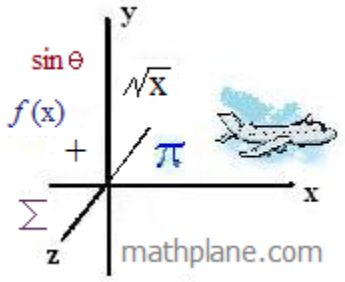
Rational Exponents and Radical Equations

Notes, Examples, and Practice Quizzes (with Answers)



Topics include exponent rules, factoring, extraneous solutions, quadratics, absolute value, and more.

Exponents & Roots

Definition of Exponent: $X^A = X_1 \cdot X_2 \cdot \ldots X_{A-2} \cdot X_{A-1} \cdot X_A$

Example: $4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$

Rules, Examples, and Notes:

Rule #1: $X^A \cdot X^B = X^{(A+B)}$

Examples: $Y^3 \times Y^5 = Y^8$

$5^3 \cdot 5^2 = 125 \times 25 = 3125 = 5^5$

Note: $Z^2 \times Z^4 = (Z \times Z) \times (Z \times Z \times Z \times Z) = Z^6$

2 + 4 = 6 total

Rule #2: $(X^A)^B = X^{(A \times B)}$

Examples: $(X^4)^3 = X^{12}$

$(4^2)^4 = 4^8 = 16^4 = 65536$

Note: $(Y^4)^3 = (Y \cdot Y \cdot Y \cdot Y) \times (Y \cdot Y \cdot Y \cdot Y) \times (Y \cdot Y \cdot Y \cdot Y) = Y^{12}$

3 groups of 4 each ----- 12 Total

Rule #3: $X^0 = 1$

Examples: $Y^0 = 1$

$8^0 = 1$

Note: $Y^4 \times Y^{-4} = \frac{Y \cdot Y \cdot Y \cdot Y}{Y \cdot Y \cdot Y \cdot Y} = 1$

Rule #4: $X^{(-A)} = \frac{1}{(X^A)}$

Example: $X^{-3} = 1/(X^3)$

$5^{-2} = 1/5^2 = \frac{1}{25}$

Note: $Y^{(-A)} = Y^{(-A)} \cdot \frac{Y^A}{Y^A} = \frac{Y^{(-A)} \times Y^A}{Y^A} = \frac{Y^0}{Y^A} = \frac{1}{Y^A}$

Rule #5: $X^{(1/2)} = \sqrt{X}$ (or, more generally: $X^{(m/n)} = \sqrt[n]{X^m}$)

Examples: $25^{(1/2)} = \sqrt{25} = 5$

$8^{(1/3)} = \sqrt[3]{8} = 2$

"cube root of 8"

Note: $Y^{(1/2)} \times Y^{(1/2)} = Y^1$ as $\sqrt{Y} \cdot \sqrt{Y} = Y$

$8^{(1/3)} \times 8^{(1/3)} \times 8^{(1/3)} = 8^{(1/3+1/3+1/3)} = 8^1 = 8$

$A^{(5/2)} = A^{(1/2)} \times A^5 = (\sqrt{A})^5$

Rule #6: $X^A \cdot Y^A = (XY)^A$

Examples: $5^3 \cdot 7^3 = 125 \times 343 = 35^3 = 42875$

$(5 \times 5 \times 5)(7 \times 7 \times 7) = (5 \times 7)(5 \times 7)(5 \times 7) = 35 \times 35 \times 35$

$4^{(1/2)} \times 16^{(1/2)} = 64^{(1/2)} = 8$

$\sqrt{4} \times \sqrt{16} = \sqrt{4 \times 16} = \sqrt{64} = 8$

Solving radical (exponent) equations

4 Steps:

1) *Isolate* radical
2) *Square* both sides
3) *Solve*
4) *Check* (for extraneous answers)

4 Steps for *fractional* exponents
1) Isolate term
2) Raise to power that eliminates the exponents
3) Solve
4) Check

Example 1: $\sqrt{5x} + 10 = 25$ — Isolate -- subtract 10 from both sides

$\sqrt{5x} = 15$ — Square both sides

$5x = 225$ — Solve -- divide 5 from both sides

$\boxed{x = 45}$ — Check

$\sqrt{5}(45) + 10 = 25$

$25 = 25$ ✓

Example 2: $\sqrt{3x} + 12 = 6$

$\sqrt{3x} = -6$

$3x = 36$

$x = 12$

Now, check the answer.

$\sqrt{3(12)} + 12 = 6$

$18 \neq 6$ ✗

There is no solution!

Example 3: $\sqrt{x + 30} = x$ — square both sides

$x + 30 = x^2$ — solve

$x^2 - x - 30 = 0$

$(x + 5)(x - 6) = 0$

$x = -5, 6$ — check

$\sqrt{(-5) + 30} = (-5)$

$5 \neq -5$

-5 is extraneous!

$\sqrt{(6) + 30} = (6)$

$6 = 6$ ✓

$\boxed{x = 6}$

Example 4: $4(x - 2)^{\frac{1}{3}} - 12 = 0$ — isolate the exponent

$4(x - 2)^{\frac{1}{3}} = 12$

$(x - 2)^{\frac{1}{3}} = 3$ — raise to 3rd power (to eliminate the exponent)

$x - 2 = 27$ — solve

$\boxed{x = 29}$ — check

$4((29) - 2)^{\frac{1}{3}} - 12 = 0$

$4(3) - 12 = 0$

$0 = 0$ ✓

Rational Exponent Equations: Negative Numbers, Absolute Values, and Eliminated Answers

Rational Exponent Equations
Domain Restrictions:
A Comparison

$y = x^{\frac{2}{3}}$ can x = -4? YES

$(-4^2)^{\frac{1}{3}}$ or $(-4^{\frac{1}{3}})^2$

$y = \sqrt[3]{16}$

$y = x^{\frac{3}{2}}$ can x = -4? NO

$(-4^3)^{\frac{1}{2}}$ or $(-4^{\frac{1}{2}})^3$

NOT REAL!!

Examples:

$2(x+4)^{\frac{2}{3}} = 8$

$(x+4)^{\frac{2}{3}} = 4$

$(x+4) = 4^{\frac{3}{2}}$

$x + 4 = 8$ $\quad x + 4 = -8$

$x = 4$ $\quad x = -12$

$2(x-3)^{\frac{2}{3}} = 50$

$\left((x-3)^{\frac{2}{3}}\right)^{\frac{3}{2}} = 25^{\frac{3}{2}}$

$x - 3 = 125$ or $x - 3 = -125$

$x = 128$ or $x = -122$

$2(x+5)^{\frac{2}{5}} = 32$

$(x+5)^{\frac{2}{5}} = 16$

$\left((x+5)^{\frac{1}{5}}\right)^2 = 16$

$\left|(x+5)^{\frac{1}{5}}\right| = 4$

since it is the "square root of a square", the term is absolute value

$x = 1019$ or $x = -1029$

$(x+3)^{\frac{3}{5}} = -8$

Since it is a 1/5 root, a negative is permitted...

$x + 3 = (-8)^{\frac{5}{3}}$

(if possible, "Go smaller first")

$x + 3 = (-8^{\frac{1}{3}})^5$

$x + 3 = (-2)^5$

$x + 3 = -32$

$x = -35$

(It's easier to find the cube root of 8 first, then 2 to the 5th power --- rather than 8 to the 5th power first, then the cube root of 32,768!)

$2(x)^{\frac{3}{2}} + 21 = 13$

$2x^{\frac{3}{2}} = -8$

$x^{\frac{3}{2}} = -4$

Since it is a 1/2 root, a negative is NOT permitted...

$x = (-4)^{\frac{2}{3}}$

$x = 16^{\frac{1}{3}}$

But, when you check the answer:

$2(x)^{\frac{3}{2}} + 21 = 13$

$2(16^{\frac{1}{3}})^{\frac{3}{2}} + 21 = 13$

$2(16)^{\frac{1}{2}} + 21 = 13$

$2(4) + 21 = 13$

$8 = -8$

There is no real solution!!

General rule: If n is even, then $\sqrt[n]{x^n} = |n|$

Why do you need to include an absolute value?

Does $\sqrt{x^2} = x$?

Test points: If x = 3: $\sqrt{3^2} = 3$

$\sqrt{9} = 3$

$3 = 3$ ✓

But, if x = -3 $\sqrt{(-3)^2} = -3$

$\sqrt{9} = -3$

$3 = -3$ ✗

However, if we include an absolute value sign:

$\sqrt{x^2} = |x|$

If x = 3: $\sqrt{3^2} = |3|$

$\sqrt{9} = |3|$

$3 = |3|$ ✓

But, if x = -3 $\sqrt{(-3)^2} = |-3|$

$\sqrt{9} = |-3|$

$3 = |-3|$ ✓

Practice Exercises -→

Exponents, Roots, & Addition Exercise

Solve the 15 problems below. Then, add all the solutions. What is the total? (rounded to 3 decimal places.)

1) $(3^3)^2 =$ _____

2) $(2)^{-2} =$ _____

3) $(4)^{3/2} =$ _____

4) $\sqrt{64} - \sqrt[3]{8} =$ _____

5) $9^2 + 9^{1/2} =$ _____

6) $(.3)^3 =$ _____

7) $(32)^{2/5} =$ _____

8) $(1/3)^{-2} =$ _____

9) $(-5)^3 =$ _____

10) $\sqrt{(3)^4} =$ _____

11) $\sqrt{2} \times \sqrt{50} =$ _____

12) $1^2 - 2^3 + 3^4 =$ _____

13) $(1/2)^3 =$ _____

14) $8^{1/3} \cdot 8^{2/3} =$ _____

15) $\sqrt[3]{(-8)} - \sqrt[3]{27} =$ _____

Now Add them up! The Total of ALL 15 solutions is _____

(rounded to 3 decimal places)

Rational Exponents and Radical Equations

I. Evaluate

a) $9^{\frac{1}{2}}$ b) $9^{-\frac{1}{2}}$ c) 1^{0} d) $27^{\frac{2}{3}}$ e) $81^{-\frac{1}{4}}$

f) $25^{1.5}$ g) $16^{.25}$ h) $4^{3.5}$ i) $64^{-.5}$ j) $9^{-2.5}$

II. Simplify the expressions

a) $\sqrt{8} \cdot \sqrt{40}$ b) $6^{\frac{1}{2}} \cdot 12^{\frac{1}{2}}$ c) $\sqrt[4]{16} + \sqrt[3]{8}$

d) $\left(5\sqrt{3}\right)^{2}$ e) $(81)^{\frac{1}{4}} \cdot (81)^{\frac{1}{2}}$ f) $\sqrt[3]{\sqrt{64}}$

g) $(9m^{4})^{\frac{1}{2}}$ h) $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$ i) $\left(\frac{9}{16}\right)^{\frac{3}{2}}$

Rational Exponents and Radical Equations

III. Solve the following.

a) $\sqrt{4x-27}-1=4$

b) $5\sqrt{x}+7=8$

c) $2+(4-x)^{\frac{3}{2}}=10$

d) $\sqrt{3x}=\sqrt{x+4}$

e) $(x+4)^{\frac{3}{4}}=27$

f) $\sqrt{(x+1)^3}-1=7$

IV. Solve . (Identify any extraneous solutions)

a) $\sqrt{x+7}+5=x$

b) $\sqrt{x+2}=x$

c) $(5x+4)^{\frac{1}{2}}-3x=0$

d) $\sqrt{4x-5}=3\sqrt{x-5}$

e) $(x-9)^{\frac{1}{2}}+1=x^{\frac{1}{2}}$

f) $(x+5)^{\frac{1}{2}}-(5-2x)^{\frac{1}{4}}=0$

V. Simplify (or factor) the following.

Rational Exponents and Radical Equations

a) $(\sqrt{b^2+1}-1)(\sqrt{b^2+1}+1)$

b) $y^{5/2}-y^{1/2}$

c) $x^{-3/2}-2x^{-1/2}+x^{1/2}$

d) $6x^{-1/2}+8x^{1/2}+2x^{3/2}$

e) $\dfrac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}$

f) $\dfrac{2(a+1)^{1/2}-a(1+a)^{-1/2}}{a+1}$

VI. More rational exponent equations

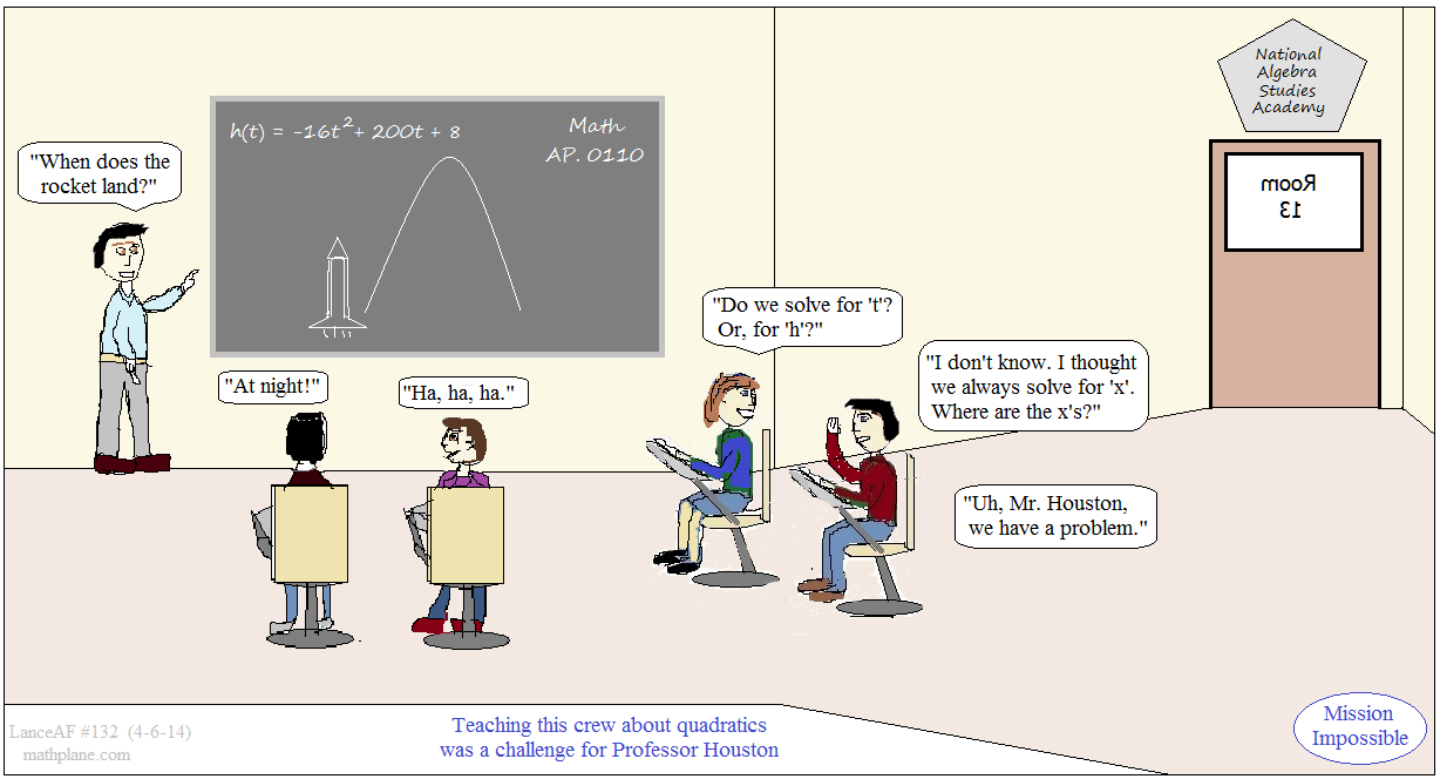
a) $2(x+5)^{\frac{3}{2}} + 128 = 0$

b) $y = 6 + \sqrt{y}$

c) $\sqrt{3-x} = \sqrt{7-2x}$

d) $3(x+5)^{\frac{2}{3}} + 2 = 50$

e) $2(x-1)^{\frac{3}{2}} - 7 = 23$


"When does the rocket land?"
h(t) = -16t² + 200t + 8
Math AP. 0110
"At night!"
"Ha, ha, ha."
"Do we solve for 't'? Or, for 'h'?"
"I don't know. I thought we always solve for 'x'. Where are the x's?"
"Uh, Mr. Houston, we have a problem."
National Algebra Studies Academy
Room 13
LanceAF #132 (4-6-14)
mathplane.com
Teaching this crew about quadratics was a challenge for Professor Houston
Mission Impossible

Solutions →

Exponents, Roots, & Addition Exercise

Solve the 15 problems below. Then, add all the solutions.
What is the total? (rounded to 3 decimal places)

SOLUTIONS

1) $(3^3)^2$ = (3 x 3 x 3)(3 x 3 x 3) = 27 x 27 = 729

2) $(2)^{-2}$ = 2^2 = 4.. therefore, 2^{-2} = 1/4 or .25

3) $(4)^{3/2}$ = 4^3 is 64, and $64^{1/2}$ = 8

4) $\sqrt{64} - \sqrt[3]{8}$ = 8 - 2 = 6

5) $9^2 + 9^{1/2}$ = 81 + 3 = 84

6) $(.3)^3$ = .3 x .3 x .3 = .09 x .3 = .027

7) $(32)^{2/5}$ = $32^{1/5}$ x $32^{1/5}$ = 2 x 2 = 4

8) $(1/3)^{-2}$ = $(1/3)^2$ = 1/9.. therefore, $(1/3)^{-2}$ = 9 (the reciprocal of 1/9)

9) $(-5)^3$ = -5 x -5 x -5 = -125

10) $\sqrt{(3)^4}$ = $(3)^4$ = 81 and $\sqrt{81}$ = 9

11) $\sqrt{2}$ x $\sqrt{50}$ = $\sqrt{100}$ = 10

12) $1^2 - 2^3 + 3^4$ = 1 - 8 + 81 = 74

13) $(1/2)^3$ = 1/2 x 1/2 x 1/2 = 1/8 = .125

14) $8^{1/3} \cdot 8^{2/3}$ = 8^1 = 8

15) $\sqrt[3]{(-8)} - \sqrt[3]{27}$ = -2 - 3 = -5

729
.25
8
6
84
827.25

.027
4
9
-125
9
-102.973

10
74
.125
8
-5
87.125

Now Add them up! The Total of ALL 15 solutions is 811.402

(rounded to 3 decimal places)

Rational Exponents and Radical Equations

SOLUTIONS

I. Evaluate

a) $9^{\frac{1}{2}}$

3

b) $9^{-\frac{1}{2}}$

$\frac{1}{3}$

c) 1^{0}

1

d) $27^{\frac{2}{3}}$

$\left(27^{\frac{1}{3}}\right)^{2}$

$3^{2} = \boxed{9}$

e) $81^{-\frac{1}{4}}$

$81^{\frac{1}{4}} = 3$

because $3 \cdot 3 \cdot 3 \cdot 3 = 81$

so, $81^{\frac{-1}{4}} = \boxed{-3}$

f) $25^{1.5}$

$25^{\frac{3}{2}} = \sqrt{25}^{3}$

$\boxed{= 125}$

g) $16^{.25}$

$\boxed{2}$

$16^{.25} \cdot 16^{.25} \cdot 16^{.25} \cdot 16^{.25} = 16^{1}$

$2 \times 2 \times 2 \times 2 = 16$

h) $4^{3.5}$

$4^{\frac{7}{2}}$

$2^{7} = \boxed{128}$

i) $64^{-.5}$

$\frac{1}{\sqrt{64}} = \boxed{\frac{1}{8}}$

j) $9^{-2.5}$

$\frac{1}{9^{2.5}} = \frac{1}{9^{\frac{5}{2}}} = \frac{1}{3^{5}}$

$\boxed{= \frac{1}{243}}$

II. Simplify the expressions

a) $\sqrt{8} \cdot \sqrt{40}$

$\sqrt{48}$

$\sqrt{16 \cdot 3} = \boxed{4\sqrt{3}}$

b) $6^{\frac{1}{2}} \cdot 12^{\frac{1}{2}}$

$\sqrt{6} \cdot \sqrt{12}$

$\sqrt{72} = \boxed{6\sqrt{2}}$

c) $\sqrt[4]{16} + \sqrt[3]{8}$

$2 + 2 = \boxed{4}$

d) $\left(5\sqrt{3}\right)^{2}$

$5\sqrt{3} \cdot 5\sqrt{3} =$

$25 \cdot 3 =$

$\boxed{75}$

e) $(81)^{\frac{1}{4}} \cdot (81)^{\frac{1}{2}}$

$3 \cdot 9 = \boxed{27}$

$81^{\left(\frac{1}{4} + \frac{1}{2}\right)} = 81^{\frac{3}{4}} = 27$

f) $\sqrt[3]{\sqrt{64}}$

$\sqrt[3]{8} = \boxed{2}$

g) $(9m^{4})^{\frac{1}{2}}$

$9^{\frac{1}{2}} \cdot m^{\frac{4}{2}}$

$\boxed{3m^{2}}$

h) $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$

$\frac{1}{\left(\frac{1}{4}\right)^{\frac{1}{2}}} = \frac{1}{\frac{1}{2}} = \boxed{2}$

i) $\left(\frac{9}{16}\right)^{\frac{3}{2}}$

$\left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$

and $\left(\frac{3}{4}\right)^{3} = \boxed{\frac{27}{64}}$

Rational Exponents and Radical Equations

SOLUTIONS

III. Solve the following.

a) $\sqrt{4x-27}-1=4$

$\sqrt{4x-27}=5$

(square both sides)

$4x-27=25$

$4x=52$

$\boxed{x=13}$

b) $5\sqrt{x}+7=8$

(isolate the radical)

$5\sqrt{x}=1$

$\sqrt{x}=\frac{1}{5}$

(square both sides)

$\boxed{x=\frac{1}{25}}$

c) $2+(4-x)^{\frac{3}{2}}=10$

$(4-x)^{\frac{3}{2}}=8$

$(4-x)^{1}=8^{\frac{2}{3}}$

$4-x=4$

$\boxed{x=0}$

d) $\sqrt{3x}=\sqrt{x+4}$

$3x=x+4$

$2x=4$

$\boxed{x=2}$

To check answer, substitute into original problem: $\sqrt{3(2)}=\sqrt{(2)+4}$

$\sqrt{6}=\sqrt{6}$ ✓

e) $(x+4)^{\frac{3}{4}}=27$

$x+4=27^{\frac{4}{3}}$

$x+4=81$

$\boxed{x=77}$

f) $\sqrt{(x+1)^3}-1=7$

$\sqrt{(x+1)^3}=8$

$(x+1)^3=64$

$x+1=4$

$\boxed{x=3}$

IV. Solve. (Identify any extraneous solutions)

a) $\sqrt{x+7}+5=x$

(isolate radical)

$\sqrt{x+7}=x-5$

(square both sides)

$x+7=x^2-10x+25$

$x^2-11x+18=0$

$(x-2)(x-9)=0$

$x=2, 9$

$\boxed{x=9}$

(check answers)

$\sqrt{(2)+7}+5=(2)$

$3+5\neq 2$ ✗

$\sqrt{(9)+7}+5=(9)$

$4+5=9$ ✓

b) $\sqrt{x+2}=x$

$x+2=x^2$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=-1, 2$ (check answers)

$\sqrt{(-1)+2}=(-1)$ NO (extraneous)

$\sqrt{(2)+2}=(2)$ YES

$\boxed{x=2}$

c) $(5x+4)^{\frac{1}{2}}-3x=0$

$\sqrt{(5x+4)}=3x$

$5x+4=9x^2$

$9x^2-5x-4=0$

$(9x+4)(x-1)=0$

$x=1, -4/9$

$\boxed{x=1}$

(check answers)

$(5(1)+4)^{\frac{1}{2}}-3(1)$

$=0$ ✓

$(5(-\frac{4}{9})+4)^{\frac{1}{2}}-3(-\frac{4}{9})$

$=\frac{8}{3}$ ✗

d) $\sqrt{4x-5}=3\sqrt{x-5}$

(square both sides)

$4x-5=9(x-5)$

$4x-5=9x-45$

$40=5x$

$\boxed{x=8}$

e) $(x-9)^{\frac{1}{2}}+1=x^{\frac{1}{2}}$

$\sqrt{(x-9)}=\sqrt{x}-1$

(square both sides)

$x-9=x-2\sqrt{x}+1$

(isolate the radical)

$2\sqrt{x}=10$

$\sqrt{x}=5$

$\boxed{x=25}$

f) $(x+5)^{\frac{1}{2}}-(5-2x)^{\frac{1}{4}}=0$

$(x+5)^{\frac{1}{2}}=(5-2x)^{\frac{1}{4}}$

(Remove the exponents by taking the '4th power' of each side -- or, squaring each side twice)

$(x+5)^2=5-2x$

$x^2+10x+25=5-2x$

$x^2+12x+20=0$

$(x+2)(x+10)=0$

$x=-2, -10$

$\boxed{x=-2}$

(check answers)

-2 : $3^{\frac{1}{2}}=9^{\frac{1}{4}}$ ✓

-10 : $(-5)^{\frac{1}{2}}=(25)^{\frac{1}{4}}$ ✗

extraneous!

V. Simplify (or factor) the following. SOLUTIONS

a) $(\sqrt{b^2+1}-1)(\sqrt{b^2+1}+1)$

$\sqrt{b^2+1}^{\,2} - 1$ (FOIL (the conjugates))

$b^2 + 1 - 1$

b^2

b) $y^{5/2} - y^{1/2}$ (GCF: the lowest exponent)

$y^{1/2}(y^2 - 1)$ (factor)

$y^{1/2}(y+1)(y-1)$

c) $x^{-3/2} - 2x^{-1/2} + x^{1/2}$ (factor out the lowest exponent)

$x^{-3/2}(1 - 2x + x^2)$ (factor the quadratic)

$x^{-3/2}(x-1)(x-1)$

$x^{-3/2}(x-1)^2$

d) $6x^{-1/2} + 8x^{1/2} + 2x^{3/2}$ (take out greatest common factor and "smallest exponent")

$2x^{-1/2}(3 + 4x + x^2)$

(factor quadratic)

$2x^{-1/2}(x+1)(x+3)$

$\frac{2(x+1)(x+3)}{x^{1/2}}$ or $\frac{2x^{1/2}(x+1)(x+3)}{x}$

e) $\frac{x^{-2} - y^{-2}}{x^{-1} + y^{-1}}$ (re-write the negative exponents)

$\frac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x} + \frac{1}{y}}$ (add numerator terms; add denominator terms)

$\frac{\frac{y^2 - x^2}{x^2 y^2}}{\frac{y + x}{xy}}$ (invert and multiply)

$\frac{y^2 - x^2}{x^2 y^2} \cdot \frac{xy}{y+x}$ (factor, cancel, and simplify)

$\frac{(y-x)(y+x)}{x^2 y^2} \cdot \frac{xy}{y+x}$

$\frac{(y-x)}{xy}$

f) $\frac{2(a+1)^{1/2} - a(1+a)^{-1/2}}{a+1}$

$\frac{(a+1)^{-1/2}[2(a+1)^1 - a]}{a+1}$ factor out term with lowest exponent $(a+1)^{-1/2}$

$\frac{(a+1)^{-1/2}[2a+2-a]}{a+1}$

$\frac{2+a}{(a+1)^{3/2}}$

VI. More rational exponent equations

SOLUTIONS

Rational Exponents and Radical Equations

a) $2(x+5)^{\frac{3}{2}} + 128 = 0$

$2(x+5)^{\frac{3}{2}} = -128$

$(x+5)^{\frac{3}{2}} = -64$

Note: square root isn't negative, so there will be no solution!!

$x+5 = (-64)^{\frac{2}{3}}$

$x+5 = (-4)^2$

$x = 11$

NO SOLUTION

if $x = 11$,

then $2(11+5)^{\frac{3}{2}} + 128 = 0$

$128 + 128 = 0$

b) $y = 6 + \sqrt{y}$

$y - y^{\frac{1}{2}} - 6 = 0$

$(y^{\frac{1}{2}} - 3)(y^{\frac{1}{2}} + 2) = 0$

$(y^{\frac{1}{2}} - 3) = 0$ $\quad$ $(y^{\frac{1}{2}} + 2) = 0$

$y = 9$ $\quad$ no real solution

c) $\sqrt{3-x} = \sqrt{7-2x}$

square both sides

$3 - x = 7 - 2x$

$x = 4$

quick check:

$\sqrt{3-4} = \sqrt{7-2(4)}$

$\sqrt{-1} = \sqrt{-1}$

NO REAL SOLUTIONS

d) $3(x+5)^{\frac{2}{3}} + 2 = 50$

isolate the exponent part

$3(x+5)^{\frac{2}{3}} = 48$

$(x+5)^{\frac{2}{3}} = 16$

Since the root is 2/3, a negative is permitted!

$\left((x+5)^{\frac{1}{3}}\right)^2 = 16$

$(x+5)^{\frac{1}{3}} = \pm 4$

$x + 5 = \pm 64$

$x = 59$ or -69

e) $2(x-1)^{\frac{3}{2}} - 7 = 23$

$2(x-1)^{\frac{3}{2}} = 30$

$(x-1)^{\frac{3}{2}} = 15$

Since it is a 1/2 root, a negative is NOT permitted...

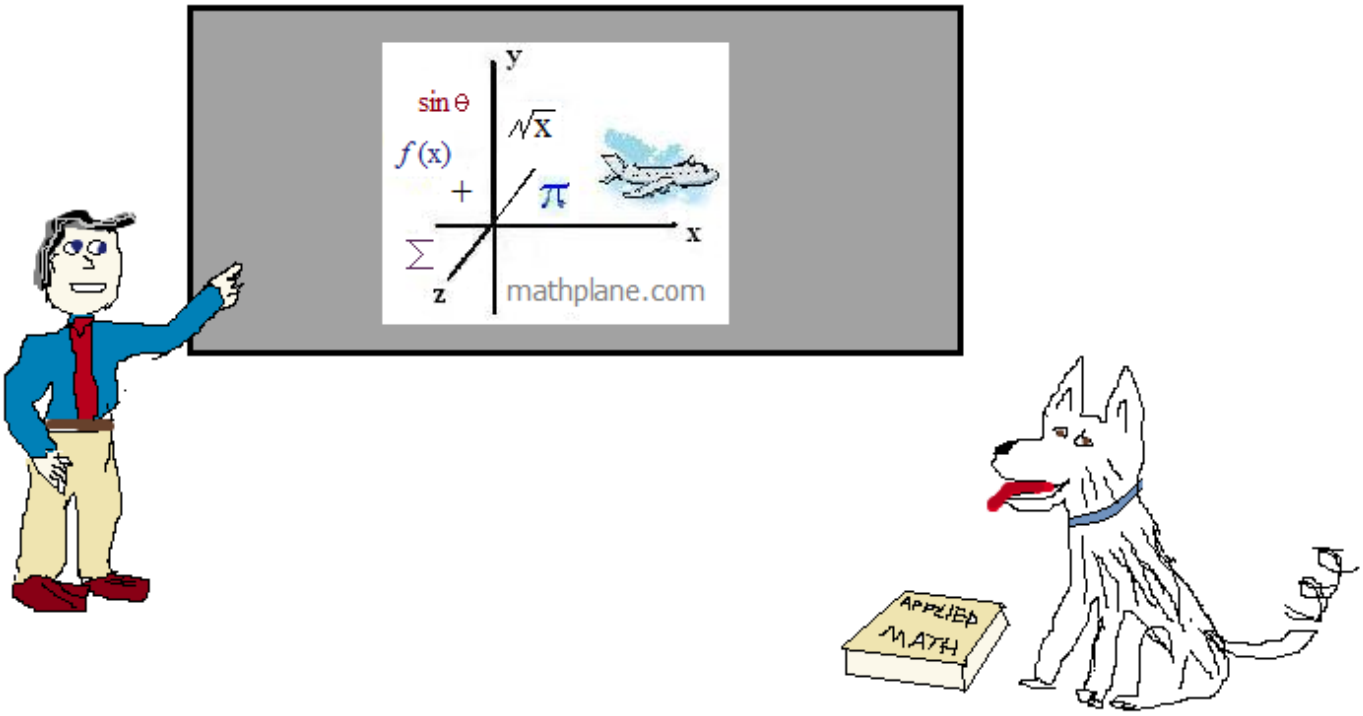
$(x-1) = (15)^{\frac{2}{3}}$

$x = \sqrt[3]{225} + 1$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy.



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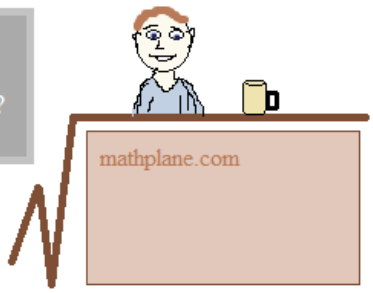
And, *Mathplane Express* for mobile and tablets at Mathplane.ORG

ONE MORE EXERCISE! -→

Hidden Message

Hint:
A math beverage?



Letter/Number Key									
A	B	E	O	P	Q	R	S	T	U
1	2	3	4	5	6	7	8	9	0

Solve the 14 equations.
Then, convert the numbers into letters to reveal the answer!

Find X:

1) $\sqrt[3]{X} = 2$

2) $X^3 = 216$

3) $N^X = 1$

4) $4 \cdot 2^{-2} = X$

5) $3^3 = X$

6) $(27)^{\frac{1}{3}} = X$

7) $\left(\frac{1}{49}\right)^{\frac{-1}{2}} = X$

8) $2^{X-2} = 4$

9) $(32)^{\frac{2}{5}} = X$

10) $3\sqrt[4]{81} = X$

11) $(125)^{\frac{-1}{3}} = X$ (express as decimal)

12) $\sqrt{49} - \sqrt{16} = X$

13) $3^{(X+3)} = 27^2$

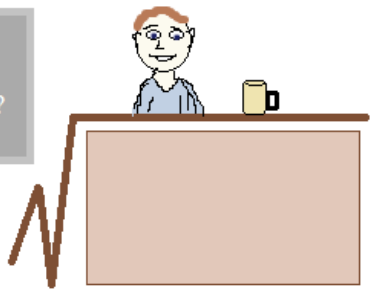
14) $\sqrt[3]{(7)^3} = X$

2 → ____

0. → ____

Hidden Message

Hint:
A math beverage?



Letter/Number Key									
A	B	E	O	P	Q	R	S	T	U
1	2	3	4	5	6	7	8	9	0

Solve the 14 equations.
Then, convert the numbers into letters to reveal the answer!

Find X:

SOLUTIONS

1) $\sqrt[3]{X} = 2$ $\quad X = 2^3 = 8$

2) $X^3 = 216$ $\quad X = \sqrt[3]{216} = 6$

3) $N^X = 1$ $\quad X = 0$

4) $4 \cdot 2^{-2} = X$ $\quad 4 \cdot \frac{1}{4} = 1$

5) $3^3 = X$ $\quad 3 \cdot 3 \cdot 3 = 27$

6) $(27)^{\frac{1}{3}} = X$ $\quad \sqrt[3]{27} = 3$

7) $\left(\frac{1}{49}\right)^{\frac{-1}{2}} = X$ $\quad \left(\frac{49}{1}\right)^{\frac{1}{2}} = 7$

8) $2^{X-2} = 4$ $\quad 2^{X-2} = 2^2$ then, $X - 2 = 2$ $\quad X = 4$

9) $(32)^{\frac{2}{5}} = X$ $\quad (32^{\frac{1}{5}})^2 = X$ $\quad (2)^2 = X$ $\quad X = 4$

10) $3\sqrt[4]{81} = X$ $\quad 3(3) = 9$

11) $(125)^{\frac{-1}{3}} = X$ (express as decimal) $\quad \left(\frac{1}{125}\right)^{\frac{1}{3}} = \frac{1}{5} = .2$

12) $\sqrt{49} - \sqrt{16} = X$ $\quad 7 - 4 = 3$

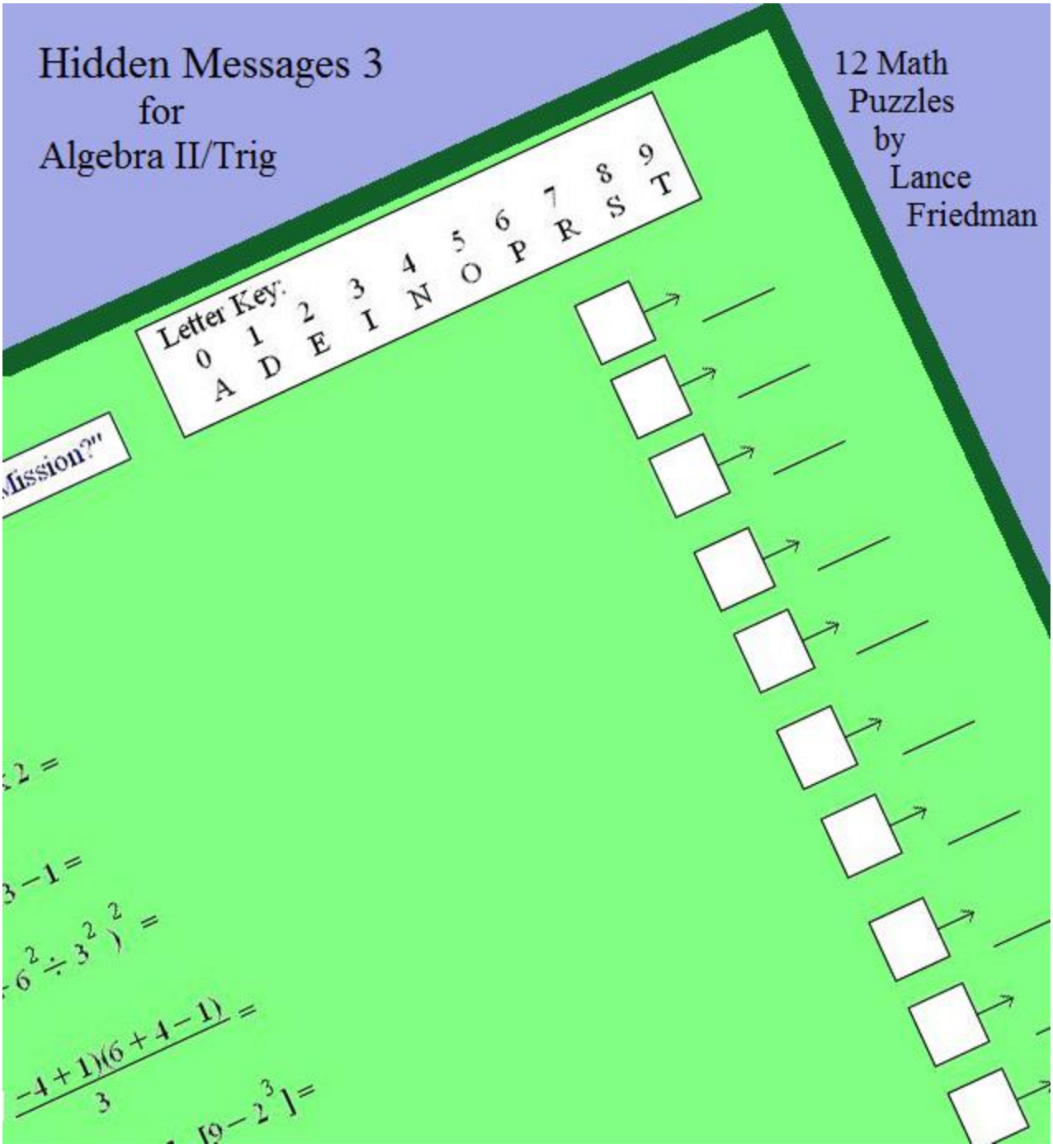
13) $3^{(X+3)} = 27^2$ $\quad 3^{(X+3)} = (3^3)^2$ $\quad 3^{(X+3)} = 3^6$ $\quad X + 3 = 6$, $X = 3$

14) $\sqrt[3]{(7)^3} = X$ $\quad (7^3)^{\frac{1}{3}} = 7^1 = 7$

A math beverage?

Square root beer!

	8	S
	6	Q
	0	U
	1	A
2	7	R
	3	E
	7	R
	4	O
	4	O
	9	T
0.	2	B
	3	E
	3	E
	7	R



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