 2 & \text{if } x < 3 \\ -|x-6| + 5 & \text{if } x \geq 3 \end{cases}$$

2)



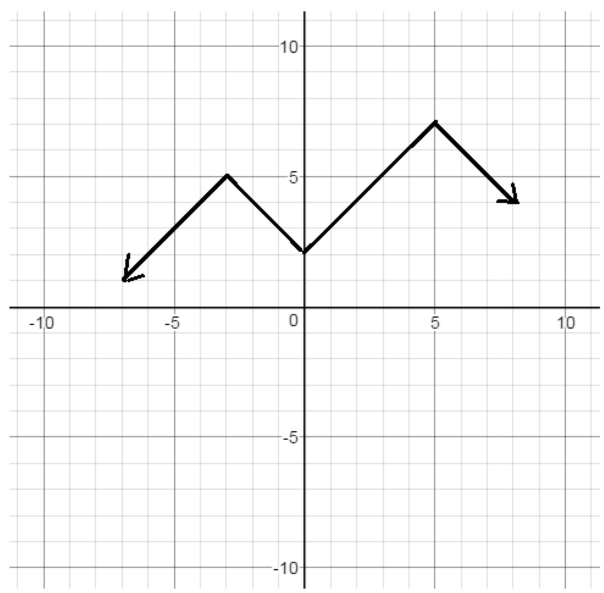
$$g(x) = \begin{cases} -3|x+6| + 3 & \text{in the interval } (-8, -5] \\ \frac{1}{2}|x-3| - 8 & \text{in the interval } [-5, \infty) \end{cases}$$

3)



$$h(x) = \begin{cases} \frac{5}{2}|x+3| - 6 & \text{if } -7 < x \leq -1 \\ \frac{2}{5}|x-4| - 3 & \text{if } x > -1 \end{cases}$$

4)



$$p(x) = \begin{cases} -|x+3| + 5 & \text{in the interval } (-\infty, 0) \\ -|x-5| + 7 & \text{in the interval } (0, \infty) \end{cases}$$

II. Write each equation as a piecewise function (i.e. using linear equations)

SOLUTIONS

Piecewise Absolute Value Functions

1) $y = 3|x + 2| + 4$

slope will be -3 on the left and $+3$ on the right

going through the point (-2, 4), we can determine the lines

$$\begin{cases} -3x + (-2) & \text{if } x < -2 \\ 3x + 10 & \text{if } x \geq -2 \end{cases}$$

2) $y = 5 - |x + 7|$

$y = -1\,|x + 7| + 5$

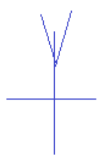
vertex is at (-7, 5)

slope is -1 for $x \leq -7$

1 for $x > -7$

$$\begin{cases} -x - 2 & \text{if } x \leq -7 \\ x + 12 & \text{if } x > -7 \end{cases}$$

3) $y = |3x| + 4$



the 3 inside the absolute value behaves the same as if it were outside the absolute value...

slope is -3 when $x < 0$ and
slope is -3 when $x > 0$

y-intercept is at (0, 4)

$$\begin{cases} -3x + 4 & \text{if } x \leq 0 \\ 3x + 4 & \text{if } x > 0 \end{cases}$$

III. Write each absolute value inequality to describe the phrase . Then, graph on the number line.

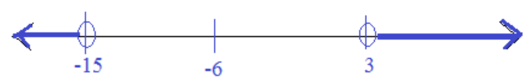
Absolute Value and Distance

1) all real numbers where "the distance from 5 is at most 12 units away"

$|x - 5| \leq 12$

2) all real numbers where "the distance from -6 is over 9 units away"

$|x - (-6)| > 9$

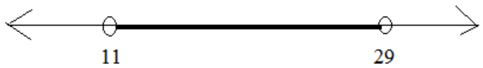


3) all real numbers "within 11 units of 13"

$|x - 13| \leq 11$

IV. Describe each number line graph/compound inequality with an absolute value equation/inequality. SOLUTIONS

1) $11 < x < 29$



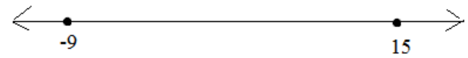
1) center (i.e. midpoint of boundaries) $\frac{11 + 29}{2} = 20$

2) distance to each boundary is 9 units

3) the set of points is less than 9 units...

$|x - 20| < 9$

2) $x = -9$ or 15



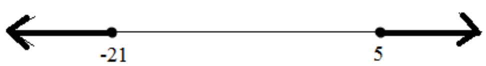
midpoint is $\frac{-9 + 15}{2} = 3$

distance to each boundary is 12

$|x - 3| = 12$

"all numbers that are 12 units from 3"

3) $x \geq 5$ or $x \leq -21$



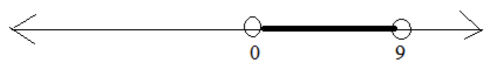
1) center of boundaries is -8

2) distance to each boundary is 13 units

3) set of all points is great than or equal to 13 units...

$|x - (-8)| \geq 13$

4) $x < 9$ and $x > 0$



1) center of boundaries: 4.5

2) distance to each boundary (from center): 4.5

3) all points are less than the distance to boundaries

$|x - 4.5| < 4.5$

V. Interpreting tables of values

x	$g(x)$
-5	5
-3	1
-1	-3
0	-5
1	-7
2	-5
5	1
9	9

a) $g(x)$ is an absolute function. Fill in the blanks.
(recognize the slopes are -2 and 2)

b) what is the minimum value of $g(x)$?
vertex is (1, -7) ------> minimum is -7

c) what is the equation of the function?

$2|x - 1| - 7$

x	$f(x)$
-5	-10
-4	-7
-3	-4
-2	-1
-1	2
0	-1
1	-4
2	-7

a) If this an absolute value function, fill in the rest of the table.
slopes are 3 and -3... vertex is at $x = -1$

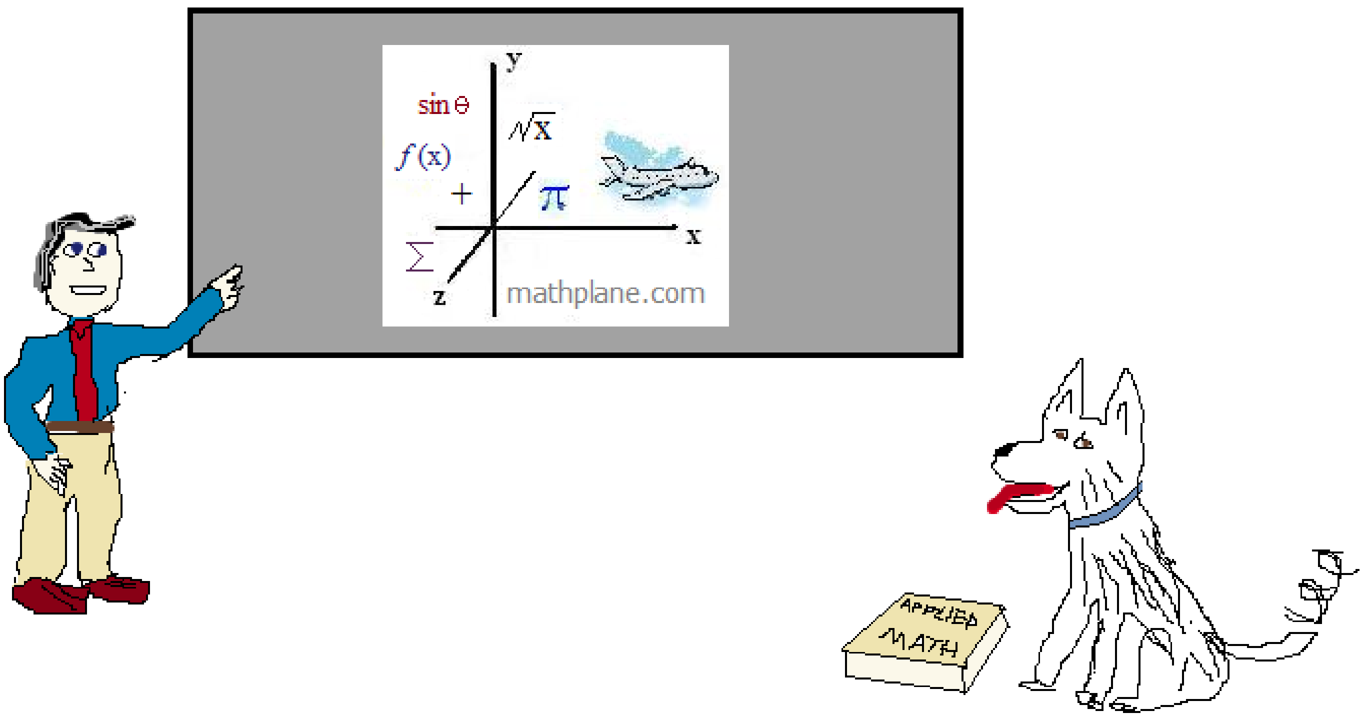
b) What is x when $f(x) = -10$? $x = -5$ or 3

c) Write the equation. $-3|x + 1| + 2$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at Facebook, Google+, TeachersPayTeachers, TES, and Pinterest…