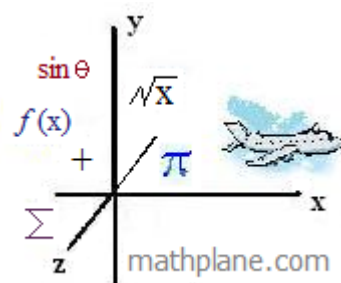# Algebra II Review Practice Test 002

25+ questions (with solutions)

Topics include exponents, graphing functions, logarithms, matrices, quadratics, and more...

mathplane.com

Algebra II Review Practice

1) Simplify

a) $9^{\frac{1}{2}}$ b) $9^{\frac{-1}{2}}$ c) $(-6)^2$ d) $27^{\frac{2}{3}}$ e) -2^2

2) For the piecewise function

$$f(x) = \begin{cases} 2x + 1 & \text{if } x \leq 0 \\ 3 & \text{if } 0 < x \leq 6 \\ x^2 & \text{if } x > 6 \end{cases}$$

$f(8) =$

$f(0) =$

$f(-3) =$

3) Identify the domain and range of each function. (write answers in interval notation)

a) $y = |x| - 2$ b) $y = \sqrt{1 + x} - 2$ c) $y = \sqrt{25 - x^2}$ d) $y = 5^x$

    D: D: D: D:

    R: R: R: R:

4) $1 - \sqrt{13 - x} = x$

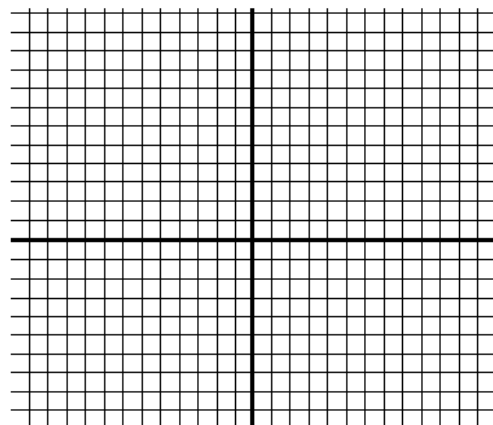5) For the quadratic $y = 2x^2 - 12x + 10$, answer the following. Then, sketch the graph.

    y-intercept:

    x-intercept(s):

    axis of symmetry:

    vertex:

    equation in *vertex form*:

6) Simplify the complex expressions (into standard form)

   a) i^{18} b) $(3 + i)(3 - i)$ c) $\dfrac{2 + i}{5 + i}$ d) $(4i + 7)^2$

7) Solve

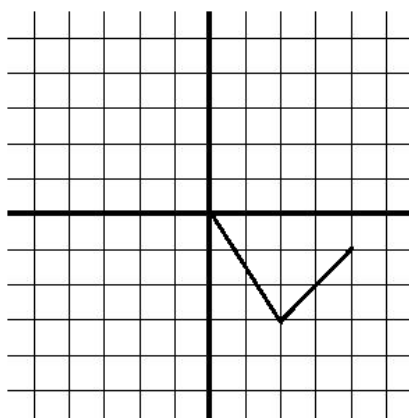   a) $x^4 - 10x^2 + 9 = 0$ b) $3x^3 + 15x^2 = 0$ c) $9x^3 + 36x^2 - x = 4$
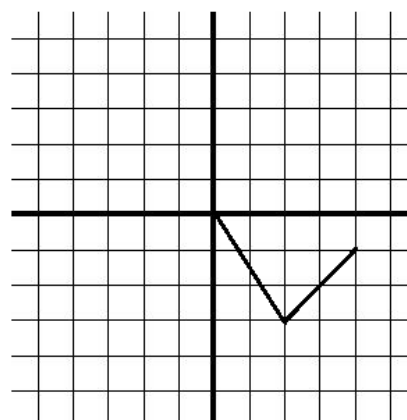
8) $f(x)$ is a function with domain $[-4, 4]$,

   Sketch the rest of the graph if   a) $f(x)$ is even   b) $f(x)$ is odd

9) What is the equation of a parabola with intercepts $(-4, 0)$, $(7, 0)$, and $(0, -14)$?

10) How many x-intercepts in the following quadratics?

   a) $3x^2 - 7x + 10$ b) $4x^2 - 9x + 1$ c) $9x^2 - 6x + 1$

11) Solve for x.

    a) $\log_x 32 = 5$ b) $\log_3 (1 - x) = 4$ c) $\log 25 + \log 4 = x$ d) $\ln e = x$

12) What are the linear equations?
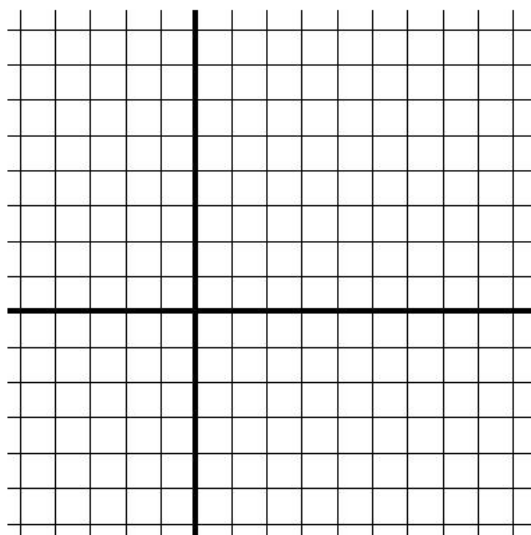
    a) x-intercept, 6   b) vertical line through (3, 6)   c) parallel to y = 2x and
        y-intercept, -3   passing through (1, -1)

13) Graph the absolute value function $y = -|x - 5| + 3$

      What is the y-intercept?

      What are the x-intercepts?

14) Find a cubic with roots -5, 1, and 6 AND the
     coefficient of x^2 is 8

15) Using long division or synthetic division, find $x^4 - 3x^2 + 2x + 10 \div (x - 1)$

16) Answer for the following matrices:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -2 \\ 5 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 3 & -3 \\ 6 & 1 & 7 \end{bmatrix}$$

a) $A - B$ b) BC c) BA d) $2A + B$

17) You deposit 10,000 dollars into a bank that offers 4% annual interest. How much money will you have *after 3 years*, if

       a) compounded semi-annually

       b) compounded monthly

       c) compounded continuously

18) Complete the square

     $x^2 + 8x - 17$ $3x^2 - 18x + 1$ $x^2 + 7x + 10$

19) Solve the system   a) Elimination/substitution method   OR   b) Matrix/solving simultaneous equations
                                    (w/o calculator)   (calculator)

$$x + 2y - z = -3$$
$$-x + 3z = 11$$
$$3x + y + 2z = 1$$

20) List all the *possible* rational roots of $3x^3 - 5x^2 + 5x - 2$

21) $x + \dfrac{2}{x-1} = \dfrac{x-3}{1-x}$

22) Answer the following for $g(x) = \dfrac{3x-6}{x+1}$
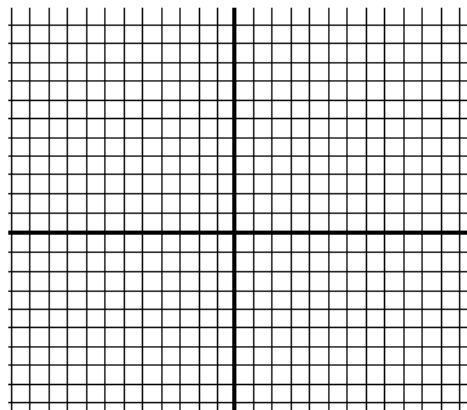
      y-intercept:

      x-intercept:

      vertical asymptote:

      horizontal asymptote:

      Sketch the graph of $g(x)$

23) A kid launches a water balloon from a balcony. The parabolic equation of the balloon is expressed as

$$h(t) = -16t^2 + 80t + 96$$

    where t = time in seconds
    h(t) = height from the ground level (feet)

    a) How high is the balcony?

    b) What is the maximum height the water balloon reaches?

    c) When does the balloon hit the ground?

24) Answer the following:

    a) What is the degree of $f(x) = 3 - 2x^2 - 5x$?

    b) For $x^{100} + x^5 - 3x^3 + 1$,   is $(x-1)$ a factor? Is $(x+1)$ a factor?

    c) What is the remainder of $2x^5 + x^2 + x + 13 \div (x+2)$

25) What is the equation of a parabola that passes through $(1, 7)$ $(-3, 67)$ and $(4, 46)$?

      Hint: $ax^2 + bx + c = y$

26) $-3|x - 8| + 7 = -20$ $\qquad\qquad$ $|2x + 6| = x + 5$ $\qquad\qquad$

27) Determine the equation of an ellipse with these characteristics:

major axis = 12

minor axis = 8

foci are on the x-axis

center is the origin

28) Sam buys 25-cent and 2-cent stamps to mail postcards.
He purchased 48 stamps and paid $7.40.
How much of each stamp did he buy?

29) A box's height is 2 feet larger than its width. And, the height is 5 feet smaller than its length.
If the volume of the box is 30 cubic feet, what are the dimensions?

SOLUTIONS-→

Algebra II Review Practice
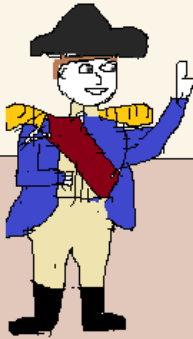
1) Simplify

a) $9^{\frac{1}{2}}$ $\boxed{3}$

b) $9^{\frac{-1}{2}}$ $\boxed{\dfrac{1}{3}}$

c) $(-6)^2$

$(-6)(-6) = \boxed{36}$

d) $27^{\frac{2}{3}}$

$\left(27^{\frac{1}{3}}\right)^2 = \boxed{9}$

e) -2^2 $\boxed{-4}$

$-1 \cdot 2^2$

(order of operations: exponents before multiplication)

2) For the piecewise function

$$f(x) = \begin{cases} 2x+1 & \text{if } x \le 0 \\ 3 & \text{if } 0 < x \le 6 \\ x^2 & \text{if } x > 6 \end{cases}$$

$f(8) = (8)^2 = 64$

$f(0) = 2(0) + 1 = 1$

$f(-3) = 2(-3) + 1 = -5$

3) Identify the domain and range of each function. (write answers in interval notation)

a) $y = |x| - 2$

D: $(-\infty, +\infty)$

R: $[-2, +\infty)$

b) $y = \sqrt{1+x} - 2$

D: $[-1, +\infty)$

R: $[-2, +\infty)$

c) $y = \sqrt{25 - x^2}$

D: $[-5, 5]$

R: $[0, 5]$

d) $y = 5^x$

D: $(-\infty, +\infty)$

R: $(0, +\infty)$

4) $1 - \sqrt{13-x} = x$ Isolate the radical: $-\sqrt{13-x} = x - 1$

$\sqrt{13-x} = -x + 1$

Square both sides: $13 - x = x^2 - x - x + 1$

$x^2 - x - 12 = 0$

factor: $(x+3)(x-4) = 0$

$\boxed{x = -3, \cancel{4}}$

**check answers!

$1 - \sqrt{13-(-3)} = -3$

$-3 = -3$ ✓

$1 - \sqrt{13 - 4} = 4$

$-2 = 4$ ✗

extraneous!

5) For the quadratic $y = 2x^2 - 12x + 10$, answer the following. Then, sketch the graph.

y-intercept: (0, 10)

let $x = 0$, then $y = 10$

x-intercept(s): (1, 0) (5, 0)

let $y = 0$, then $0 = 2x^2 - 12x + 10$

$0 = 2(x-1)(x-5)$

$x = 1, 5$

axis of symmetry: $x = 3$

AOS: midpoint of zeros is 3..

or, $\dfrac{-b}{2a} = \dfrac{-(-12)}{2(2)} = 3$

vertex: (3, -8)

Vertex is on axis of symmetry

let $x = 3$, then $y = -8$

equation in *vertex form*:

(complete the square)

$y = 2(x^2 - 6x + 9) + 10 - 2(9)$

$\boxed{y = 2(x-3)^2 - 8}$

SOLUTIONS
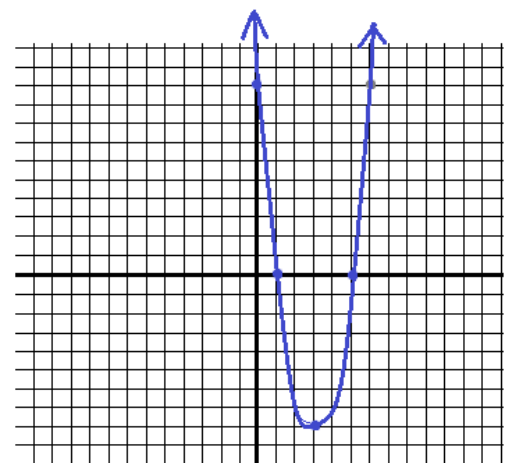
6) Simplify the complex expressions (into standard form)

a) i^{18}

$i^2 \cdot i^{16}$

$\boxed{-1}$

b) $(3+i)(3-i)$

$9 + 3i - 3i - i^2$

$\boxed{10}$

c) $\dfrac{2+i}{5+i} \cdot \dfrac{5-i}{5-i}$

$\dfrac{10 + 5i - 2i - i^2}{25 - i^2}$

$\boxed{\dfrac{11}{26} + \dfrac{3i}{26}}$

d) $(4i+7)^2$

$16i^2 + 28i + 28i + 49$

$-16 + 56i + 49$

$\boxed{33 + 56i}$

7) Solve

a) $x^4 - 10x^2 + 9 = 0$

$(x^2 - 1)(x^2 - 9) = 0$

$(x+1)(x-1)(x+3)(x-3) = 0$

$\boxed{x = -1, 1, -3, 3}$

b) $3x^3 + 15x^2 = 0$

$3x^2(x+5) = 0$

$\boxed{x = -5, 0}$

c) $9x^3 + 36x^2 - x = 4$

$9x^3 + 36x^2 - x - 4 = 0$ (factor by grouping)
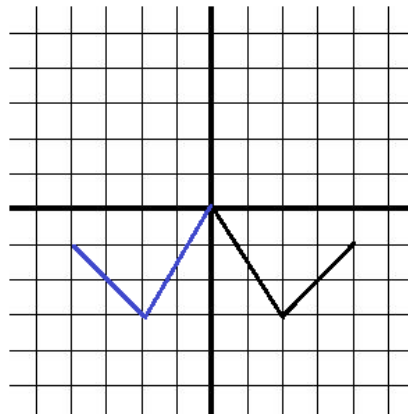
$9x^2(x+4) + (-1)(x+4) = 0$

$(9x^2 - 1)(x+4) = 0$

$(3x+1)(3x-1)(x+4) = 0$

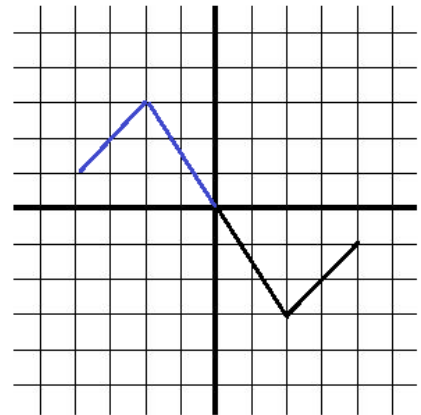$\boxed{x = -4, \dfrac{1}{3}, \dfrac{-1}{3}}$

8) $f(x)$ is a function with domain $[-4, 4]$,

Sketch the rest of the graph if

a) $f(x)$ is even

b) $f(x)$ is odd

9) What is the equation of a parabola with intercepts $(-4, 0)$, $(7, 0)$, and $(0, -14)$?

$y = a(x - x_1)(x - x_2)$

$y = a(x + 4)(x - 7)$

(plug in 3rd point to find 'a')

$-14 = a(0 + 4)(0 - 7)$

$-14 = -28a$

$a = \dfrac{1}{2}$

$\boxed{y = \dfrac{1}{2}(x + 4)(x - 7)}$

10) How many x-intercepts in the following quadratics? **find discriminants

a) $3x^2 - 7x + 10$

$b^2 - 4ac = 49 - 4(3)(10) = -71$

$\boxed{\text{zero intercepts}}$

b) $4x^2 - 9x + 1$

$b^2 - 4ac = 81 - 4(4)(1) = 65$

$\boxed{\text{two intercepts}}$

c) $9x^2 - 6x + 1$

$b^2 - 4ac = 36 - 4(9)(1) = 0$

$\boxed{\text{one intercept}}$

11) Solve for x.

a) $\log_x 32 = 5$

$x^5 = 32$

$\boxed{x = 2}$

b) $\log_3 (1 - x) = 4$

$3^4 = 1 - x$

$\boxed{x = -80}$

c) $\log 25 + \log 4 = x$

$\log(25 \times 4) = \log 100$

$\boxed{x = 2}$

d) $\ln e = x$

$\log_e e = x$
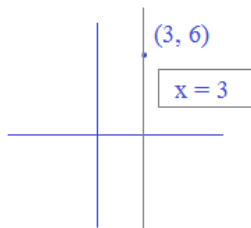
$\boxed{x = 1}$

12) What are the linear equations?

a) x-intercept, 6   (6, 0)
   y-intercept, -3   (0, -3)

slope $= \dfrac{0 - (-3)}{6 - 0} = \dfrac{1}{2}$

since we know the y-intercept, we'll use slope intercept form:

$\boxed{y = \dfrac{1}{2}x - 3}$

b) vertical line through (3, 6)

(3, 6)

$\boxed{x = 3}$

c) parallel to y = 2x and passing through (1, -1)

slope is 2
point is (1, -1)

use point slope form:

$\boxed{y + 1 = 2(x - 1)}$

13) Graph the absolute value function $y = -|x - 5| + 3$

What is the y-intercept? $y = -|(0) - 5| + 3$

$\boxed{(0, -2)}$

$= -5 + 3 = -2$

What are the x-intercepts?

$(0) = -|x - 5| + 3$ (isolate the absolute value)

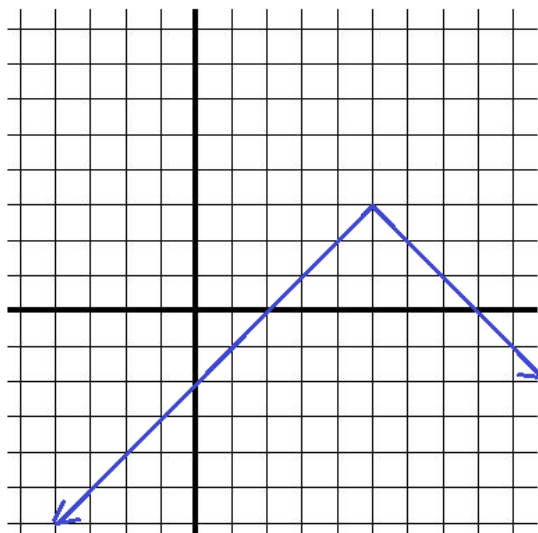$-3 = -|x - 5|$

$3 = |x - 5|$ ("split" the absolute value)

$\boxed{(8, 0) \text{ and } (2, 0)}$

x - 5 = 3   x - 5 = -3
x = 8   x = 2

14) Find a cubic with roots -5, 1, and 6 AND the coefficient of x^2 is 8

$(x + 5)(x - 1)(x - 6)$

multiply by -4 to change coefficients

$(x + 5)(x^2 - 7x + 6) = $

$\begin{array}{r} x^3 - 7x^2 + 6x \\ + \quad 5x^2 - 35x + 30 \\ \hline x^3 - 2x^2 - 29x + 30 \end{array}$

$\boxed{-4x^3 + 8x^2 + 116x - 120}$

15) Using long division or synthetic division, find $x^4 - 3x^2 + 2x + 10 \div (x - 1)$

$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 2 & 10 \\ & & 1 & 1 & -2 & 0 \\ \hline & 1 & 1 & -2 & 0 & 10 \end{array}$

$\boxed{x^3 + x^2 - 2x + \dfrac{10}{x - 1}}$

$\begin{array}{r} x^3 + x^2 - 2x + 0 \\ x - 1 \overline{)\, x^4 + \ -3x^2 + 2x + 10} \end{array}$

subtract $x^4 - x^3$

$x^3 - 3x^2$

subtract $x^3 - x^2$

$-2x^2 + 2x$

subtract $-2x^2 + 2x$

$0 \quad + 10$

SOLUTIONS

16) Answer for the following matrices:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -2 \\ 5 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 3 & -3 \\ 6 & 1 & 7 \end{bmatrix}$$

a) $A - B$

$$\begin{bmatrix} 1-4 & 2-(-2) \\ -1-5 & 0-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -6 & -1 \end{bmatrix}$$

b) BC

$$\begin{bmatrix} 4 & -2 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 0 & 3 & -3 \\ 6 & 1 & 7 \end{bmatrix} = \begin{bmatrix} -12 & 10 & -26 \\ 6 & 16 & -8 \end{bmatrix}$$

c) BA Note: $BA \neq AB$

$$\begin{bmatrix} 4 & -2 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 4 & 10 \end{bmatrix}$$

d) $2A + B$

$$\begin{bmatrix} 2(1) & 2(2) \\ 2(-1) & 2(0) \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$$

17) You deposit 10,000 dollars into a bank that offers 4% annual interest. How much money will you have *after 3 years*, if

a) compounded semi-annually

$$A = 10,000(1 + .02)^6 = 11,261.62$$

(6 total compounds; 2% per compound)

b) compounded monthly

$$A = 10,000(1 + \frac{.04}{12})^{36} = 11,272.72$$

(.04/12 per month; 36 months)

c) compounded continuously

$$A = 10,000e^{.04(3)} = 11,274.97$$

(4% per year; 3 years)

$$A = P(1 + \frac{r}{n})^{nt}$$

A = future amount
P = principal amount
r = interest rate
n = number of times the amount is compounded per year
t = number of years

$$A = Pe^{rt} \quad \text{(continuous compounding)}$$

18) Complete the square

$$x^2 + 8x - 17$$

$x^2 + 8x \qquad - 17$ "separate the x terms"

$x^2 + 8x + 16 - 17$ "add $\left(\frac{b}{2}\right)^2$"

$x^2 + 8x + 16 - 17 - 16$ "subtract to balance"

$$\boxed{(x + 4)^2 - 33}$$ "factor and simplify"

$$3x^2 - 18x + 1$$

$3x^2 - 18x \qquad + 1$ "factor to get coefficient of 1"

$3(x^2 - 6x \qquad) + 1$

$3(x^2 - 6x + 9) + 1 - 3(9)$

$$\boxed{3(x - 3)^2 - 26}$$

$$x^2 + 7x + 10$$

$x^2 + 7x \qquad + 10$

$x^2 + 7x + \frac{49}{4} \qquad + 10 - \frac{49}{4}$

$$\boxed{(x + \frac{7}{2})^2 - \frac{9}{4}}$$

19) Solve the system

a) Elimination/substitution method (w/o calculator)   OR   b) Matrix/solving simultaneous equations (calculator)

$$x + 2y - z = -3$$
$$-x + 3z = 11$$
$$3x + y + 2z = 1$$

combine 1st and 3rd:

$x + 2y - z = -3$
$-6x - 2y - 4z = -2$
_____
$-5x \qquad -5z = -5$

combine 2nd and result of 1st/3rd:

$-5x - 5z = -5$
$x + z = 1$
$-x + 3z = 11$
_____
$4z = 12$
$z = 3$

substitute 2nd:

$-x + 3z = 11$
$-x + 9 = 11$
$x = -2$

substitute 1st:

$x + 2y - z = -3$
$-2 + 2y - 3 = -3$
$y = 1$

(Plug in answers to check)

or $$\begin{bmatrix} 1 & 2 & -1 \\ -1 & 0 & 3 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -3 \\ -1 & 0 & 3 & 11 \\ 3 & 1 & 2 & 1 \end{array}\right]$$

SOLUTION

$$\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

20) List all the *possible* rational roots of $3x^3 - 5x^2 + 5x - 2$

Constant: -2 "factors of p" 1, 2
Lead coefficient: 3   "factors of q" 1, 3

possible rational roots:

$$\boxed{\pm 1 \quad \pm 2 \quad \pm\frac{1}{3} \quad \pm\frac{2}{3}}$$

SOLUTIONS

21) $x + \dfrac{2}{x-1} = \dfrac{x-3}{1-x}$

$x = \dfrac{x-3}{1-x} \quad \dfrac{-2}{x-1} \quad \dfrac{(-1)}{(-1)}$

$x = \dfrac{x-3}{1-x} + \dfrac{2}{1-x}$

$x = \dfrac{x-1}{1-x}$ (cross multiply)

$x(1-x) = x-1$

$x - x^2 - x + 1 = 0$

$-x^2 + 1 = 0$

$\boxed{x = -1 \text{ or } \cancel{1}}$

test solutions:

-1: $-1 + \dfrac{2}{-2} = \dfrac{-4}{2}$

$-2 = -2$ ✓

1: $1 + \dfrac{2}{0} = \dfrac{-2}{0}$

undefined! (extraneous)

22) Answer the following for $g(x) = \dfrac{3x-6}{x+1}$

y-intercept: $(0, -6)$

x-intercept: $(2, 0)$

vertical asymptote: $x = -1$

horizontal asymptote: $y = 3$

Sketch the graph of $g(x)$

$g(0) = \dfrac{-6}{1} = -6$

$0 = \dfrac{3x-6}{x+1} \quad x = 2$

function is undefined
when $x + 1 = 0$
$x = -1$

function is neither top heavy
nor bottom heavy;
coefficients are $3/1 = 3$
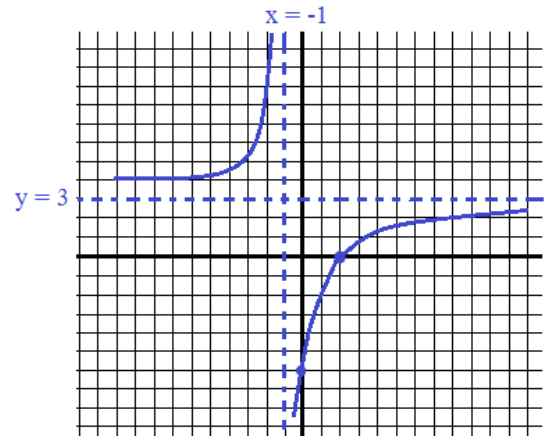
23) A kid launches a water balloon from a balcony. The parabolic equation of the balloon is expressed as

$h(t) = -16t^2 + 80t + 96$

where t = time in seconds
$h(t)$ = height from the ground level (feet)
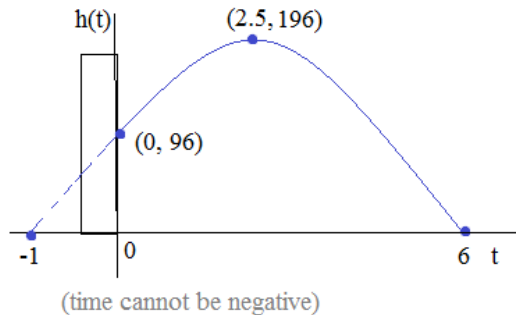
a) How high is the balcony? at time 0, (0, 96) $\boxed{96 \text{ feet}}$

b) What is the maximum height the water balloon reaches? vertex $\boxed{196 \text{ feet}}$

c) When does the balloon hit the ground? $\boxed{6 \text{ seconds}}$
x-intercept: $-16t^2 + 80t + 96 = 0$

$\dfrac{-b}{2a} = \dfrac{-80}{-32} = 2.5$

$h(2.5) = 196$

(time cannot be negative)

24) Answer the following: $-16(t+1)(t-6) = 0 \quad t = -\cancel{1} \text{ or } 6$

a) What is the degree of $f(x) = 3 - 2x^2 - 5x$? $\boxed{\text{the degree is 2}}$

b) For $x^{100} + x^5 - 3x^3 + 1$, is $(x-1)$ a factor? Is $(x+1)$ a factor?
$\boxed{\text{YES}}$ $\boxed{\text{NO}}$

(using factor theorem) plug in the opposite of the constant ----
$(1)^{100} + (1)^5 - 3(1)^3 + 1 = 0$
$(-1)^{100} + (-1)^5 - 3(-1)^3 + 1 = 4$

c) What is the remainder of $2x^5 + x^2 + x + 13 \div (x+2)$

(Use remainder theorem) plug in -2: $2(-2)^5 + (-2)^2 + (-2) + 13 = -49$ $\boxed{\text{Remainder is } -49}$

25) What is the equation of a parabola that passes through $(1, 7)$ $(-3, 67)$ and $(4, 46)$?

Hint: $ax^2 + bx + c = y$

$\boxed{4x^2 - 7x + 10}$

(test 3 points to check answer)

$(1, 7)$: $a(1)^2 + b(1) + c = 7$

$(-3, 67)$: $a(-3)^2 + b(-3) + c = 67$

$(4, 46)$: $a(4)^2 + b(4) + c = 46$

$a + b + c = 7$
$9a - 3b + c = 67$
$16a + 4b + c = 46$

solve the 3 equations with 3 unknowns:

$a = 4$
$b = -7$
$c = 10$

26) $-3|x - 8| + 7 = -20$

$-3|x - 8| = -27$

$|x - 8| = 9$

$\boxed{x = 17 \text{ or } -1}$

$|2x + 6| = x + 5$

$2x + 6 = x + 5$ OR $2x + 6 = -(x + 5)$
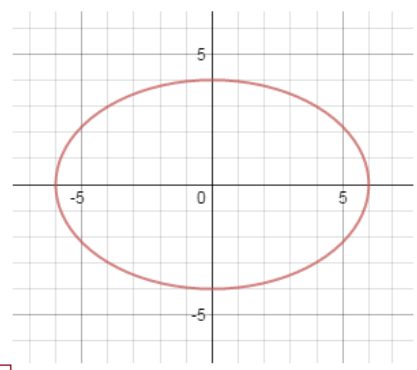
$\boxed{x = -1}$ $2x + 6 = -x - 5$

$3x = -11$

$\boxed{x = -11/3}$

SOLUTIONS

27) Determine the equation of an ellipse with these characteristics:

major axis = 12   If major axis is 12, the a value is 6.

minor axis = 8   If the minor axis is 8, the b value is 4

foci are on the x-axis   Since foci are on the x-axis, the ellipse is 'horizontal'

center is the origin

$$\frac{(x - 0)^2}{6^2} + \frac{(y - 0)^2}{4^2} = 1$$

$$\boxed{\frac{x^2}{36} + \frac{y^2}{16} = 1}$$

28) Sam buys 25-cent and 2-cent stamps to mail postcards.
He purchased 48 stamps and paid $7.40.
How much of each stamp did he buy?

let x = # of 25-cent stamps
    y = # of 2-cent stamps

$x + y = 48$

$.25(x) + .02(y) = 7.40$

(using substitution)
$x = 48 - y$

then, $.25(48 - y) + .02y = 7.40$

$12 - .25y + .02y = 7.40$

$-.23y = -4.6$

$y = 20...$

then, $x + 20 = 48$

$x = 28$

$\boxed{\begin{array}{l} 28 \text{ 25-cent stamps} \\ 20 \text{ 2-cent stamps} \end{array}}$

29) A box's height is 2 feet larger than its width. And, the height is 5 feet smaller than its length.
If the volume of the box is 30 cubic feet, what are the dimensions?

Volume = (length)(width)(height)

$30 = (h + 5)(h - 2)(h)$

$30 = (h + 5)(h^2 - 2h)$

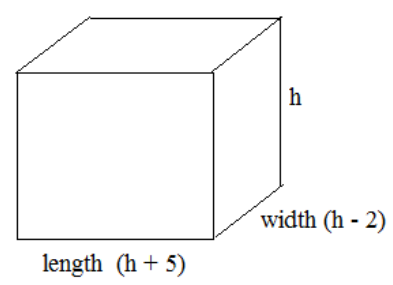$30 = h^3 + 5h^2 - 2h^2 - 10h$

$h^3 + 3h^2 - 10h - 30 = 0$

$h^2(h + 3) - 10(h + 3) = 0$

$(h^2 - 10)(h + 3) = 0$

$h = \sqrt{10}, -\sqrt{10}, -3$

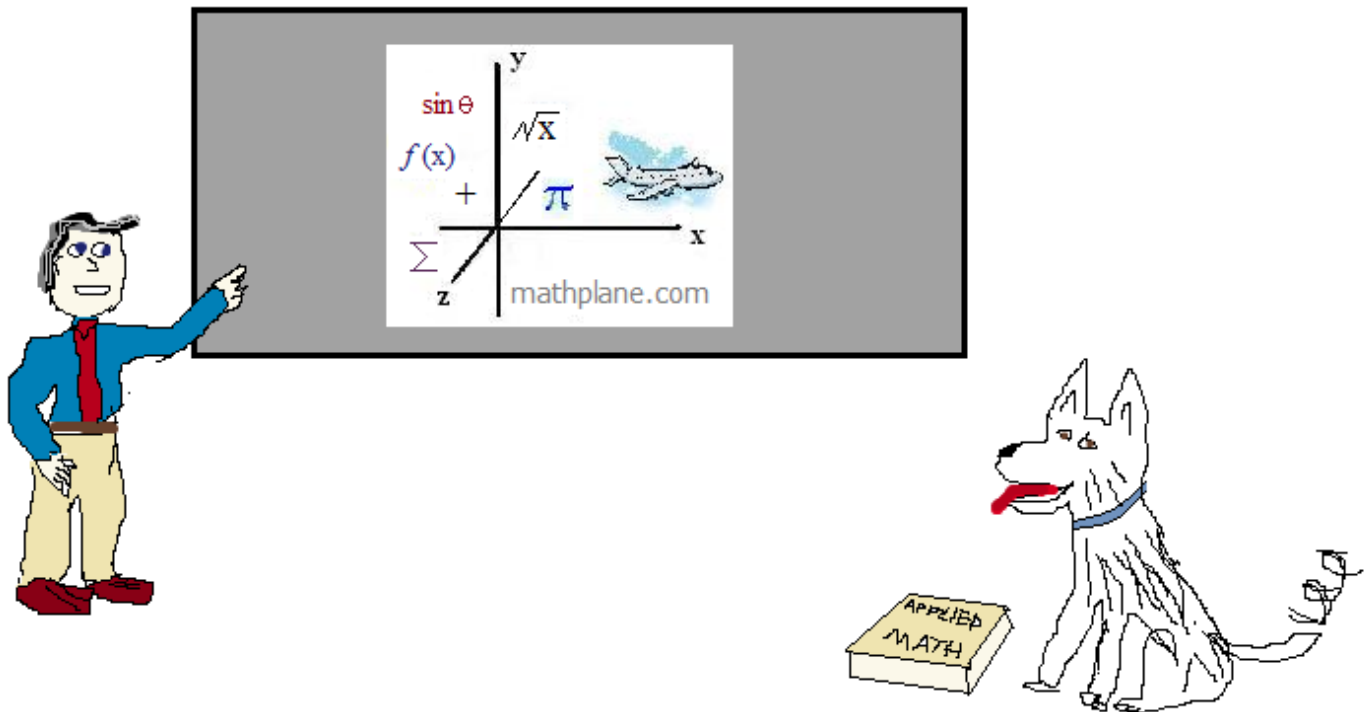**since height (h) must be greater than 0, the only solution is $\sqrt{10}$

To check: $(\sqrt{10} + 5)(\sqrt{10})(\sqrt{10} - 2) =$ $\boxed{(8.16)(3.16)(1.16) = 30}$ ✓

h

width (h - 2)

length (h + 5)

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy…

Also, at Facebook, Google+, Pinterest, TES, and TeachersPayTeachers

Plus, Mathplane *Express* for mobile at mathplane.ORG