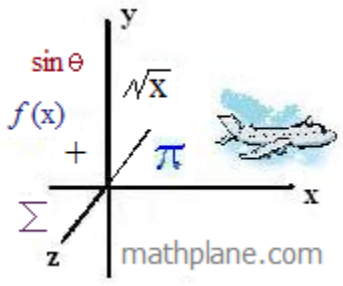
Algebra II

Review Test 005

(and, solutions!)

25 Questions include half-life, asymptotes, roots & intercepts, absolute value, inequalities, sequences, linear systems, and more…



Algebra II Review Test 005

1) Solve:

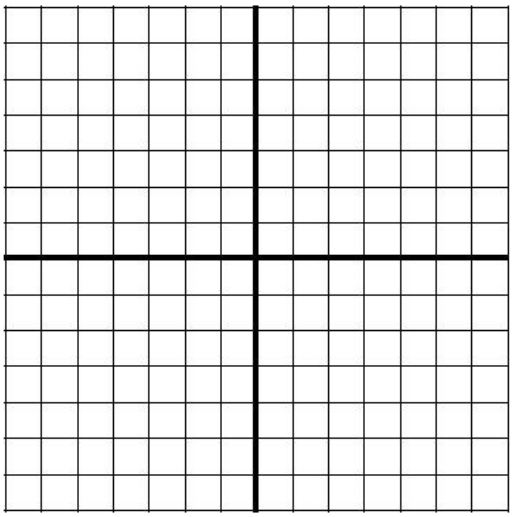
$|3x + 7| = 1$ $\quad$ $3|d| + 4 = 16$ $\quad$ $2|y + 4| + 8 = 4$

2) $y = \dfrac{x - 18}{x + 6}$

What is the x-intercept?

What is the y-intercept?

3) Graph the function $g(x) = \dfrac{x^2 + x - 2}{x^2 - x - 6}$



Identify the following:

Horizontal Asymptote:

Vertical Asymptote:

x-intercept:

y-intercept:

Removable discontinuity:
("holes")

4) Simplify: $\dfrac{x^2 + xy}{xy - y^2} \cdot \dfrac{xy^2 - y^3}{x^2 - y^2}$

5) $100,000 is deposited in a bank that offers 5.2% annual interest *compounded daily*.
How long will it take to accumulate $10,000 in interest?

Algebra II Review Test 005

6) log 3 = .477
log 4 = .602

(Without a calculator) Find the following:

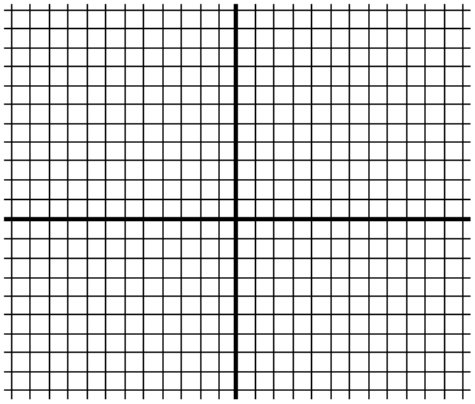
log 12 = log .75 = log 400 = log 16 =

7) $\begin{bmatrix} x & y \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 23 \\ 5 & 17 \end{bmatrix}$ x = y =

8) At the Ye Olde Snack Shop, raisins cost $3.40 per pound and nuts cost $2.50 per pound. If a 50-pound mixture of nuts and raisins costs $2.86 per pound, how much of each are in the mixture?

9) $(5\sqrt{3}+\sqrt{10})(5\sqrt{3}-\sqrt{10}) =$

10) Graph the function $h(x) = x^2 - 6$ where the *domain* is {0, 1, 2, 3, 4}



Algebra II Review Test 005

11) Simplify:

$7a^{-5}b^{6} \div 21a^{4}b^{-2}$

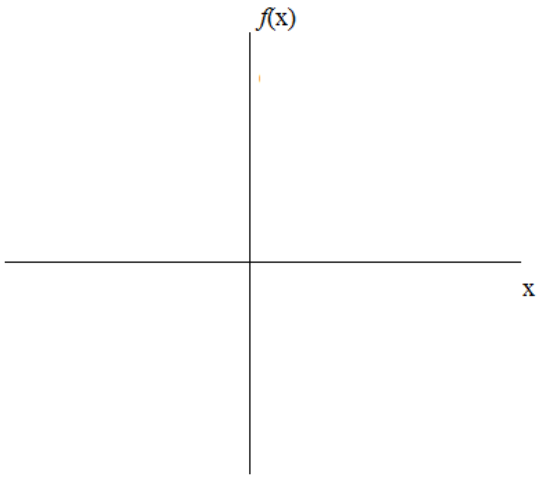
$\left(\frac{9}{49}\right)^{\frac{-3}{2}}$

$(2^{9})^{\frac{1}{3}} \cdot \sqrt{32}$

12) What is the equation of a line *perpendicular* to x = 3 that goes through (5, 7)?

13) $f(x) = x^{4} - 2x^{3} - 7x^{2} + 8x + 12$

Find the zeros, identify the end behavior, and sketch the function:



14) $g(x) = 4 - \sqrt{3x - 6}$

a) $g(5) =$

b) $g(a + 2) =$

c) $g(2 - x) =$

15) In the quadratic $y = 5x - 7x^{2} + 8$

what is a) the linear term?

b) the degree?

c) the constant?

Algebra II Review Test 005

16) Identify the center and radius of the circle:

$$x^2 + y^2 - 8x + 6y = -16$$

center:

radius:

17) Find the solution to the linear system

$$\begin{aligned} 3x + 7y + 2z &= 2 \\ 2x - 6y &= 22 \\ -x + 4y - 4z &= -15 \end{aligned}$$

18) Solve: $(x+2)^2 = (x-4)^2$ $\qquad$ $\dfrac{x+1}{x-1} = \dfrac{3x}{3x-6}$

19) 500 mg of a radioactive material has a half-life of 8 years.

a) How much material remains after 24 years?

b) How much material remains after 36 years?

c) When will <u>less than 5 mg</u> of the radioactive material remain?

20) Sequences:

a) What is the 5th term in the following *arithmetic* sequence?

$A_1 = 2 \quad A_2 = 4 \qquad A_5 =$

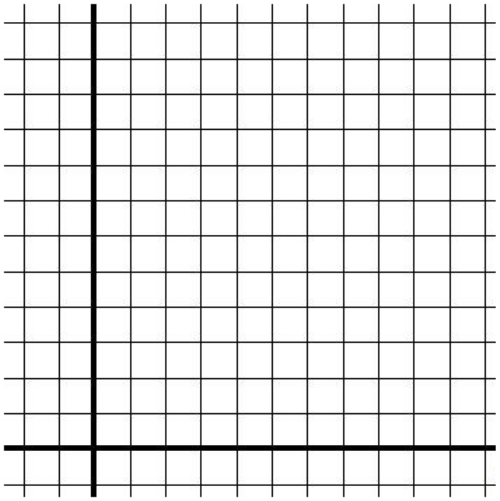
b) What is the 5th term in the following *geometric* sequence?

$G_1 = 2 \quad G_2 = 4 \qquad G_5 =$

Algebra II Review Test 005

21) Find the maximum value of $P = 2x + y$ subject to the constraints

$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 7$$
$$5x + 2y \leq 20$$



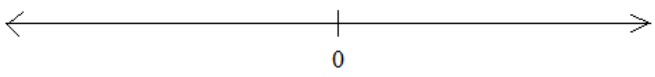
Graph the constraints (and identify the feasibility region).

22) Factor: $x^6 - 1$

23) $f(x) = 3x + 4$
$g(x) = x^2 - 5$

a) $f(g(4)) =$ b) $g(f(4)) =$ c) $g^{-1}(x) =$

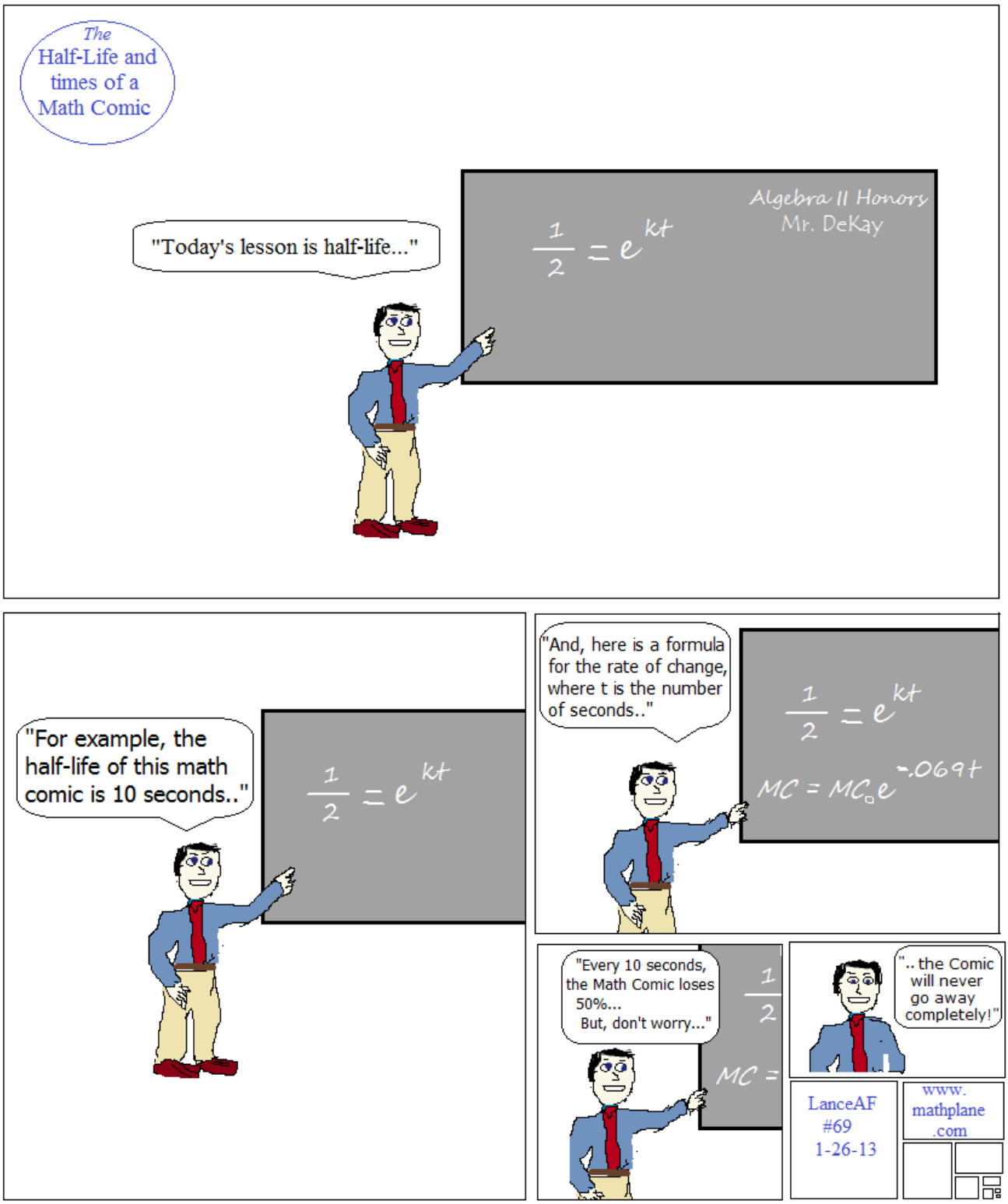
24) $\dfrac{(x + 3)(2 - 5x)}{x + 1} \leq 0$ Express the answer in interval notation. Then, graph on the number line.



25) Find X:

a) $9^{(X+1)} = 27^{2X}$

b) $\sqrt{5} = 125^{3X}$


The Half-Life and times of a Math Comic
"Today's lesson is half-life..."
Algebra II Honors
Mr. DeKay
$\frac{1}{2} = e^{kt}$
"For example, the half-life of this math comic is 10 seconds.."
$\frac{1}{2} = e^{kt}$
"And, here is a formula for the rate of change, where t is the number of seconds.."
$\frac{1}{2} = e^{kt}$
$MC = MC_0 e^{-.069t}$
"Every 10 seconds, the Math Comic loses 50%... But, don't worry..."
$\frac{1}{2}$
$MC =$
".. the Comic will never go away completely!"
LanceAF
#69
1-26-13
www.
mathplane
.com

Algebra II Review Test 005

SOLUTIONS

1) Solve:

$|3x + 7| = 1$

$3x + 7 = -1$ → $x = -8/3$

$3x + 7 = 1$ → $x = -2$

$3|d| + 4 = 16$

$3|d| = 12$

$|d| = 4$

$d = 4$ or -4

$2|y + 4| + 8 = 4$

$2|y + 4| = -4$

$|y + 4| = -2$

NO SOLUTIONS

absolute value cannot equal a negative

2) $y = \dfrac{x - 18}{x + 6}$

What is the x-intercept? x-intercept is (?, 0)

$0 = \dfrac{x - 18}{x + 6}$ $x = 18$ (18, 0)

What is the y-intercept? y-intercept is (0, ?)

$y = \dfrac{0 - 18}{0 + 6}$ $y = -3$ (0, −3)

3) Graph the function $g(x) = \dfrac{x^2 + x - 2}{x^2 - x - 6} = \dfrac{(x + 2)(x - 1)}{(x + 2)(x - 3)}$

Identify the following:

Horizontal Asymptote: $y = 1$

Vertical Asymptote: $x = 3$

x-intercept: $(1, 0)$

y-intercept: $(0, \frac{1}{3})$

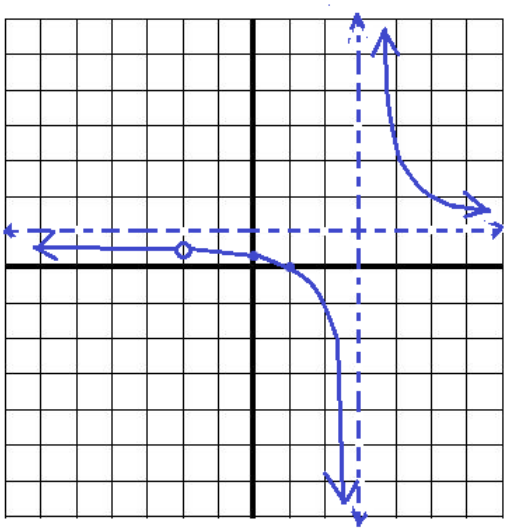
Removable discontinuity: $(-2, \frac{3}{5})$ ("holes")

HA: degree of numerator equals degree of denominator lead coefficients are 1.. 1/1 = 1

VA: at x = 3, the function is 1/0 --- undefined.. (note: we exclude the (x + 2))

x-intercept: $g(x)$ equals 0 when x is 1

y-intercept: $g(0) = -2/-6$ 1/3



$g(-2) = 0/0$ so, there is a hole at $x = -2$

excluding the $(x + 2)$, $g(-2)$ would equal $\dfrac{(-2 - 1)}{(-2 - 3)} = 3/5$

4) Simplify: $\dfrac{x^2 + xy}{xy - y^2} \cdot \dfrac{xy^2 - y^3}{x^2 - y^2}$

(factor) $\dfrac{x(x + y)}{y(x - y)} \cdot \dfrac{y^2(x - y)}{(x + y)(x - y)}$

(cancel) $\dfrac{x(x + y)}{y(x - y)} \cdot \dfrac{y^2(x - y)}{(x + y)(x - y)}$

$\dfrac{xy^2}{y(x - y)}$ (reduce)

$\dfrac{xy}{(x - y)}$

5) $100,000 is deposited in a bank that offers 5.2% annual interest *compounded daily*. How long will it take to accumulate $10,000 in interest?

$A = P(1 + \frac{r}{n})^{nt}$

A = future amount
P = principal amount
r = interest rate
n = number of times the amount is compounded per year
t = number of years

$110,000 = 100,000(1 + \frac{.052}{365})^t$

$1.1 = (1 + \frac{.052}{365})^t$

$\log 1.1 = \log (1 + \frac{.052}{365})^t$

$\log 1.1 = t \cdot \log(1 + \frac{.052}{365})$

$.0414 = t(.00006187)$

t = 669 days... (approximately)

t = # of times compounded ... then, since the bank compounds the money daily, the number of years is

669/365 = 1.83 years

Algebra II Review Test 005

SOLUTIONS

6) log 3 = .477
log 4 = .602

(Without a calculator) Find the following:

log 12 = log(3 • 4)
log3 + log4 =
.477 + .602 = 1.079

log .75 = $\log\left(\frac{3}{4}\right)$
log3 − log4 =
.477 − .602 = −.125

log 400 = log(100 • 4)
log100 + log4 =
2 + .602 = 2.602

log 16 = $\log 4^2$
2log4 =
2 • (.602) = 1.204

note: $10^{1.079} \cong 12$

7)
$$\begin{bmatrix} x & y \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 23 \\ 5 & 17 \end{bmatrix}$$

x = 2
y = −3

2x + 1y = 1
4x + (-5)y = 23

3(2) + (-1)1 = 5 ✓
3(4) + (-1)(-5) = 17 ✓

(solve 2 equations, 2 unknowns)

4x + 2y = 2
−(4x − 5y = 23)
7y = −21
y = −3
x = 2

8) At the Ye Olde Snack Shop, raisins cost $3.40 per pound and nuts cost $2.50 per pound. If a 50-pound mixture of nuts and raisins costs $2.86 per pound, how much of each are in the mixture?

let R = amount of raisins
N = amount of nuts

R + N = 50 Quantity
$3.40R + $2.50N = $143 Cost

30 pounds of nuts
20 pounds of raisins

Since the mixture cost $2.86 per pound,
50 pounds of mixture cost 50 x 2.86 = $143

Solve 2 equations, 2 unknowns:

3.40(50 - N) + 2.50N = 143
170 − 3.4N + 2.5N = 143
-.9N = -27 N = 30
(and, R = 20)

check answer:
30 x $2.50 = $75
20 x $3.40 = $68
and, $143 (50 pounds) is $2.86 per lb ✓

9) $(5\sqrt{3} + \sqrt{10})(5\sqrt{3} - \sqrt{10}) = 75 - 5\sqrt{30} + 5\sqrt{30} - 10$

FOIL

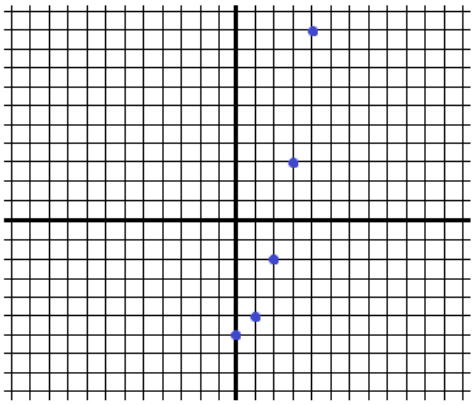
65

10) Graph the function $h(x) = x^2 - 6$
where the *domain* is {0, 1, 2, 3, 4}

the domain is just 5 points, and the range is the 5 corresponding values:

$h(0) = -6$
$h(1) = -5$
$h(2) = -2$
$h(3) = 3$
$h(4) = 10$

so, plot the 5 points to graph the function with the given domain...



SOLUTIONS

Algebra II Review Test 005

11) Simplify:

$7a^{-5}b^{6} \div 21a^{4}b^{-2}$

$\frac{7a^{-5}b^{6}}{21a^{4}b^{-2}} = \frac{7\,b^{2}b^{6}}{21\,a^{5}a^{4}} = \frac{1\,b^{8}}{3\,a^{9}}$

$= \frac{b^{8}}{3a^{9}}$

$\left(\frac{9}{49}\right)^{\frac{-3}{2}}$

$\left(\frac{49}{9}\right)^{\frac{3}{2}} = \left(\frac{7}{3}\right)^{3}$

$= \frac{343}{27}$

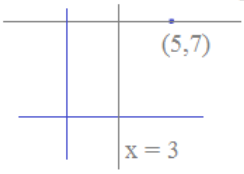
$(2^{9})^{\frac{1}{3}} \cdot \sqrt{32}$

$2^{3} \cdot 4\sqrt{2} = 2^{3} \cdot 2^{2} \cdot 2^{\frac{1}{2}}$

$2^{\frac{11}{2}}$ or $32\sqrt{2}$

12) What is the equation of a line *perpendicular* to x = 3 that goes through (5, 7)?

since x = 3 is vertical, any perpendicular line would be horizontal..



a horizontal line through (5, 7) is

y = 7

13) $f(x) = x^{4} - 2x^{3} - 7x^{2} + 8x + 12$

Find the zeros, identify the end behavior, and sketch the function:

zeros: -1, -2, 2, 3

(rational root theorem) possible rational roots:

$\pm 1 \ \pm 2 \ \pm 3 \ \pm 4 \ \pm 6 \ \pm 12$

try 1: $(1)^{4} - 2(1)^{3} - 7(1)^{2} + 8(1) + 12 = 12$ ✗

try -1: $(-1)^{4} - 2(-1)^{3} - 7(-1)^{2} + 8(-1) + 12 = 0$ ✓

-1	1	-2	-7	8	12
		-1	3	4	-12
	1	-3	-4	12	0

$x^{3} - 3x^{2} - 4x + 12$

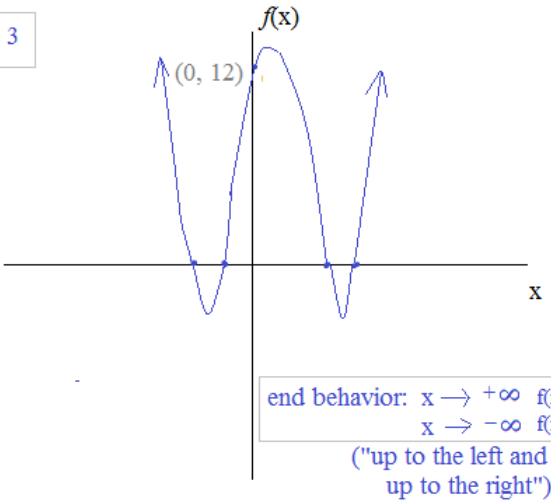
factor by grouping

$x^{2}(x - 3) - 4(x - 3)$

$(x^{2} - 4)(x - 3)$

$(x + 2)(x - 2)(x - 3)$

-2, 2, 3 are zeros



end behavior: $x \to +\infty \ \ f(x) \to +\infty$

$x \to -\infty \ \ f(x) \to +\infty$

("up to the left and up to the right")

14) $g(x) = 4 - \sqrt{3x - 6}$

a) $g(5) =$ 1

$4 - \sqrt{3(5) - 6}$

$4 - 3 = 1$

b) $g(a + 2) =$ $4 - \sqrt{3a}$

$4 - \sqrt{3(a + 2) - 6}$

$4 - \sqrt{3a + 6 - 6}$

$4 - \sqrt{3a}$

c) $g(2 - x) =$ $4 - \sqrt{-3x}$

$4 - \sqrt{3(2 - x) - 6}$

$4 - \sqrt{6 - 3x - 6}$

$4 - \sqrt{-3x}$

15) In the quadratic $y = 5x - 7x^{2} + 8$

what is a) the linear term? 5x

b) the degree? the lead degree is 2

c) the constant? 8

Algebra II Review Test 005

SOLUTIONS

16) Identify the center and radius of the circle:

$x^2 + y^2 - 8x + 6y = -16$

$x^2 - 8x + y^2 + 6y = -16$ (complete the square to put into standard form)

$x^2 - 8x + 16 + y^2 + 6y + 9 = -16 + 16 + 9$

$(x - 4)(x - 4) + (y + 3)(y + 3) = 9$

$(x - 4)^2 + (y + 3)^2 = 9$

center: $(4, -3)$ $\quad h = 4 \quad k = -3$

radius: 3 $\quad r = 3$

17) Find the solution to the linear system

1 $3x + 7y + 2z = 2$
2 $2x - 6y = 22$
3 $-x + 4y - 4z = -15$

use matrix/calculator

$$\left[\begin{array}{ccc|c} 3 & 7 & 2 & 2 \\ 2 & -6 & 0 & 22 \\ -1 & 4 & -4 & -15 \end{array}\right] \text{ or } \begin{bmatrix} 3 & 7 & 2 \\ 2 & -6 & 0 \\ -1 & 4 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 22 \\ -15 \end{bmatrix} \qquad \begin{bmatrix} 5 \\ -2 \\ \frac{1}{2} \end{bmatrix}$$

use elimination/substitution

1 $6x + 14y + 4z = 4$
3 $-x + 4y - 4z = -15$
4 $5x + 18y = -11$

2 $6x - 18y = 66$
$11x = 55$
$x = 5$

2 $2(5) - 6y = 22$
$y = -2$

1 $3(5) + 7(-2) + 2z = 2$
$2z = 1$
$z = 1/2$

18) Solve: $(x + 2)^2 = (x - 4)^2$

$x^2 + 4x + 4 = x^2 - 8x + 16$

$12x = 12$

$x = 1$

$\frac{x + 1}{x - 1} = \frac{3x}{3x - 6}$ (cross multiply)

$(x + 1)(3x - 6) = 3x(x - 1)$

$3x^2 - 3x + 6 = 3x^2 - 3x$

$-3x + 6 = -3x$

$6 = 0$

NO SOLUTIONS

19) 500 mg of a radioactive material has a half-life of 8 years.

a) How much material remains after 24 years? 62.5 (see chart)

b) How much material remains after 36 years?

c) When will less than 5 mg of the radioactive material remain?

t (years)	size (mg)
0	500
8	250
16	125
24	62.5
32	31.25
36	?
40	15.625
48	7.8125
?	5

Need to find rate of decay:

$250 = 500e^{r(8)}$

$\frac{1}{2} = e^{8r}$ $\quad r = -.087$

$\ln \frac{1}{2} = \ln e^{8r}$

$-.693 = 8r$

b) $A = 500e^{-.087(36)}$

$= 500(.0436) = 21.82$

c) $5 = 500e^{-.087(t)}$

$.01 = e^{-.087(t)}$

$\ln .01 = -.087(t)\ln e$

$-4.6 = -.087t$

$t = 52.933$

$t > 52.933$ years

20) Sequences:

a) What is the 5th term in the following *arithmetic* sequence?

"common difference" = 2 $\quad$ 2, 4, 6, 8, 10, ... $\quad A_1 = 2 \quad A_2 = 4$ $\quad A_5 = 10$

b) What is the 5th term in the following *geometric* sequence?

"common ratio" = 4/2 = 2 $\quad$ 2, 4, 8, 16, 32, ... $\quad G_1 = 2 \quad G_2 = 4$ $\quad G_5 = 32$

SOLUTIONS

Algebra II Review Test 005

21) Find the maximum value of $P = 2x + y$ subject to the constraints

$x \geq 0$
$y \geq 0$
$x + y \leq 7$
$5x + 2y \leq 20$

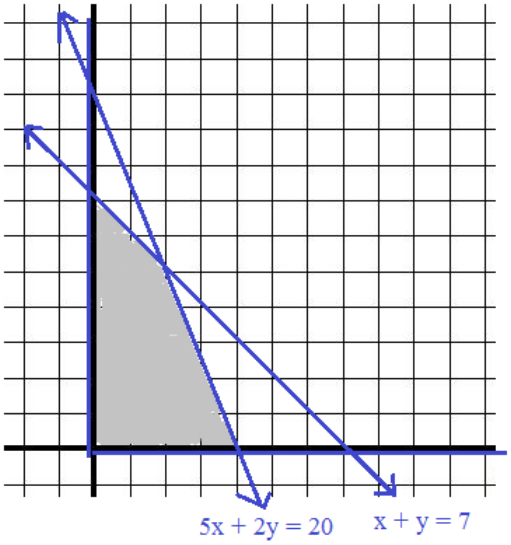
Gray area is the feasibility region...

And, the vertices represent *possible* maximum values..

test vertices: (0, 0): $P = 2(0) + (0) = 0$
(0, 7): $P = 2(0) + (7) = 7$
(2, 5): $P = 2(2) + (5) = 9$
(4, 0): $P = 2(4) + (0) = 8$

The maximum value is 9...

Graph the constraints (and identify the feasibility region).



22) Factor: $x^6 - 1$

difference of squares $(x^3 + 1)(x^3 - 1)$
then
sum/difference of cubes $(x+1)(x^2 - x + 1)(x - 1)(x^2 + x + 1)$

23) $f(x) = 3x + 4$
$g(x) = x^2 - 5$

a) $f(g(4)) =$

$g(4) = (4)^2 - 5 = 11$
$f(11) = 3(11) + 4 = 37$

b) $g(f(4)) =$

$f(4) = 3(4) + 4 = 16$
$g(16) = (16)^2 - 5 = 251$

c) $g^{-1}(x) =$ to find the inverse of $g(x)$,

$y = x^2 - 5$ switch x/y
$x = y^2 - 5$ solve for y
$y^2 = x + 5$
$y = \sqrt{x + 5}$

24) $\dfrac{(x + 3)(2 - 5x)}{x + 1} \leq 0$

Express the answer in interval notation. Then, graph on the number line.

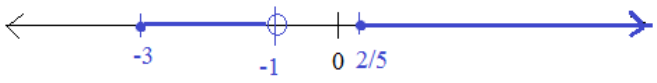
Find critical values:

$(x + 3) = 0 \quad x = -3$
$(2 + 5x) = 0 \quad x = 2/5$
$(x + 1) \neq 0 \quad x \neq -1$

then, test regions:

$x = -4 \quad > 0$ no
$x = -2 \quad < 0$ yes
$x = 0 \quad > 0$ no
$x = 2 \quad < 0$ yes

$[3, -1) \cup [2/5, +\infty)$



25) Find X:

a) $9^{(X+1)} = 27^{2X}$

$(3^2)^{(X+1)} = (3^3)^{2X}$
$3^{2X+2} = 3^{6X}$
$2X + 2 = 6X$
$X = \frac{1}{2}$

b) $\sqrt{5} = 125^{3X}$

$5^{\frac{1}{2}} = (5^3)^{3X}$
$\frac{1}{2} = 9X$
$X = \frac{1}{18}$

Thanks for checking out this review test. (Hope it helped!)

If you have any questions, suggestions, or feedback, let me know.

Cheers,
Lance..