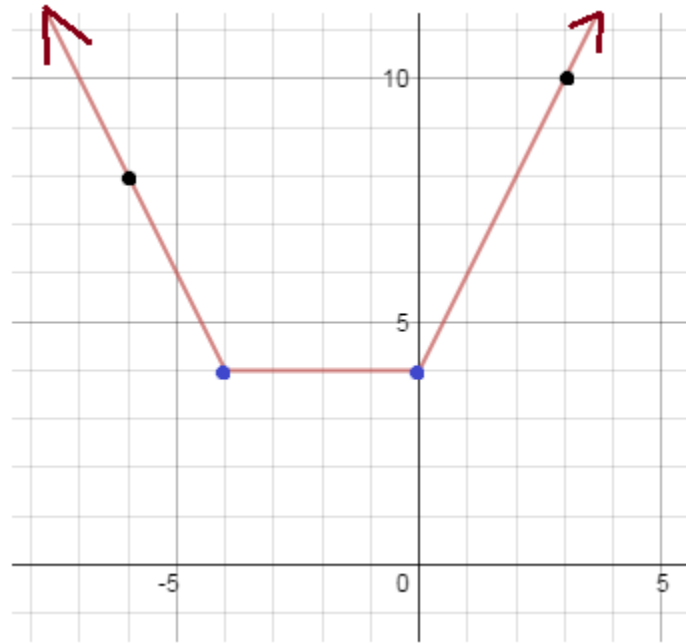
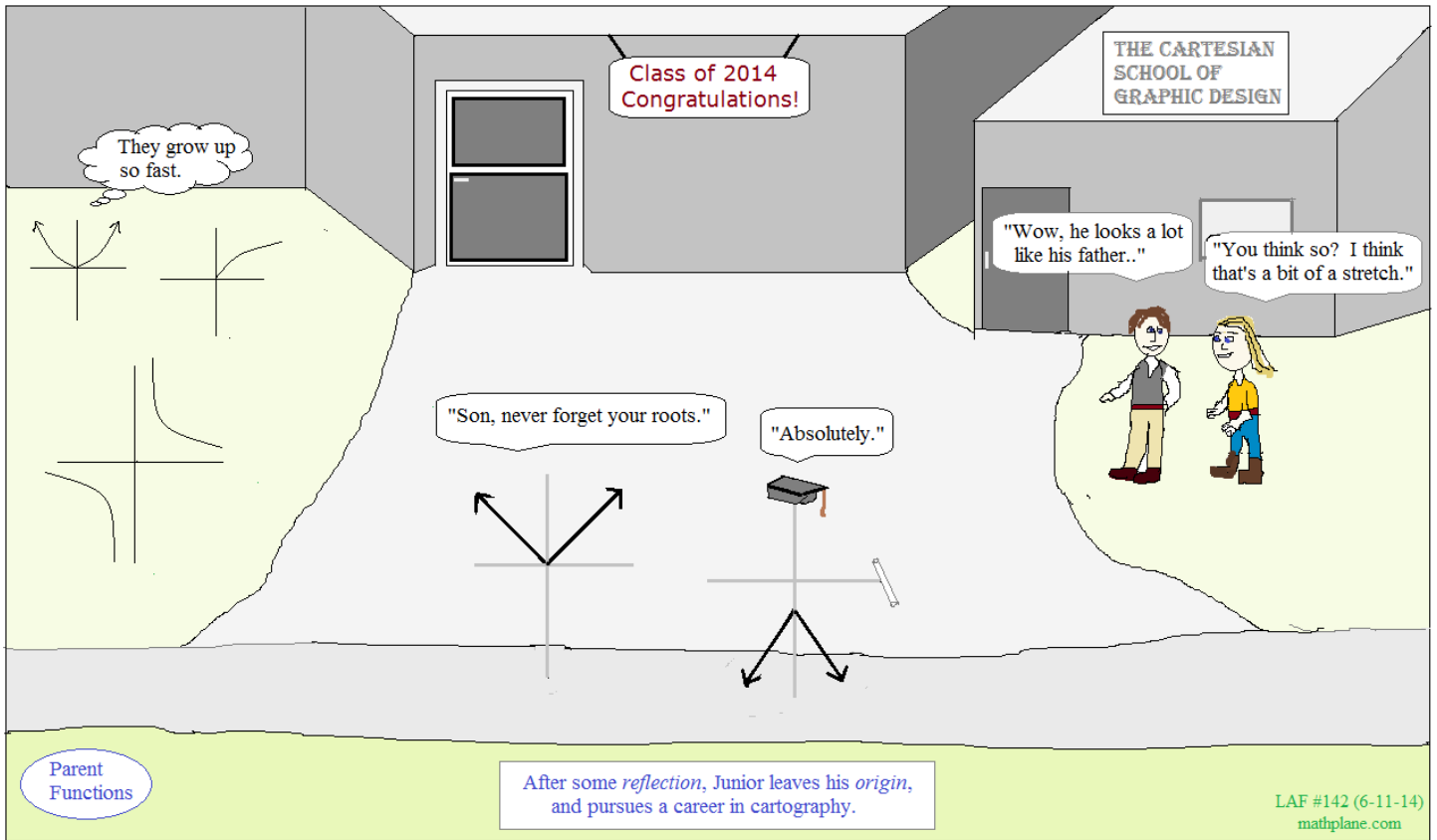


Double Absolute Values

Notes, Examples, & Practice Test (with Solutions)



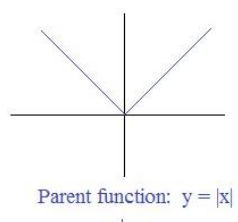
Topics include absolute value, slope, “kinks and corners”, graphing, inequalities, standard form, vertex, and more.



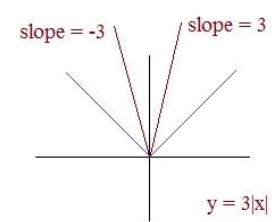
Absolute Value Review Notes and Examples→

Standard Form of an Absolute Value Function
 $y = a|x - h| + k$ where (h, k) is the vertex...

Examples: The vertex is on the origin $(0, 0)$
 $y = a|x|$
 where a is the amount the parent function is "multiplied" or "stretched".
 (see graphs and tables)



x	y
-2	2
-1	1
0	0
1	1
2	2



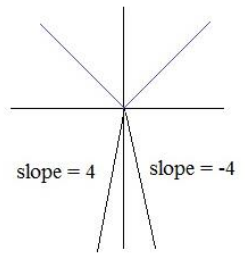
x	y
-2	6
-1	3
0	0
1	3
2	6

"Slope of the Absolute Value Line"
 Also, 'a' indicates the slope:
 a is the slope of the absolute value line on the right side of the vertex.
 And,
 $-a$ is the slope of the absolute value line on the left side of the vertex.

** If $a < 0$, then the graph 'faces down'
 Slope on the left side of the vertex will be positive. And, slope on the right side of the vertex will be negative.

$y = -4|x|$

x	y
-2	-8
-1	-4
0	0
1	-4
2	-8



"Absolute Value is a Piecewise Function"

Example: $y = |x - 3| + 6$

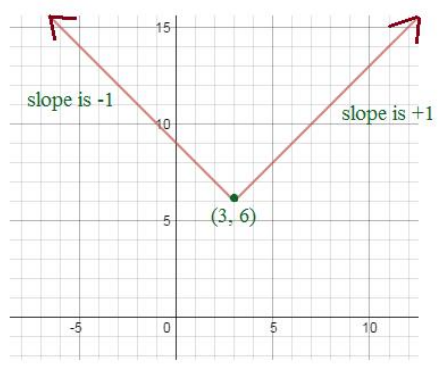
$a = 1$ $(h, k) = (3, 6)$

Since $a > 0$, the function opens upward (faces up)...
 Slope is 1 to the right of the vertex
 Slope is -1 to the left of the vertex

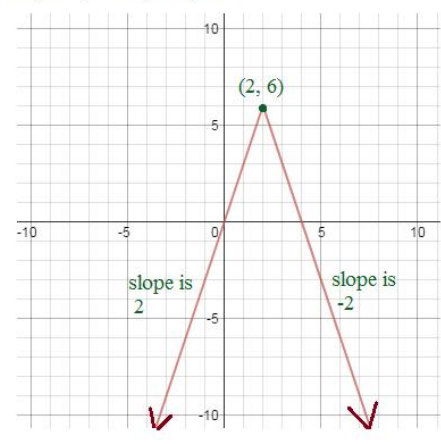
This graph can be described in this form:

$$f(x) = \begin{cases} x + 3, & \text{if } x > 3 \\ -x + 9, & \text{if } x \leq 3 \end{cases}$$

The vertex is a "kink" or "corner"
 Vertex is at $x = 3$ $(3, 6)$



Example: $y = -3|x - 2| + 6$



Vertex: $(2, 6)$
 "a" value: -3

Note: the absolute value "V" is upside down.. (opens downward)
 So, the slope left of $x = 2$ is positive
 and, the slope right of $x = 2$ is negative...

The slope at $x = 2$ (the vertex) does not exist
 (because it's rate of change is ambiguous)

The rate of change coming from the left of $(2, 6)$ is -2 ... And, the rate of change approaching from the right of $(2, 6)$ is -2 ..
 So, at 2, we don't know!

Absolute Value Equations: How many solutions?

Steps for solving absolute value equations:

- 1) Isolate the absolute value
- 2) "Split into negative and positive equations"
- 3) Solve
- 4) Check your answer(s)!

Two solutions:

$$|5x + 4| - 10 = -5$$

(isolate the absolute value)

$$|5x + 4| = 5$$

<p>('positive equation')</p> $5x + 4 = 5$ $5x = 1$ $x = \frac{1}{5}$	<p>('negative equation')</p> $5x + 4 = -5$ $5x = -9$ $x = \frac{-9}{5}$
--	---

(Check)

$ 5(1/5) + 4 - 10 = -5$ $5 - 10 = -5$ $\checkmark -5 = -5$	$ 5(-9/5) + 4 - 10 = -5$ $ -5 - 10 = -5$ $\checkmark 5 - 10 = -5$
---	---

One solution:

$$2|x + 3| + 8 = 8$$

$$2|x + 3| = 0$$

$$|x + 3| = 0$$

$$x + 3 = 0$$

$$x = -3$$

$$2|(-3) + 3| + 8 = 8$$

$$2|0| + 8 = 8$$

$$\checkmark 8 = 8$$

After isolating the absolute value,
for $|ax + b| = c$

- if $c > 0$, then 2 solutions
- $c = 0$, then 1 solution
- $c < 0$, then no solutions

No solutions:

$$5|3x + 2| + 20 = 10$$

(isolate the absolute value)

$$5|3x + 2| = -10$$

$$|3x + 2| = -2$$

← Absolute value output is never negative!

<p>('positive equation')</p> $3x + 2 = -2$ $3x = 4$ $x = 4/3$	<p>('negative equation')</p> $3x + 2 = 2$ $3x = 0$ $x = 0$
---	--

$5 3(4/3) + 2 + 20 = 10$ $5 6 + 20 = 10$ $\times 30 + 20 = 10$	$5 3(0) + 2 + 20 = 10$ $5 2 + 20 = 10$ $\times 30 = 10$
--	---

Solving Absolute Value/Inequality Equations

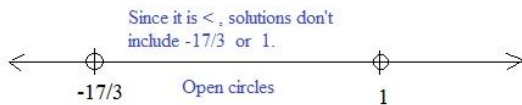
Example: $|3x + 7| < 10$

(solve) $|3x + 7| = 10$

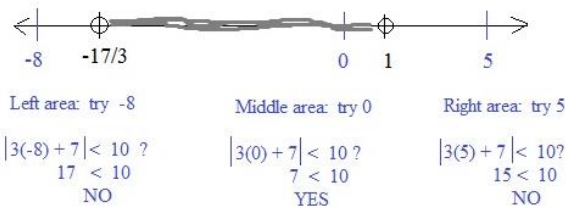
$$\begin{aligned} 3x + 7 &= 10 \\ 3x &= 3 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} 3x + 7 &= -10 \\ 3x &= -17 \\ x &= -17/3 \end{aligned}$$

(graph)



(test regions)



- Steps:
- 1) Solve absolute value equation (for both 'negative' and 'positive' values) Determine "critical points"
 - 2) Graph --- "open circles" or "closed circles"?
 - 3) Test regions and Check answers (plugging points into the ORIGINAL equation)

(check answers -- "critical points")

$$\begin{aligned} |3(-17/3) + 7| &= |-17 + 7| = 10 \checkmark \\ |3(1) + 7| &= |3 + 7| = 10 \checkmark \end{aligned}$$

Example: $3|x - 7| + 4 \geq 10$

(solve)

Isolate the absolute value terms.
Then, solve equal to 'positive answer' and equal to 'negative answer'

$$3|x - 7| + 4 = 10$$

$$3|x - 7| = 6$$

$$|x - 7| = 2$$

'positive' $x - 7 = 2$
 $x = 9$

'negative' $x - 7 = -2$
 $x = 5$

(graph)

"critical values" are $x = 5$ and $x = 9$



"closed circles" because solution includes 5 and 9

(test regions)



Use:

0 (left) $3|(0) - 7| + 4 = 25 \geq 10$ YES

7 (middle) $3|(7) - 7| + 4 = 4 \not\geq 10$ NO

12 (right) $3|(12) - 7| + 4 = 19 \geq 10$ YES

(check answers)

$$3|(5) - 7| + 4 = 10 \checkmark$$

$$3|(9) - 7| + 4 = 10 \checkmark$$

(**Remember: plug values into original equation)

Double Absolute Value Equations & Inequalities

Example: $|x + 5| = |x - 1|$

positive positive

$$x + 5 = x - 1$$

NO SOLUTION

OR

positive negative

$$x + 5 = -(x - 1)$$

$$x + 5 = -x + 1$$

$$2x = -4$$

$$x = -2$$

Notice, the other 2 possibilities offer the same results..

negative positive

$$-(x + 5) = x - 1$$

$$-x - 5 = x - 1$$

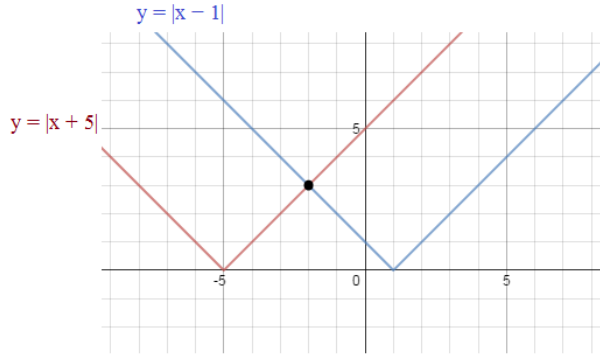
$$x = -2$$

negative negative

OR $-(x + 5) = -(x - 1)$

$$-x - 5 = -x + 1$$

NO SOLUTION



check your answer....

$$x = -2 \quad |(-2) + 5| = |(-2) - 1|$$

$$|3| = |-3|$$

$$3 = 3$$

The two absolute value functions intersect at $x = -2$

Example: $|2x + 6| = 3|x + 4| - 5$

In this case, the slope (and shape) of the absolute values are different, and there are 2 solutions!

negative positive

$$-(2x + 6) = 3(x + 4) - 5$$

$$-2x - 6 = 3x + 12 - 5$$

$$-13 = 5x$$

$$x = -13/5$$

negative negative

$$-(2x + 6) = 3 \cdot -(x + 4) - 5$$

$$-2x - 6 = -3x - 12 - 5$$

$$x = -11$$

positive positive

$$2x + 6 = 3(x + 4) - 5$$

$$2x + 6 = 3x + 12 - 5$$

$$x = -1$$

positive negative

$$(2x + 6) = 3 \cdot -(x + 4) - 5$$

$$2x + 6 = -3x - 12 - 5$$

$$5x = -23$$

$$x = -23/5$$

then, check your answers....

$$|2(-13/5) + 6| = 3|(-13/5) + 4| - 5$$

$$x = -13/5 \quad \frac{4}{5} = \frac{21}{5} - 5$$

$$\frac{4}{5} \neq \frac{-4}{5}$$

$$|2(-23/5) + 6| = 3|(-23/5) + 4| - 5$$

$$x = -23/5 \quad \frac{16}{5} = 3 \cdot \frac{3}{5} - 5$$

$$\frac{16}{5} \neq \frac{-16}{5}$$

$$|2(-11) + 6| = 3|(-11) + 4| - 5$$

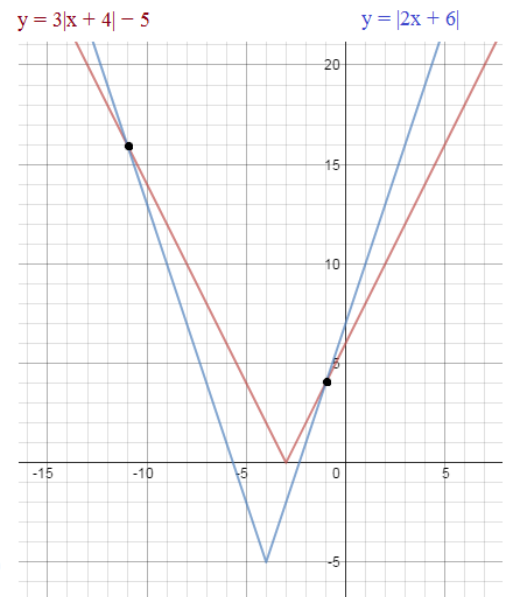
$$x = -11 \quad 16 = 3(7) - 5$$

$$16 = 16$$

$$|2(-1) + 6| = 3|(-1) + 4| - 5$$

$$x = -1 \quad 4 = 9 - 5$$

$$4 = 4$$



The two absolute functions intersect at $(-1, 4)$ and $(-11, 16)$

Double Absolute Value Equations & Inequalities

Example: $|2x + 4| > |x + 3|$

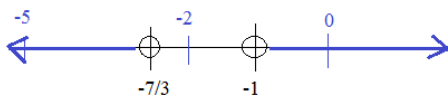
Set equations equal to each other to find the critical value(s), where the absolute values intersect.

<i>positive positive</i>	<i>positive negative</i>	<i>negative positive</i>	<i>negative negative</i>
$2x + 4 = x + 3$	$2x + 4 = -(x + 3)$	$-(2x + 4) = x + 3$	$-(2x + 4) = -(x + 3)$
$x = -1$	$2x + 4 = -x - 3$	$-2x - 4 = x + 3$	$-2x - 4 = -x - 3$
	$3x = -7$	$-3x = 7$	$x = -1$
	$x = -7/3$	$x = -7/3$	

At $x = -1$: $|2(-1) + 4| = |(-1) + 3|$ At $x = -7/3$: $|2(-7/3) + 4| = |(-7/3) + 3|$

$2 = 2$ ✓ $2/3 = 2/3$ ✓

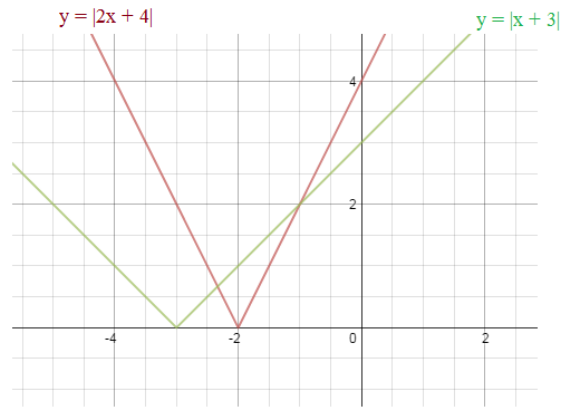
Then, test the regions (intervals) to solve the inequality...



at $x = -5$: $|2(-5) + 4| > |(-5) + 3|$
 $6 > 2$ YES

at $x = -2$: $|2(-2) + 4| > |(-2) + 3|$
 $0 > 1$ NO

at $x = 0$: $|2(0) + 4| > |(0) + 3|$
 $4 > 3$ YES



In the intervals $(-\infty, -7/3)$ and $(-1, \infty)$,
 $|2x + 4|$ is above $|x + 3|$

Example: $-|x + 3| + 7 \leq 5|x + 2| - 10$

1) Find possible critical values:

<i>positive positive</i>	<i>negative negative</i>
$-(x + 3) + 7 = 5(x + 2) - 10$	$-(x + 3) + 7 = 5 \cdot -(x + 2) - 10$
$-x + 4 = 5x + 0$	$-x + 4 = -5x - 20$
$x = 2/3$	$x = -6$
<i>negative positive</i>	<i>negative negative</i>
$-(-(x + 3) + 7) = 5(x + 2) - 10$	$-(-(x + 3) + 7) = 5 \cdot -(x + 2) - 10$
$x + 10 = 5x + 0$	$x + 10 = -5x - 20$
$x = 5/2$	$x = -5$

2) Check answers

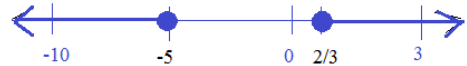
if $x = 2/3$, then
 $-|(2/3) + 3| + 7 = 5|(2/3) + 2| - 10$
 $10/3 = 10/3$ ✓

if $x = -6$, then
 $-|(-6) + 3| + 7 = 5|(-6) + 2| - 10$
 $4 = 10$ ✗

if $x = 5/2$, then
 $-|(5/2) + 3| + 7 = 5|(5/2) + 2| - 10$
 $3/2 = 25/2$ ✗

if $x = -5$, then
 $-|(-5) + 3| + 7 = 5|(-5) + 2| - 10$
 $5 = 5$ ✓

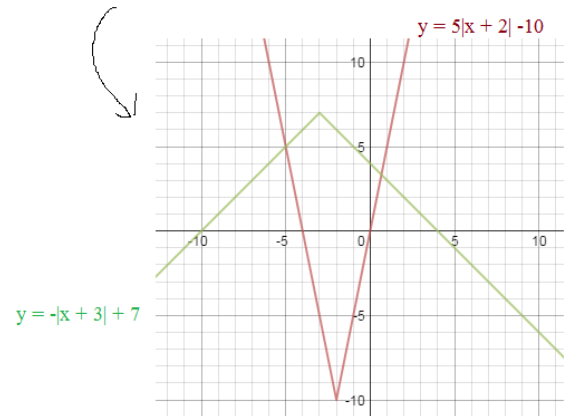
3) Test the regions...



at $x = 0$, $-|(0) + 3| + 7 \leq 5|(0) + 2| - 10$
 $4 \leq 0$ NO

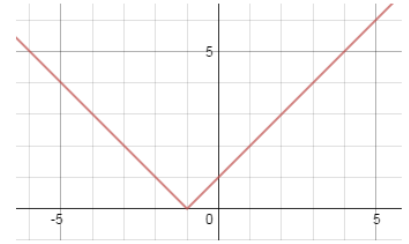
at $x = -10$, YES... at $x = 3$, YES...

In the intervals $(-\infty, -5]$ and $[2/3, \infty)$,
 $-|x + 3| + 7$ is under or at $5|x + 2| - 10$

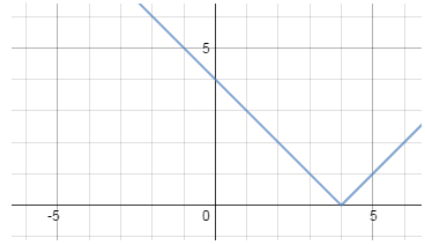


Example: $y = |x + 1| + |x - 4|$

$y = |x + 1|$ The graph "turns" or "kinks" at -1



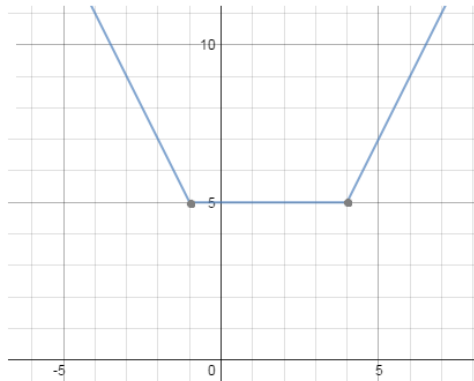
$y = |x - 4|$ The graph turns at $x = 4$



$y = |x + 1| + |x - 4|$

The graph turns at $x = -1$ and $x = 4$
(the "corners" are at -1 and 4)

at $x = -1, y = +5$ $(-1, 5)$
at $x = 4, y = +5$ $(4, 5)$



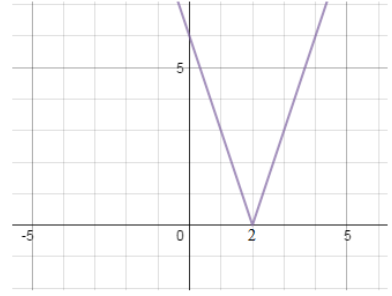
The slope in the interval $(-\infty, -1)$ is -2
slope of $|x + 1|$ is -1 and slope of $|x - 4|$ is -1
and, the sum of the slopes/rates of change is -2

The slope in the interval $(-1, 4)$ is 0
slope of $|x + 1|$ is $+1$ and slope of $|x - 4|$ is -1
and, the sum of the slopes is 0

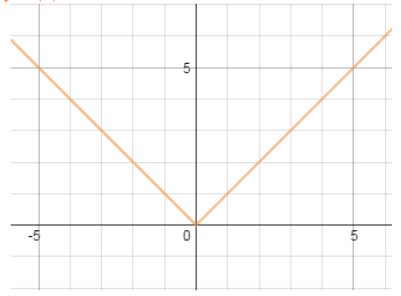
The slope in the interval $(4, \infty)$ is 2
slope of $|x + 1|$ is $+1$ and slope of $|x - 4|$ is $+1$
and, the sum of the slopes is $+2$

Example: $y = 3|x - 2| + |x|$

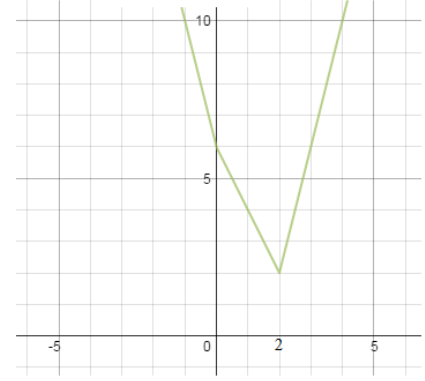
$y = 3|x - 2|$



$y = |x|$



$y = 3|x - 2| + |x|$



NOTE: slope does not exist at kinks and corners. Why? Because the slope from the left is different than the slope from the right. It's ambiguous and cannot be defined...

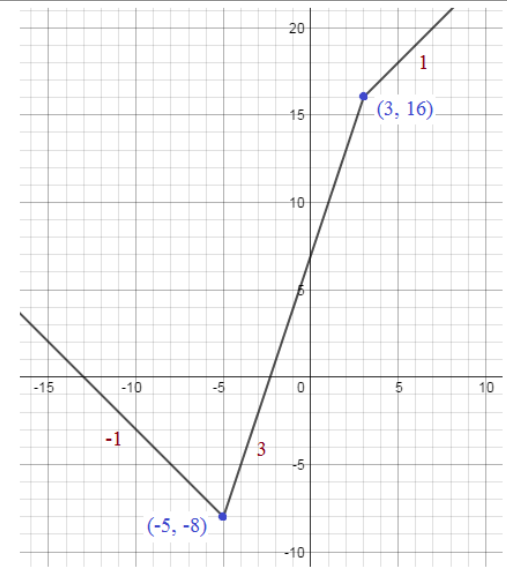
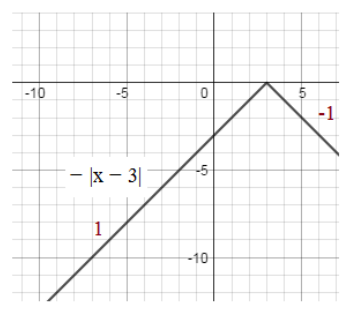
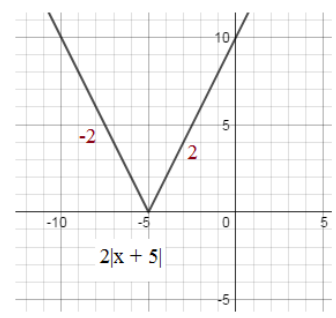
The "kinks" occur at $x = 2$ and $x = 0$
The slopes of each segment are the sums of the separate absolute values!
ex: slope at $x = 1$ is $-3 + 1 = -2$

Example: $y = 2|x + 5| - |x - 3|$

the "corners/kinks" will occur at $x = -5$ and $x = 3$
at $x = -5, y = -8$ at $x = 3, y = 16$

The slope of the line left of $(-5, -8)$ is -1
between $(-5, -8)$ and $(3, 16)$ is $+3$
right of $(3, 16)$ is $+1$

slopes in red



How to sketch a double absolute value graph

Example: Sketch $y = 2|x + 4| - |x - 7|$

Step 1: Identify the "kinks" or "corners" of the composite graph

Take *each* segment and find the maximum or minimum

$2|x + 4|$ ----> minimum occurs at $x = -4$

$-|x - 7|$ ----> maximum occurs at $x = 7$

Step 2: Draw segment(s) connecting the corners...

If $x = -4$, then $y = -11$ So, $(-4, -11)$ is a 'corner'

If $x = 7$, then $y = 22$ So, $(7, 22)$ is a 'corner'

Step 3: Extend the graph

Method 1: Use the slopes to extend lines...

$2|x + 4|$: slope for $x < -4$ is -2

for $x > -4$ is 2

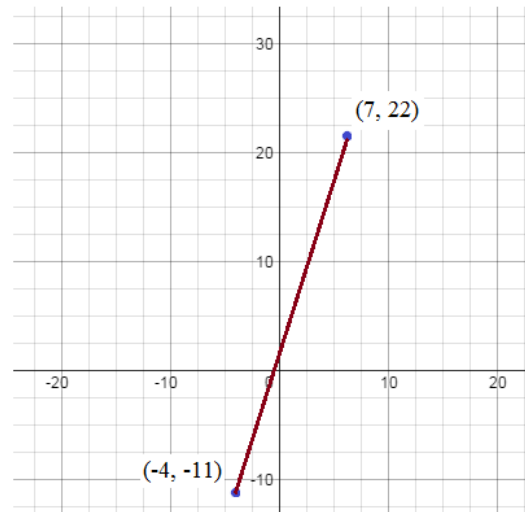
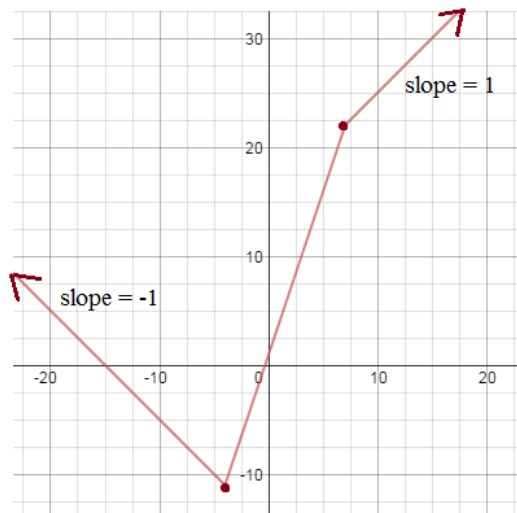
$-|x - 7|$: slope for $x < 7$ is 1

slope for $x > 7$ is -1

so, combined: slope for $x < -4$ is -1 ($-2 + 1$)

slope for $-4 < x < 7$ is 3 ($2 + 1$)

slope for $x > 7$ is 1 ($2 + (-1)$)



Method 2: Identify a point in each region. Then, draw lines...

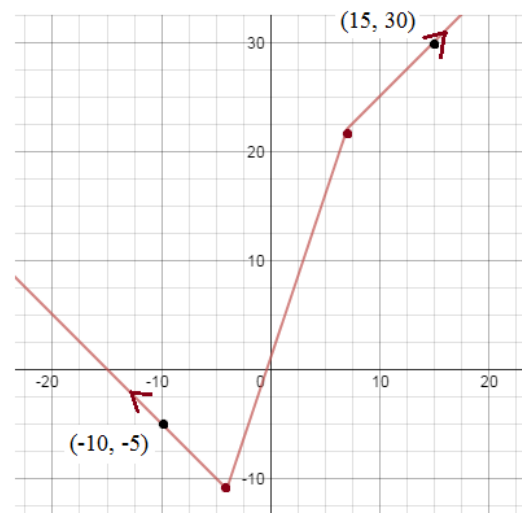
The corners are at $(-4, -11)$ and $(7, 22)$

We need to extend a ray from $x = -4$ to the left...
so, pick a point less than -4 ...

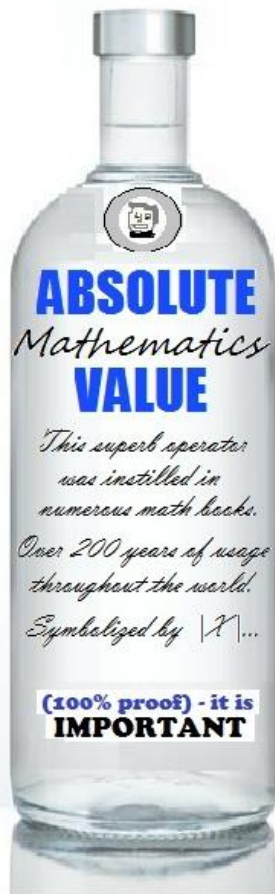
If $x = -10$, then $y = -5$

and, we need to extend a ray from $x = 7$ to the right...
so, pick a point greater than 7 ...

If $x = 15$, then $y = 30$



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Practice Quiz (and Solutions)-→

I. Solve the equations

a) $|x + 6| = |x - 10|$

b) $\frac{|x + 1|}{|x - 1|} = 5$

II. Solve the inequalities

a) $|x + 3| + |2x - 4| > 6$

b) $|x - 4| < -|x + 5| + 2$

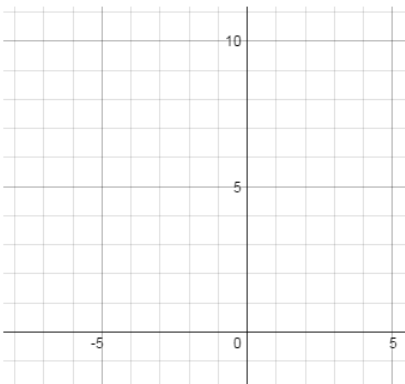
c) $\frac{|2x + 3|}{|x| - 2} \geq 10$

d) $2|x + 1| \leq 3|x + 5|$

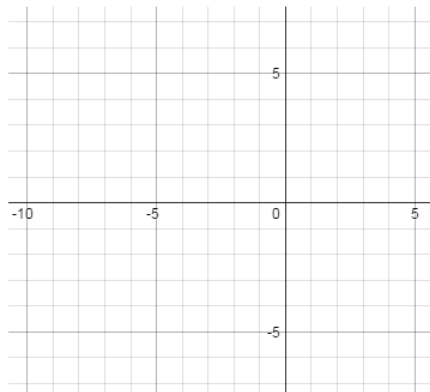
III. Graph

Double Absolute Values Quiz

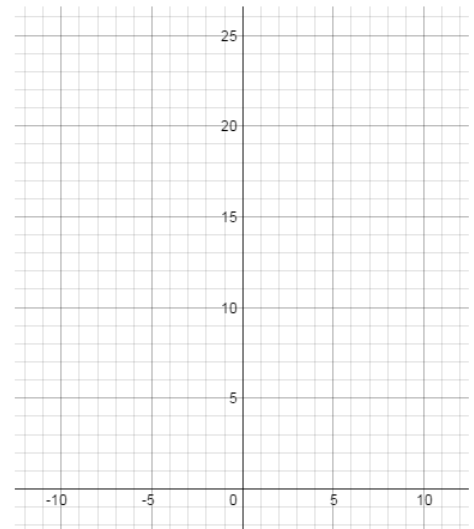
a) $y = |x| + |x + 4|$



b) $y = |x + 6| - |x + 1|$

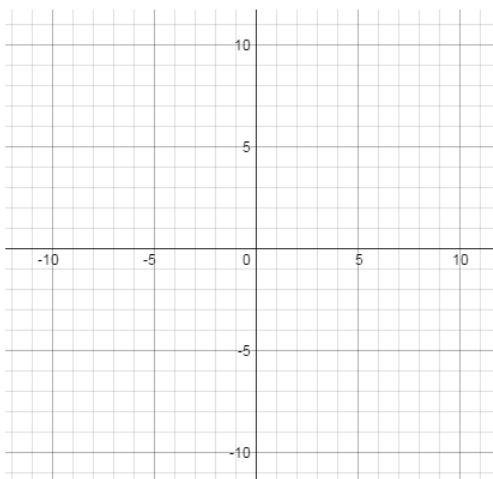


c) $y = 2|x - 3| + 3|x + 1|$

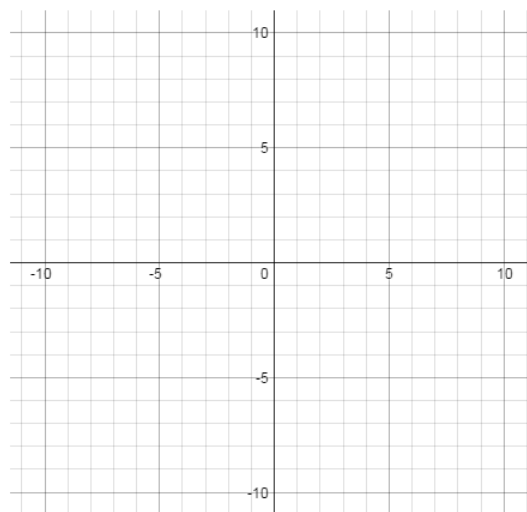


IV. Graph and Solve

a) $7 = |x + 5| + |x - 3|$



b) $|x + 4| = -2|x + 4| + 6$



I. Solve the equations

a) $|x + 6| = |x - 10|$
 $(x + 6) = (x - 10)$ OR $(x + 6) = -(x - 10)$
 no solution $x = 2$
 $-(x + 6) = (x - 10)$ OR $-(x + 6) = -(x - 10)$
 $x = 2$ no solution
 check: $x = 2: |(2) + 6| = |(2) - 10|$
 $8 = 8$ ✓

b) $\frac{|x + 1|}{|x - 1|} = 5$ cross multiply: $5|x - 1| = |x + 1|$

find possible solutions:
 $5(x - 1) = (x + 1)$ OR $5(x - 1) = -(x + 1)$
 $x = 3/2$ $x = 2/3$
 $5 \cdot -(x - 1) = (x + 1)$ OR $5 \cdot -(x - 1) = -(x + 1)$
 $-5x + 5 = x + 1$ $-5x + 5 = -x - 1$
 $x = 2/3$ $x = 3/2$

check your answers: check: $x = 2/3: \frac{|(2/3) + 1|}{|(2/3) - 1|} = 5$ $x = 3/2: \frac{|(3/2) + 1|}{|(3/2) - 1|} = 5$
 $\frac{|5/3|}{|-1/3|} = 5$ ✓ $\frac{|5/2|}{|1/2|} = 5$ ✓

Double Absolute Values Quiz

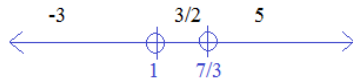
II. Solve the inequalities

a) $|x + 3| + |2x - 4| > 6$

Find "critical values" where equations equal 6

Test each region

$(x + 3) + (2x - 4) = 6$
 $x = 7/3$



$(x + 3) + -(2x - 4) = 6$
 $x = 1$

at $x = -3: |0| + |-10| > 6$ YES

at $x = 3/2: |9/2| + |-1| > 6$ NO

~~$-(x + 3) + (2x - 4) = 6$~~

at $x = 5: |8| + |6| > 6$ YES

~~extraneous not solutions~~
 ~~$-(x + 3) + -(2x - 4) = 6$~~
 ~~$x = 13$~~
 ~~$x = -5/3$~~

$(-\infty, 1) \cup (7/3, \infty)$

b) $|x - 4| < -|x + 5| + 2$

~~$(x - 4) = -(x + 5) + 2$~~

Find any "critical values"

~~$x = 1/2$~~

**After checking each possibility, we find there are no critical values....

~~$(x - 4) = --(x + 5) + 2$~~

~~$x - 4 = x + 7$ no solution~~

So, the solution is either all real numbers or no real numbers...

~~$-(x - 4) = -(x + 5) + 2$~~

~~$-x + 4 = -x - 3$ no solution~~

If we test $x = 0,$

~~$-(x - 4) = --(x + 5) + 2$~~

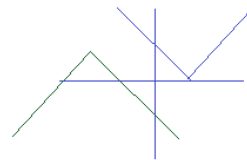
~~$-2x = 3$~~

$|(0) - 4| < -|(0) + 5| + 2$

~~$x = -3/2$~~

$4 < -3$ NO

NO SOLUTIONS



If you graph each equation, you see there are no intersections...

c) $\frac{|2x + 3|}{|x - 2|} \geq 10$

If $x = 2$ or $-2,$ the left equation is undefined. Then, to find other values, cross multiply...

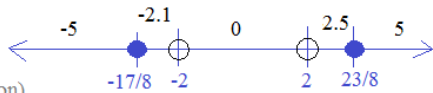
$|2x + 3| = 10|x| - 20$

$(2x + 3) = 10(x) - 20$
 $x = 23/8$

~~$(2x + 3) = 10(-x) - 20$~~
 ~~$x = -23/12$~~

~~$-(2x + 3) = 10(x) - 20$~~
 ~~$x = 17/12$~~

~~$-(2x + 3) = 10(-x) - 20$~~
 ~~$x = -17/8$~~



(test each region)

at $x = -5: \frac{|-7|}{|-5| - 2} = \frac{7}{3} \geq 10$
 NO

at $x = 2.5: \frac{|8|}{|2.5| - 2} = 16 \geq 10$
 YES!

at $x = 0: \frac{|3|}{|0| - 2} = -3/2 \geq 10$
 NO

at $x = -2.1: \frac{|-1.2|}{|-2.1| - 2} = 12 \geq 10$
 YES!

at $x = 5: \frac{|13|}{|5| - 2} = 13/3 \geq 10$
 NO

$[-17/8, -2) \cup (2, 23/8]$

d) $2|x + 1| \leq 3|x + 5|$

$2(x + 1) = 3(x + 5)$

$x = -13$

$2 \cdot -(x + 1) = 3(x + 5)$

$-2x - 2 = 3x + 15$

$x = -17/5$

$2(x + 1) = 3 \cdot -(x + 5)$

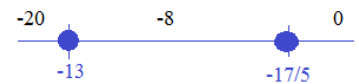
$2x + 2 = -3x - 15$

$x = -17/5$

$2 \cdot -(x + 1) = 3 \cdot -(x + 5)$

$-2x - 2 = -3x - 15$

$x = -13$



(test each region)

at $x = -20: 2|-19| < 3|-15|$
 $38 < 45$ YES

at $x = -8: 2|-7| < 3|-3|$
 $14 < 9$ NO

at $x = 0: 2|1| < 3|5|$
 $2 < 15$ YES

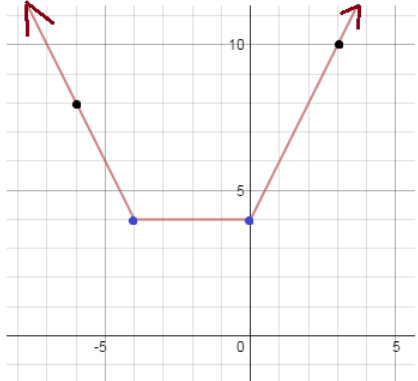
$(-\infty, -13] \cup [-17/5, \infty)$

Double Absolute Values Quiz

III. Graph

a) $y = |x| + |x + 4|$

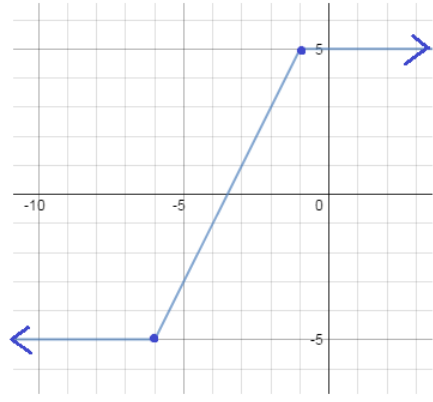
The "kinks" or "corners" are at $(0, 4)$ and $(-4, 4)$



then, pick a point left -4: $(-6, 8)$
pick a point right of 0: $(3, 10)$

b) $y = |x + 6| - |x + 1|$

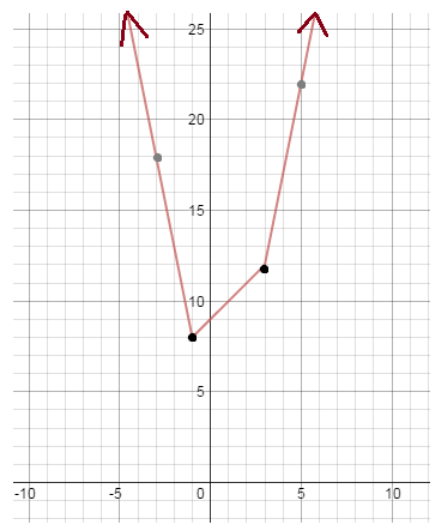
The corners are at $(-6, -5)$ and $(-1, 5)$
Connect them with a segment...



The slope for $x < -6$ or $x > -1$ is zero.. (horizontal lines)

c) $y = 2|x - 3| + 3|x + 1|$

Kinks at $x = -1$ and $x = 3$ $(-1, 8)$ and $(3, 12)$

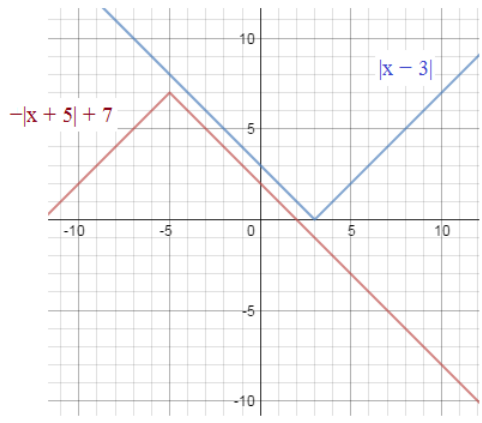


Then, plot points at $x = -3$ and $x = 5$
 $(-3, 18)$ and $(5, 22)$

IV. Graph and Solve

a) $7 = |x + 5| + |x - 3|$

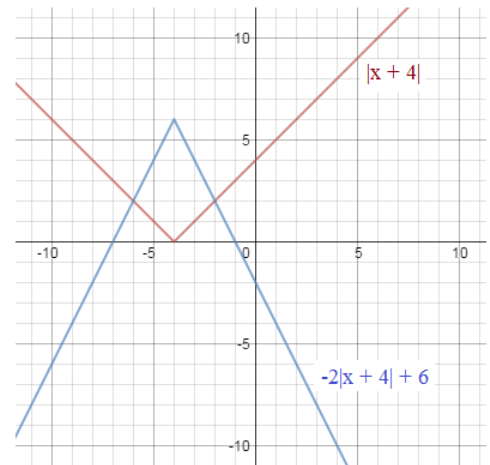
Rewrite as $-|x + 5| + 7 = |x - 3|$



No intersection, so NO SOLUTIONS!

b) $|x + 4| = -2|x + 4| + 6$

Graphically, we see the intersections are at $(-6, 2)$ and $(-2, 2)$

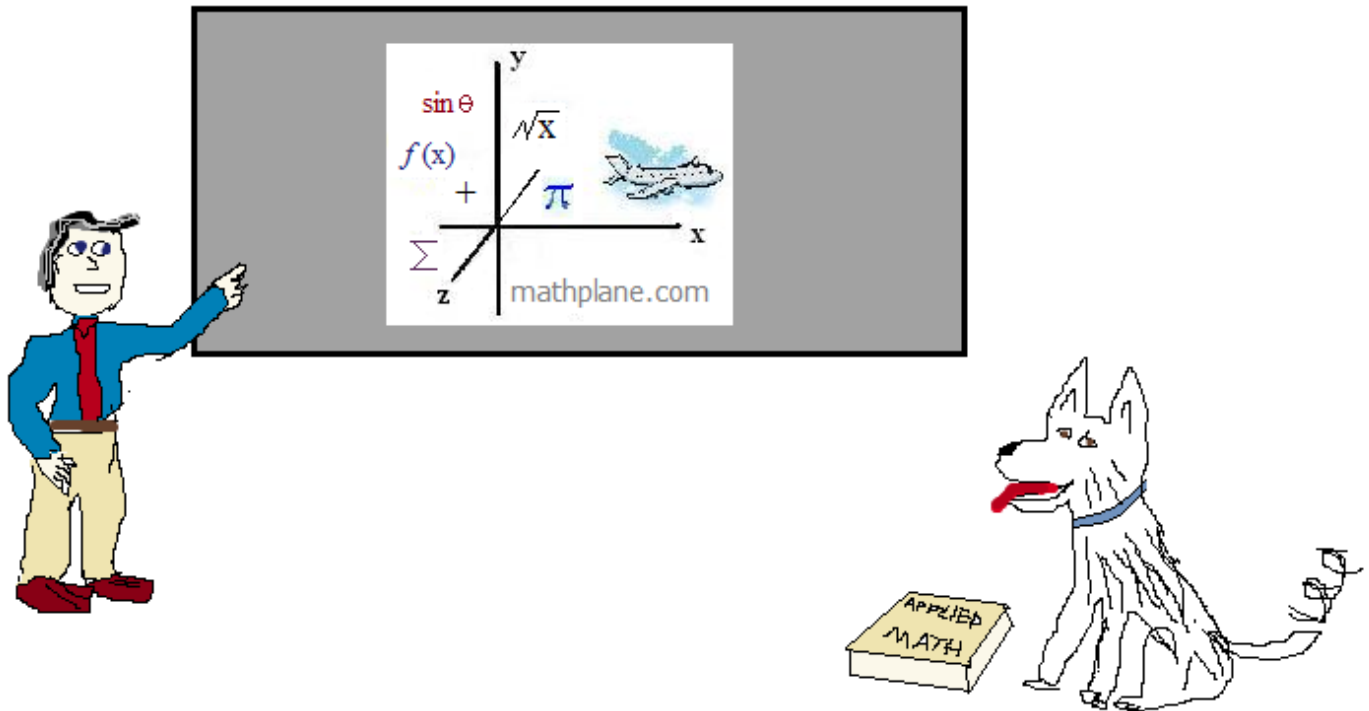


Algebraically:
 positive positive negative negative
 $(x + 4) = -2(x + 4) + 6$ $-(x + 4) = -2 \cdot -(x + 4) + 6$
 $3x = -6$ $-x - 4 = 2x + 14$
 $x = -2$ $x = -6$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



We're also at Facebook, Google+, Pinterest, and TeachersPayTeachers

One More topic →

"Triple Absolute Value Equations"

Example: Sketch $y = |x + 3| + 2|x - 5| - 4|x|$

Step 1: Find the "corners" or "kinks" where the slope changes....

$|x + 3|$ -----> at $x = -3$ $(-3, 4)$

$2|x - 5|$ -----> at $x = +5$ $(5, -12)$

$-4|x|$ -----> at $x = 0$ $(0, 13)$

Step 2: Connect the corners...

Notice, the slopes of each segment:

Left: $(-1) + (-2) - (-4) = 1$

Left/middle: $(1) + (-2) - (-4) = 3$

Right/middle: $(1) + (-2) - (-4) = -5$

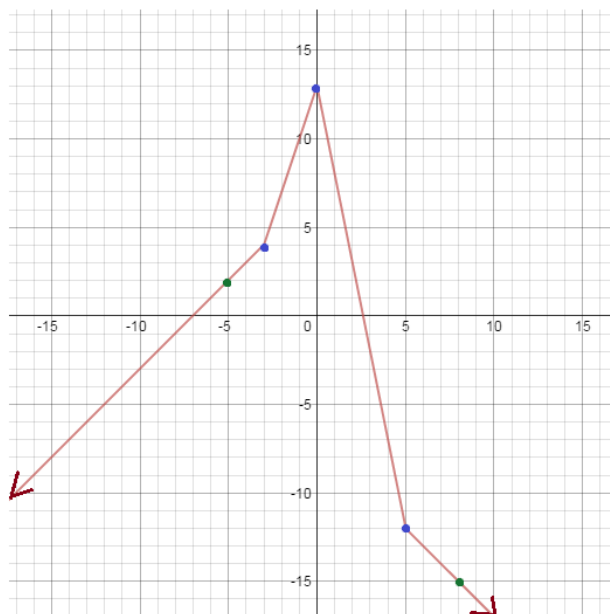
Right: $(1) + (2) - (-4) = -1$

They are the sums of the individual absolute value slopes in each interval!

Step 3: Pick a point in the left region/interval and a point in the right region/interval... Then, extend the graph!

Left: pick a point < -3 ... If $x = -5$, then $y = 2$... $(-5, 2)$

Right: pick a point > 5 ... If $x = 8$, then $y = -15$ $(8, -15)$



Example: Solve $|x| + 3|x + 3| - |x + 7| = 5$

Step 1: Try all possibilities

positive positive positive
 $x + 3(x + 3) - (x + 7) = 5$
 $3x = 3$
 $x = 1$

positive positive negative
 $x + 3(x + 3) - -(x + 7) = 5$
 $5x = -11$
 $x = -11/5$

positive negative positive
 $x + 3 \cdot -(x + 3) - (x + 7) = 5$
 $-3x = -11$
 $x = 11/3$

positive negative negative
 $x + 3 \cdot -(x + 3) - -(x + 7) = 5$
 $-x = 7$
 $x = -7$

negative positive positive
 $-x + 3(x + 3) - (x + 7) = 5$
 $x = 3$

negative positive negative
 $-x + 3(x + 3) - -(x + 7) = 5$
 $3x = 3$
 $x = 1$

negative negative positive
 $-x + 3 \cdot -(x + 3) - (x + 7) = 5$
 $-5x = 21$
 $x = -21/5$

negative negative negative
 $-x + 3 \cdot -(x + 3) - -(x + 7) = 5$
 $-3x = 7$
 $x = -7/3$

Step 2: Check answers (to eliminate extraneous solutions)

After checking the solutions above, we find that

$x = 1$ or $x = -21/5$

$|1| + 3|1 + 3| - |1 + 7| = 5$

$1 + 12 - 8 = 5$ ✓

$|-21/5| + 3|-21/5 + 3| - |-21/5 + 7| = 5$

$21/5 + 18/5 - 14/5 = 5$ ✓

Step 3: Optional: Graph the equatoin...

