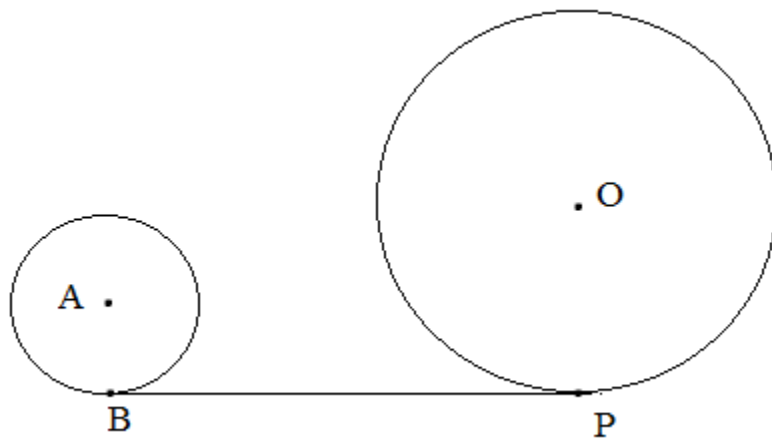
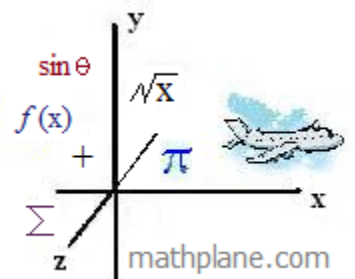


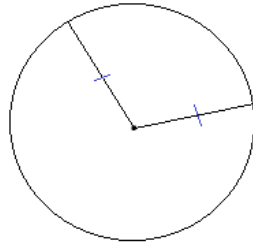
# Circles and Geometry



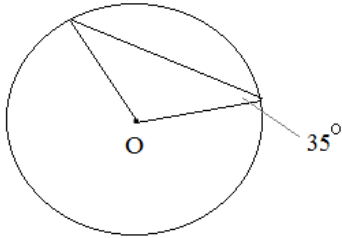
A collection of notes, examples, and practice questions (with answers).



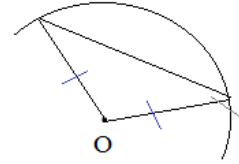
All radii are congruent...



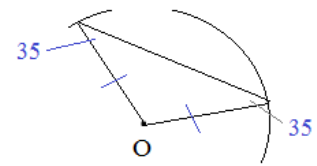
Example: what is the measure of central angle O?



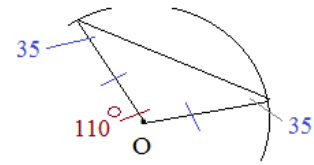
All radii are congruent...



In a triangle, 'if congruent sides, then congruent angles'

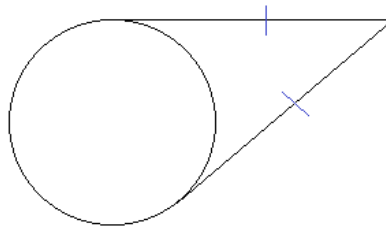


In a triangle, sum of the angles is 180



central angle is 110 degrees...

Tangent line segments are congruent when common outside endpoint...

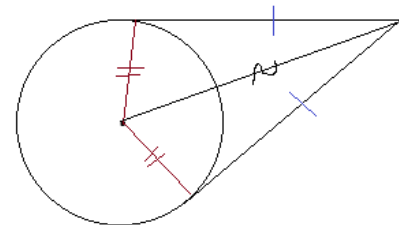
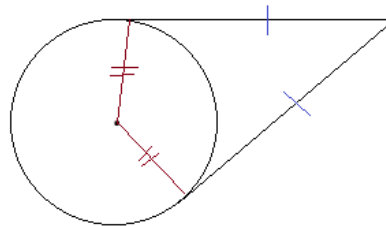


Quick Proof:

Draw 2 auxillary lines (radii)..  
All radii are congruent...

then, draw another auxillary line  
from center of circle to outside endpoint

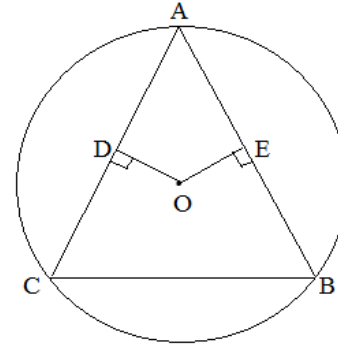
Reflexive property... Now, we have  
2 congruent triangles (Side-side-side)  
Then, CPCTC....



Chords and Proofs

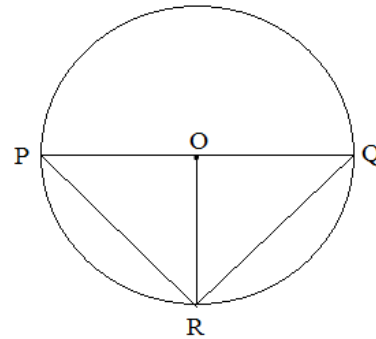
Example: Given:  $\overline{OD} \cong \overline{OE}$   
 $\overline{OD} \perp \overline{AC}$ ,  $\overline{OE} \perp \overline{AB}$

Prove:  $\angle B \cong \angle C$

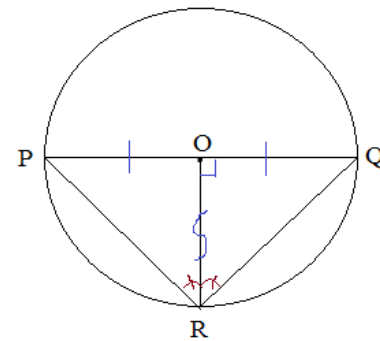


Statements	Reasons
1) $\overline{OD} \cong \overline{OE}$	1) Given
2) $\overline{AC} \cong \overline{AB}$	2) If chords are equidistant to the center of a circle, then the chords are congruent
3) $\angle B \cong \angle C$	3) If sides (of triangle) are congruent, then the angles are congruent

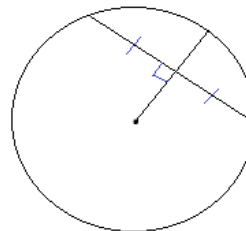
Example: Given:  $\odot O$   
 $\overline{OR}$  bisects  $\overline{PQ}$   
 Prove:  $\overrightarrow{RO}$  bisects  $\angle PRQ$



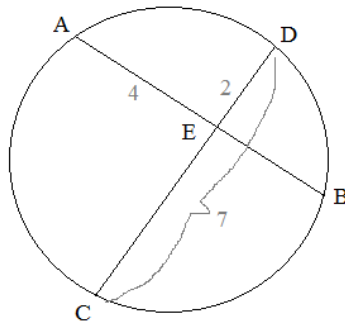
Statements	Reasons
1) $\overline{OR}$ bisects $\overline{PQ}$	1) Given
2) $\overline{PO} \cong \overline{QO}$	2) Definition of bisector
3) $\overline{OR} \cong \overline{OR}$	3) Reflexive property
4) $\overline{OR} \perp \overline{PQ}$	4) If radius bisects a chord, it is perpendicular to the chord
5) $\angle POR$ & $\angle QOR$ are right angles	5) Definition of perpendicular
6) $\triangle POR \cong \triangle QOR$	6) Side-Angle-Side (SAS) (2, 5, 3)
7) $\angle PRO = \angle QRO$	7) Corresponding Parts of Congruent Triangles are Congruent (CPCTC)
8) $\overrightarrow{RO}$ bisects $\angle PRQ$	8) Definition of Bisector



Radius is a perpendicular bisector to chord...



Example:  $\overline{ED} = 2$   
 $\overline{CD} = 7$   
 $\overline{AE} = 4$   
 Find  $\overline{EB}$



"Chord-chord Theorem" : When 2 chords inside a circle intersect, the product of their segment parts are congruent.

$$(AE)(EB) = (ED)(CE)$$

$$ED + CE = CD$$

$$2 + CE = 7$$

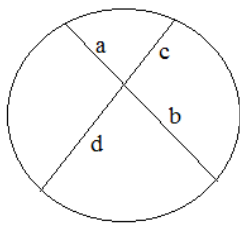
$$CE = 5$$

$$(4)(EB) = (2)(5)$$

$$EB = 5/2$$

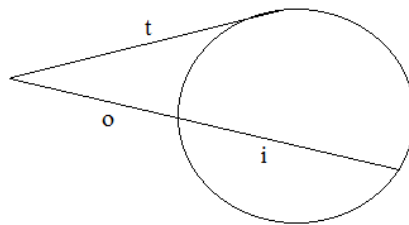
Power Theorems of Circles summary:

*Chord-Chord:* When 2 chords intersect, the products of their segment parts are equal.



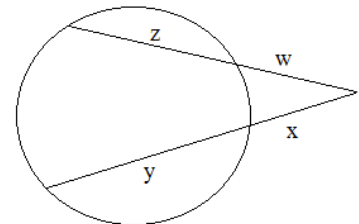
$$ab = cd$$

*Secant-Tangent:* When a tangent and secant meet at an external point, the measure of the tangent squared equals the product of the secant's outer part and total.



$$t^2 = o(i + o)$$

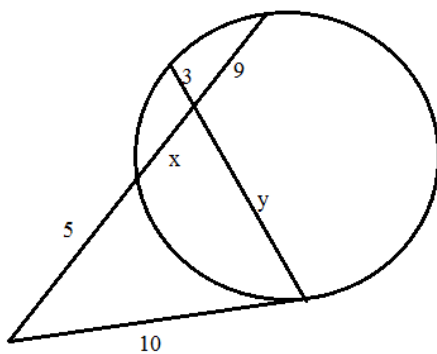
*Secant-Secant:* When 2 secants meet at an external point, the products of their external parts and total lengths are congruent.



$$w(z + w) = x(x + y)$$

"outer" x "whole" = "outer" x "whole"

Example: Find x and y:



$$10^2 = 5 \cdot (5 + (x + 9)) \quad \text{Using Secant-Tangent Power Theorem}$$

$$20 = x + 14$$

$$x = 6$$

$$3 \cdot y = x \cdot 9 \quad \text{Using Chord-Chord Power Theorem}$$

$$3y = 54$$

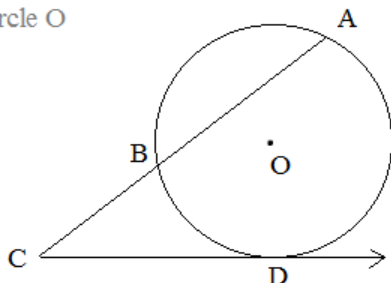
$$y = 18$$

Circles and Geometry: Notes and Examples

Example:  $\overline{BC} = 5$   $\overline{CD} = 8$

Find the length of chord  $\overline{AB}$

Note:  $\overrightarrow{CD}$  is tangent to circle O



"Secant - Tangent Theorem" : When a secant and a tangent share an endpoint outside the circle, the length of the tangent squared equals the product of the secant and the external segment.

$$(\overline{CD})^2 = (\overline{BC})(\overline{AC})$$

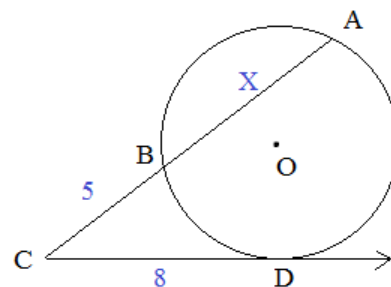
$$(8)^2 = (5)(X + 5)$$

$$64 = 5X + 25$$

$$39 = 5X$$

$$X = \frac{39}{5}$$

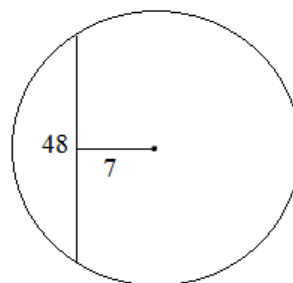
so, chord  $\overline{AB} = 7.8$



Example: Find the *circumference* of a circle in which a 48cm chord is 7 cm from the center.

Step 1: Draw a picture and utilize geometry concepts

Given a chord, and the distance to the center:

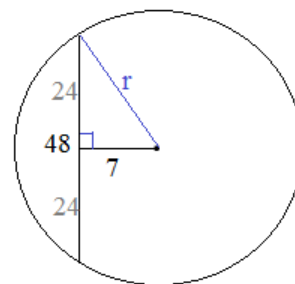


Step 2: Recognize geometry properties you'll need to solve the problem.

**\*\*The perpendicular bisector of a chord passes through the center of the circle!**

Then, draw a radius from the center to the endpoint of the chord --- this forms a right triangle....

Since this is a 7-24-25 triangle, the radius = 25

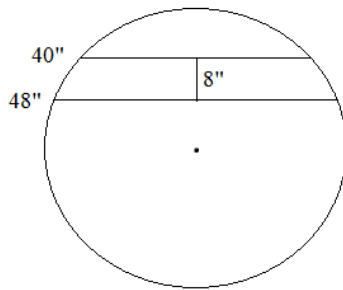


Step 3: Answer the question.

Since the radius is 25, the circumference =  $2\pi 25 = 50\pi$

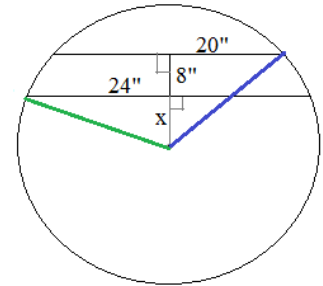
**Example:** Find the radius of a circle in which a 48 inch chord is 8 inches closer to the center than a 40 inch chord.

Step 1: Draw a diagram

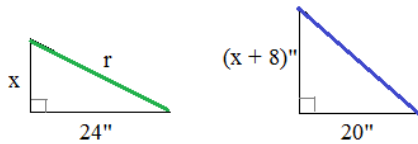


Step 2: Recognize geometry properties and identify useful parts of diagram

- 1) Radius is perpendicular bisector of a chord.
- 2) Pythagorean Theorem
- 3) All radii are congruent



Step 3: Solve



$$x^2 + 24^2 = r^2$$

$$(x + 8)^2 + 20^2 = r^2$$

$$x^2 + 24^2 = (x + 8)^2 + 20^2$$

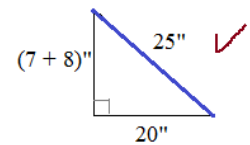
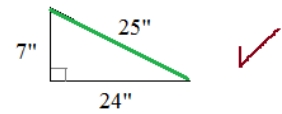
$$x^2 + 576 = x^2 + 16x + 64 + 400$$

$$112 = 16x$$

$$x = 7$$

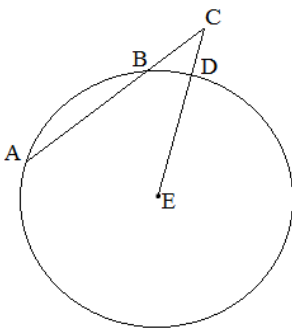
If  $x = 7$ , then the radii are 25 inches

Step 4: Check

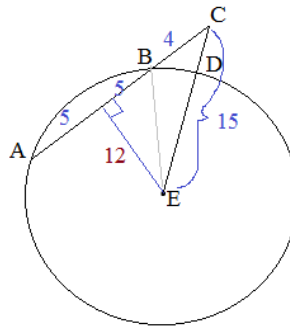


**Example:** Given:  $\overline{AB} = 10$   
 $\overline{CE} = 15$   
 $\overline{BC} = 4$

Find:  $\overline{CD}$



Method 1: Recognize the properties of chords and radii



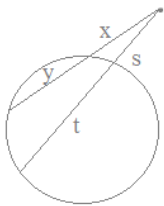
radius bisects chord, dividing AB in half

Also, this creates a right triangle with sides 9 - 12 - 15

then, we know BE is 13 (because of 5 - 12 - 13 right triangle)

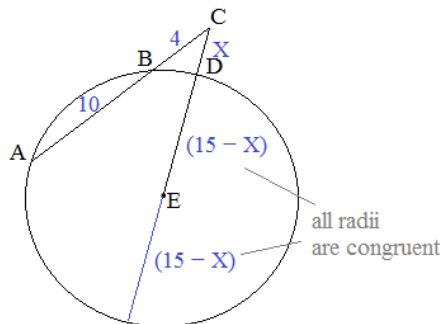
Since BE is 13, then DE is 13.... Therefore, CE is 2

"Secant-Secant Theorem" : When 2 secant segments share an endpoint outside the circle, (length of the external part)(entire part) of first equals (length of the external part)(entire part) of the second.



$$x(x + y) = s(s + t)$$

Method 2: Recognize the relationship between two intersecting secants



$$(\text{external})(\text{whole}) = (\text{external})(\text{whole})$$

$$(4)(14) = (X)(X + (15 - X) + (15 - X))$$

$$56 = (X)(30 - X)$$

$$X^2 - 30X + 56 = 0$$

$$(X - 2)(X - 28) = 0$$

$$X = 2, 28$$

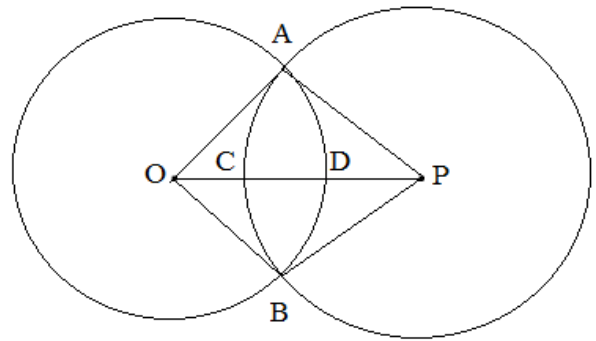
(28 is extraneous, because lengths cannot be negative!)

Given: Circles O and P

$$\overline{CD} + \overline{DP} = 23$$

$$\text{Perimeter of } \triangle OAP = 94$$

$\overline{CD}$  is 3 units longer than  $\overline{OC}$



Find: measures of  $\overline{OB}$  and  $\overline{BP}$

**SOLUTION:**

$$\text{Let } x = \overline{OC}$$

$\overline{CD}$  is 3 units longer than  $\overline{OC}$

$$\text{Then, } \overline{CD} = x + 3$$

$$\text{Since } \overline{CD} + \overline{DP} = 23,$$

$$\overline{DP} = 23 - \overline{CD}$$

$$\overline{DP} = 23 - (x + 3) = 20 - x$$

\*\*\*All radii are congruent

$$\overline{OD} = 2x + 3, \text{ so } \overline{OA} = 2x + 3$$

$$\overline{OB} = 2x + 3$$

$$\overline{CP} = (20 - x) + (x + 3) = 23, \text{ so } \overline{PA} = 23$$

$$\overline{PB} = 23$$

$$\triangle OAP = 94$$

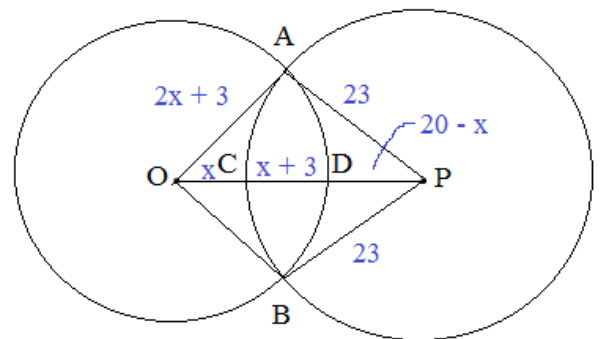
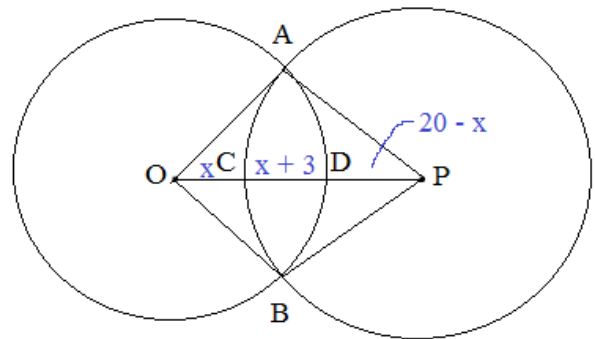
$$\overline{OA} + \overline{PA} + \overline{OP} = 94$$

$$(2x + 3) + 23 + (x + 23) = 94$$

$$3x = 45$$

$$x = 15$$

$$\text{Then, since } x = 15, \overline{OA} = \overline{OB} = 33$$

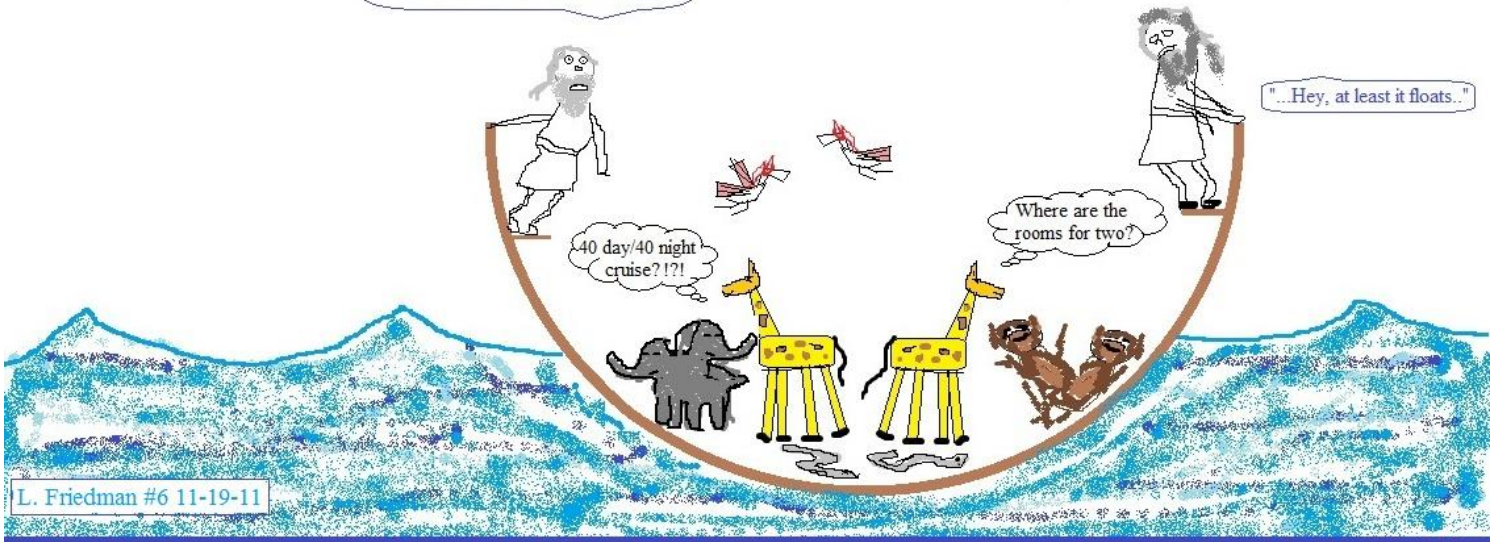


"Noah's Arc"

"Perhaps you misunderstood the command to 'build an ark'?"

"I suppose I did...."

"...Hey, at least it floats.."



Eventually, Noah realizes that this assignment was NOT a geometry construction

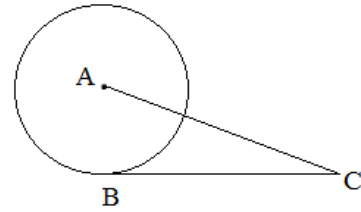
# Practice Quiz



Geometry and Circles Quiz

- 1) Given: Radius of circle A is 16 feet  
 $\overline{BC}$  is tangent to circle A  
 $\overline{BC} = 30$  feet

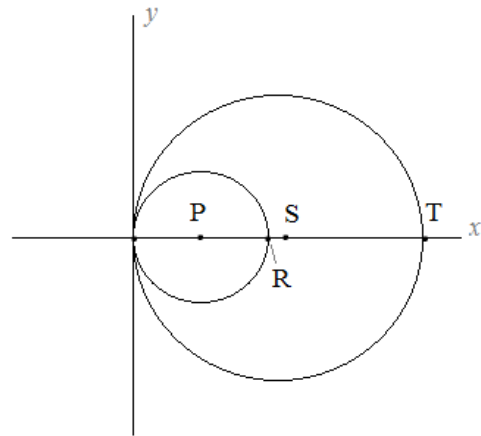
What is the length of  $\overline{AC}$  ?



- 2)  $\odot P$  and  $\odot S$  are internally tangent

P is (12, 0)  
 S is (28, 0)

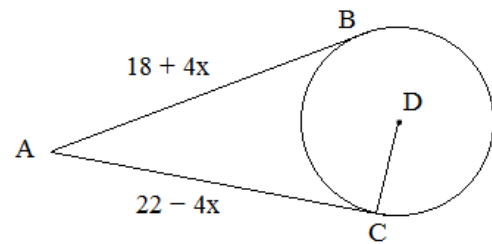
- a) What is the coordinates of R and T?  
 b) What is the length of  $\overline{PT}$ ?



- 3)  $\overline{AB}$  and  $\overline{AC}$  are tangents to circle D

If  $\overline{CD} = 10x$ ,

what is the length of  $\overline{CD}$  ?



- 4) 2 concentric circles have radii 4 and 9..

What is the length of the *larger circle's chord* that is *tangent to the smaller circle*?

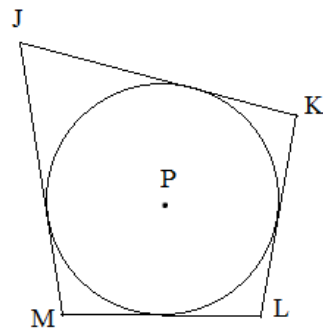
- 5) Circle P is inscribed in quadrilateral JKLM, (i.e. each side is tangent to circle P)

$$\overline{JK} = 22$$

$$\overline{KL} = 15$$

$$\overline{LM} = 13$$

What is  $\overline{JM}$ ?



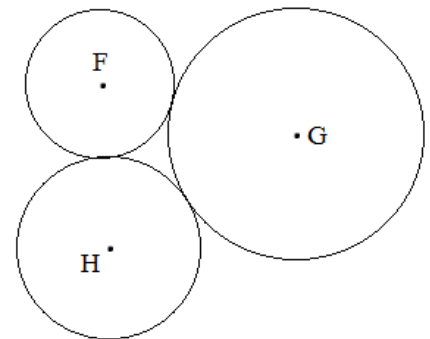
- 6) Circles F, G, and H are tangent.

The distance between F and G: 15

between G and H: 18

between F and H: 13

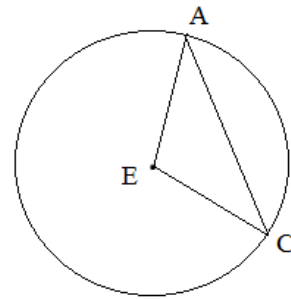
What is the radius of circle G?



7)  $\overline{AC}$  is a chord in circle E

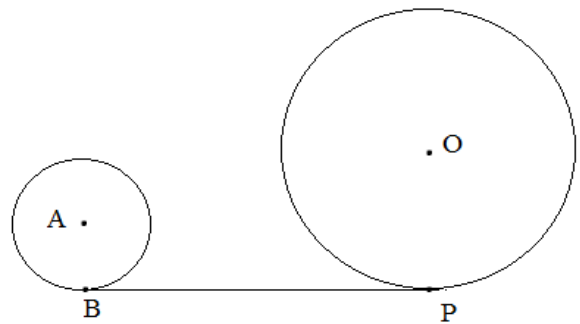
If  $\angle ACE = 32^\circ$ ,

what is the measure of central angle E ?

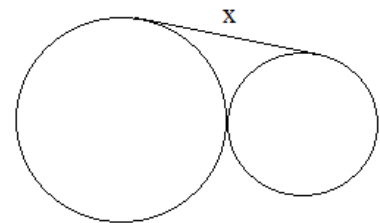


- 8) Given: Circle A radius = 5  
 Circle O radius = 12  
 Distance between circles A and O = 8  
 $\overline{BP}$  is a common external tangent

Find: Length of  $\overline{BP}$

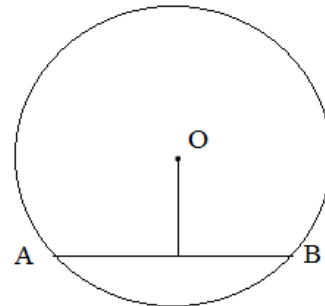


- 9) The radii of the two *tangent* circles are 3cm and 4cm.  
 What is the length of the external common tangent?

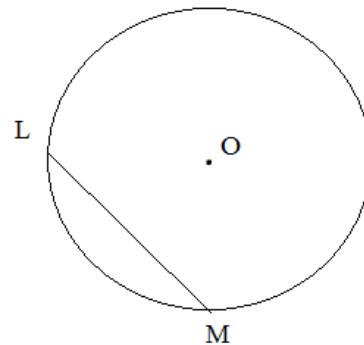


- 10) Given: Length of chord  $\overline{AB}$  is 12  
Distance from center O to the chord is 6

What is the radius of the circle?



- 11) The radius of  $\odot O$  is 100 cm.  
If the distance from center O to the chord is 70 cm,  
what is the length of the chord?



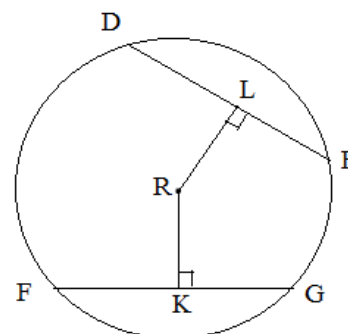
- 12)  $\overline{DE}$  and  $\overline{FG}$  are chords in circle R

$$\overline{DE} = 3x + 1$$

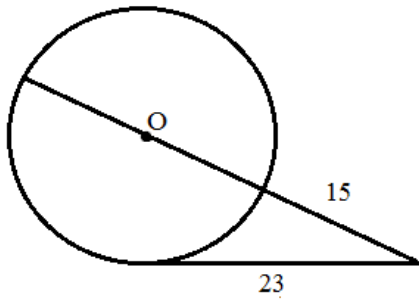
$$\overline{FG} = 4x - 5$$

$$m\overline{RL} = m\overline{RK}$$

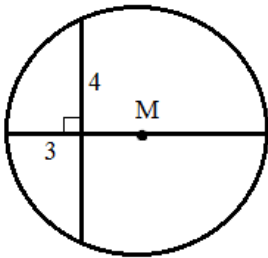
What is the measure of  $\overline{DL}$  ?



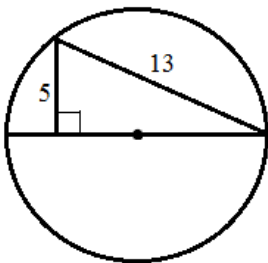
13) What is the diameter of circle O?



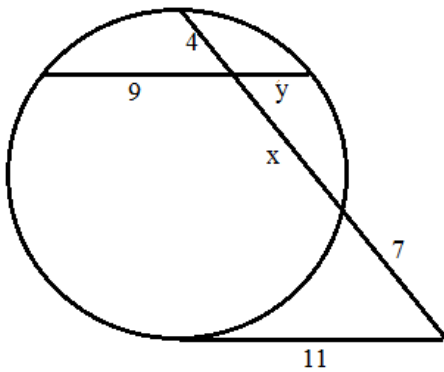
14) What is the diameter of circle M?

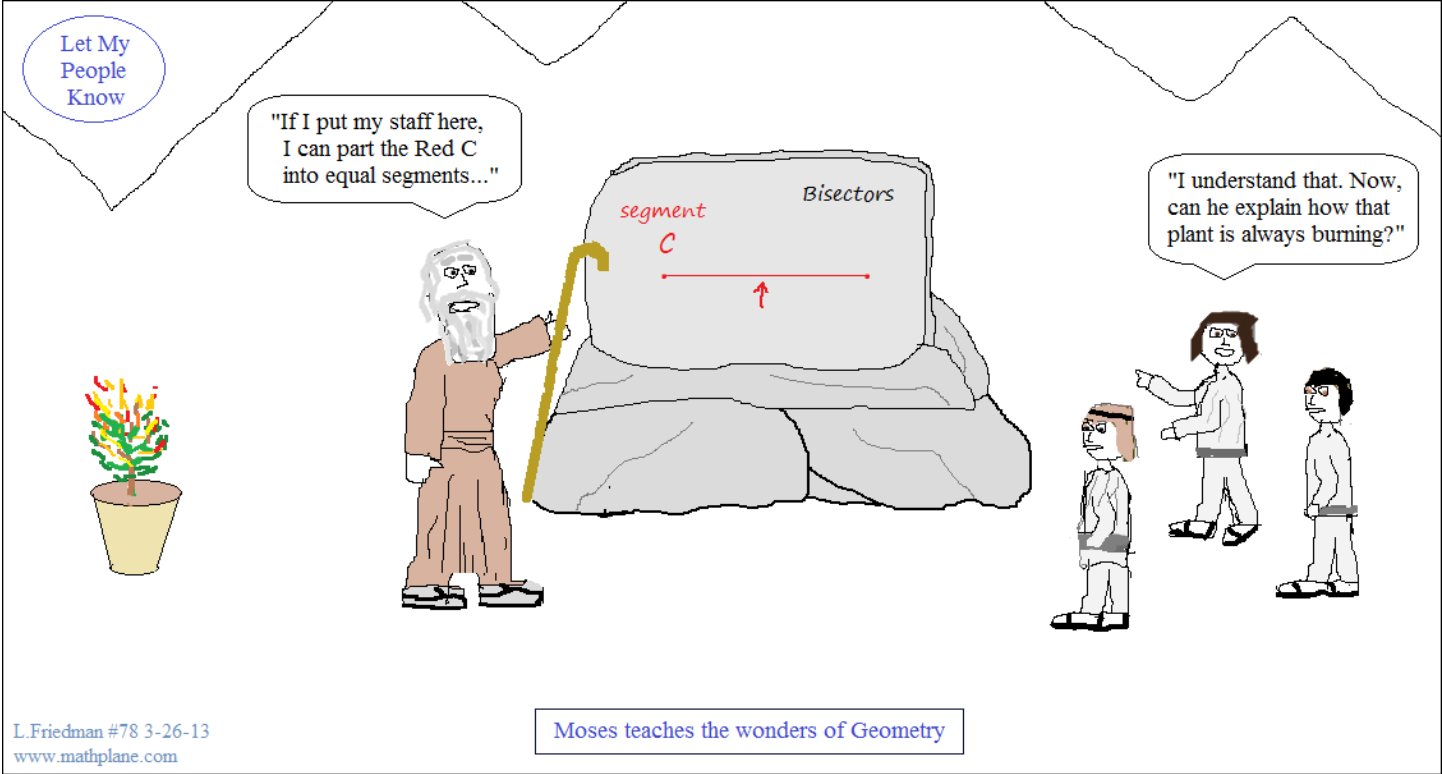


15) What is the diameter of the circle?



16) Find  $x$  and  $y$ :





Answers ->

Geometry and Circles Quiz

SOLUTIONS

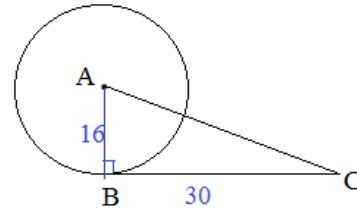
- 1) Given: Radius of circle A is 16 feet  
 $\overline{BC}$  is tangent to circle A  
 $\overline{BC} = 30$  feet

What is the length of  $\overline{AC}$  ?

("Radius is perpendicular to tangent line")

pythagorean theorem:  $(16)^2 + (30)^2 = \overline{AC}^2$   
 $1156 = \overline{AC}^2$

$\overline{AC} = 34$  feet



note: special right triangle  
 $2 \times (8-15-17)$   
 $16-30-34$

- 2)  $\odot P$  and  $\odot S$  are internally tangent

P is (12, 0)  
 S is (28, 0)

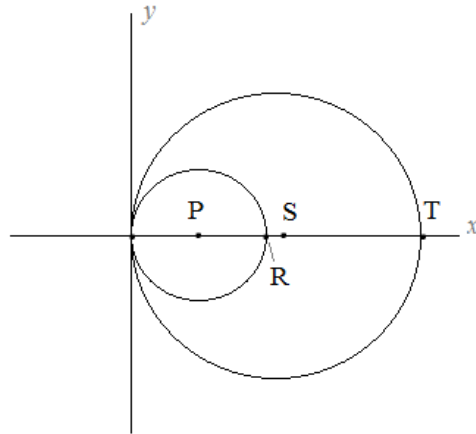
- a) What is the coordinates of R and T?  
 b) What is the length of  $\overline{PT}$ ?

distance from origin to point P = 12 units  
 therefore, the radius of circle P is 12...

Coordinate R = (24, 0)

distance from origin to point S = 28 units..  
 therefore, the radius of circle S is 28...

Coordinate T = (56, 0)



If P is (12, 0) and T is (56, 0),  
 then the length of  $\overline{PT}$  is 44 units..

- 3)  $\overline{AB}$  and  $\overline{AC}$  are tangents to circle D

If  $\overline{CD} = 10x$ ,

what is the length of  $\overline{CD}$  ?

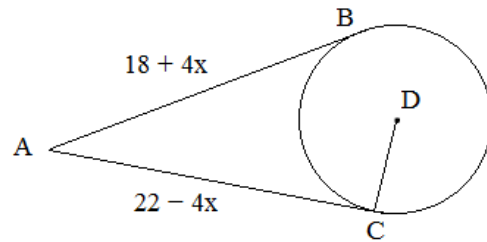
("Two tangent lines from the same point have the same length")

$\overline{AB} = \overline{AC}$

$18 + 4x = 22 - 4x$

$8x = 4$

$x = 1/2$



if  $x = 1/2$ , then  $\overline{AB} = 20$  ✓  
 $\overline{AC} = 20$

and, the length of  $\overline{CD} = 5$

SOLUTIONS

- 4) 2 concentric circles have radii 4 and 9.

What is the length of the *larger circle's chord* that is *tangent to the smaller circle*?

First, draw a sketch --- identify the radii and chord..

Note: we have a right triangle!

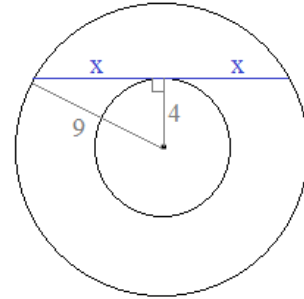
$$(4)^2 + (x)^2 = (9)^2$$

$$(x)^2 = 65$$

$$x = 8.06$$

("radius is perpendicular to chord and point of tangency")

so, the chord is (approx.)  
16.12



- 5) Circle P is inscribed in quadrilateral JKLM, (i.e. each side is tangent to circle P)

$$\overline{JK} = 22$$

$$\overline{KL} = 15$$

$$\overline{LM} = 13$$

What is  $\overline{JM}$  ?

("walk-around problem")

("Tangent segments from points of tangency to the same outside point have the same length")

since  $JD = x$ ,

$DK$  must be  $22 - x$

since  $KE$  is  $22 - x$ ,

$EL$  must be  $15 - (22 - x)$

$$= x - 7$$

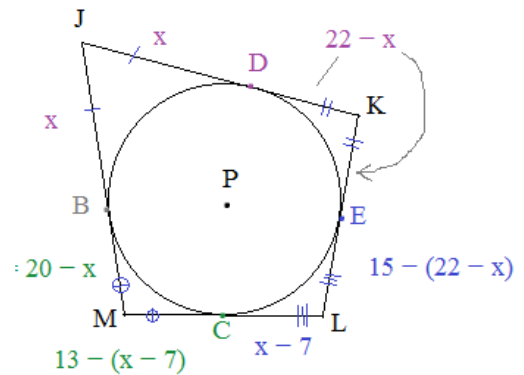
since  $\overline{LM}$  is 13, then

$$\overline{MC}$$
 is  $13 - (x - 7)$

$$= 20 - x$$

finally,  $\overline{MB} = 20 - x$

and  $\overline{JB} = x$ ..... therefore,  $\overline{JM} = 20$



- 6) Circles F, G, and H are tangent.

The distance between F and G: 15

between G and H: 18

between F and H: 13

What is the radius of circle G?

(another "walk-around" or "walk-about" problem)

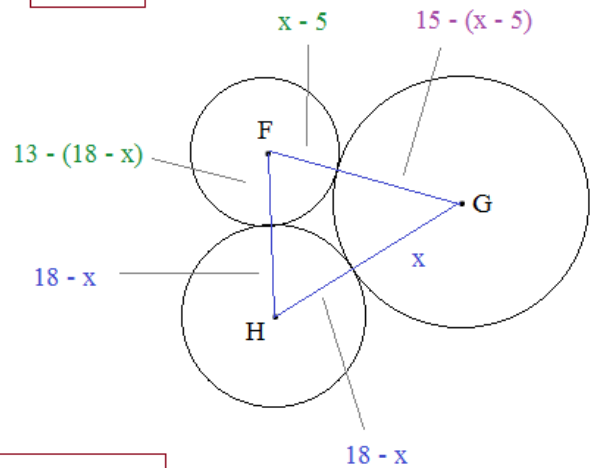
In this case, use the congruent radii to walk around the triangle!

$$x = 15 - (x - 5)$$

$$x = 20 - x$$

$$x = 10 \text{ is the radius of circle G}$$

(note: to check answer, substitute 10 into every x value and check the radii of each circle)





SOLUTIONS

7)  $\overline{AC}$  is a chord in circle E

If  $\angle ACE = 32^\circ$ ,

what is the measure of central angle E ?

Step 1: label the diagram...

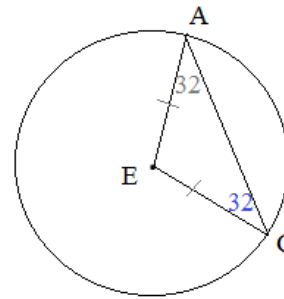
angle ACE is 32 degrees...

("all radii are congruent")

Step 2: recognize that angle CAE must be 32 degrees  
(because opposite sides of triangle congruent, then  
opposite angles congruent...)

$$32 + 32 + \angle E = 180$$

central angle E  
is  $116^\circ$



8) Given: Circle A radius = 5

Circle O radius = 12

Distance between circles A and O = 8

$\overline{BP}$  is a common external tangent

Find: Length of  $\overline{BP}$

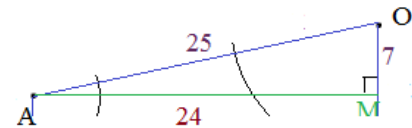
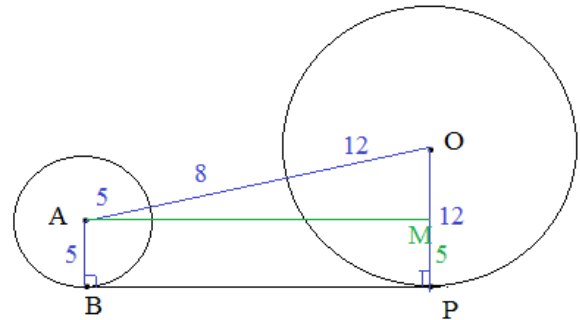
("radius from center to point of tangency  
forms a right angle")

Step 1: Using geometry theorems/properties,  
label the diagram..

Step 2: Draw parallel line to create rectangle AMPB

Step 3: Extract the right triangle to find the base/BP (AM = BP)

Since the base AM is 24, then  $BP = 24$



(7-24-25 special right triangle)

9) The radii of the two *tangent* circles are 3cm and 4cm.  
What is the length of the external common tangent?

Step 1: Label the parts (and utilize the external tangent)

Step 2: Draw a parallel line to x (to establish a rectangle  
and right triangle)

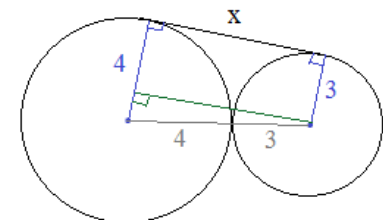
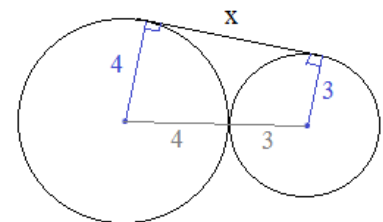
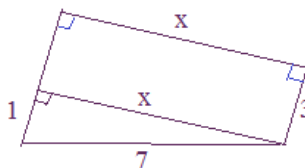
Step 3: Extract the right triangle and solve

using pythagorean theorem:

$$x^2 + (1)^2 = (7)^2$$

$$x^2 = 48$$

$$x = 4\sqrt{3} \text{ cm}$$



SOLUTIONS

- 10) Given: Length of chord  $\overline{AB}$  is 12  
Distance from center O to the chord is 6

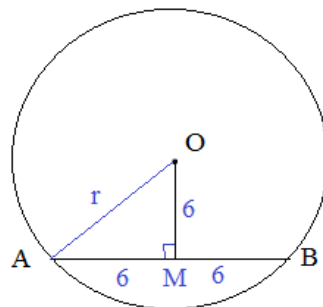
What is the radius of the circle?

("all radii are congruent")

Step 1: Label and construct useful right triangle

Step 2: recognize that OM is perpendicular bisector

Step 3: solve



$\triangle AOM$  is a 45-45-90 right triangle  
(or, use pythagorean theorem)

radius  $r = 6\sqrt{2}$

- 11) The radius of  $\odot O$  is 100 cm.  
If the distance from center O to the chord is 70 cm,  
what is the length of the chord?

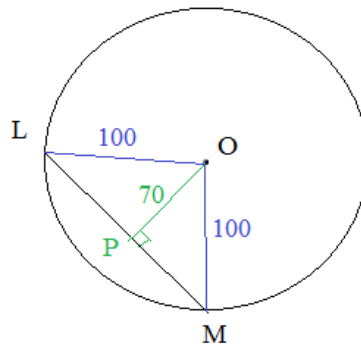
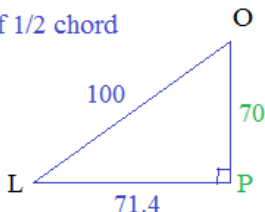
("all radii are congruent")

Step 1: Draw auxiliary lines/radii and label

Step 2: Use right triangle to find length of 1/2 chord

Step 3: double the result to  
length of entire chord..

approx. 142.8cm



- 12)  $\overline{DE}$  and  $\overline{FG}$  are chords in circle R

$\overline{DE} = 3x + 1$

$\overline{FG} = 4x - 5$

$m\overline{RL} = m\overline{RK}$

What is the measure of  $\overline{DL}$  ?

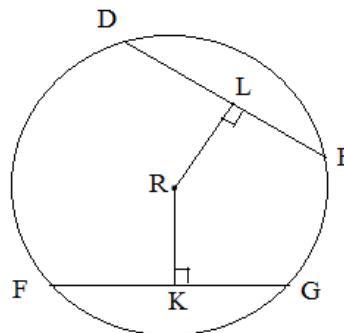
\*\*since the distance from the center to each chord  
is the same, the chords are congruent..

$DE = FG$

$3x + 1 = 4x - 5$

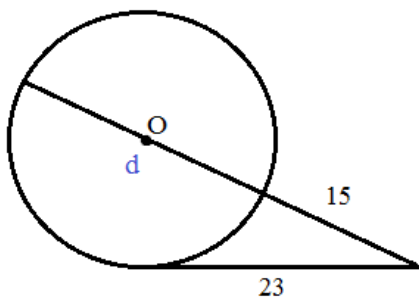
$6 = x$

If  $x = 6$ , then  $\overline{DE} = \overline{FG} = 19$



Since RL bisects DE, the measure of DL is 9.5

13) What is the diameter of circle O?



SOLUTIONS

Secant-Tangent (meet at exterior point)

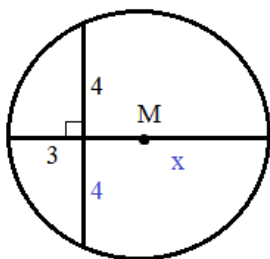
$$23^2 = 15(d + 15)$$

$$529 = 15d + 225$$

$$304 = 15d$$

$$\text{diameter} \approx 20.27$$

14) What is the diameter of circle M?



When a radius (or diameter) cuts a chord, it is the perpendicular bisector...

Therefore, segments are 3, x, 4, and 4..

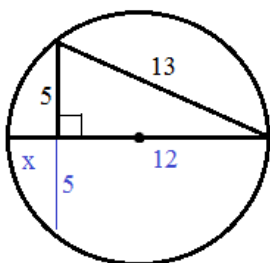
Chord-Chord Theorem

$$3(x) = 4(4)$$

$$x = 16/3$$

$$\text{diameter} = 8 \frac{1}{3}$$

15) What is the diameter of the circle?



\*\*Draw an auxiliary line. This creates chord-chord!

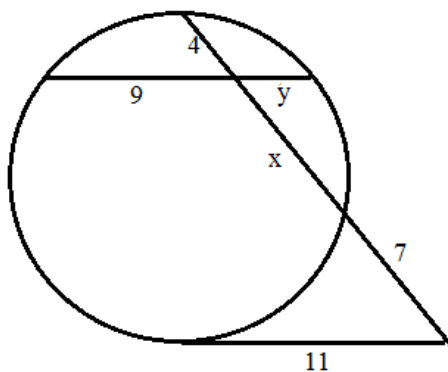
Using Pythagorean Theorem, the segment lengths are x, 12, 5 and 5

$$12(x) = 5(5)$$

$$x = 25/12$$

$$\text{diameter} = 169/12$$

16) Find x and y:



First, find x (using secant-tangent theorem)

$$11^2 = 7(7 + x + 4)$$

$$121 = 77 + 7x$$

$$x = 44/7$$

then, find y (using chord-chord theorem)

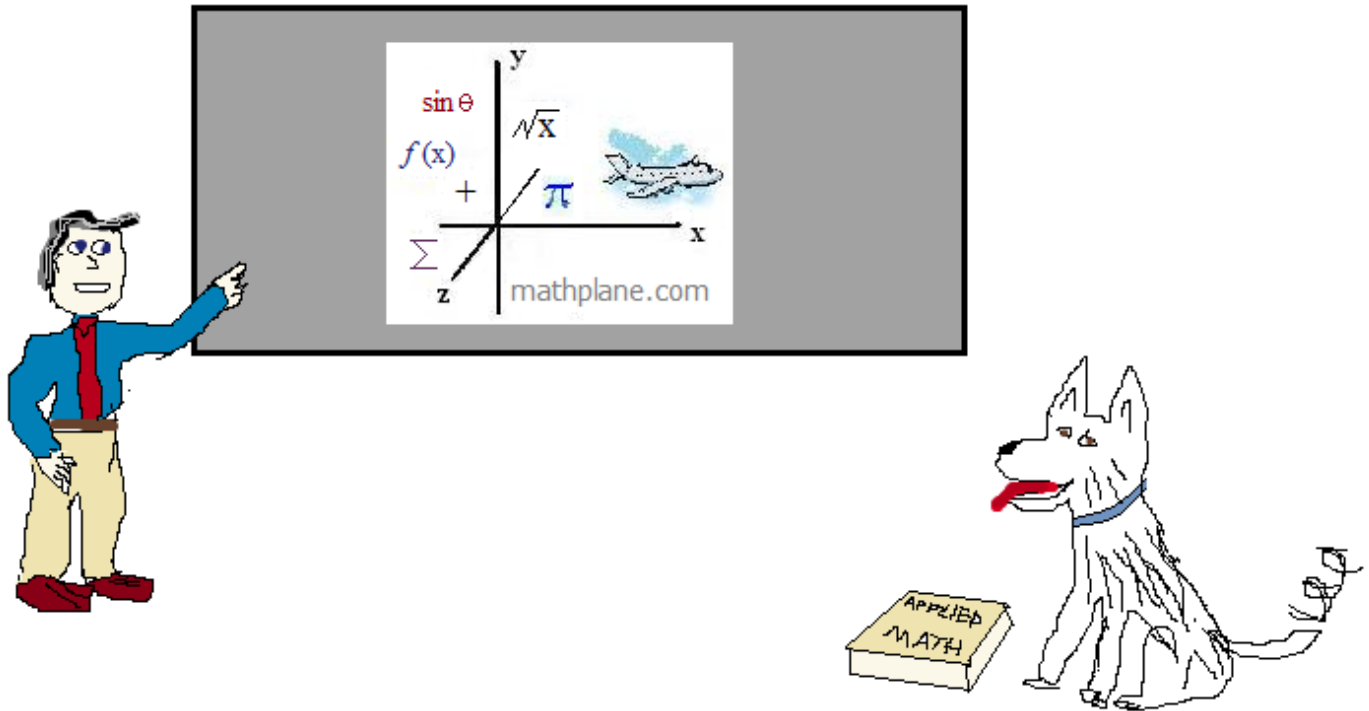
$$9(y) = 4(44/7)$$

$$y = 176/63 \text{ (approx. 2.8)}$$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy.



**ONE MORE QUESTION:**

Two intersecting circles have a common chord of length 24 feet.

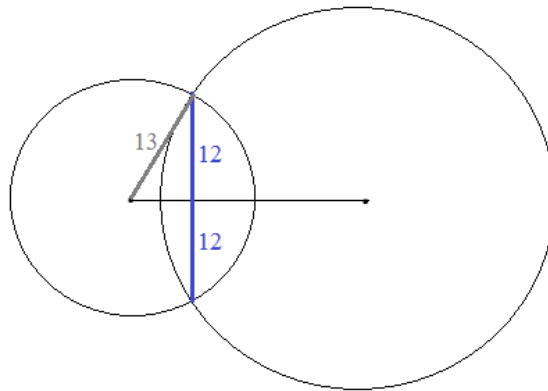
The centers of the circles are 21 feet apart.

If the radius of one circle is 13 feet, what is the length of the other radius?

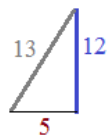
**Answer on the next page ->**

Two intersecting circles have a common chord of length 24 feet. If the centers of the circles are 21 feet apart, and the radius of one circle is 13, what is the radius of the other circle?

Step 1: Draw a diagram



Step 2: Recognize Geometry properties



Pythagorean Theorem

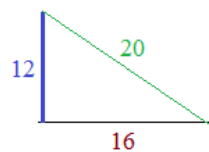
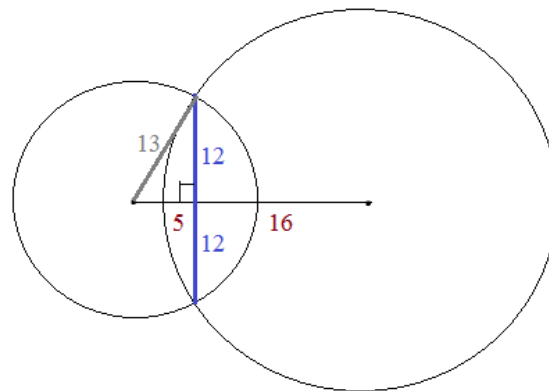
A radius is the perpendicular bisector of a circle's chord

If the leg of the right triangle is 5, then the other part is  $21 - 5 = 16$

Step 3: Solve

So, we found the distance from chord to center is 16.. then, using Pythagorean Theorem, we find the radius is 20

(12-16-20 right triangle)



The radius of the other circle is 20 feet