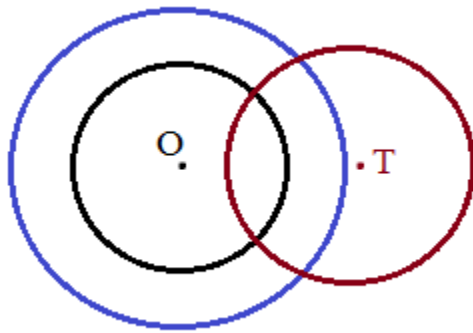


Circles Introduction

Notes, examples, formulas, and practice questions



Topics include parts of a circle, arc length, sector area, and more.

Circle : definitions, notes and formulas

What is it? A circle is a shape consisting of points -- in the same plane -- that are equidistant from a center point.

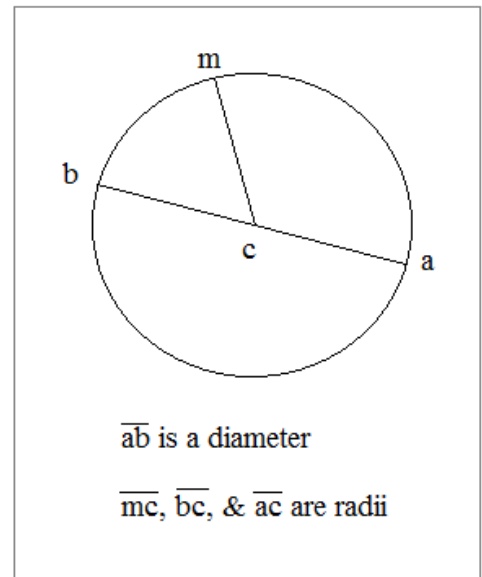
Parts of a circle:

Radius (r): Distance from any point on the circle to the Center (c)

Diameter (d): Length of any line segment that connects two points on the circle AND passes through the center

$$\text{Diameter} = 2 \times \text{Radius}$$

The center is the midpoint of any diameter.



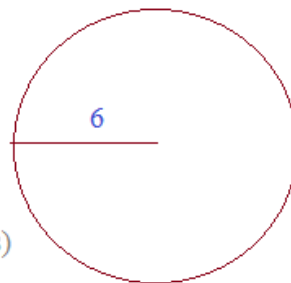
Area of the circle: $\pi (\text{radius})^2$
 πr^2

Circumference: the "perimeter" of the circle: $\pi (\text{diameter})$ OR $2\pi (\text{radius})$
 πd OR $2\pi r$

π is an irrational number approximately 3.14

Example: Given a circle with radius 6 inches

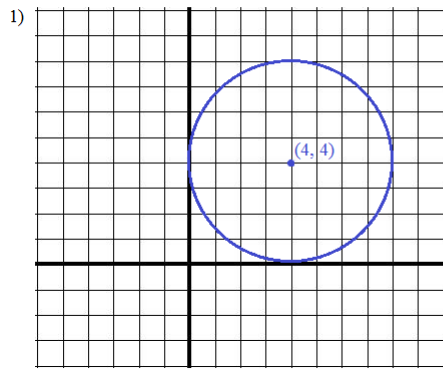
$$\begin{aligned} \text{area} &= \pi r^2 = \pi (6 \text{ inches})^2 \\ &= 36\pi \text{ square inches} \\ &\text{(approximately 113.1 sq. inches)} \end{aligned}$$



$$\begin{aligned} \text{circumference} &= \pi d = \pi (12 \text{ inches}) \\ &= 12\pi \text{ inches} \\ &\text{(approximately 37.7 inches)} \end{aligned}$$

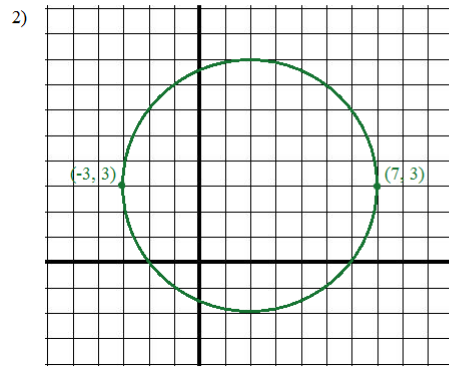
Practice Exercise →

Find the area and circumference of each circle.



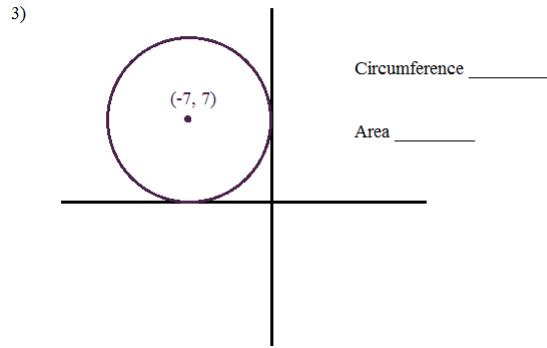
Circumference _____

Area _____



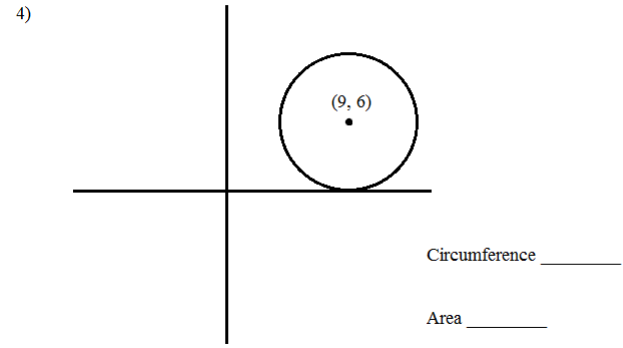
Circumference _____

Area _____



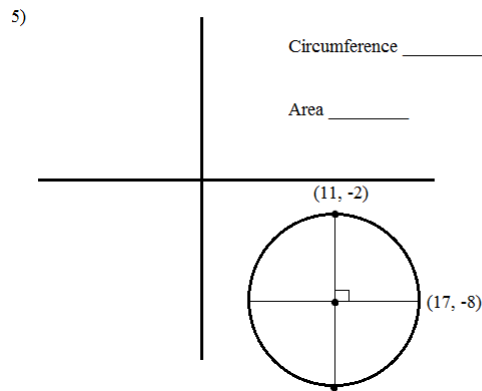
Circumference _____

Area _____



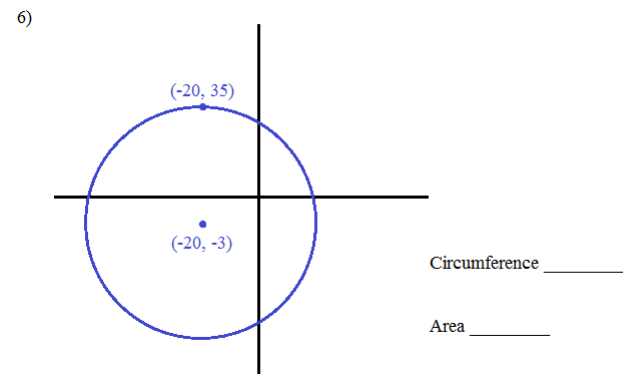
Circumference _____

Area _____



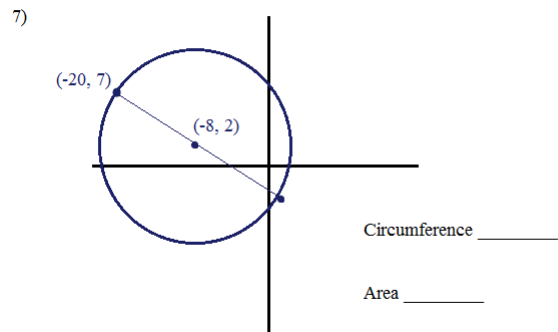
Circumference _____

Area _____



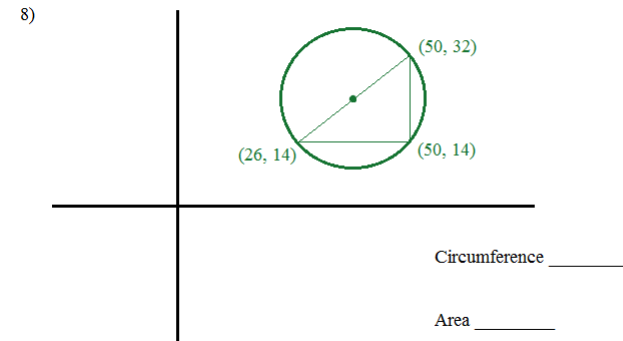
Circumference _____

Area _____



Circumference _____

Area _____



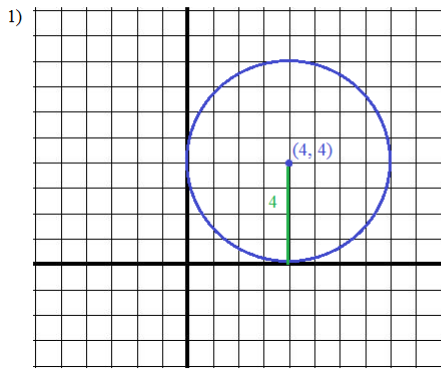
Circumference _____

Area _____

Find the area and circumference of each circle.

SOLUTIONS

Circles Area and Circumference



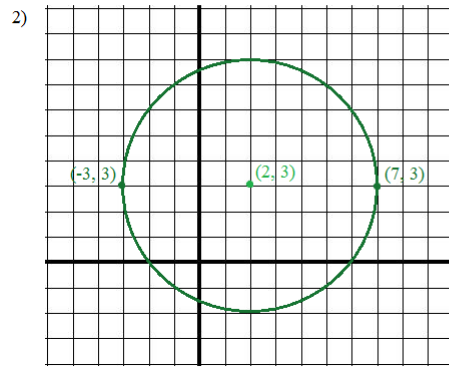
radius: 4

$$C = 2\pi r$$

$$A = \pi r^2$$

Circumference 8π

Area 16π



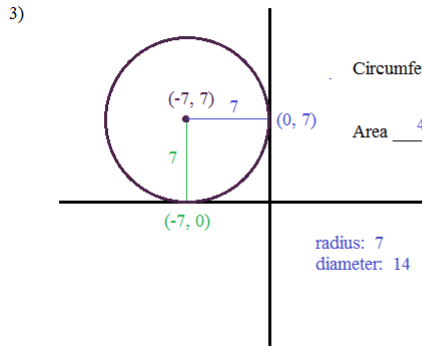
center of circle is midpoint: (2, 3)

radius: 5 diameter: 10

Circumference 10π

Area 25π

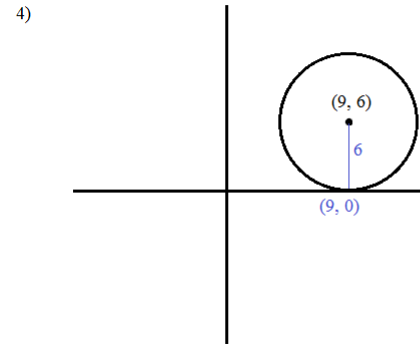
$$A = \pi r^2 \quad C = \pi d$$



Circumference 14π units

Area 49π square units

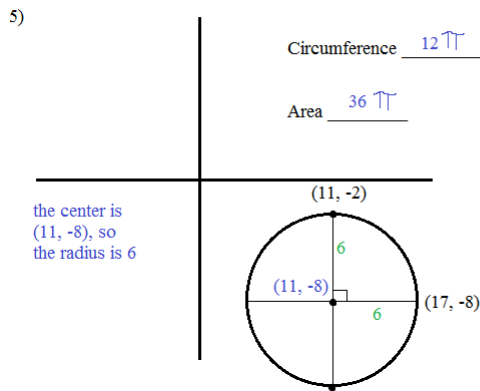
radius: 7
diameter: 14



radius: 6 units

Circumference 12π

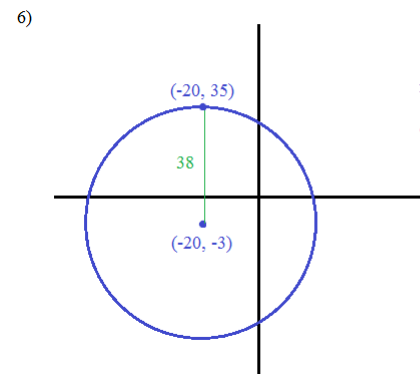
Area 36π



the center is (11, -8), so the radius is 6

Circumference 12π

Area 36π

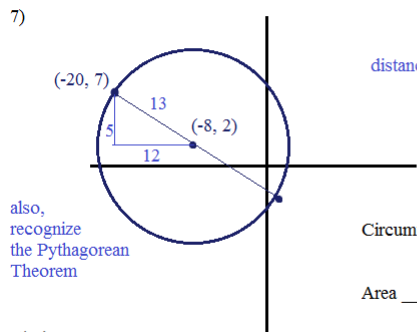


radius: $35 - (-3) = 38$

diameter: 76

Circumference 76π units

Area 1444π square units



also, recognize the Pythagorean Theorem

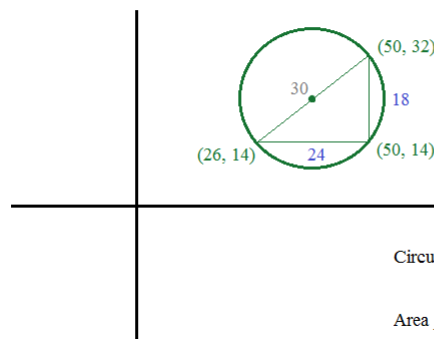
distance formula: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$\sqrt{(-20 - (-8))^2 + (7 - 2)^2}$$

$$\sqrt{(12)^2 + (5)^2} = 13$$

Circumference 26π

Area 169π



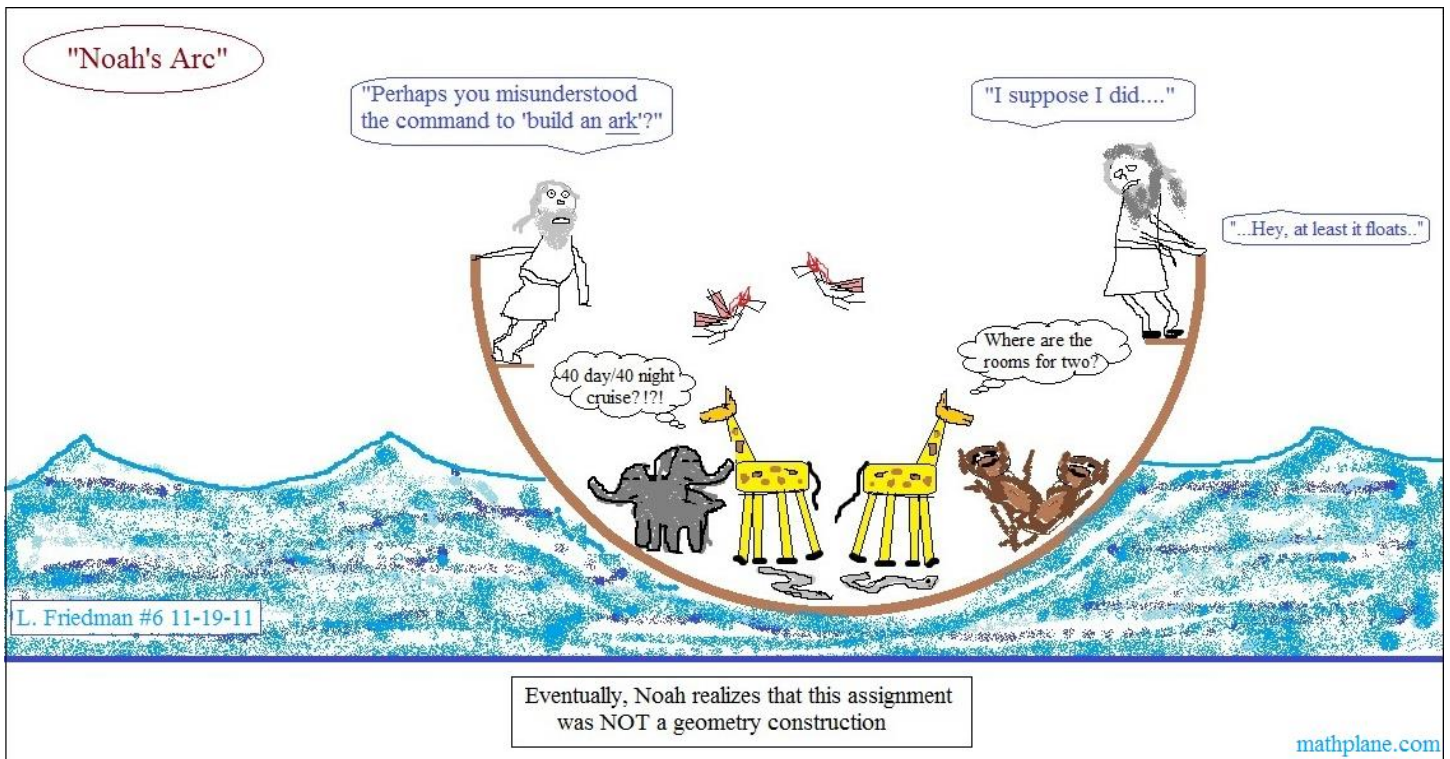
Use Pythagorean Theorem, or recognize this is a "3-4-5 Pythagorean Triple"

$18 - 24 - 30$
 $3 \times 6 \quad 4 \times 6 \quad 5 \times 6$

diameter: 30
radius: 15

Circumference 30π

Area 225π



More notes, examples, and exercises →

Parts of a circle:

Sector: An area inside the circle bounded by 2 radii and an arc.
A portion of the circle (area)

Arc: A curved segment of the circle.
A portion of the circle (perimeter)

$$\text{Sector Area} = (\text{portion of circle})(\text{area of entire circle})$$

$$\text{Arc Length} = (\text{portion of circle})(\text{circumference of entire circle})$$

Example: radius $MO = 10$ units
central angle $MOL = 80^\circ$

$$\text{area of circle } O = 100\pi$$

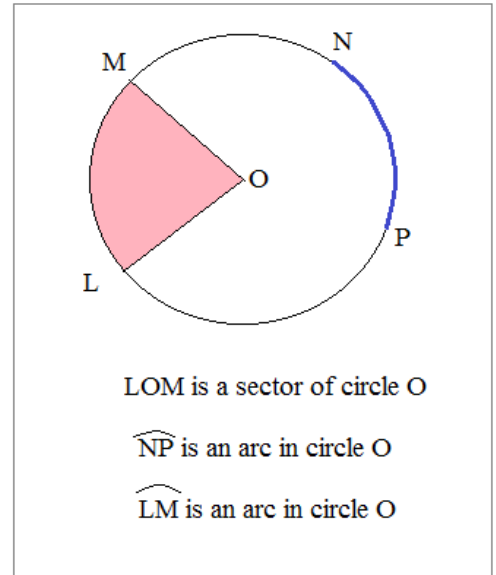
$$\text{circumference of circle } O = 20\pi$$

$$\text{"portion of the circle": } \frac{80^\circ}{360^\circ} = \frac{2}{9}$$

$$\begin{aligned} \text{therefore, sector area} &= \frac{2}{9} \cdot 100\pi \\ &= 69.8 \text{ sq. units} \\ &\quad (\text{approximately}) \end{aligned}$$

$$\begin{aligned} \text{arc length} &= \frac{2}{9} \cdot 20\pi \\ &= 13.96 \text{ units (approximately)} \end{aligned}$$

Circle : definitions, notes and formulas



$$\frac{\text{measure of central } \angle \text{MOL}}{360^\circ} = \frac{\text{arc length } \widehat{ML}}{\text{circumference of circle O}} = \frac{\text{area of sector MOL}}{\text{area of circle O}}$$

Arcs of the Circle

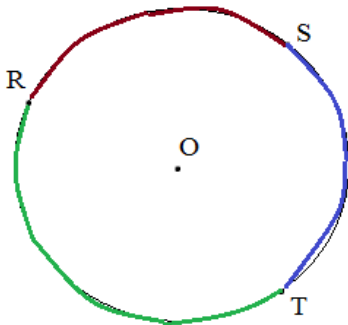
"Minor Arcs"

Lengths less than 1/2 the perimeter of the circle

\widehat{RS} (or \widehat{SR})

\widehat{ST} (or \widehat{TS})

\widehat{TR} (or \widehat{RT})



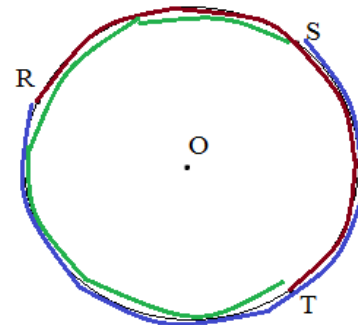
"Major Arcs"

Arc lengths greater than 1/2 the perimeter of the circle

\widehat{RST} (or \widehat{TSR})

\widehat{STR} (or \widehat{RTS})

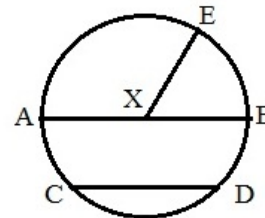
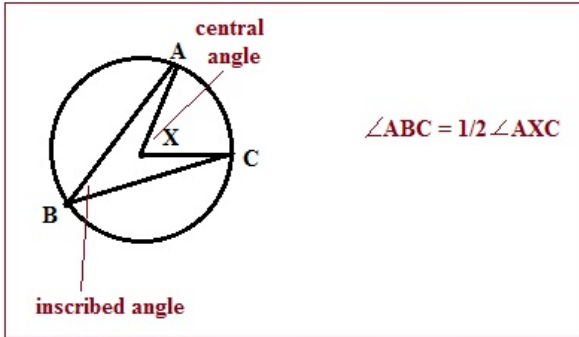
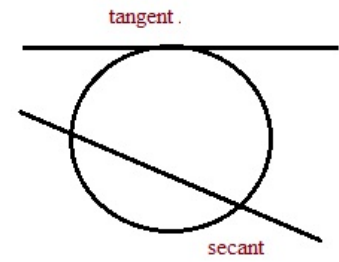
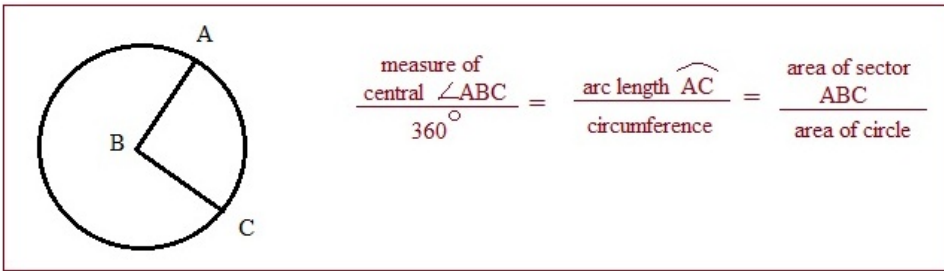
\widehat{TRS} (or \widehat{SRT})



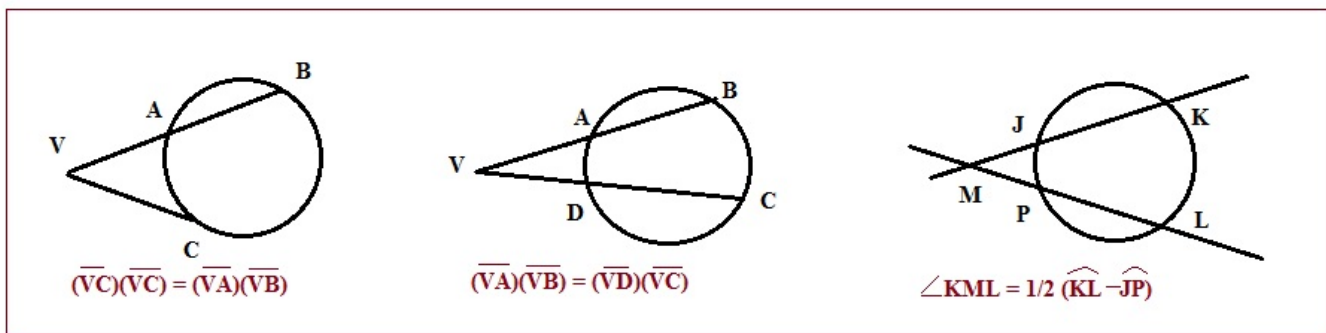
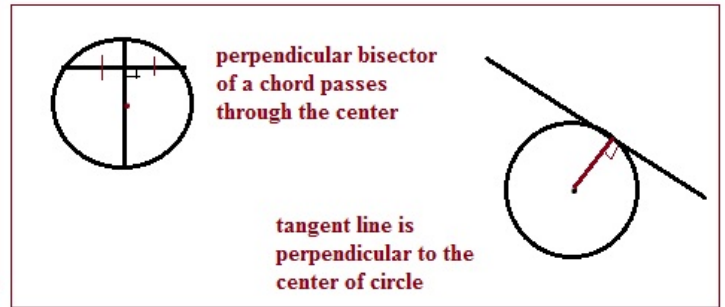
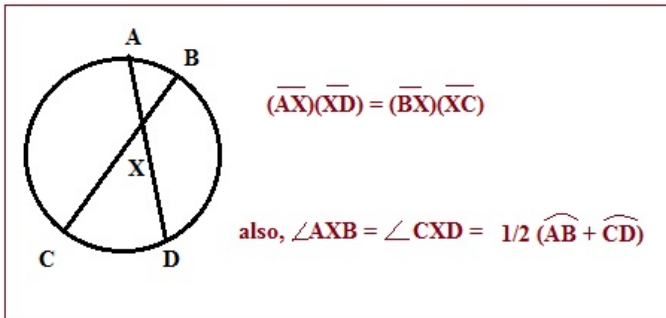
Note: Indicating the points in a specific order identifies the arc

Arc lengths = 1/2 the perimeter are *semi-circles*

Circle Formulas, ratios and relationships



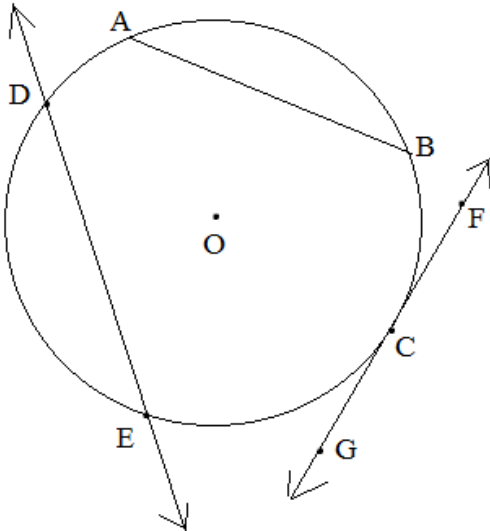
- \widehat{AB} ----> semi-circle
- \widehat{BE} ----> minor arc
- \widehat{BDA} ----> major arc
- \overline{AB} ----> diameter
- \overline{XE} ----> radius
- \overline{CD} ----> chord



Tangent: a line (or curve or plane) that touches the circle at exactly one point

Chord: a line segment with endpoints on the circle

Secant: a line with two points on the circle



Example: Given circle O:

\overline{FG} is a tangent at point C

\overline{AB} is a chord

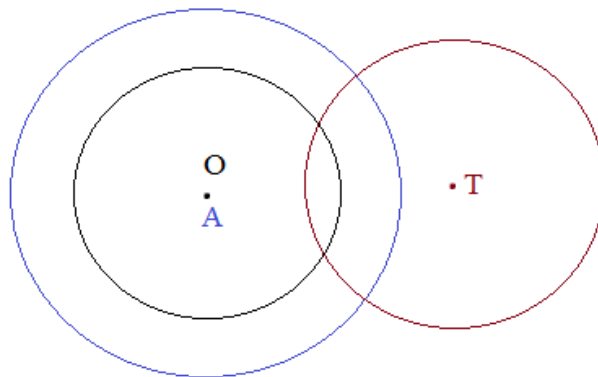
\overleftrightarrow{DE} is a secant (through points D and E)

\overline{DE} is a chord

Concentric Circles

What are they? Circles that share a common center.

Example:



O and A are concentric

also,
O and T are overlapping
(intersecting) circles

A and T are overlapping circles

note: any diameter of circle A would go through circle O
and contain one diameter of circle O

Standard Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

(h, k) is the center
r is the radius

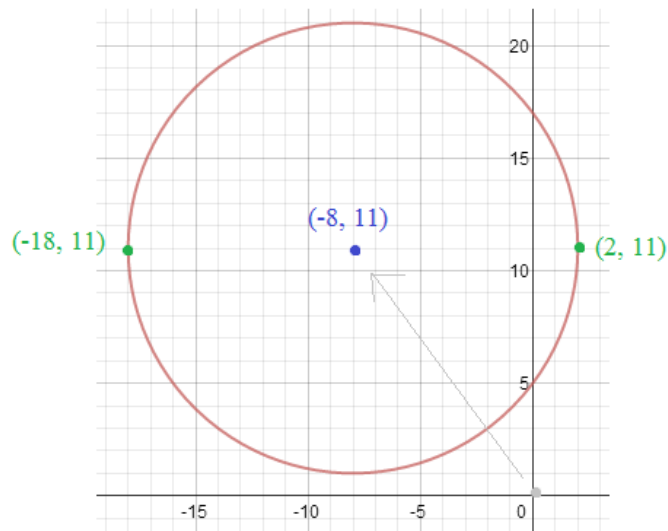
Example: What is the equation of a circle with diameter 20 and its center translated 8 units to the left and 11 units up from the origin?

since diameter is 20, radius is 10...

origin shifted 8 units to the left: (-8
origin shifted 11 units up: (-8, 11)

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + 8)^2 + (y - 11)^2 = 100$$



Example: A circle's diameter has endpoints (1, 4) and (7, -4). What is the equation of the circle?

To determine the equation of a circle, we need the center and the radius.

Center is the midpoint of the diameter's endpoints. (4, 0)

The radius is 1/2 the length of the diameter...

Using the distance formula to find the length:

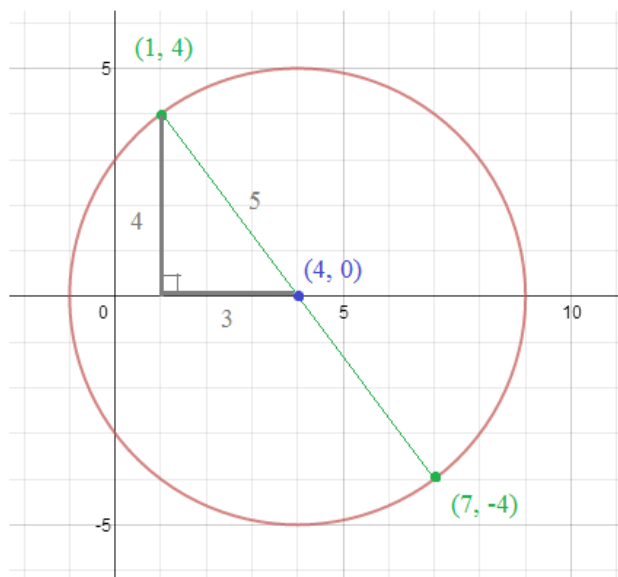
$$\sqrt{36 + 64} = 10$$

Radius: 5

Center (h, k): (4, 0)

$$(x - 4)^2 + y^2 = 25$$

(to check, plug in both endpoints and see if they work in the equation)

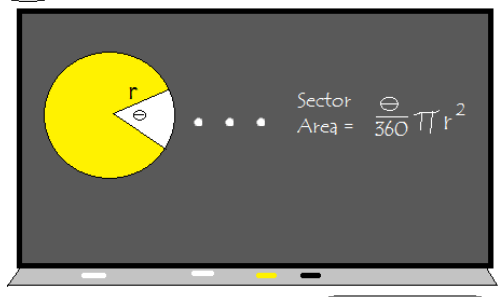
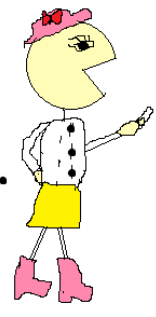
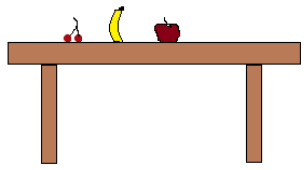


80's
Geometry
Class

Midway
Afterschool
Tutoring
Help

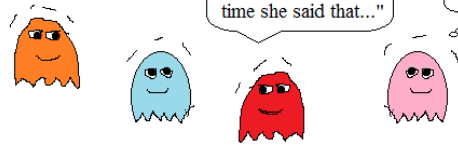
"... therefore, the sector area
is equal to ---
Hey! Clyde, pay attention...
And, sit still!"

"I swear these little monsters
are trying to kill me."



"Like, if we got a
quarter for every
time she said that..."

"Totally..."



L. Friedman #76 3-16-13
www.mathplane.com

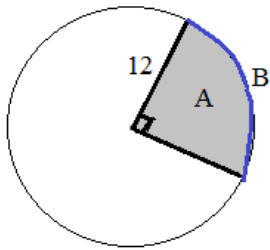
Day after day, Ms. Pacman
faces this unruly group

Practice Questions ->

III. Arc length and Sector area

Determine the arc length and/or sector area of the following:

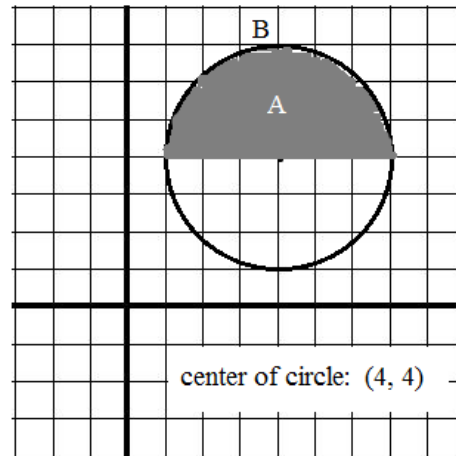
a)



Sector area A =

Arc length B =

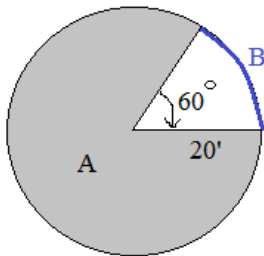
b)



Sector area A =

Arc length B =

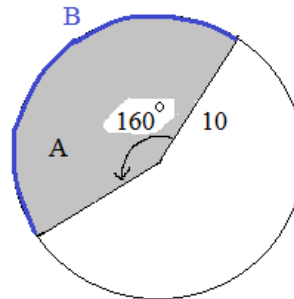
c)



Sector area A =

Arc length B =

d)



Sector area A =

Arc length B =

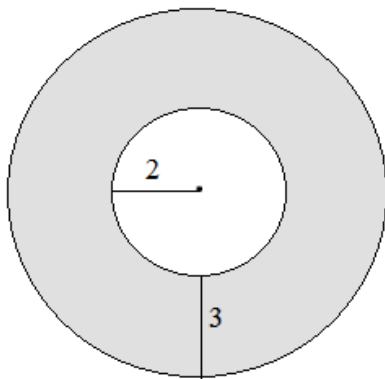
IV. Miscellaneous

- a) Given: circle with center O containing points A and B
arc length of $\widehat{AB} = 7$ meters
circumference of circle = 70 meters

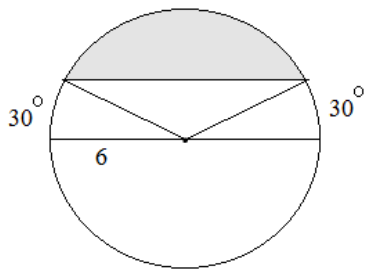
What is the measure of central angle AOB ?

- b) If the (sector) area of $1/6$ of a circle is 24π square feet,
what is the radius of the circle?

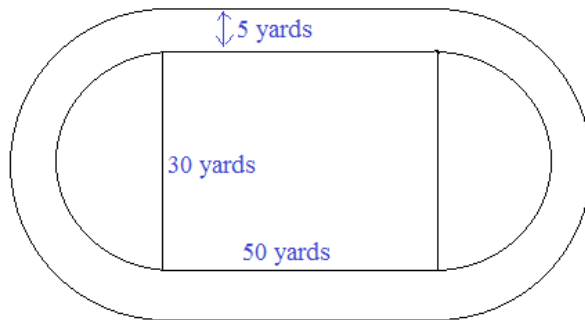
- c) In the following concentric circles,
what is the area of the shaded region?



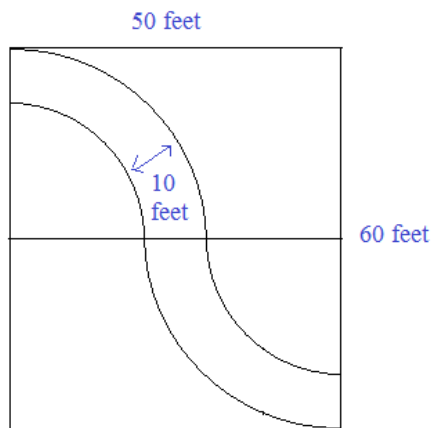
d) Find the measure of the shaded area:



e) Find the distance of the outer track:



f) The city wants to pave both sides of this winding road.
If the road is 10 feet wide, how long will this stretch of side pavement be?



V: Geometry Properties

a) Diameter \overline{AB} of circle O has the following points:

$$A = (2, 12)$$

$$B = (-4, 4)$$

What is the radius?

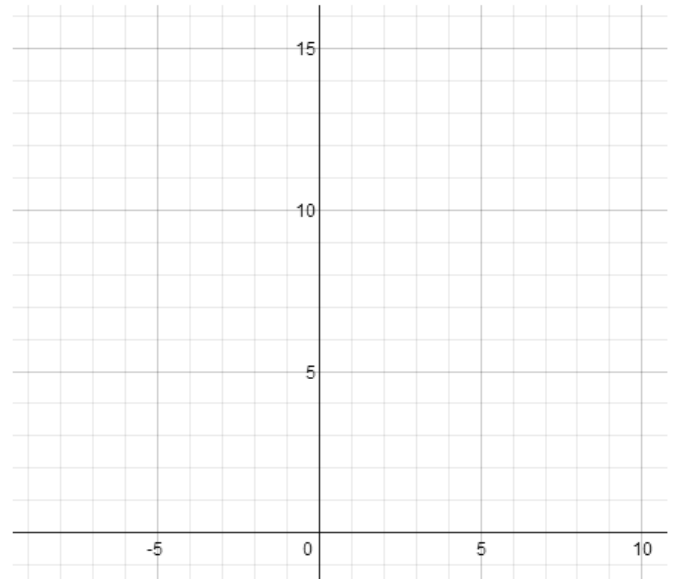
What is the center?

Area of the circle?

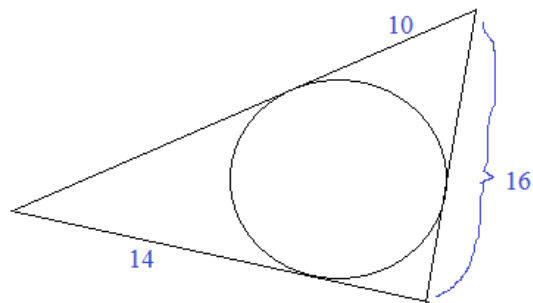
Circumference of the circle?

Equation of the circle?

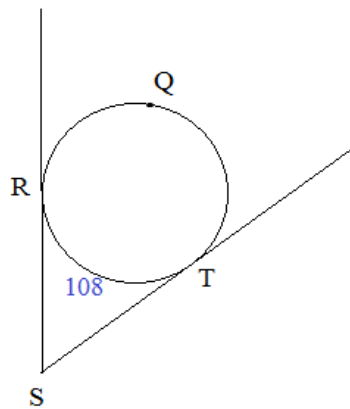
Sketch the graph.

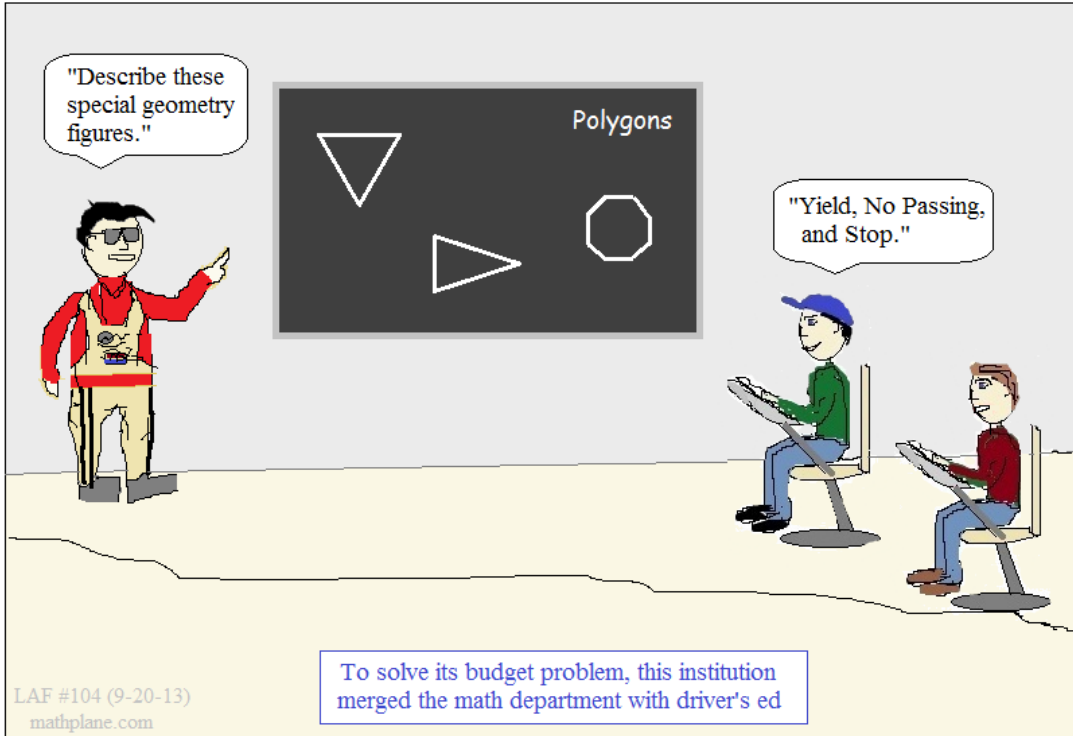


b) What is the perimeter of the triangle?

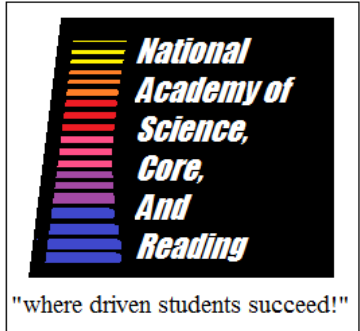


c) What is the measure of $\angle S$?





Accelerated
Math

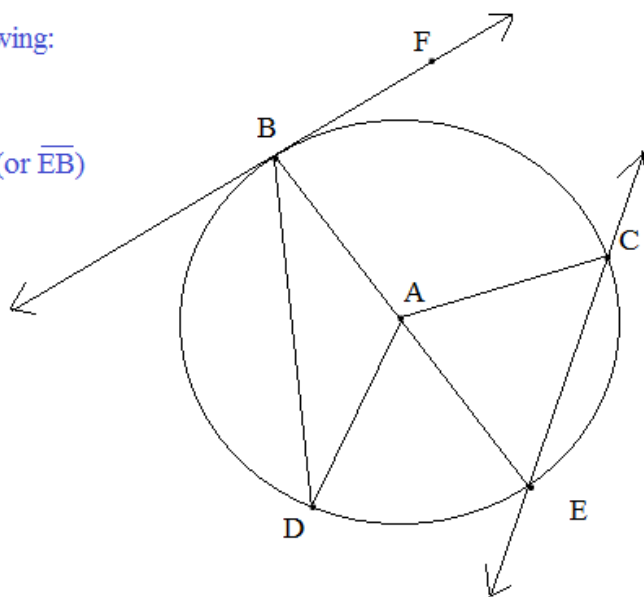


SOLUTIONS ->

I. Identifying parts

Write the letter(s) that describe the parts of the circle: answers include the following:

- a) Center: A
- b) Chord: \overline{BD} (or \overline{DB}); \overline{CE} (or \overline{EC}); \overline{BE} (or \overline{EB})
- c) Secant: \overleftrightarrow{EC} (or \overleftrightarrow{CE})
- d) Tangent: \overleftrightarrow{BF} (or \overleftrightarrow{FB})
- e) A radius: \overline{AB} \overline{AC} \overline{AD} \overline{AE}
- f) Diameter: \overline{BE} (or \overline{EB})
(note: it's the longest chord)
- g) A minor arc: \widehat{BC} ; \widehat{CE} ; \widehat{ED} ; \widehat{DB}
- h) A major arc: \widehat{BCD} \widehat{EBC} \widehat{CDB}
(note: \widehat{BE} is a semi-circle)



II. Circumference and Area

Determine the circumference and area of each circle:

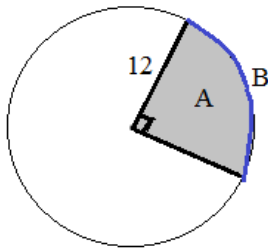
- | | | |
|--|--|--|
| <p>a) radius = 4 feet</p> <p>Area = $\pi(\text{radius})^2$
 $\pi \cdot (4 \text{ feet})^2$
 16π square feet</p> <p>Circumference = $2\pi(\text{radius})$
 $2\pi(4 \text{ feet})$
 8π feet
 (approximately 25.1 feet)</p> | <p>b) diameter = 10 inches
radius = 5 inches</p> <p>Area = $\pi(r)^2$
 $\pi(5 \text{ inches})^2$
 25π square inches</p> <p>Circumference = $\pi(\text{diameter})$
 $\pi(10 \text{ inches})$
 10π inches
 (approximately 31.4 inches)</p> | <p>c) endpoints of the diameter:
(2, 0) and (-4, 0)</p> <p>The length of the diameter is the distance between the endpoints: 6 units</p> <p>$d = 6$
$r = 3$</p> <div style="border: 1px solid black; padding: 5px;"> <p>Area = $\pi(r)^2$
 $= 9\pi$
 Circumference = $\pi(d)$
 $= 6\pi$</p> </div> |
|--|--|--|

III. Arc length and Sector area

SOLUTIONS

Determine the arc length and/or sector area of the following:

a)



central angle is 90 degrees...

$$\text{Portion: } \frac{90}{360} = \frac{1}{4}$$

1/4 of the entire circle

$$\text{Sector area A} = 36\pi$$

$$\text{Arc length B} = 6\pi$$

Circumference:

$$2\pi(12) = 24\pi$$

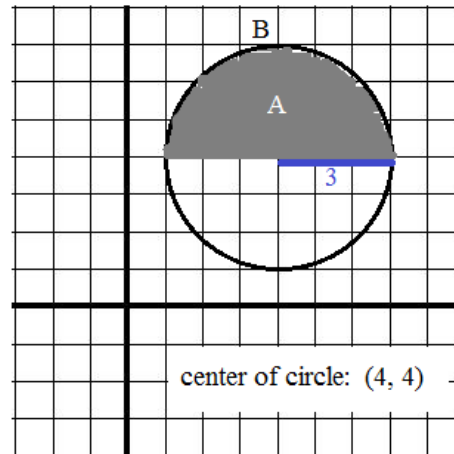
$$\text{Arc Length: } \frac{1}{4} \cdot 24\pi = 6\pi$$

Area of circle:

$$\pi(12)^2 = 144\pi$$

$$\text{Sector Area: } \frac{1}{4} \cdot 144\pi = 36\pi$$

b)



radius is 3 units

Arc B is a semi-circle

$$\text{Portion: } \frac{1}{2}$$

Area of entire circle: 9π

Sector area:

$$\frac{1}{2} \cdot 9\pi =$$

$$\frac{9}{2}\pi$$

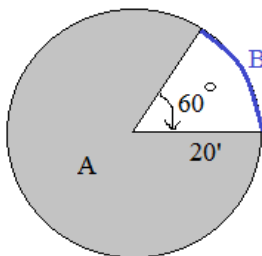
$$\text{Sector area A} = \frac{9}{2}\pi \text{ sq. units}$$

$$\text{Arc length B} = 3\pi \text{ units}$$

$$\text{Circumference of circle: } 2(\pi)3 = 6\pi$$

$$\text{Arc length: } \frac{1}{2} \cdot 6\pi = 3\pi$$

c)



$$\text{Sector area A} = \frac{200}{3}\pi \text{ sq. feet}$$

$$\text{Arc length B} = \frac{20}{3}\pi \text{ feet}$$

central angle = 60 degrees

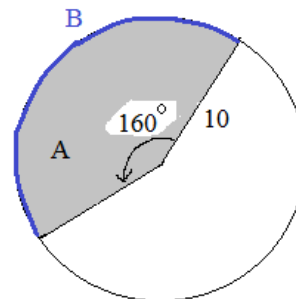
$$\text{portion of the circle: } \frac{60}{360} = \frac{1}{6}$$

$$\text{area of circle: } \pi(r)^2 = 400\pi \text{ sq. feet}$$

$$\text{circumference: } 2\pi(r) = 40\pi \text{ feet}$$

**then, sector portions are 1/6 of the values

d)



$$\text{portion: } \frac{160^\circ}{360^\circ} = \frac{4}{9}$$

$$\text{area} = 100\pi$$

sector area:

$$100\pi \cdot \frac{4}{9} = \frac{400}{9}\pi$$

$$\text{circumference} = 20\pi$$

arc length:

$$20\pi \cdot \frac{4}{9} = \frac{80}{9}\pi$$

$$\text{Sector area A} = \frac{400}{9}\pi$$

$$\text{Arc length B} = \frac{80}{9}\pi$$

- a) Given: circle with center O containing points A and B
 arc length of $\widehat{AB} = 7$ meters
 circumference of circle = 70 meters

What is the measure of central angle AOB ?

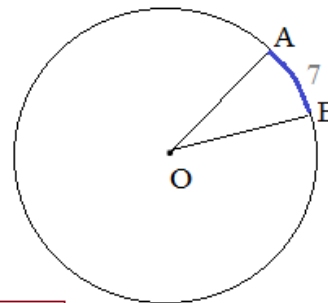
since the entire circumference is 70 meters,
 the arc AB (7 meters) is

$$\frac{7}{70} = \frac{1}{10} \text{ of the entire circumference..}$$

therefore, the central angle must be $\frac{1}{10}$ of the entire circle...

$$\frac{1}{10} \text{ of } 360 \text{ degrees is } 36^\circ$$

entire circumference: 70



- b) If the (sector) area of $\frac{1}{6}$ of a circle is 24π square feet,
 what is the radius of the circle?

The sector is 24π

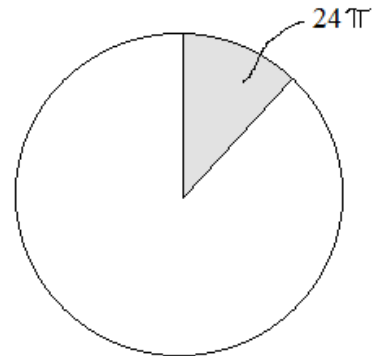
If $\frac{1}{6}$ of the circle is 24π , then the area of
 the entire circle is

$$6 \times 24\pi = 144\pi$$

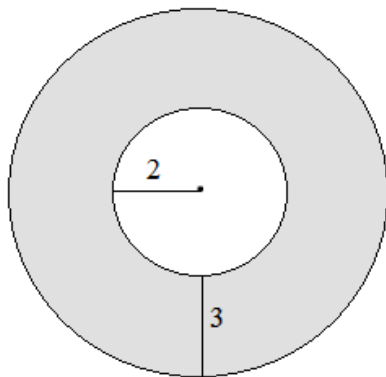
$$\text{Area of circle} = \pi r^2 = 144\pi$$

$$r^2 = 144$$

$$\text{radius} = 12 \text{ feet}$$



- c) In the following concentric circles,
 what is the area of the shaded region?



$$\text{radius of big circle: } 2 + 3 = 5$$

$$\text{area of big circle: } \pi r^2 = 25\pi$$

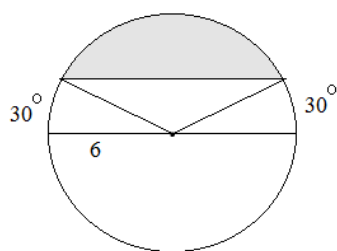
$$\text{radius of small circle: } 2$$

$$\text{area of small circle: } \pi r^2 = 4\pi$$

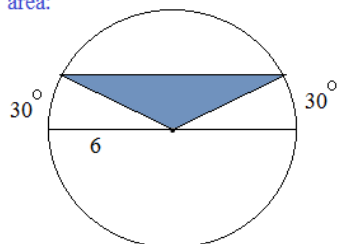
$$\text{area of shaded region} = \text{area}_{\text{big}} + \text{area}_{\text{small}}$$

$$25\pi - 4\pi = 21\pi$$

d) Find the measure of the shaded area:

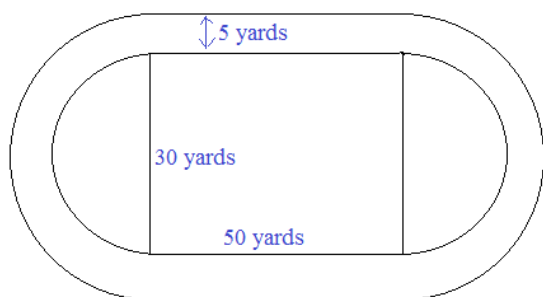


"middle triangle" area:



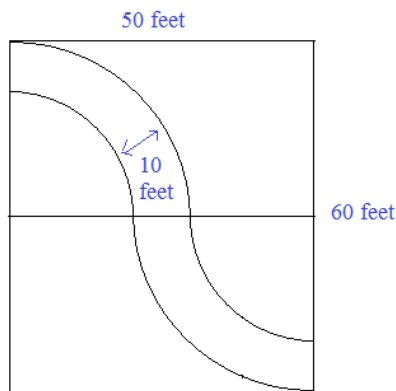
$$12\pi - 9\sqrt{3}$$

e) Find the distance of the outer track:

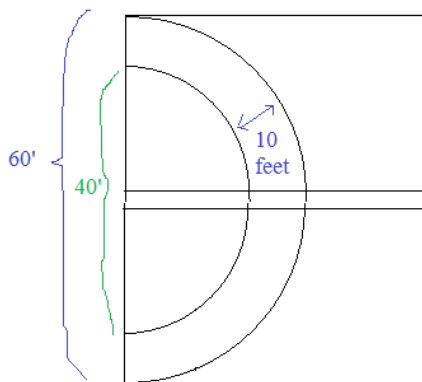


$$\begin{aligned} \text{Total circumference is} \\ 100 + 40\pi \text{ yards} \\ \text{approx. } 225.7 \text{ yards} \end{aligned}$$

f) The city wants to pave both sides of this winding road. If the road is 10 feet wide, how long will this stretch of side pavement be?



If you rearrange the arcs in the road, they form a semicircle!



"outer" sidewalk..

$$\frac{1}{2} (\text{circumference}) = 30\pi$$

"inner" sidewalk

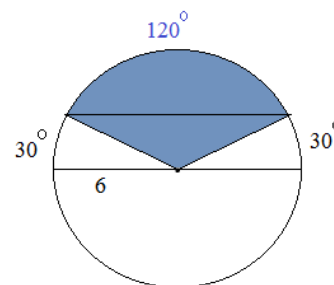
$$\frac{1}{2} (\text{circumference}) = 20\pi$$

$$\begin{aligned} \text{Total paved sides:} \\ 50\pi \end{aligned}$$

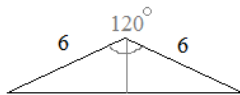
SOLUTIONS

"piece of pie" sector area:

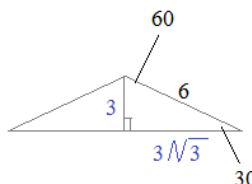
$$\frac{120^\circ}{360^\circ} \pi (6)^2 = 12\pi$$



(sides are 6, because all radii are congruent)



obtuse angle is 120, because $30 + 30 + 120 = 180$

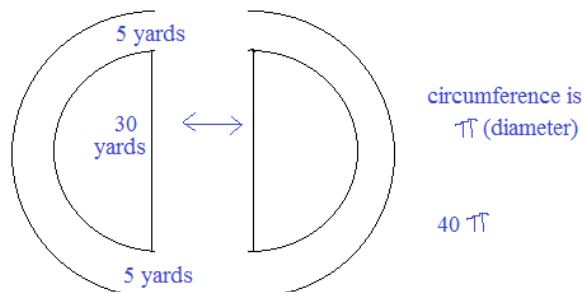


$$\frac{1}{2} (6\sqrt{3})(3) = 9\sqrt{3}$$

The distance of the OUTER track is the lengths of 2 line segments and the circumference of a circle.

The 2 straight parts of the track: 50 yards each... 100 yards total

The combined turns form a circle with diameter 40 yards



V. Geometry Properties

SOLUTIONS

a) Diameter \overline{AB} of circle O has the following points:

$$A = (2, 12)$$

$$B = (-4, 4)$$

What is the radius? Distance/length of diameter: $\sqrt{(-4 - 2)^2 + (4 - 12)^2} = 10$ so, radius is 5

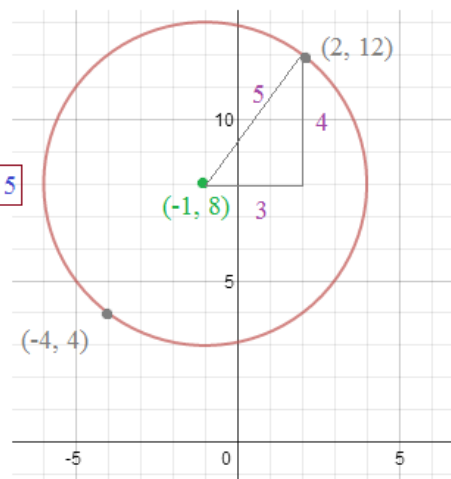
What is the center? Center is midpoint of the diameter: $(-1, 8)$

Area of the circle? $\uparrow\uparrow (\text{radius})^2 = 25 \uparrow\uparrow$

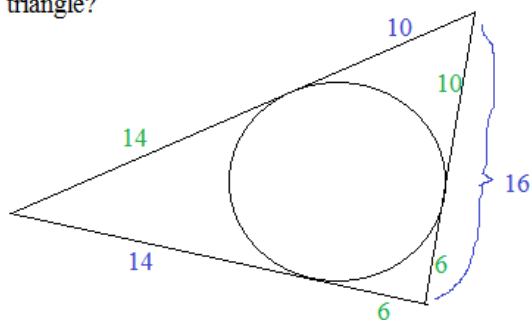
Circumference of the circle? $\uparrow\uparrow (\text{diameter}) = 10 \uparrow\uparrow$

Equation of the circle? $(x - h)^2 + (y - k)^2 = r^2$

Sketch the graph. $(x + 1)^2 + (y - 8)^2 = 25$



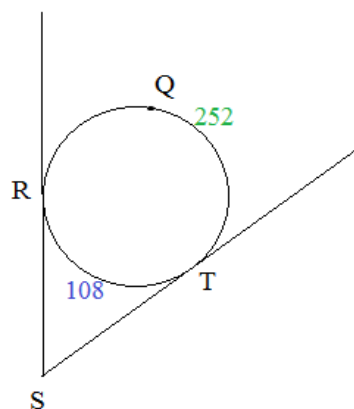
b) What is the perimeter of the triangle?



Since 2 tangent segment meet at the same point, they are congruent. Therefore, we can fill in the values for the triangle

The perimeter is 60 units

c) What is the measure of $\angle S$?



Since the degree measure of the minor arc RT is 108, the degree measure of major arc RQT is $360 - 108 = 252$.

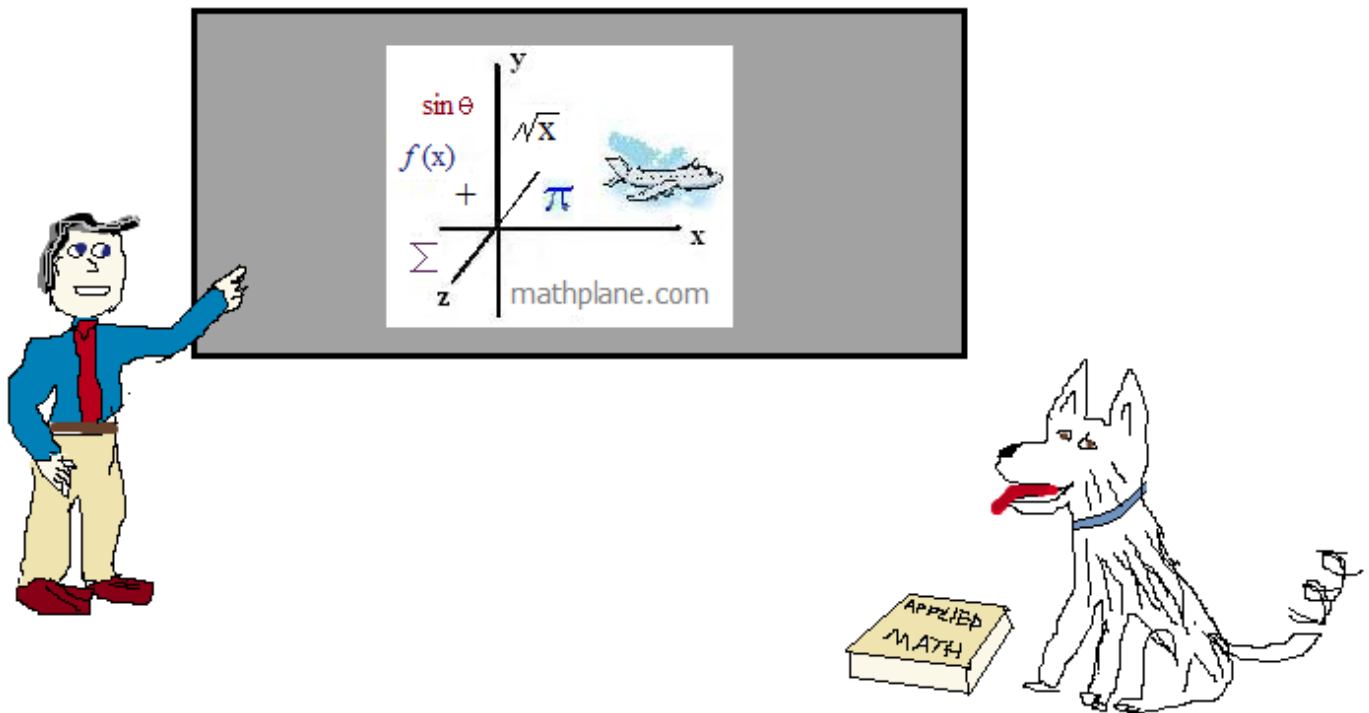
Then, the measure of angle S =

$$\frac{1}{2} (252 - 108) = 72 \text{ degrees}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

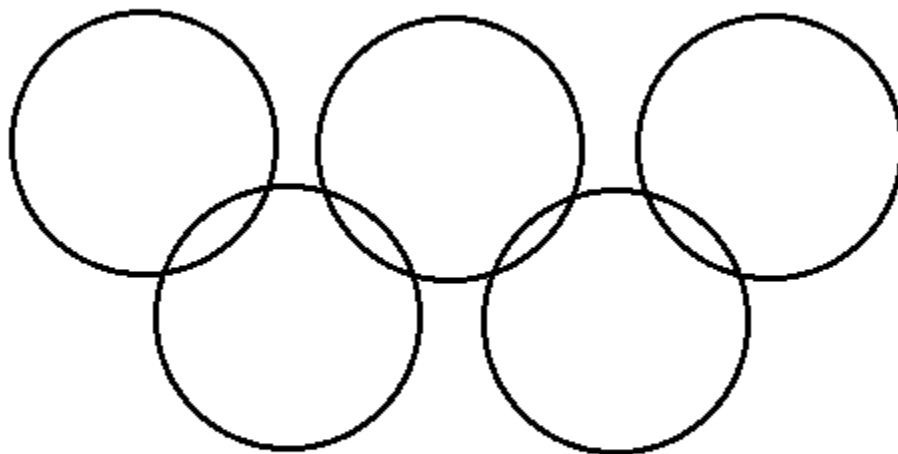
Enjoy



Also, at Facebook, Google+, TeachersPayTeachers, and Pinterest

One more question!

How many different *minor arcs* are in the diagram?



(The intersection of 2 rings creates a potential endpoint of an arc)

How many different *minor arcs* are in the diagram?
 (The intersection of 2 rings creates a potential endpoint of an arc)

